1.)

(Pre-calc review) Each of the following prompts corresponds to exactly one of the options from the "answer bank." Each answer in the answer bank corresponds to at most one prompt. Match answers to prompts! **!!!DO NOT USE A CALCULATOR OR LOOK UP ANY FORMULAS BESIDES THE ONES GIVEN!!!**

Formulas & Definitions

$\tan(x) = \frac{\sin(x)}{\cos(x)}$, $\sec(x) = \frac{1}{\cos(x)}$, $\csc(x) = \frac{1}{\sin(x)}$, $\cot(x) = \frac{1}{\tan(x)}$, $1 = \cos^2 x + \sin^2 x$

Round 1

1. An equation giving a circle of radius 2 centered at $(2, 3)$: **E**
2. An equation giving a line perpendicular to the line between $(1, 0)$ and $(5, 3)$:**A**
3. $\tan(\pi/6) - \cot(\pi/6)$:**I**
4. An equation giving a line which intersects the parabola $y = x^2$ *exactly* once: **N/A**
5. $\dfrac{\ln 32 + \ln 2}{\ln 4}$:**D**
6. An equation giving a circle of radius 4 centered at $(-2, -3)$:**K**
7. A solution to the equation $y = x^2 + \frac{2}{3}x - \frac{53}{6}$: **N/A** (should be $y = \frac{4}{3}x - \frac{4}{9}$)
8. $\dfrac{\sin \pi/2}{\cos \pi/3}$: **H**

Answer Bank:

A) $y = -\dfrac{4}{3}x + \dfrac{6}{5}$	***B)*** $(x-2)^2 + (y-3)^2 = 16$	***C)*** $y = -\dfrac{3}{4}x + \dfrac{6}{8}$	***D)*** 3	
E) $(x-2)^2 + (y-3)^2 = 4$	***F)*** $\dfrac{\sqrt{3}}{3}$	***G)*** $(x+2)^2 + (y+3)^2 = 2$	***H)*** 2	***I)*** $\dfrac{-2}{\sqrt{3}}$
J) $y = \dfrac{4}{3}x - \dfrac{1}{9}$	***K)*** $(x+2)^2 + (y+3)^2 = 16$	***L)*** $\dfrac{3}{\sqrt{3}}$	***M)*** $\sqrt{6} - \dfrac{1}{3}$	

Explanations

1. The equation for a circle centered at (x_0, y_0) of radius r is given by an equation involving the *distance formula*:
$$(y - y_0)^2 + (x - x_0)^2 = r^2.$$
Plugging in $x_0 = 2, y_0 = 3, r = 2$, we get
$$(y - 3)^2 + (x - 2)^2 = 4$$
as in E).
2. Note the line between $(1, 0)$ and $(5, 3)$ is of slope $\frac{3}{4}$. For a line of slope m, the slope of any line perpendicular to it is $\frac{-1}{m}$. Thus, the slope of the line we're looking for is $\frac{-4}{3}$. A is the only answer which gives a line of that slope.
3. Use special right triangles (in particular the 30-60-90 triangle) to compute $\tan(\pi/6) = \frac{1}{\sqrt{3}}$. Then, $\tan(\pi/6) - \cot(\pi/6) = \frac{1}{\sqrt{3}} - \sqrt{3}$. Finding a common denominator, we have $\tan(\pi/6) - \cot(\pi/6) = \frac{1-3}{\sqrt{3}} = -\frac{2}{\sqrt{3}}$.
4. This one can only really be done by testing any lines we haven't eliminated and solving for an intersection point. If we try solving for intersections between the line in C and $y = x^2$, we need to solve the equation $x^2 = -\frac{3}{4}x + \frac{6}{8}$, or $x^2 + \frac{3}{4}x - \frac{6}{8} = 0$. The quadratic formula gives
$$x = \frac{1}{8}\left(-3 \pm \sqrt{57}\right),$$
which, in particular, is two different x-coordinates (we could have also seen this by noting $b^2 - 4ac \neq 0$ for the equation $x^2 + bx + c = 0$). Unfortunately the same happens with the equation in J) because I screwed up making this (oops, sorry.). However, replacing it with what it *should* be, solving $x^2 = \frac{4}{3}x - \frac{4}{9}$ gives only one solution, $\frac{2}{3}$.

5. Start with $x = \frac{\ln 32+\ln 2}{\ln 4}$. Then, $x \ln 4 = \ln 32+\ln 2$. Exponentiate each side with base e. Now, $e^{x\ln 4} = e^{\ln 32+\ln 2}$. Recall that by definition of natural logarithm, for any a, $e^{\ln a} = \ln(e^a) = a$. Two other useful facts are that (for any z, a, b) $z^{a*b} = (z^a)^b$ and $z^{a+b} = z^a z^b$. Now, we can use these to first rewrite $e^{x\ln 4}$ as $(e^{\ln 4})^x = 4^x$, and $e^{\ln 32+\ln 2} = e^{\ln 32}e^{\ln 2} = 32 * 2 = 64$. Now, our equation from above is $4^x = 64$. This is solved by $x = 3$.

6. See explanation 1.

7. oops

8. Use the unit circle and/or special right triangles to find $\sin(\pi/2) = 1$ and $\cos(\pi/3) = 1/2$.

Round 2

As it turns out, there are more mistakes here then I realized, so this is pretty messy. My apologies, this is sloppy work on my part.

1. $|\tan x|$: **N/A** (but ALMOST D)
2. $\dfrac{1-2\cos^2 x}{\cos^2 x - \sin^2 x}$: **-1**
3. $\dfrac{\cos^2(x) - \sin^2(x)}{2\cos^2(x) - 1}$: **C**
4. $|\cot x|$: **E**
5. $\dfrac{-\cos^2 x}{\sin x - 1}$: **F**
6. $\cot x - \sin x + \sin^3 x$: **N/A**
7. $|\csc x|$: **B**

Answer Bank: ***A)*** $\dfrac{1-\cos x}{\sin x \sec x}$ ***B)*** $\sqrt{\dfrac{1+\tan^2 x}{\tan^2 x}}$ ***C)*** 1 ***D)*** $\sqrt{1-\sec^2(x)}$ ***E)*** $\dfrac{\sqrt{1-\sin^2(x)}}{\sqrt{1-\cos^2(x)}}$ ***F)*** $\sin x + 1$

Explanations

1. D should have actually been $\sqrt{\sec^2 x - 1}$. If it were that, we would get that

$$\begin{aligned}\sec^2 x - 1 &= \frac{1}{\cos^2 x} - 1\\ &= \frac{1-\cos^2 x}{\cos^2 x}\\ &= \frac{\sin^2 x}{\cos^2 x}\\ &= \tan^2 x.\end{aligned}$$

2. The identity on the first page implies $\cos^2 x + \sin^2 x - 1 = 0$. We may apply this freely. We add $0 = \cos^2 x + \sin^2 x - 1$ to the numerator to yield $1 - 2\cos^2 x = \sin^2 x - \cos^2 x$.

3. Note that the expression for #3 is simply the negative of the reciprocal of the expression in #2!!

4. $1 - \sin^2 x = \cos^2 x$ and $1 - \cos^2 x = \sin^2 x$, so the expression for E) can be written $\sqrt{\frac{\cos^2 x}{\sin^2 x}} = \sqrt{\cot^2 x} = |\cot x|$.

5. $-\cos^2 x = \sin^2 x - 1 = (\sin x - 1)(\sin x + 1)$. Now, dividing by $\sin x - 1$ cancels one of those to yield $\sin x + 1$.

6. Oy vey. Yeah, I screwed this one up. Start with the expression in A and interpret $1/\sec x = \cos x$:

$$\begin{aligned}\frac{1-\cos x}{\sin x \sec x} &= \frac{\cos x - \cos^2 x}{\sin x}\\ &= \cot x - \frac{\cos^2 x}{\sin x}\end{aligned}$$

Now, use $-\cos^2 x = \sin^2 x - 1$. We yield $\cot x - \frac{\sin^2 x - 1}{\sin x}$. Unfortunately, rather than dividing by sin, I multiplied to get the monstrosity above. Item 6 should have been $\cot x - \sin x + \csc x$.

7. Note $\frac{1+\tan^2 x}{\tan^2 x} = 1 + \cot^2 x = 1 + \frac{\cos^2 x}{\sin^2 x}$. Now, find a common denominator and yield $\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} = \csc^2 x$.

2.)

a.)

Let $x_0 = 1$, and $f(x) = x^2 - 2x$. Compute the slope of the secant line between x_0 and $x_1 = 5$, $x_2 = 2$, $x_3 = 1.5$, $x_4 = 1.1$ and $x_5 = 1.01$. Use this to find an estimate for the slope of the tangent line to $y = f(x)$ at x_0.

Response. The following gives a table of secant-slopes using the formula Slope $= \frac{f(x)-f(1)}{x-1}$:

x	Slope
5	4
2	1
1.5	0.5
1.1	0.1
1.01	0.01

The slope appears to be approaching 0. □

b.)

Use the same techniques to estimate the slope of the tangent line for $f(x) = x^3$ at $x_0 = 1$.

Response. We use the same technique:

x	Slope
5	31
2	7
1.5	4.75
1.1	3.31
1.01	3.0301
1.001	3.003

The slope appears to be approaching 3.

□