Math 1271 - Lectures 010 and 030 **Name (Print):** David DeMark
Fall 2017
Quiz 6A
10/20/17
Time Limit: 25 Minutes Section 012 & 016

You may *not* use your books, notes, graphing calculator, phones or any other internet devices on this exam.

You are **required** to show your work on each problem on this quiz.

Problem	Points	Score
1	3	
2	3	
3	4	
4	0	
Total:	10	

1. (3 points) Find the absolute minimum and absolute maximum on the given interval

$$x^3 - 65x + 10, \quad [-5, 6]$$

Two steps here. 1) Identify critical points and 2) plug those and our endpoints into the function

(which I'll call f).

1) $f'(x) = 3x^2 - 75$. $f'(x) = 0$ for $x = \pm 5$. f' exists everywhere.

2) $f(-5) = 260$, $f(5) = -240$, $f(6) = -224$. Thus, we have an absolute minimum at $(5, -240)$ and absolute maximum at $(-5, 260)$.

2. With the given function

$$y = \sqrt{13 - x^2}$$

   (a) (2 points) Find the differential $\mathrm{d}y$.

   (b) (1 point) Evaluate $\mathrm{d}y$ for $x = 2$ and $\mathrm{d}x = .1$.

   (a) We have that for $y = f(x)$, $\mathrm{d}y = \mathrm{d}f(x) = f'(x)\mathrm{d}x$. Thus,

$$\mathrm{d}y = \mathrm{d}(\sqrt{13 - x^2}) = \frac{-x\mathrm{d}x}{\sqrt{13 - x^2}}$$

   (b) Substitute in $x = 2$, $\mathrm{d}x = .1$ to the line above.

$$\mathrm{d}y = \frac{-(2)(.1)}{\sqrt{13 - (2)^2}} = \frac{-1}{15}$$

3. (4 points) A ladder 12 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 m/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 m away from the wall? Our set-up is as follows: We draw a triangle with vertices at the ladder's resting place on the wall, the corner between the floor and the wall, and the ladder's resting place on the floor. Let's draw a picture and label some stuff

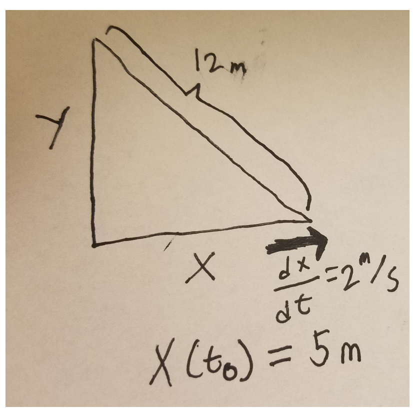

Figure 1: A triangle, labeled with the relevant data from the problem.

Known: $x(t_0) = 5\text{m}$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 2\text{m/s}$, $x^2 + y^2 = 12^2$. **Unknown**: $y'(t_0)$.

We can relate y' and x' by differentiating:

$$
\begin{aligned}
x(t)^2 + y(t)^2 &= 12^2 \\
\frac{\mathrm{d}}{\mathrm{d}t}\left(x(t)^2 + y(t)^2\right) &= \frac{\mathrm{d}}{\mathrm{d}t}\left(12^2\right) \\
2x(t) * x'(t) + 2y(t) * y'(t) &= 0 \\
\implies y'(t) &= \frac{-x(t) * x'(t)}{y(t)}
\end{aligned}
$$

To find $y'(t_0)$, we need $x(t_0)$ (which we have already), $x'(t_0)$ (ditto) and $y(t_0)$. But $12^2 = x(t_0)^2 + y(t_0)^2 = 25 + y(t_0)^2 \implies y(t_0) = \sqrt{144 - 25} = \sqrt{119}$. Now,

$$
y'(t_0) = \frac{-x(t_0)x'(t_0)}{y(t_0)} = \frac{-(5)(2)}{\sqrt{119}} = \frac{-10}{\sqrt{119}}
$$

4. (Bonus 2 points) Write the definition of a critical number.

   Relative to a function f, a critical number c is one at which $f'(c) = 0$ or $f'(c)$ does not exist (note: this is all that is needed—anything about maxima or minima or endpoints is superfluous.)