# Digital image processing

An image is a two-dimensional (2D) signal in spatial dimension with a unit sample called pixel [1] [2].

Fourier analysis plays a crucial role in image compression and to store them.

* Linearity, convolution (linear operators only mildly useful)
* Fourier transform (but the analysis is less relevant compared to 1D)
* Interpolation, sampling
* Affine transform
* Image is a finite-support signals, available in their entirety (causality losses meaning)

In figure 1 it can be seen the original image used in the code (random pattern with D=0.05, N=2M), the axis represents the dimensions of the image (823X822 [pixels]), the colors represents the intensity of the brightness in each unit of pixels (x,y), the numbers are a numerical values between 0-255 black to white (dimensionless, no units)

### Auto cross-correlation function

are the image dimensions.

We are interested to find the displacement , from it we can extract the density and index of refraction variation.

to determine the displacement from the cross-correlation function between two images we will use the method described in [5] as follows:

A diagram of a complex function

Description automatically generated

Figure 2: work plan to determine the displacement.

In figure 2 the work plan to find the displacement is instead of direct Auto-correlation we will transform the image from the spatial spectrum to fourier spectrum, applying an equation to calculate the difference by transform to the power spectrum then transform back to the spatial spectrum to get the cross-correlation function (Wiener-Khinchin theorem).

: image intensity numerical values 0-255 (2D spatial signal)

: mean subtraction of image intensity numerical values -155-155.

The mean subtraction operation is often used for data normalization. This transformation does not change the relative relationships between pixel values but brings the overall intensity distribution closer to zero. It's a common preprocessing step in image analysis to ensure that the data has a consistent scale and distribution, making it easier for algorithms to learn and generalize.

mean subtraction operation is used to normalize the data along the axis [4].

### Spatial spectrum to complex spectrum

Fourier transform of the image intensity [5]:

: Fourier Spectrum of the image

: spatial frequencies in orthogonal directions (coordinates of the complex values in the transformed space) [cycles per pixel]

For example, if has a value of 0.1, it means that there is one-tenth of a cycle of the corresponding frequency within one pixel. Similarly, if has a value of 0.2, it means that there are two-tenths (or one-fifth) of a cycle of the corresponding frequency within one pixel in the vertical direction.

From the transform we can calculate the phase and frequency distribution in the image as following [6]:

: phase, dimensionless

: frequency magnitude in cycle per pixel

### Fourier spectrum to power spectrum

To transform from the Fourier spectrum to the power spectrum will use the equation:

: power spectrum

The normalization of the coherence compensates for large values in the cross spectrum that result solely from large power, and thus isolates the influence of coupling.

If images 1 and 2 are identical, then their power spectra are equal to each other and to the cross spectrum. In this case, the coherence is 1 at all frequencies. If 1 and 2 are completely independent, then the cross spectrum is 0 for all frequencies and so is the coherence.

### Cross-correlation function

by an inverse Fourier transform of the power spectrum [5]:

: the Fourier transform of the cross-correlation function.

### shift

The phase shift, obtained from the phase cross-correlation, is related to the pixel displacement as follows [9]:

Where M, N are the dimensions of the images [pixel].

: phase shift between the images, from equation 12, it is dimensionless.

### Velocity

From basics of motion equations:

t: time between two images

Since the units are pixels, we can perform the conversion to cm, simply divide the number of pixels by the image resolution in PPI (Dots or pixels per inch) [11]. In general, a printing resolution of 300 PPI is considered the standard for achieving optimal print quality.

### index of refraction

from BOS analysis [9]:

N= number of layers in the shifted image.

: the refraction index of each layer.

: layer thickness.

For density finally use the Gladstone-Dale relation:

Where:

Since we are using a ULC-2 LED light according to the manual the wavelength can be calculates as follows [10]:

C:\ProgramData\Anaconda3\Lib\site-packages\skimage\registration\\_phase\_cross\_correlation.py

# References

[1] <http://www.ece.northwestern.edu/local-apps/matlabhelp/toolbox/images/intro6.html>

[2] <https://datacarpentry.org/image-processing/aio.html>

[3] <http://www.fast.u-psud.fr/pivmat/html/makebospattern.html>

[4] Book: PIV Practical Guide 2018, chapter 4

[5] Book: PIV Practical Guide 2018, chapter 5

[6] <https://homepages.inf.ed.ac.uk/rbf/HIPR2/fourier.htm>

[7] <https://www.analog.com/media/en/technical-documentation/dsp-book/dsp_book_Ch30.pdf>

[8] <https://ccs.fau.edu/~bressler/EDU/STSA/Modules/X.pdf>

[9] Verso, L. and Liberzon, A. (2015) ‘Background oriented schlieren in a density stratified fluid’, *Review of Scientific Instruments*, 86(10). doi:10.1063/1.4934576.

[10] <https://www.metaphase-tech.com/controllers/ulc-2/>

[11] <https://appmeup.fr/calc/en/pixel_to_cm>