Math 1271 - Lecture 050 **Name (Print):** ____________________
Spring 2018
Quiz VII
3/22/18
Time Limit: 20 Minutes Section ____________________

You may *not* use your books, notes, graphing calculator, phones or any other internet devices on this exam. Please show all work clearly and legibly.

Problem	Points	Score
1	8	
2	12	
Total:	20	

1. (8 points) A shoebox (with no lid) is to have a rectangular base with length double its width and an open top. You have 450 in^2 of cardboard from which to make the box. What dimensions maximize its volume?

Response. We draw the picture of figure one (note that the box is three-dimensional):

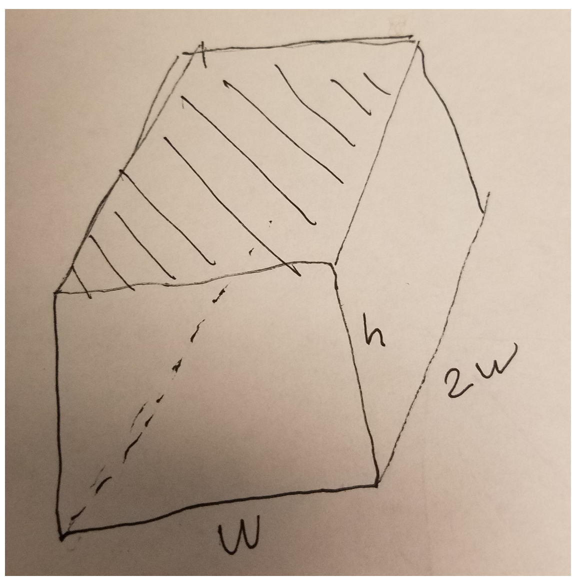

Figure 1: A box with base-length twice its width and an open top

The quantity we must maximize is $V = 2w^2h$. Our given constraint is that the surface area is 450 in^2. Going side-by-side, we calculate that the surface area is $2w^2 + 2wh + 2 * 2wh = 2w^2 + 6wh$, so we have the relation $450 = 2w^2 + 6wh$. We also have the endpoints $h \geq 0$, $w \geq 0$. We use our relation to find

$$h = \frac{450 - 2w^2}{6w} = 75w^{-1} - \frac{1}{3}w$$

Then, we use this to adjust our endpoint:

$$\begin{aligned} h &\geq 0 \\ \implies 75w^{-1} - \frac{1}{3}w &\geq 0 \\ \implies 75 - \frac{1}{3}w^2 &\geq 0 \\ \implies 75 &\geq \frac{1}{3}w^2 \\ \implies 225 &\geq w^2 \\ \implies 15 &\geq w \end{aligned}$$

Thus, we are optimizing on the interval $w \in [0, 15]$. Finally, we use our form for h to put V in terms of w:

$$V(w) = 2w^2h = 2w^2(75w^{-1} - \frac{1}{3}w) = 150w - \frac{2}{3}w^3$$

To find critical points, we find $V'(w) = 150 - 2w^2$. We set it equal to 0:

$$\begin{aligned} 0 &= 150 - 2w^2 \\ \implies 2w^2 &= 150 \\ \implies w^2 &= 75 \\ \implies w = \sqrt{75} &= 5\sqrt{3} \end{aligned}$$

We note as well that $V(0) = V(15) = 0$. Since $w = 5\sqrt{3}$ is our only critical point and $V(5\sqrt{3}) > 0$, we have that it is our maximizing width. We then calculate for our maximizing h:

$$\begin{aligned} h &= \frac{75}{w} - \frac{w}{3} \\ &= 5\sqrt{3} - \frac{5}{3}\sqrt{3} = \frac{10\sqrt{3}}{3} \end{aligned}$$

$\square$

2. Let $f(x) = x^3 + x^2 - 2x - 2$

   (a) (6 points) Determine the intervals of concavity and inflection points for f.

   Response. We find that $f''(x) = 6x + 2 = 6(x + \frac{1}{3})$. Thus, f is concave down on $(-\infty, -\frac{1}{3})$ and concave up on $(-\frac{1}{3}, \infty)$ with an inflection point at $x = -\frac{1}{3}$. □

   (b) (6 points) Use Newton's method to estimate a **critical point** of f to five decimal places with starting point $x_1 = 1/2$.
   (Hint: stop and read this question again before you start working on your answer).

   Response. Note that finding a **critical point** equates to finding a **zero** of the **derivative.** Thus, our set-up should be

$$x_i = x_{i-1} - \frac{f'(x_i)}{f''(x_i)}$$

   Using this, we find:

$$\begin{aligned} x_1 &= .5 \\ x_2 &= .55 \\ x_3 &= .548585 \\ x_4 &= .548584 \end{aligned}$$

   Thus, the answer we are looking for is $x \approx .54858$. □