Practice Test for Midterm III

David DeMark

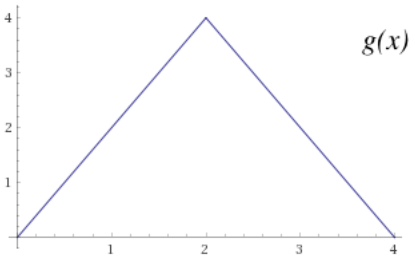
30 November 2017

Note: this is more problems than will be on the test, but it should give you a pretty good idea of what to expect in terms of difficulty.

1.)

The graph of a function $g(x)$ is below. Find $f(2), f'(2), h(2)$, and $h'(2)$ if

$$f(x) = \int_0^x g(t)\ dt \text{ and } h(x) = \int_0^{x^2} g(t)\ dt.$$



Answer. $f(2) = 4,\ f'(2) = 4,\ h(2) = 8,\ h'(2) = 0$ □

2.)

Determine a definite integral which has value given by the below limit. Do not evaluate either the limit or the integral.

a.)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \cos(1 + i\frac{4}{n})$$

Answer. $\int_1^5 \cos(x)\,\mathrm{d}x$ □

b.)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} e^{3+i\frac{6}{n}} \sin(1 + i\frac{3}{n})$$

Answer. $\int_1^4 e^{1+2x} \sin(x)\,\mathrm{d}x$.
Note for problem 2, it is possible to find other (equivalent) answers. □

3.)

Let $g(x) = \int_0^x f(x)\,\mathrm{d}x$. What conditions must f fulfill in order to yield the conclusion from the first part of the fundamental theorem? What is that conclusion?

Answer. $f(x)$ must be continuous on $[0, b]$ for some $b > 0$. When this is fulfilled, $g'(x) = f(x)$ when $0 \leq x \leq b$. □

4.)

Use the ***limit*** definition of an integral and/or evaluation "in terms of areas" to evaluate the following integral without taking an anti-derivative.

$$\int_0^5 2x^2 - x + 1\,\mathrm{d}x$$

Answer. $\frac{455}{6} \approx 75.8333$ □

4.)

Evaluate the following integrals:

a.)

$$\int_2^{e^5-1} \frac{1-x}{1-x^2}\,\mathrm{d}x$$

Answer. $5 - \ln(3) \approx 3.90139$ □

b.)

$$\int \frac{5+x^2}{1+x^2}\,\mathrm{d}x$$

Answer. $x + 4\tan^{-1}(x) + c$ □

c.)

$$\int_1^2 \frac{t^2-4}{2t}\,\mathrm{d}t$$

Answer. $\frac{3}{4} - \ln(4) \approx -0.636294$ □

6.)

600 gallons of water are stored in a cylindric tank with an inverted-dome bottom. A small hole breaks open at the very bottom at $t = 0$ s and grows wider as more water flows through so that the rate at which water flows through the hole is given by $r(t) = 3 + .5t$ in gallons. How long will it take before the last of the water leaks out of the tank?

Answer. $2\sqrt{609} - 6 \approx 43.3559$ □

7.)

Evaluate the following integrals

a.)

$$\int \frac{\sin(\ln \theta)}{\theta}\, \mathrm{d}\theta$$

Answer. $-\cos(\log(t)) + c$ □

b.)

$$\int_0^{\pi/3} \left(\frac{(1+\tan t)^{3/2}}{\cos t}\right)^2 \mathrm{d}t$$

Answer. $4\sqrt{3} + \frac{27}{4} \approx 13.6782$ □

c.)

$$\int_{-\pi}^{2017} \frac{\sin^2 x \sec^2 x}{\tan^2 x}\, \mathrm{d}x$$

Answer. $2017 + \pi$ □

d.)

$$\int_0^a x\sqrt{a - x^2}\, \mathrm{d}x$$

Answer. $\frac{1}{3}\left(1 - (1-a)^{3/2}\right) a^{3/2}$ □