1. Show that the connected sum of a torus T and the projective plane $\mathbb{R}\mathbf{P}^2$ is homeomorphic to the connected sum of three copies of $\mathbb{R}\mathbf{P}^2$.

   Proof. Let $T\#\mathbb{R}\mathbf{P}^2$ be given in the first picture below. Then by a chain of cuttings and gluings: We see that $T\#\mathbb{R}\mathbf{P}^2 \simeq \mathbb{R}\mathbf{P}^2\#\mathbb{R}\mathbf{P}^2\#\mathbb{R}\mathbf{P}^2$ as desired. □

2. Let X be a surface obtained by pasting edges of an 8-sided polygon with labeling scheme

$$a_1a_2a_3a_4a_1a_4^{-1}a_3a_2^{-1}.$$

   To which standard surface is X homeomorphic?

   Proof. We consider the polygon with the given labeling scheme, and follow a chain of cuttings and gluings: Hence, X is homeomorphic to the connected sum of three copies of $\mathbb{R}\mathbf{P}^2$.

   □

3. I'm not sure what problem I was working on this code for, but here it is anyway.