Math 1271 - Lectures 010 and 030 **Name (Print):** ______________
Fall 2017
Quiz 8C
11/07/17
Time Limit: 25 Minutes Teaching Assistant ______________

You may *not* use your books, notes, graphing calculator, phones or any other internet devices on this exam.

You are required to show your work on each problem on this quiz.

Problem	Points	Score
1	3	
2	4	
3	3	
Total:	10	

1. (3 points) Starting with the initial guess $x_1 = -2$, use Newton's method to approximate a root to the equation $e^x + x^2 - 3 = 0$ to eight decimal places.

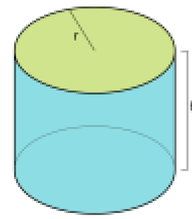
Answer.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{e^{x_n} + {x_n}^2 - 3}{e^{x_n} + 2x_n}$$

$$\begin{aligned} x_1 &= -2.00000000 \\ x_2 &= -1.70622671 \\ x_3 &= -1.67751675 \\ x_4 &= -1.67723274 \\ x_5 &= -1.67723271 \\ x_6 &= -1.67723271 \end{aligned}$$

□

2. (4 points) If 600π cm^2 material is available to make a cylinder with an open top, find the largest possible volume of the cylinder.
Hint: The surface area of a cylinder with an open top is $\pi r^2 + 2\pi rh$, where r is the base radius, h is the height.



Answer. $V = \pi r^2 h$
$A = 600\pi = \pi r^2 + 2\pi rh$ Solving the above for h, we find

$$h = \frac{600 - r^2}{2r}$$

Then,

$$\begin{aligned} V(r) &= \pi r^2 \left(\frac{600 - r^2}{2r} \right) \\ &= 300\pi r - \frac{\pi}{2} r^3 \\ \implies V'(r) &= 300\pi - \frac{3\pi}{2} r^2 \end{aligned}$$

We note that the endpoints $r = 0$ and $h = 0$ are physically absurd and the derivative exists everywhere, so the maximum *must* be a critical point of type derivative=0. We find that:

$$\begin{aligned} 0 &= 300\pi - \frac{3\pi}{2} r_{\text{max}}^2 \\ &= \frac{2}{3\pi} 300\pi - r_{\text{max}}^2 \\ &= 200 - r_{\text{max}}^2 \\ \implies r_{\text{max}} &= \pm\sqrt{200} = \pm 10\sqrt{2} \end{aligned}$$

We note that negative radius is physically absurd and conclude $r_{\text{max}} = 10\sqrt{2}$. Then,

$$\begin{aligned} h_{\text{max}} &= \frac{600 - r_{\text{max}}^2}{2r_{\text{max}}} \\ &= \frac{600 - (10\sqrt{2})^2}{20\sqrt{2}} = \frac{600 - 200}{2\sqrt{200}} = \sqrt{200} = 10\sqrt{2} \end{aligned}$$

cool. great. rad. Putting it together, $V = \pi r^2 h = \pi(10\sqrt{2})^3 = 2000\sqrt{2}\pi\text{cm}^2$. □

3. (3 points) Show that the curve $y = \sqrt{x^2+5} + 2x$ has one slant asymptote at $y = 3x$ and one horizontal asymptote at $y = 0$.

   Answer. Slant asymptote: we notice immediately that y grows without bound as x grows without bound. Thus, the slant asymptote must be on the right. Hence, we take the following limit:

$$\begin{aligned}
\lim_{x\to\infty}\left((\sqrt{x^2+5}+2x)-3x\right) &= \lim_{x\to\infty}\left((\sqrt{x^2+5}-x)\frac{\sqrt{x^2+5}+x}{\sqrt{x^2+5}+x}\right) \\
&= \lim_{x\to\infty}\frac{x^2+5-x^2}{\sqrt{x^2+5}+x} \\
&= \lim_{x\to\infty}\frac{5}{\sqrt{x^2+5}+x} \\
&= \lim_{x\to\infty}\frac{5/x}{\sqrt{1+5/x^2}+1} = 0
\end{aligned}$$

   By process of elimination, the horizontal asymptote must be on the left:

$$\lim_{x\to-\infty}(\sqrt{x^2+5}+2x) = \lim_{x\to-\infty}\left((\sqrt{x^2+5}+2x)\frac{\sqrt{x^2+5}-2x}{\sqrt{x^2+5}-2x}\right)$$

   Oh! Wait a minute! This problem is false! there isn't a horizontal asymptote, it's a slant asymptote at $y = x$! I'll let you know what the implications for grading are later... □