# MATH 8302 Homework III

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## 1.)

### a.)

**Proposition.** *We let $W$ be a compact 2-manifold embedded in $\mathbb{R}^3$ with smooth boundary $\partial W$ a closed curve. Then,*

$$\int_W \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) \mathrm{d}x\, \mathrm{d}y = \int_{\partial W} f\, \mathrm{d}x + g\, \mathrm{d}y$$

*Proof.* We let $\omega = f\,\mathrm{d}x + g\,\mathrm{d}y$. Then, $\mathrm{d}\omega = \frac{\partial f}{\partial x}\,\mathrm{d}x \wedge \mathrm{d}x + \frac{\partial f}{\partial y}\,\mathrm{d}y \wedge \mathrm{d}x + \frac{\partial g}{\partial x}\,\mathrm{d}x \wedge \mathrm{d}y + \frac{\partial g}{\partial y}\,\mathrm{d}y \wedge \mathrm{d}y = \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)$. A straightforward application of Stoke's theorem then yields the desired result. □

### c.)

## 3.)

**Proposition.** *We let $X$ be a compact $m$-manifold, and $Z$ a $k$-submanifold with no boundary. If $\omega \equiv \omega' \mod \operatorname{im} d$ (i.e. $\omega$ and $\omega'$ are cohomologous), then $\int_Z \omega = \int_Z \omega'$.*

*Proof.* We have that there is some $\alpha \in \Omega^{k-1}(X)$ such that $\omega - \omega' = d\alpha$. Thus, $\int_Z \omega - \int_Z \omega' = \int_Z(\omega - \omega') = \int_Z \mathrm{d}\alpha$. By Stokes' theorem, we have that $\int_Z \mathrm{d}\alpha = \int_{\partial Z} \alpha$. However, any form integrated over $\partial Z = \varnothing$ has integral 0, so we now have that $\int_Z(\omega - \omega') = 0$ thus yielding our desired result. □