

И.У.

$$\phi(\vec{r}, t) = \int \frac{\rho(\vec{r}, t')}{4\pi\epsilon_0 \vec{r}} dV$$

$$\vec{A}(\vec{r}, t) = \int \frac{\vec{j}(\vec{r}, t')}{4\pi\epsilon_0 c^2 \vec{r}} dV$$

Выведем отдельно скалярный потенциал

$$\phi(\vec{r}, t) = \int \frac{\rho(\vec{r}, t')}{4\pi\epsilon_0 \vec{r}} dV ,$$

где $\vec{r} = |\vec{r} - \vec{r}_0(t')|$

$$\phi(\vec{r}, t) = \int \frac{\rho(\vec{r}, t')}{4\pi\epsilon_0 |\vec{r} - \vec{r}_0(t')|} dV$$

$$\phi(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}, t')}{|\vec{r} - \vec{r}_0(t')|} dV$$

отбросим на время конст – часть.

$$\phi(\vec{r}, t) = \int \frac{\rho(\vec{r}, t')}{|\vec{r} - \vec{r}_0(t')|} dV$$

Приведём к интегралу по 4 переменным

$$\phi(\vec{r}, t) = \int \int \frac{\rho(\vec{r}, t')}{|\vec{r} - \vec{r}_0(t')|} \delta(t' - \tau) d\tau dV ,$$

где $\tau = t - \frac{1}{c} |\vec{r} - \vec{r}_0(t')|$, $\rho(\vec{r}, t) = q \delta(\vec{r} - \vec{r}_0(t'))$

$$\phi(\vec{r}, t) = \int \int \frac{q \delta(\vec{r} - \vec{r}_0(t'))}{|\vec{r} - \vec{r}_0(t')|} \delta(t' - \tau) d\tau dV$$

Поменяем порядок интегрирования и вынесем заряд, т.к. он конст.

$$\phi(\vec{r}, t) = q \int \frac{\delta(t' - \tau)}{|\vec{r} - \vec{r}_0(t')|} d\tau$$

Воспользуемся правилом $\int g(x) \delta(f(x) - \alpha) dx = \frac{g(x_0)}{f'(x_0)}$, $f(x_0) = \alpha$

$$\delta(t' - \tau) = \frac{\delta(t' - \tau)}{\frac{\partial}{\partial t'} (t' - (t - \frac{1}{c} |\vec{r} - \vec{r}_0(t')|))} = \frac{\delta(t' - \tau)}{\frac{\partial}{\partial t'} t' - \frac{\partial}{\partial t'} t + \frac{1}{c} (\frac{\partial}{\partial t'} |\vec{r} - \vec{r}_0(t')|)}$$

Исходя из прил.1, имеем:

$$\delta(t' - \tau) = \frac{\delta(t' - \tau)}{1 - \vec{n} \vec{\beta}} , \text{ где } \vec{\beta} = \vec{v}(t')$$

Переходим во время $t' = \tau$

Тогда скалярный потенциал примет вид:

$$\phi(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1}{|\vec{r} - \vec{r}_0(t')| (1 - \vec{n} \vec{\beta})}$$

Найдём теперь решение уравнения для векторного потенциала

$$\vec{A}(\vec{r}, t) = \int \frac{\vec{j}(\vec{r}, t')}{4\pi\epsilon_0 c^2 |\vec{r} - \vec{r}_0(t')|} dV,$$

где $|\vec{r}| = |\vec{r} - \vec{r}_0(t')|$

$$\vec{A}(\vec{r}, t) = \int \frac{\vec{j}(\vec{r}, t')}{4\pi\epsilon_0 c^2 |\vec{r} - \vec{r}_0(t')|} dV$$

$$\vec{A}(\vec{r}, t) = \frac{1}{4\pi\epsilon_0 c^2} \int \frac{\vec{j}(\vec{r}, t')}{|\vec{r} - \vec{r}_0(t')|} dV$$

отбросим на время конст – часть.

$$\phi(\vec{r}, t) = \int \frac{\rho(\vec{r}, t')}{|\vec{r} - \vec{r}_0(t')|} dV$$

Приведём к интегралу по 4 переменным

$$\phi(\vec{r}, t) = \int \int \frac{\rho(\vec{r}, t')}{|\vec{r} - \vec{r}_0(t')|} \delta(t' - \tau) d\tau dV,$$

где $\tau = t - \frac{1}{c}|\vec{r} - \vec{r}_0(t')|$, $\vec{j}(\vec{r}, t) = q\vec{v}(t')\delta(\vec{r} - \vec{r}_0(t'))$

$$\vec{A}(\vec{r}, t) = \int \int \frac{q\vec{v}(t')\delta(\vec{r} - \vec{r}_0(t'))}{|\vec{r} - \vec{r}_0(t')|} \delta(t' - \tau) d\tau dV$$

Поменяем порядок интегрирования и вынесем заряд, т.к. он конст.

$$\vec{A}(\vec{r}, t) = q \int \frac{\vec{v}(t')\delta(t' - \tau)}{|\vec{r} - \vec{r}_0(t')|} d\tau$$

Воспользуемся правилом $\int g(x)\delta(f(x) - \alpha)dx = \frac{g(x_0)}{f'(x_0)}$, $f(x_0) = \alpha$

$$\delta(t' - \tau) = \frac{\delta(t' - \tau)}{\frac{\partial}{\partial t'}(t' - (t - \frac{1}{c}|\vec{r} - \vec{r}_0(t')|))} = \frac{\delta(t' - \tau)}{\frac{\partial}{\partial t'}t' - \frac{\partial}{\partial t'}t + \frac{1}{c}(\frac{\partial}{\partial t'}|\vec{r} - \vec{r}_0(t')|)}$$

Исходя из прил.1, имеем:

$$\delta(t' - \tau) = \frac{\delta(t' - \tau)}{1 - \vec{n}\vec{\beta}}, \text{ где } \vec{\beta} = \vec{v}(t')$$

Переходим во время $t' = \tau$

Тогда скалярный потенциал примет вид:

$$\vec{A}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0 c} \frac{\vec{\beta}}{|\vec{r} - \vec{r}_0(t')|(1 - \vec{n}\vec{\beta})}$$

Используя прил.2 и прил.4, убеждаемся в правильности данных формул, найдя калибровку

Лоренца $\frac{d}{dt}\left(\frac{q}{4\pi} + c^2 \nabla \vec{A}\right) = 0$:

$$\frac{d\phi}{dt} = \frac{d}{dt}\left(\frac{q}{4\pi\epsilon_0} \frac{1}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})}\right) = \frac{q}{4\pi\epsilon_0} \frac{c\vec{n}\vec{\beta} - c\vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau))\dot{\vec{\beta}}}{(|\vec{r} - \vec{r}_0(\tau)|)^2(1 - \vec{n}\vec{\beta})^3}$$

$$\nabla \vec{A} = \nabla\left(\frac{-q}{4\pi\epsilon_0 c} \frac{\vec{\beta}}{(|\vec{r} - \vec{r}_0(\tau)|)^2(1 - \vec{n}\vec{\beta})^3}\right) = \frac{-q}{4\pi\epsilon_0 c} \frac{\vec{n}\vec{\beta} - \vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}}{(|\vec{r} - \vec{r}_0(\tau)|)^2(1 - \vec{n}\vec{\beta})^3}$$

Как можем заметить, равенство выполняется.

Перейдём к нахождению электрического поля.

Воспользуемся формулой $-\nabla\phi - \frac{d\vec{A}}{dt}$

Для нахождения поля обратимся к вспомогательным уравнениям 3 и 5.

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{1}{(|\vec{r} - \vec{r}_0(\tau)|)^2 (1 - \vec{n}\vec{\beta})^3} \left[\vec{n} \left(1 - \vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \right) - \vec{\beta} (1 - \vec{n}\vec{\beta}) \right] - \frac{q}{4\pi\epsilon_0 c} \frac{c}{(|\vec{r} - \vec{r}_0(\tau)|)^2 (1 - \vec{n}\vec{\beta})^3} \times \left[\vec{\beta} \left(\vec{n}\vec{\beta} - \vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \right) + |\vec{r} - \vec{r}_0(\tau)| \frac{\dot{\vec{\beta}}}{c} (1 - \vec{n}\vec{\beta}) \right] =$$

вынесем за скобки $\frac{q}{4\pi\epsilon_0} \frac{1}{(|\vec{r} - \vec{r}_0(\tau)|)^2 (1 - \vec{n}\vec{\beta})^3}$, тогда записанное выше уравнение примет

вид:

$$\begin{aligned} &= \left[\vec{n} \left(1 - \vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \right) - \vec{\beta} (1 - \vec{n}\vec{\beta}) \right] - \left[\vec{\beta} \left(\vec{n}\vec{\beta} - \vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \right) + |\vec{r} - \vec{r}_0(\tau)| \frac{\dot{\vec{\beta}}}{c} (1 - \vec{n}\vec{\beta}) \right] = \\ &= \vec{n} \left(1 - \vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \right) - \vec{\beta} + \vec{n}\vec{\beta}^2 - \vec{n}\vec{\beta}^2 + \vec{\beta}^3 - (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \vec{\beta} - |\vec{r} - \vec{r}_0(\tau)| \frac{\dot{\vec{\beta}}}{c} (1 - \vec{n}\vec{\beta}) = \\ &= \vec{n} - \vec{\beta}^2 \vec{n} + (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \vec{n} - \vec{\beta} + \vec{\beta}^3 - (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \vec{\beta} - |\vec{r} - \vec{r}_0(\tau)| \frac{\dot{\vec{\beta}}}{c} (1 - \vec{n}\vec{\beta}) = \\ &= (\vec{n} - \vec{\beta}) \left((\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \right) + \vec{n} (1 - \vec{\beta}^2) - \vec{\beta} (1 - \vec{\beta}^2) - |\vec{r} - \vec{r}_0(\tau)| \frac{\dot{\vec{\beta}}}{c} (1 - \vec{n}\vec{\beta}) = \\ &= (\vec{n} - \vec{\beta}) \left((\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \right) + (\vec{n} - \vec{\beta}) (1 - \vec{\beta}^2) - |\vec{r} - \vec{r}_0(\tau)| \frac{\dot{\vec{\beta}}}{c} (1 - \vec{n}\vec{\beta}) = \\ &= (\vec{n} - \vec{\beta}) (1 - \vec{\beta}^2) + |\vec{r} - \vec{r}_0(\tau)| (\vec{n} - \vec{\beta}) - |\vec{r} - \vec{r}_0(\tau)| (\vec{n}(\vec{n} - \vec{\beta})) \frac{\dot{\vec{\beta}}}{c} = \end{aligned}$$

Возвращаясь к константам, получим:

$$= \frac{q}{4\pi\epsilon_0 |\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n}\vec{\beta})^3} \left[(\vec{n} - \vec{\beta}) (1 - \vec{\beta}^2) + |\vec{r} - \vec{r}_0(\tau)| \left(\vec{n} \frac{\dot{\vec{\beta}}}{c} \right) (\vec{n} - \vec{\beta}) - |\vec{r} - \vec{r}_0(\tau)| (\vec{n}(\vec{n} - \vec{\beta})) \frac{\dot{\vec{\beta}}}{c} \right]$$

Прил. 1.

$$\begin{aligned} \frac{\partial}{\partial t'} |\vec{r} - \vec{r}_0(t')| &= \frac{\partial}{\partial t'} \sqrt{\vec{r}^2 - 2\vec{r}\vec{r}_0(t') + \vec{r}_0^2(t')} = \frac{\partial}{\partial t'} (\vec{r}^2 - 2\vec{r}\vec{r}_0(t') + \vec{r}_0^2(t'))^{\frac{1}{2}} = \\ &= \frac{\frac{\partial}{\partial t'} (\vec{r}^2 - 2\vec{r}\vec{r}_0(t') + \vec{r}_0^2(t'))}{(\vec{r}^2 - 2\vec{r}\vec{r}_0(t') + \vec{r}_0^2(t'))^{\frac{1}{2}}} = \frac{2(\vec{r}_0(t') - \vec{r})\dot{\vec{r}}_0(\tau)}{2|\vec{r} - \vec{r}_0(\tau)|} = -\vec{n}\vec{v}(t') \end{aligned}$$

Прил. 2.

$$\frac{d}{dt} \frac{1}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} = \frac{d\tau}{dt} \frac{d}{d\tau} \left(\frac{1}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} \right)$$

$$\begin{aligned} \frac{d}{d\tau} [|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})] &= (|\vec{r}-\vec{r}_0(\tau)| - (\vec{r}-\vec{r}_0(\tau))\vec{\beta}) \frac{d}{d\tau} |\vec{r}-\vec{r}_0(\tau)| - \frac{d}{d\tau} ((\vec{r}-\vec{r}_0(\tau))\vec{\beta}) = \\ &= -\vec{n}\vec{v}(\tau) - ((\vec{r}-\vec{r}_0(\tau))\vec{\beta} + \dot{\vec{\beta}}(\vec{r}-\vec{r}_0(\tau))) = -\vec{n}\vec{v}c - (\vec{r}-\vec{r}_0(\tau))\dot{\vec{\beta}} + c\vec{\beta}^2 \end{aligned}$$

$$\frac{d}{dt} \frac{1}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} = \frac{d}{dt} \frac{1}{|\vec{r}-\vec{r}_0(\tau)|^2 (1-\vec{n}\vec{\beta})^3} (\vec{n}\vec{\beta}c - c\beta^2 + (\vec{r}-\vec{r}_0(\tau))\dot{\vec{\beta}})$$

Прил.3

$$\begin{aligned}
\nabla \frac{1}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} &= \frac{\partial}{\partial \vec{r}} \left(\frac{1}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} \right) + \frac{\partial}{\partial \tau} \left(\frac{1}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} \right) \nabla \tau \\
\frac{\partial}{\partial \vec{r}} \frac{1}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} &= \frac{\frac{\partial}{\partial \vec{r}} (|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta}))}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^2} = \frac{\frac{\partial}{\partial \vec{r}} (|\vec{r}-\vec{r}_0(\tau)| - (\vec{r}-\vec{r}_0(\tau))\vec{\beta})}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^2} \\
\frac{\partial}{\partial \vec{r}} |\vec{r}-\vec{r}_0(\tau)| &= \frac{\partial}{\partial \vec{r}} \sqrt{\vec{r}^2 - 2\vec{r}\vec{r}_0(\tau) + \vec{r}_0^2(\tau)} = \frac{\frac{\partial}{\partial \vec{r}} (\vec{r}^2 - 2\vec{r}\vec{r}_0(\tau) + \vec{r}_0^2(\tau))}{2|\vec{r}-\vec{r}_0(\tau)|} = \frac{2(\vec{r}-\vec{r}_0(\tau))}{2|\vec{r}-\vec{r}_0(\tau)|} = \vec{n} \\
\frac{\partial}{\partial \vec{r}} ((\vec{r}-\vec{r}_0(\tau))\vec{\beta}) &= \vec{\beta} \left(\frac{\partial}{\partial \vec{r}} (\vec{r}-\vec{r}_0(\tau)) \right) + (\vec{r}-\vec{r}_0(\tau)) \frac{\partial}{\partial \vec{r}} \vec{\beta} = \vec{\beta} \\
\frac{\partial}{\partial \vec{r}} \frac{1}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} &= \frac{\vec{n}-\vec{\beta}}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^2} = \frac{(\vec{n}-\vec{\beta})(1-\vec{n}\vec{\beta})}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} \\
\frac{\partial}{\partial \tau} \frac{1}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} \nabla \tau &= \frac{\partial}{\partial \tau} (|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta}))^{-1} \nabla \tau = \frac{-\frac{\partial}{\partial \tau} (|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta}))}{(|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta}))^2} \nabla \tau = \\
&= \frac{\frac{\partial}{\partial \tau} (|\vec{r}-\vec{r}_0(\tau)| - (\vec{r}-\vec{r}_0(\tau))\vec{\beta}) \nabla \tau}{(|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta}))^2} \\
\frac{\partial}{\partial \tau} |\vec{r}-\vec{r}_0(\tau)| &= -\vec{n}\vec{v} \\
\frac{\partial}{\partial \tau} ((\vec{r}-\vec{r}_0(\tau))\vec{\beta}) &= \vec{\beta} \frac{\partial}{\partial \tau} (\vec{r}-\vec{r}_0(\tau)) + (\vec{r}-\vec{r}_0(\tau)) \frac{\partial \vec{\beta}}{\partial \tau} = -\vec{\beta}\vec{v} + (\vec{r}-\vec{r}_0(\tau))\dot{\vec{\beta}} \\
\nabla \tau &= \frac{\frac{-\vec{n}}{c}}{(1-\vec{n}\vec{\beta})} \\
\frac{\partial}{\partial \tau} \frac{1}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} \nabla \tau &= \frac{\vec{n}\vec{\beta}c + c\vec{\beta}^2 - (\vec{r}-\vec{r}_0(\tau))\dot{\vec{\beta}}}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^2} \frac{\frac{-\vec{n}}{c}}{(1-\vec{n}\vec{\beta})} = \\
&= \frac{\vec{\beta} - \vec{n}\vec{\beta}^2 + (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}\vec{n}}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} = \frac{\vec{\beta} + \vec{n}\vec{\beta}^2 - (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}\vec{n}}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} = \frac{-\vec{\beta} + \vec{n}\vec{\beta}^2 - (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}\vec{n}}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3}
\end{aligned}$$

$$\begin{aligned}
& \frac{(\vec{\beta}-\vec{n})(1-\vec{n}\vec{\beta})}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} + \frac{-\vec{\beta}+\vec{n}\vec{\beta}^2-(\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}\vec{n}}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} = \\
& = \frac{1}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} \left((\vec{\beta}-\vec{n})(1-\vec{n}\vec{\beta}) - \vec{\beta} + \vec{n}\vec{\beta}^2 - (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}\vec{n} \right) = \\
& = \frac{1}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} \left(\vec{\beta} - \vec{n} - \vec{n}\vec{\beta}^2 + \vec{n}^2\vec{\beta} - \vec{\beta} + \vec{n}\vec{\beta}^2 - (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}\vec{n} \right) = \\
& = \frac{1}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} \left(\vec{\beta} - \vec{n} - \vec{n}\vec{\beta}^2 + \vec{\beta} - \vec{\beta} + \vec{n}\vec{\beta}^2 - (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}\vec{n} \right) = \\
& = \frac{1}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} \left(-\vec{n} + \vec{n}\vec{\beta}^2 - (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}\vec{n} + \vec{\beta} - \vec{n}\vec{\beta}^2 \right) = \\
& = \frac{1}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} \left(\vec{n} \left(2 - \vec{\beta}^2 + (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c} \right) + \vec{\beta}(1-\vec{n}\vec{\beta}) \right)
\end{aligned}$$

Прил.4

$$\begin{aligned}
& \nabla \left(\frac{\vec{\beta}(\tau)}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} \right) = \frac{-1}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^2} \times \\
& \times (\vec{\beta} \nabla (|\vec{r}-\vec{r}_0(\tau)| - (\vec{r}-\vec{r}_0(\tau))\vec{\beta}) - (|\vec{r}-\vec{r}_0(\tau)| - (\vec{r}-\vec{r}_0(\tau))\vec{\beta}) \nabla \vec{\beta}) \\
& \nabla (|\vec{r}-\vec{r}_0(\tau)| - (\vec{r}-\vec{r}_0(\tau))\vec{\beta}) = \frac{1}{(1-\vec{n}\vec{\beta})} \left[\vec{n}(1-\vec{\beta}^2 + (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}) - \vec{\beta}(1-\vec{n}\vec{\beta}) \right] \\
& \nabla \vec{\beta} = \frac{\partial}{\partial \vec{r}} \vec{\beta} + \frac{\partial}{\partial \tau} \vec{\beta} \nabla \tau = 0 - \dot{\vec{\beta}} \frac{\vec{n}}{(1-\vec{n}\vec{\beta})} = -\dot{\vec{\beta}} \frac{\vec{n}}{(1-\vec{n}\vec{\beta})} \\
& (|\vec{r}-\vec{r}_0(\tau)| - (\vec{r}-\vec{r}_0(\tau))\vec{\beta}) \nabla \vec{\beta} = \frac{-1}{(1-\vec{n}\vec{\beta})} \left[\vec{n}(|\vec{r}-\vec{r}_0(\tau)| - (\vec{r}-\vec{r}_0(\tau))\vec{\beta}) \right] \frac{\dot{\vec{\beta}}}{c} = \\
& = \frac{-1}{(1-\vec{n}\vec{\beta})} \left[(\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c} - (\vec{r}-\vec{r}_0(\tau))\vec{\beta} \frac{\dot{\vec{\beta}}}{c} \vec{n} \right] \\
& \nabla \left(\frac{\vec{\beta}(\tau)}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} \right) = \frac{-1}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} \times \\
& \times \left[\vec{n}(1-\vec{\beta}^2 + (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}) - \vec{\beta}(1-\vec{n}\vec{\beta}) \right] \vec{\beta} - \left(- \left[(\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c} - (\vec{r}-\vec{r}_0(\tau))\vec{\beta} \frac{\dot{\vec{\beta}}}{c} \vec{n} \right] \right) = \\
& = \frac{-1}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} \left[\vec{n}\vec{\beta} - \vec{\beta}^2 + (\vec{r}-\vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c} \right]
\end{aligned}$$

Прил.5

$$\begin{aligned}
& \frac{d}{dt} \left(\frac{\dot{\vec{\beta}}}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})} \right) = \frac{d\tau}{dt} \frac{d}{d\tau} \left(\frac{\dot{\vec{\beta}}}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})} \right) = \\
& = \frac{-1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n}\vec{\beta})^2} \frac{d\tau}{dt} \left[\frac{d}{d\tau} \vec{\beta} |\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta}) - \vec{\beta} \frac{d}{d\tau} |\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta}) \right] \\
& \frac{d}{d\tau} \vec{\beta} = \dot{\vec{\beta}} \\
& (|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta}) \dot{\vec{\beta}}) = (|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau))\vec{\beta}) \dot{\vec{\beta}} \\
& \frac{d}{d\tau} (|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})) = \frac{d}{d\tau} (|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau))\vec{\beta}) = [-\vec{n}\vec{\beta}c + \vec{\beta}^2c - (\vec{r} - \vec{r}_0(\tau))\dot{\vec{\beta}}]\vec{\beta} \\
& \frac{d}{dt} \left(\frac{\vec{\beta}}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})} \right) = \frac{-1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n}\vec{\beta})^3} \times \\
& \times \left((|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau))\vec{\beta}) \dot{\vec{\beta}} - (-\vec{n}\vec{\beta}^2c + \vec{\beta}^2 - (\vec{r} - \vec{r}_0(\tau))\dot{\vec{\beta}}\vec{\beta}) \right) = \\
& = \frac{d}{d\tau} (|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})) = \frac{d}{d\tau} (|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau))\vec{\beta}) = [-\vec{n}\vec{\beta}c + \vec{\beta}^2c - (\vec{r} - \vec{r}_0(\tau))\dot{\vec{\beta}}]\vec{\beta} \\
& \frac{d}{dt} \left(\frac{\vec{\beta}}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})} \right) = \frac{c}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n}\vec{\beta})^3} \times \\
& \times \left(\vec{\beta}(\vec{n}\vec{\beta} - \vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau))\frac{\dot{\vec{\beta}}}{c}) + |\vec{r} - \vec{r}_0(\tau)|\frac{\dot{\vec{\beta}}}{c}(1 - \vec{n}\vec{\beta}) \right)
\end{aligned}$$

Прил.6

$$\begin{aligned}
\frac{d}{dt} \frac{q(\tau)}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} &= \frac{d\tau}{dt} \frac{d}{d\tau} \frac{q(\tau)}{|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})} = \\
&= \frac{-1}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^2} \frac{d\tau}{dt} \left((|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})) \frac{d}{d\tau} q(\tau) - q(\tau) \frac{d}{d\tau} (|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})) \right) \\
\frac{d}{dt} q(\tau) &= \dot{q}(\tau) \\
\dot{q}(\tau) (|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})) &= \dot{q}(\tau) (|\vec{r}-\vec{r}_0(\tau)| - (\vec{r}-\vec{r}_0(\tau))\vec{\beta}) \\
\frac{d}{d\tau} (|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})) &= [-\vec{n}\vec{\beta}c + \vec{\beta}^2c - (\vec{r}-\vec{r}_0(\tau))\dot{\vec{\beta}}] \\
\frac{-1}{|\vec{r}-\vec{r}_0(\tau)|^2(1-\vec{n}\vec{\beta})^3} &\left(\dot{q}(\tau) [|\vec{r}-\vec{r}_0(\tau)|(1-\vec{n}\vec{\beta})] - q(\tau) [-\vec{n}\vec{\beta}c + \vec{\beta}^2c - (\vec{r}-\vec{r}_0(\tau))\dot{\vec{\beta}}] \right)
\end{aligned}$$

Прил 7

$$\begin{aligned}
\nabla \left(\frac{q(\tau)}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})} \right) &= \frac{\partial}{\partial \vec{r}} \left(\frac{q(\tau)}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})} \right) + \frac{\partial}{\partial \tau} \left(\frac{q(\tau)}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})} \right) \nabla \tau \\
\frac{\partial}{\partial \vec{r}} \left(\frac{q(\tau)}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})} \right) &= \frac{1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n}\vec{\beta})^2} [0 - q(\tau)(\vec{n} - \vec{\beta})] = \\
&= \frac{1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n}\vec{\beta})^3} [-q(\tau)(\vec{n} - \vec{\beta})] (1 - \vec{n}\vec{\beta}) \\
\frac{\partial}{\partial \tau} \left(\frac{q(\tau)}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})} \right) &= \frac{1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n}\vec{\beta})^2} \times \\
\times \left[|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta}) \frac{\partial}{\partial \tau} q(\tau) - q(\tau) \frac{\partial}{\partial \tau} |\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta}) \right] &= \\
= \frac{1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n}\vec{\beta})^2} \left[|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta}) \dot{q}(\tau) - q(\tau) [-\vec{n}\vec{v} + \vec{\beta}\vec{v} - (\vec{r} - \vec{r}_0(\tau))\dot{\vec{\beta}}] \right] \\
\nabla \left(\frac{q(\tau)}{|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta})} \right) &= \frac{1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n}\vec{\beta})^3} \times \\
\times \left[-q(\tau)(\vec{n} - \vec{\beta})(1 - \vec{n}\vec{\beta}) - (\vec{r} - \vec{r}_0(\tau))(1 - \vec{n}\vec{\beta}) \frac{\dot{q}(\tau)}{c} + q(\tau) [-\vec{\beta} + \vec{n}\vec{\beta}^2 - (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \vec{n}] \right] =
\end{aligned}$$

отбросим на время общий множитель

$$\begin{aligned}
&= -q(\tau)(\vec{n} - \vec{\beta})(1 - \vec{n}\vec{\beta}) - |\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta}) \dot{q}(\tau) \frac{\vec{n}}{c} + q(\tau) [-\vec{\beta} + \vec{n}\vec{\beta}^2 - (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \vec{n}] = \\
&= -q(\tau)(\vec{n} - \vec{\beta} - \vec{\beta} + \vec{n}\vec{\beta}^2) - (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{q}(\tau)}{c} + (\vec{r} - \vec{r}_0(\tau)) \vec{n}\vec{\beta} \frac{\dot{q}(\tau)}{c} - q(\tau)\vec{\beta} + q(\tau)\vec{\beta}^2 \vec{n} - \\
&- q(\tau)(\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \vec{n} = -|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta}) \dot{q}(\tau) \frac{\vec{n}}{c} - q(\tau)\vec{n} + 2q(\tau)\vec{\beta} - \vec{n}\vec{\beta}^2 q(\tau) - q(\tau)\vec{\beta} + \\
&+ q(\tau)\vec{\beta}^2 \vec{n} - q(\tau)(\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \vec{n} = -|\vec{r} - \vec{r}_0(\tau)|(1 - \vec{n}\vec{\beta}) \dot{q}(\tau) \frac{\vec{n}}{c} - q(\tau)\vec{n} + q(\tau)\vec{\beta} - \\
&- q(\tau)(\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \vec{n} = \frac{-\dot{q}(\tau)}{c} \vec{n} |\vec{r} - \vec{r}_0(\tau)|(1 - \vec{\beta}\vec{n}) - q(\tau)(\vec{n} + \vec{\beta} - (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \vec{n})
\end{aligned}$$

Прил. 8

$$\begin{aligned}
& \frac{d}{dt} \left(\frac{q(\tau) \vec{\beta}}{|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})} \right) = \frac{1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n} \vec{\beta})^2} \times \\
& \times \left[(|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})) \frac{d}{dt} (q(\tau) \vec{\beta}) - (q(\tau) \vec{\beta}) \frac{d}{dt} (|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})) \right] \\
& \frac{d}{dt} (q(\tau) \vec{\beta}) = \vec{\beta} \frac{d}{dt} q(\tau) + q(\tau) \frac{d}{dt} \vec{\beta} = [\vec{\beta} \dot{q}(\tau) + \dot{\vec{\beta}} q(\tau)] \left(\frac{1}{(1 - \vec{n} \vec{\beta})} \right) \\
& \frac{1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n} \vec{\beta})^3} [(\vec{\beta} \dot{q}(\tau) + \dot{\vec{\beta}} q(\tau)) (|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta}))] = \\
& = \frac{1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n} \vec{\beta})^3} [\vec{\beta} \dot{q}(\tau) (|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau)) \vec{\beta}) + \dot{\vec{\beta}} q(\tau) (|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau)) \vec{\beta})] \\
& \frac{d}{dt} (|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})) = \frac{d\tau}{dt} \frac{d}{d\tau} (|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau)) \vec{\beta}) = \\
& = \frac{1}{(1 - \vec{n} \vec{\beta})} [\vec{n} \vec{\beta} c + \vec{\beta}^2 c - (\vec{r} - \vec{r}_0(\tau)) \dot{\vec{\beta}}] \\
& \frac{d}{dt} \left(\frac{q(\tau) \vec{\beta}}{|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})} \right) = \frac{1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n} \vec{\beta})^3} \times \\
& \times [\vec{\beta} \dot{q}(\tau) (|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau)) \vec{\beta}) + \dot{\vec{\beta}} q(\tau) (|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})) - q(\tau) \vec{\beta} (\vec{n} \vec{\beta} c + \vec{\beta}^2 c - (\vec{r} - \vec{r}_0(\tau)) \dot{\vec{\beta}})] = \\
& = \frac{1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n} \vec{\beta})^3} [(|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})) (\vec{\beta} \dot{q}(\tau) + \dot{\vec{\beta}} q(\tau)) - q(\tau) \vec{\beta} (\vec{n} \vec{\beta} c + \vec{\beta}^2 c - (\vec{r} - \vec{r}_0(\tau)) \dot{\vec{\beta}})]
\end{aligned}$$

Прил.9

$$\begin{aligned} \nabla \left(\frac{q(\tau) \vec{\beta}(\tau)}{|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})} \right) &= \frac{-1}{|\vec{r} - \vec{r}_0(\tau)|^2 (1 - \vec{n} \vec{\beta})^2} \times \\ &\times (q(\tau) \vec{\beta} \nabla (|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau)) \vec{\beta}) - (|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau)) \vec{\beta}) \nabla q(\tau) \vec{\beta}) \\ \nabla (|\vec{r} - \vec{r}_0(\tau)| - (\vec{r} - \vec{r}_0(\tau)) \vec{\beta}) &= \frac{1}{(1 - \vec{n} \vec{\beta})} \left[\vec{n} (1 - \vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c}) - \vec{\beta} (1 - \vec{n} \vec{\beta}) \right] \\ \nabla \vec{q}(\tau) \vec{\beta} &= \frac{\partial}{\partial \vec{r}} (q(\tau) \vec{\beta}) + \frac{\partial}{\partial \tau} (\vec{\beta} q(\tau)) \nabla \tau = 0 - \frac{\dot{\vec{\beta}}}{\partial \tau} (\vec{\beta} q(\tau)) \nabla \tau = -(\dot{q}(\tau) \vec{\beta} + \vec{\beta} \dot{q}(\tau)) \frac{\frac{\vec{n}}{c}}{(1 - \vec{n} \vec{\beta})} \end{aligned}$$

$$\nabla \left(\frac{q(\tau) \vec{\beta}(\tau)}{|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})} \right) =$$

отбросив общую дробную часть

$$\begin{aligned} &= \left[\vec{n} (1 - \vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c}) - \vec{\beta} (1 - \vec{n} \vec{\beta}) \right] q(\tau) \vec{\beta} - \frac{\vec{n}}{c} \left(-(\dot{q}(\tau) \vec{\beta} + q(\tau) \dot{\vec{\beta}}) [|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})] \right) = \\ &= q(\tau) \left[\vec{n} \vec{\beta} - \vec{\beta}^2 + (\vec{r} - \vec{r}_0(\tau)) \frac{\dot{\vec{\beta}}}{c} \right] + \dot{q}(\tau) \frac{\vec{\beta}}{c} \vec{n} [|\vec{r} - \vec{r}_0(\tau)| (1 - \vec{n} \vec{\beta})] \end{aligned}$$