Math 1271
Fall 2014
Exam 1a
Thursday 2 October 2014
Time Limit: 50 minutes

Name (Print): Solutions & grading guide
Student ID: ____________
Section Number: ______
Teaching Assistant: ____________
Signature: ____________

This exam contains 6 numbered problems on four sheets of paper. Check to see if any pages are missing. Point values are in parentheses.

No books or notes are allowed. Scientific calculators – that is, calculators that cannot display graphs of functions, cannot perform symbolic differentiation, and cannot solve algebraic equations – are allowed but not required.

Show your work, except where indicated.

1	20 pts	
2	20 pts	
3	15 pts	
4	10 pts	
5	15 pts	
6	20 pts	
TOTAL	100 pts	

1. (20 points) (5 points each) Multiple choice. CIRCLE the letter – A, B, C, or D – of the correct answer. No justification necessary. No partial credit.

Exam 1b

(a) Which of the following is the derivative $f'(x)$ of the function $f(x) = \cos(e^{(x^2)})$?

(A) $-\sin(e^{2x})$
(B) $-2x\sin(e^{(x^2)})$
(C) $-\sin(e^{(x^2)}) + \cos(2xe^{(x^2)})$
(D) $-2xe^{(x^2)}\sin(e^{(x^2)})$

A

(b) Which of the following is the derivative $g'(x)$ of the function

$$g(x) = 3^{(x^2+1)}?$$

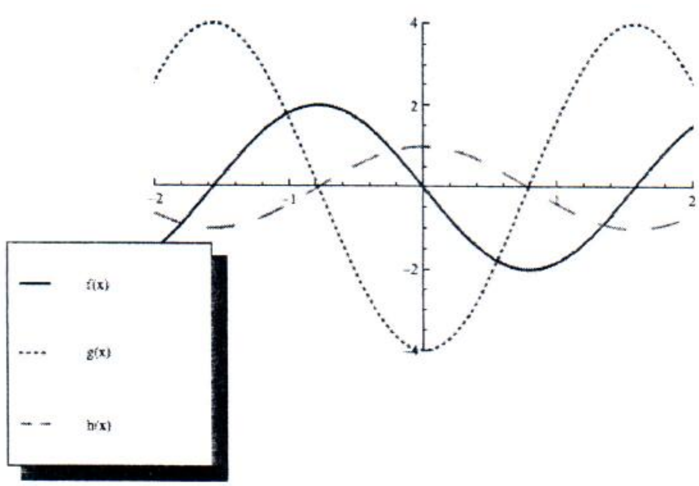
(A) $(x^2+1)\cdot 3^{(x^2)}\cdot(2x)$
(B) $(\ln 3)\cdot 3^{(x^2+1)}$
(C) $(\ln 3)\cdot(2x)\cdot 3^{(x^2+1)}$
(D) $(x^2+1)\cdot 3^{(x^2)}$

B

(c) Pictured below are the graph of $f(x)$ with a solid line, the graph of $g(x)$ with a dotted line, and the graph of $h(x)$ with a dashed line. Which of the following is most likely true?

(A) $f'(x) = g(x)$ and $g'(x) = h(x)$
(B) $h'(x) = f(x)$ and $g'(x) = h(x)$
(C) $h'(x) = f(x)$ and $f'(x) = g(x)$
(D) $f'(x) = h(x)$ and $h'(x) = g(x)$

B

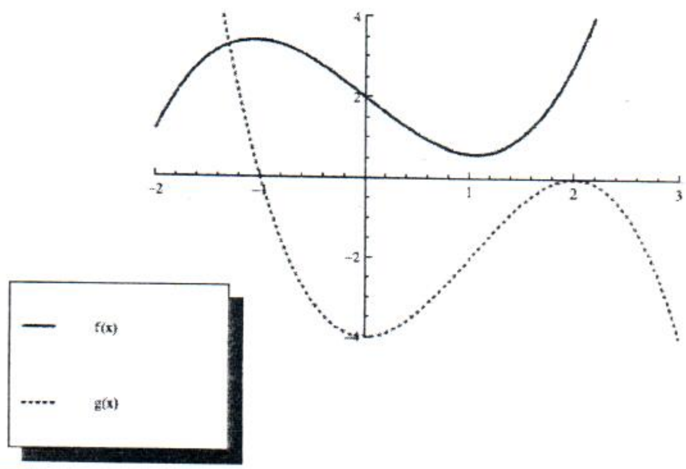


(d) Pictured below are the graph of $f(x)$ with a solid line and the graph of $g(x)$ with a dotted line. Which of the following quantities is positive?

Exam 1b

C

(A) $(fg)'(2)$
(B) $(fg)'(0)$
(C) $(fg)(2)$
(D) $(fg)'(-1)$



2. (20 points) The two parts of this problem refer to the function

$$f(t) = \frac{1}{\sqrt{t+1}} + 1.$$

no lim (−1)

(a) (15) Find $f'(t)$ directly from the definition of the derivative. That is, do not use derivative shortcuts, except to check your work.

$$f'(t) = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h} = \lim_{h\to 0} \frac{\left(\frac{1}{\sqrt{t+h+1}}+1\right) - \left(\frac{1}{\sqrt{t+1}}+1\right)}{h}$$

(5) correct definition with function in place correctly if present

$$= \lim_{h\to 0} \frac{\sqrt{t+1} - \sqrt{t+h+1}}{h\sqrt{t+1}\sqrt{t+h+1}} = \lim_{h\to 0} \frac{-h}{h\sqrt{t+1}\sqrt{t+h+1}\left(\sqrt{t+1}+\sqrt{t+h+1}\right)}$$

(8) correct algebra

$$= \frac{-1}{2(t+1)^{3/2}}.$$

(5) correct derivative, no matter how obtained.

(b) (5) Now suppose that $f(t)$ models the position, measured in meters, of a particle on a line, at time t, measured in seconds. Find the average velocity of the particle during the time interval between $t = 8$ and $t = 15$ seconds. Include correct units.

The average velocity on the interval $8 \le t \le 15$ is equal to

$$\underbrace{\frac{f(15)-f(8)}{15-8}}_{(2)} = \frac{\frac{5}{4}-\frac{4}{3}}{7} = \underbrace{\frac{-1}{84}}_{(2)}\ \underbrace{m/s}_{(1)}.$$

3. (15 points) (5 each) Suppose that f, g, and h are differentiable functions. Values for the functions and their derivatives are shown in the table. Find each quantity below, or state that the quantity cannot be determined from the information given.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
-1	-1	2	4	0	2	-1
0	1	-2	0	6	1	-2
1	2	-3	-1	-3	2	-1

(a) $(h \circ g \circ f)'(0)$

$$= h'(g(f(0))) \cdot g'(f(0)) \cdot f'(0)$$ 2

$$= h'(g(1)) \cdot g'(1) \cdot (-2)$$

$$= h'(-1) \cdot (-3) \cdot (-2)$$ 3

$$= (-1) \cdot (-3) \cdot (-2) = \boxed{-6}$$

(b) $\left(\frac{h}{g}\right)'(1)$

$$= \frac{h'(1)g(1) - h(1)g'(1)}{(g(1))^2}$$ 2

$$= \frac{(-1)(-1) - (2)(-3)}{(-1)^2} = \boxed{7}$$ 3

(c) $\frac{d}{dx}\left[g(x \cdot f(x))\right]$ at $x = -1$

2

$$\frac{d}{dx}[g(x \cdot f(x))] = g'(x \cdot f(x)) \cdot (1 \cdot f(x) + x \cdot f'(x))$$

At $x=-1$: $g'(-f(-1)) \cdot (f(-1) - f'(-1))$

$$= g'(1) \cdot (-1 - 2)$$ 3

$$= (-3) \cdot (-3) = \boxed{9}$$

4. (10 points) (5 each) Both parts of this problem refer to the piecewise-defined function

$$f(x) = \begin{cases} cx^2 - 3x & \text{if } x < 2 \\ 4 - cx & \text{if } x \geq 2, \end{cases}$$

where c is a real number.

(a) Find the value(s) of c that make(s) f a continuous function at every real number.

$$c(2)^2 - 3(2) = 4 - c(2)$$ 3

evaluate at $x = 2$ and set equal

$$4c - 6 = 4 - 2c$$

$$6c = 10$$

$$c = \frac{5}{3}.$$

2

(b) Find the value(s) of c so that the graph of f has a horizontal tangent line at $x = -1$.

$$f'(x) = 2cx - 3, \text{ for } x < 2.$$

$$f'(-1) = -2c - 3$$

The graph of f has a horizontal tangent line at $x=-1$ when $f'(-1) = -2c - 3 = 0$

3

set $f'(-1) = 0$

$$c = \frac{-3}{2}.$$

2

5. (15 points) (5 each) All three parts of this problem refer to the function

$$f(x) = \frac{3x^2 - 4x + 1}{2x^2 + 2x - 4}. \quad = \frac{(3x-1)(x-1)}{2(x+2)(x-1)} = \frac{3x-1}{2(x+2)}, \text{ for } x \neq 1.$$

A note about terminology: "$+\infty$", "$-\infty$", and "does not exist" are distinct possible answers.

(a) Find

$$\lim_{x\to+\infty} f(x). \quad = \boxed{\frac{3}{2}}$$

2

$$f(x) = \frac{3 - \frac{4}{x} + \frac{1}{x^2}}{2 + \frac{2}{x} - \frac{4}{x^2}} \longrightarrow \frac{3}{2}, \text{ as } x \to +\infty.$$

3 algebra

(b) Find

$$\lim_{x\to-2^+} f(x). \quad = \boxed{-\infty}$$

$$\left.\begin{array}{l} \text{as } x \to -2^+, \quad 3x - 1 \to -7, \text{ and} \\ \text{as } x \to -2^+, \quad 2(x+2) \to 0^+. \end{array}\right\}$$

3, or similar observations concerning sign

Thus

$$\lim_{x\to-2^+} f(x) = -\infty.$$

2

(c) Find

$$\lim_{x\to 1} f(x).$$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{3x-1}{2(x+2)} = \boxed{\frac{1}{3}}$$

2

3 algebra above

6. (20 points) Both parts of this problem refer to the equation $e^x = \cos x$.

(a) (15) Show that the equation $e^x = \cos x$ has at least one negative real root. Be sure to include the name of a theorem and the reason that the theorem applies to this situation.

Let $f(x) = \cos x - e^x$.

$f(-\pi) = -1 - e^{-\pi} < 0$.

$f(-2\pi) = 1 - e^{-2\pi} > 0$ (since $e^{-2\pi} < 1$)

5

NOTE: $x=0$ CANNOT be one of the endpoints!

5

Then, by the Intermediate Value Theorem,

5

since f is continuous, there exists c with $-2\pi < c < -\pi$, such that $f(c) = 0$. That is, $e^c = \cos c$.

(b) (5) The equation in (a) also has a root at $x = 0$; that is, the graphs of e^x and $\cos x$ intersect at $x = 0$. Do the two graphs also have the same tangent line at that point of intersection? Explain why or why not.

$\boxed{\text{NO:}}$ the derivative of e^x at $x=0$ is $e^0 = 1$,

and the derivative of $\cos x$ at $x=0$ is $-\sin 0 = 0 \neq 1$.

2

3

4/5 for stating slopes are different w/o calculating derivative.