Implicit Differentiation and Applications (including rel8ed r8s)

1.)

Compute $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y (Hint: try simplifying or otherwise manipulating the equation to make things less messy). Then, find the tangent line at the given point (x_0, y_0)

$$\frac{2^y}{x} = 2yx \qquad (x_0, y_0) = (1, 1)$$

2.)

Use implicit differentiation to compute $f'(x)$ where $f(x) = \arccos x^2$ *without looking up the derivative of* $\arccos$. (Hint: you should get an answer in terms of x and $f(x)$ (or y if you so choose). Substitute in $y = \arccos x$, then draw a triangle to simplify it to the well-known version).

3.)

Compute $\frac{\mathrm{d}y}{\mathrm{d}x}$ for $y = x^{\sin x}$

4.)

A weather balloon is floating at a constant altitude of 50m above level ground. It is connected by a length of rope connecting it and a spool on the ground. The balloon is floating horizontally away from its tether at a constant rate of 3 m/s. Find the rate at which the angle (in radians!) its rope makes with the ground is changing when the rope is extended to a length of 626 m. You may assume that the rope is stretched fully taut.

Approximations: Differentials & Newton's Method

5.)

Use Newton's method to estimate a root to the following functions to five decimal places from the given starting point (if one is given).

a.)

$f(x) = e^x - 3x$; $x_1 = 1.5$

b.)

$f(x) = x^5 + x^4 - x^3 + x^2 - x + 1$

6.)

Use Newton's method to estimate the quantity $\sqrt{3+2\sqrt[3]{4}}$

7.)

Find $\mathrm{d}y$ **at** x_0 **given** $\mathrm{d}x$**.**

a.)

$y = x^3 + 1$; $x_0 = 2$, $\mathrm{d}x = .01$ (Do this without a calculator.)

b.)

$y = \cos(2\pi x^2)$; $x = .4$, $\mathrm{d}x = .1$

Differential Behavior: Curve Sketching & The Mean Value Theorem

8.)

Show that $f(x) = \frac{e^x - e^{-x}}{2}$ **does not have any root** x_0 **on the interval** $(0, \infty)$**. State which theorem you are using (if any) and justify that it may be applied.**

9.)

Given f, sketch the curve and list any asymptotes of any type. State whether the curve has even or odd symmetry or neither. State as well the domains on which f is (i) continuous (ii) increasing/decreasing (iii) concave up/down. (Hint: the functions given should factor and/or simplify surprisingly nicely.)

a.)

$f(x) = x^5 - 2x^3 + x$

b.)

$$f(x) = \frac{x^2 + xe^{-x} + 2x + e^{-x} + 1}{x + 1}$$

Extrema and Optimization

10.)

Let $L(x) = 2x + 1$ and $P(x) = -15 + 8x - x^2$.

a.)

Find the x-coordinate for the point on the graph of $y = L(x)$ which has the shortest *vertical* distance to the curve $y = P(x)$.

b.)

Find the x-coordinate for the point on the graph of $y = L(x)$ which has the shortest *total* distance to the curve $y = P(x)$.

11.)

(Go ahead and use a calculator on this one...)
Hans is planning his workout. He plans on using a stationary bike for some time, then going for a jog. He wants to burn 600 calories over the course of his workout. He burns $50 \log t_1$ calories in t_1 minutes on the stationary bicycle, and $75 \log t_2$ calories in t_2 minutes jogging. What is the least amount of time he could spend working out?[1]

[1] physiology/exercise science/nutrition majors: yes, I know this question is physically wildly unrealistic.

Odds & Ends: Indeterminates and Antiderivatives

12.)

Compute the following limits.

a.)

$$\lim_{x\to 1} \frac{x^6 - 1}{\sin(2\pi x)}$$

b.)

$$\lim_{x\to a} \frac{\cos x \ln(x-a)}{\ln(e^x - e^a)}$$

c.)

$$\lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^{4x}$$

13.)

Compute the antiderivative of $\dfrac{x - x^3}{\sqrt[3]{x}}$