

# Tropical Grassmannians and Clusters: Associating $\text{Trop}^+\text{Gr}_{3,6}$ with cluster algebras of type $D_4$ .

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In this paper, we intend to give a detailed explanation (but not a full proof) of Theorem 4.1 in [BCL17], which gives an explicit bijection between the rays of the Speyer-Williams fan  $F_{3,6}$  and the almost positive roots of type  $D_4$ . Much attention will be focused on defining and motivating the definitions above. [MS15] will be used as a “bedrock” resource for all things tropical, most pertinently, sections 4.3 (*Grassmannians*) and 5.4 (*Arrangements of Trees*) will be drawn upon heavily in describing the tropical grassmanian and its positive part. Similarly, the draft ([FWZ16],[FWZ17]) of Fomin, Williams and (posthumously) Zelevinsky’s text on Cluster Algebras will serve as a “bedrock” source for all things related to cluster algebras; in particular, chapter 5 (*Finite type classification*) will be instrumental in defining clusters of type  $D_n$ . In addition to the sources already named, [SW05] (in which it is originally defined) will be crucial in defining  $F_{k,n}$  and illustrating its connection to  $\text{Trop}^+\text{Gr}_{k,n}$ . To illustrate this relationship, we shall reconstruct the computations of [BCL17, §5], in which the authors compute which (combinatorial types of) planes in the tropical projective space  $\mathbb{TP}^5$  are realized in  $\text{Trop}^+\text{Gr}_{3,6}$  as according to the classification of such planes by [Her+09].

## References

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