Practice Test for Midterm III

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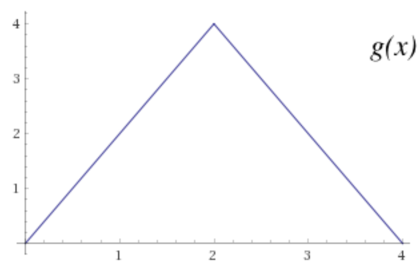
30 November 2017

Note: this is more problems than will be on the test, but it should give you a pretty good idea of what to expect in terms of difficulty.

1.)

The graph of a function $g(x)$ is below. Find $f(2), f'(2), h(2)$, and $h'(2)$ if

$$f(x) = \int_0^x g(t)\ dt \text{ and } h(x) = \int_0^{x^2} g(t)\ dt.$$



2.)

Determine a definite integral which has value given by the below limit. Do not evaluate either the limit or the integral.

a.)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \cos(1 + i\frac{4}{n})$$

b.)

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}e^{3+i\frac{6}{n}}\sin(1+i\frac{3}{n})$$

3.)

Let $g(x) = \int_0^x f(x)\,\mathrm{d}x$. What conditions must f fulfill in order to yield the conclusion from the first part of the fundamental theorem? What is that conclusion?

4.)

Use the ***limit*** definition of an integral and/or evaluation "in terms of areas" to evaluate the following integral without taking an anti-derivative.

$$\int_0^5 2x^2 - x + 1\,\mathrm{d}x$$

4.)

Evaluate the following integrals:

a.)

$$\int_2^{e^5-1} \frac{1-x}{1-x^2}\,\mathrm{d}x$$

b.)

$$\int \frac{5+x^2}{1+x^2}\,\mathrm{d}x$$

c.)

$$\int_1^2 \frac{t^2-4}{2t}\,\mathrm{d}t$$

6.)

600 gallons of water are stored in a cylindric tank with an inverted-dome bottom. A small hole breaks open at the very bottom at $t = 0$ s and grows wider as more water flows through so that the rate at which water flows through the hole is given by $r(t) = 3 + .5t$ in gallons. How long will it take before the last of the water leaks out of the tank?

7.)

Evaluate the following integrals

a.)

$$\int \frac{\sin(\ln\theta)}{\theta}\,\mathrm{d}\theta$$

b.)

$$\int_0^{\pi/3} \left(\frac{(1+\tan t)^{3/2}}{\cos t}\right)^2 \mathrm{d}t$$

c.)

$$\int_{-\pi}^{2017} \frac{\sin^2 x \sec^2 x}{\tan^2 x}\,\mathrm{d}x$$

d.)

$$\int_0^a x\sqrt{a-x^2}\,\mathrm{d}x$$