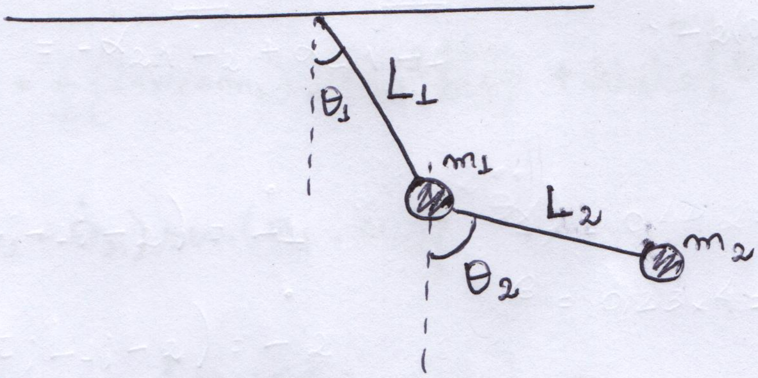
Sistema caótico - Pêndulo duplo



Massa 1:

$x_1 = L_1 \, sen\theta_1$

$y_1 = -L_1 \cos\theta_1$

Massa 2:

$x_2 = L_2 \, sen\theta_2 + x_1$

$y_2 = -L_2 \cos\theta_2 - y_1$

Energia cinética/Potencial:

$T_1 = \frac{1}{2} m_1 (L_1 \dot\theta_1)^2$

$U_1 = -m_1 g L_1 \cos\theta_1$

Energia cinética/Potencial:

$$T_2 = \frac{1}{2} m_2 \Big[(L_2 \dot\theta_2 \cos\theta_2)^2 + (L_1 \dot\theta_1 \cos\theta_1)^2 + 2L_1L_2\dot\theta_1\dot\theta_2 \cos\theta_1 \cos\theta_2 + (L_2\dot\theta_2 \, sen\theta_2)^2 + (L_1\theta_1 \, sen\theta_1)^2 + 2L_1L_2\dot\theta_1\dot\theta_2 \, sen\theta_1 \, sen\theta_2 \Big]$$

$U_2 = -m_2 g (L_2 \cos\theta_2 + L_1 \cos\theta_1)$

Lagrangeana do sistema

$$\mathcal{L} = \frac{1}{2}\Big\{ (m_1+m_2)(L_1\dot\theta_1)^2 + m_2(L_2\dot\theta_2)^2 + \underset{m_2}{2L_1L_2}\dot\theta_1\dot\theta_2 \underbrace{(\cos\theta_1\cos\theta_2 + sen\theta_1 \, sen\theta_2)}_{\cos(\theta_1-\theta_2)} \Big\}$$

$$+ (m_1+m_2) g L_1 \cos\theta_1 + m_2 g L_2 \cos\theta_2$$

$$\boxed{\mathcal{L} = \frac{1}{2}\Big\{ (m_1+m_2)(L_1\dot\theta_1)^2 + m_2(L_2\dot\theta_2)^2 + 2L_1L_2\dot\theta_1\dot\theta_2 \overset{m_2}{\cos}(\theta_1-\theta_2) \Big\} + (m_1+m_2) g L_1 \cos\theta_1 + m_2 g L_2 \cos\theta_2}$$

θ_1 :

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = -\frac{1}{2}\left\{2L_1L_2\dot{\theta}_1\dot{\theta}_2^{\,m_2} sen(\theta_1-\theta_2)\right\} - (m_1+m_2)gL_1 sen\theta_1$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}\right) = \frac{1}{2}\left\{(m_1+m_2)2(L_1^2\ddot{\theta}_1) + 2L_1L_2^{m_2}\left[\ddot{\theta}_2\cos(\theta_1-\theta_2) - \dot{\theta}_2\right.\right.$$

$$\left. - \dot{\theta}_2(\dot{\theta}_1-\dot{\theta}_2) sen(\theta_1-\theta_2)\right]$$

$$\left[\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}\right) - \frac{\partial \mathcal{L}}{\partial \theta_1}\right] = 0$$

$$(m_1+m_2)\cancel{2}(\sqrt{2}L_1)^2\ddot{\theta}_1 + \cancel{2}L_1L_2\ddot{\theta}_2^{\,m_2}\cos(\theta_1-\theta_2) \overbrace{- 2L_1L_2\dot{\theta}_1\dot{\theta}_2 sen(\theta_1-\theta_2)}^{\overset{m_2}{\divideontimes}}$$

$$+ \dot{\theta}_2^{2\,m_2} sen(\theta_1-\theta_2) + \underbrace{2L_1L_2\dot{\theta}_1\dot{\theta}_2 sen(\theta_1-\theta_2)}_{\underset{m_2}{\divideontimes}} + (m_1+m_2)\frac{gL_1}{2} sen\theta_1 = 0$$
$\cancel{2}\cancel{2}L_1L_2$

$$\ddot{\theta}_1 + \frac{\cancel{L_1}L_2\ddot{\theta}_2 m_2\cos(\theta_1+\theta_2)}{2L_1^2(m_1+m_2)} + \frac{\cancel{L_1}L_2\dot{\theta}_2 m_2 sen(\theta_1-\theta_2)}{2L_1^2(m_1+m_2)}$$

$$+\frac{g}{2L_1} sen\theta_1 = 0$$

$$\boxed{\ddot{\theta}_1 = -\frac{m_2L_2}{2(m_1+m_2)L_1}\left[\ddot{\theta}_2\cos(\theta_1-\theta_2) + \dot{\theta}_2^2 sen(\theta_1-\theta_2)\right] - \left(\frac{g}{2L_1}\right) sen\theta_1 = 0}$$

a θ_2:

$$\frac{\partial \mathcal{L}}{\partial \theta_2} = \frac{1}{\cancel{2}}\left\{ \frac{\cancel{2} L_1 \cancel{L_2} \dot{\theta}_1 \dot{\theta}_2 \cancel{m_2} \operatorname{sen}(\theta_1 - \theta_2)}{L_2} \right\} - \frac{\cancel{m_2} g \cancel{L_2} \cos \theta_2}{L_2}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right) = \frac{1}{\cancel{2}}\left\{ \cancel{2}\cancel{m_2}\cancel{L_2^2}\,\ddot{\theta}_2 + \frac{\cancel{2}\cancel{m_2} L_1 \cancel{L_2}}{L_2}\left[\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \dot{\theta}_1(\dot{\theta}_1 - \dot{\theta}_2)\operatorname{sen}(\theta_1 - \theta_2)\right]\right\}$$

$$\left[\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right) - \frac{\partial \mathcal{L}}{\partial \theta_2}\right] = 0$$

$$\ddot{\theta}_2 + \left(\frac{L_1}{L_2}\right)\cos(\theta_1 - \theta_2)\,\ddot{\theta}_1 - \left(\frac{L_1}{L_2}\right)\operatorname{sen}(\theta_1 - \theta_2)\,\dot{\theta}_1^2 + \overbrace{\left(\frac{L_1}{L_2}\right)\operatorname{sen}(\theta_1 - \theta_2)\,\dot{\theta}_1 \dot{\theta}_2}^{※}$$

$$\underbrace{-\left(\frac{L_1}{L_2}\right)\operatorname{sen}(\theta_1 - \theta_2)\,\dot{\theta}_1 \dot{\theta}_2}_{※} + \frac{g}{L_2}\cos\theta_2 = 0$$

$$\boxed{\ddot{\theta}_2 = -\left(\frac{L_1}{L_2}\right)\left[\cos(\theta_1 - \theta_2)\,\ddot{\theta}_1 + \operatorname{sen}(\theta_1 - \theta_2)\,\dot{\theta}_1^2\right] - \left(\frac{g}{L_2}\right)\operatorname{sen}\theta_2}$$