Prologue/Rapid-fire round: Midterm 1 Review

Do not take much time on this section—if this stuff isn't automatic for you, go back to chapters 2 and 3!!

-2.)

Compute the following limits or state they do not exist.

a.)

$$\lim_{x\to 3} \frac{x^2-9}{x^2+x-12}$$

b.)

$$\lim_{x\to\infty} \frac{6x^{-2}+2x^{-1}-1}{8x^{-5}+2}$$

c.)

$$\lim_{x\to -2} \frac{|x^2-3x+10|}{x^2+5x+6}$$

d.)

$$\lim_{x\to\infty} \frac{(x\cos(\sqrt{x}))^2}{x^3+7}$$

-1.)

Given $f(x)$, find $f'(x)$ and use it to compute $T(x)$, the tangent line to $y = f(x)$ and $x = x_0$.

a.)

$f(x) = \dfrac{x^3+10x}{e^x}$
$x_0 = 1$

b.)

$f(x) = \sqrt{e^x + \tan x}$
$x_0 = 0$

c.)

$f(x) = \ln(x^3 + 3^x)$
$x_0 = 2$

0.)

Use the definition of derivative to find $f'(x)$. Check your work using rules of differentiation. Also, state the domain of f

$f(x) = \sqrt{2x^2 - 1}$

Implicit Differentiation and Applications (including rel8ed r8s)

1.)

Compute $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y (Hint: try simplifying or otherwise manipulating the equation to make things less messy). Then, find the tangent line at the given point (x_0, y_0)

$$\frac{2^y}{x} = 2yx \qquad (x_0, y_0) = (1, 1)$$

2.)

Use implicit differentiation to compute $f'(x)$ where $f(x) = \arccos(x^2)$ *without looking up the derivative of* $\arccos$. (Hint: you should get an answer in terms of x and $f(x)$ (or y if you so choose). Substitute in $y = \arccos x$, then draw a triangle to simplify it to the well-known version).

3.)

Compute $\frac{\mathrm{d}y}{\mathrm{d}x}$ **for** $y = x^{\sin x}$

4.)

A weather balloon is floating at a constant altitude of 50m above level ground. It is connected by a length of rope connecting it and a spool on the ground. The balloon is floating horizontally away from its tether at a constant rate of 3 m/s. Find the rate at which the angle (in radians!) its rope makes with the ground is changing when the rope is extended to a length of 626 m. You may assume that the rope is stretched fully taut.

Approximations: Differentials & Newton's Method

5.)

Use Newton's method to estimate a root to the following functions to five decimal places from the given starting point (if one is given).

a.)

$f(x) = e^x - 3x$; $x_1 = 1.5$

b.)

$f(x) = x^5 + x^4 - x^3 + x^2 - x + 1$

6.)

Use Newton's method to estimate the quantity $\sqrt{3+2\sqrt[3]{4}}$

7.)

Find $\mathrm{d}y$ **at** x_0 **given** $\mathrm{d}x$**.**

a.)

$y = x^3 + 1$; $x_0 = 2$, $\mathrm{d}x = .01$ (Do this without a calculator.)

b.)

$y = \cos(2\pi x^2)$; $x = .4$, $\mathrm{d}x = .1$

Differential Behavior: Curve Sketching & The Mean Value Theorem

8.)

Show that $f(x) = \frac{e^x - e^{-x}}{2}$ does not have any root x_0 on the interval $(0, \infty)$. State which theorem you are using (if any) and justify that it may be applied.

9.)

Given f, sketch the curve and list any asymptotes of any type. State whether the curve has even or odd symmetry or neither. State as well the domains on which f is (i) continuous (ii) increasing/decreasing (iii) concave up/down. (Hint: the functions given should factor and/or simplify surprisingly nicely.)

a.)

$f(x) = x^5 - 2x^3 + x$

b.)

$$f(x) = \frac{x^2 + xe^{-x} + 2x + e^{-x} + 1}{x + 1}$$

Extrema and Optimization

10.)

Let $L(x) = 2x + 1$ and $P(x) = -15 + 8x - x^2$. **Find the x-coordinate for the point on the graph of $y = L(x)$ which has the shortest *vertical* distance to the curve $y = P(x)$.**

11.)

(Go ahead and use a calculator on this one...)
Hans is planning his workout. He plans on using a stationary bike for some time, then going for a jog. He wants to burn 600 calories over the course of his workout. He burns $50 \log t_1$ calories in t_1 minutes on the stationary bicycle, and $75 \log t_2$ calories in t_2 minutes jogging. What is the least amount of time he could spend working out?[1]

[1] physiology/exercise science/nutrition majors: yes, I know this question is physically wildly unrealistic.

Odds & Ends: Indeterminates and Antiderivatives

12.)

Compute the following limits.

a.)

$$\lim_{x \to 1} \frac{x^6 - 1}{\sin(2\pi x)}$$

b.)

$$\lim_{x \to a} \frac{\cos x \ln(x - a)}{\ln(e^x - e^a)}$$

c.)

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{4x}$$

13.)

Let $f'(x) = \dfrac{x - x^3}{\sqrt[3]{x}}$. Suppose $f(1) = \frac{4}{11}$. Find $f(x)$.