Math 1271 - Lecture 050 **Name (Print):** David DeMark (KEY)
Spring 2018
Quiz VI
3/8/18
Time Limit: 20 Minutes Section ______________

You may *not* use your books, notes, graphing calculator, phones or any other internet devices on this exam. Please show all work clearly and legibly.

Problem	Points	Score
1	7	
2	6	
3	7	
Total:	20	

1. (a) (4 points) State the mean value theorem.

Response. For real numbers $a < b$, **if**[1] f is a function which is continuous on $[a, b]$ and differentiable on (a, b), **then** there is a real number c with $a < c < b$ such that $f'(c) = \frac{f(a)-f(b)}{a-b}$. □

(b) (3 points) For $f(x) = x^3+x+4$ find all c in the interval $(-2, 0)$ which satisfy the statement of the mean value theorem for $f(x)$ on the interval $[-2, 0]$.

Response. We compute that $f(0) = 4$ and $f(-2) = -6$. Thus, we are looking for c such that $f'(c) = \frac{f(0)-f(-2)}{0-(-2)} = \frac{4-(-6)}{2} = 5$. We set $f'(c) = 3c^2 + 1 = 5$. Solving that yields $c = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$. As only $-\frac{2}{\sqrt{3}}$ is in our interval $(-2, 0)$, that is our only such c value. □

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[1] Note the **If... Then...** structure to my response. The **if** is the hypothesis and the **then** is the conclusion. Both parts are necessary to any theorem!!!

2. (6 points) Let $g(x) = x^5 - 10x$. Find where g is increasing and decreasing and find any local maxima or minima.

   Response. As we will refer to both, we compute $g'(x)$ and $g''(x)$ upfront:

$$g'(x) = 5x^4 - 10 = 5(x^4 - 2)$$
$$g''(x) = 20x^3$$

   To find intervals of increase & decrease, we need to determine when $f'(x) > 0$. To do that, we solve for critical points. g is differentiable everywhere, so all critical points are of the type $g'(x) = 0$. We set $g'(x) = x^4 - 2 = (x^2 - \sqrt{2})(x^2 + \sqrt{2}) = (x - \sqrt[4]{2})(x + \sqrt[4]{2})(x^2 + \sqrt{2}) = 0$ and get $x = \pm\sqrt[4]{2}$. Thus, our intervals of interest are $(-\infty, -\sqrt[4]{2})$, $(-\sqrt[4]{2}, \sqrt[4]{2})$ and $(\sqrt[4]{2}, \infty)$.

   Plugging in $x = -2$ gives $g'(-2) = 5(2^4 - 2) = 70$. Plugging in $x = 0$ gives $g'(0) = -10$. Finally, by even symmetry, plugging in $x = 2$ gives $g'(2) = 70$. Thus, g is increasing on $(-\infty, -\sqrt[4]{2}) \cup (\sqrt[4]{2}, \infty)$ and decreasing on $(-\sqrt[4]{2}, \sqrt[4]{2})$. We note $g''(x)$ is positive when $x > 0$ and negative when $x < 0$. Thus, $x = -\sqrt[4]{2}$ is a local maximum and $x = \sqrt[4]{2}$ is a local minimum by the second derivative test. □

3. Use L'hopital's rule to compute the following limits:

   (a) (3 points)

$$\lim_{x\to 0} \frac{e^x - 1}{x} =$$

   Response. Note $\lim_{x\to 0} e^x - 1 = 0 = \lim_{x\to 0} x$. Thus,

$$\lim_{x\to 0} \frac{e^x - 1}{x} = \lim_{x\to 0} \frac{\frac{\mathrm{d}}{\mathrm{d}x}(e^x - 1)}{\frac{\mathrm{d}}{\mathrm{d}x}(x)} = \lim_{x\to 0} \frac{e^x}{1} = \frac{e^0}{1} = 1$$

   □

   (b) (4 points)

$$\lim_{x\to 0} \frac{\sin(3x)}{\tan(7x)} =$$

   Response. Note $\lim_{x\to 0} \sin(3x) = 0 = \lim_{x\to 0} \tan(7x)$. Thus,

$$\lim_{x\to 0} \frac{\sin(3x)}{\tan(7x)} = \lim_{x\to 0} \frac{\frac{\mathrm{d}}{\mathrm{d}x}\sin(3x)}{\frac{\mathrm{d}}{\mathrm{d}x}\tan(7x)} = \lim_{x\to 0} \frac{3\cos(3x)}{7\sec^2(7x)} = \frac{3}{7}\lim_{x\to 0} \frac{\cos(3x)}{\sec^2(7x)} = \frac{3}{7} * \frac{1}{1} = \frac{3}{7}$$

   □