Math 1271 - Lectures 010 and 030 **Name (Print):** ____________________

Fall 2017

Quiz 5

10/12/17

Time Limit: 25 Minutes Teaching Assistant ____________________

You may *not* use your books, notes, graphing calculator, phones or any other internet devices on this exam.

You are required to show your work on each problem on this quiz.

Problem	Points	Score
1	2	
2	4	
3	4	
Total:	10	

1. (2 points) Differentiate the function $f(x) = e^{x^3}$.

   [Response] Chain rule! Let $g(x) = e^x$ and $h(x) = x^3$. Then, $g'(x) = e^x$ and $h'(x) = 3x^2$. Since $f(x) = g(h(x))$, the chain rule says $f'(x) = h'(x)g(h'(x)) = (3x^2)e^{x^3}$. It was also possible to start by taking the log of both sides and work from there—this would get you the right answer if you did it right, but there's a lot more to screw up there...

   One other thing to note: $e^{x^3} \neq e^{3x}$, or more generally $a^{(b^c)} \neq (a^b)^c$. To illustrate: $2^{2^3} = 2^8 = 256$, but $(2^2)^3 = 2^6 = 64$.

2. (4 points) Differentiate $y = x^{e^x}$ using logarithmic differentiation (hint: take the natural logarithm of both sides, simplify, and find y' using implicit differentiation).

   [Response] There are two ways to start this: take ln of both sides to get $\ln y = \ln(x^{e^x})$ and then recall $\ln(a^b) = b\ln(a)$ (so $\ln y = e^x \ln x$), or just use the identity $x = e^{\ln x}$ to re-write $x^{e^x} = (e^{\ln x})^{e^x} = e^{e^x \ln x}$, then take logarithm of both sides to get $\ln y = \ln(e^{e^x \ln x}) = e^x \ln x$.

   Anyway, we should wind up with the equation

   $$\ln y = e^x \ln x$$

   Now, let's differentiate and use the product rule on the right. Write: $f(x) = g(x)h(x)$ with $g(x) = e^x$ and $h(x) = \ln x$. The product rule then gives

   $$\frac{y'}{y} = e^x \ln x + \frac{e^x}{x}$$

   Now multiplying through by y, we get

   $$y' = ye^x(\ln x + \frac{1}{x})$$

   Substituting in $y = x^{e^x}$ gives us our answer

   $$y' = e^x x^{e^x}(\ln x + \frac{1}{x})$$

3. (4 points) If $f(x)^3 + \sin(f(x)) = x$ and $f(0) = 0$, find $f'(0)$.

   [Response] When we see some messy function of $f(x)$ like the left and need to find something involving f', our first thought should be implicit differentiation. I'll make the aesthetic choice to call $f(x)$ by y. Let's re-write it like that and then take $\frac{\mathrm{d}}{\mathrm{d}x}$ of both sides.

   $$\begin{aligned} y^3 + \sin(y) &= x \\ \frac{\mathrm{d}}{\mathrm{d}x}(y^3 + \sin(y)) &= \frac{\mathrm{d}}{\mathrm{d}x}x \\ 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} + \cos(y)\frac{\mathrm{d}y}{\mathrm{d}x} &= 1 \end{aligned}$$

   Notice the left simplifies to $\frac{\mathrm{d}y}{\mathrm{d}x}(3y^2 + \cos y)$. Now we can divide through to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y and x. Once we have that, let's re-write it in terms of f.

   $$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{3y^2 + \cos(y)} \\ f'(x) &= \frac{1}{3f(x)^2 + \cos(f(x))} \end{aligned}$$

   (continued on third page)

We need $f'(0)$. We know what $f(0)$ is so we can use it!

$$
\begin{aligned}
f'(0) &= \frac{1}{3f(0)^2 + \cos(f(0))} \\
&= \frac{1}{3(0)^2 + \cos(0)} \\
&= \frac{1}{0+1} = 1
\end{aligned}
$$