3.)

Evaluate the following limits.

a.)

$$\lim_{x\to 2}\frac{1}{x-2}$$

Response. $\displaystyle\lim_{x\to 2^-}\frac{1}{x-2} = -\infty$, $\displaystyle\lim_{x\to 2^+}\frac{1}{x-2} = \infty$, so $\displaystyle\lim_{x\to 2}\frac{1}{x-2}$ does not exist. □

b.)

$$\lim_{x\to 2}\frac{1}{(x-2)^2}$$

Response. $\displaystyle\lim_{x\to 2^-}\frac{1}{(x-2)^2} = \infty$, $\displaystyle\lim_{x\to 2^+}\frac{1}{(x-2)^2} = \infty$, so $\displaystyle\lim_{x\to 2}\frac{1}{(x-2)^2} = \infty$ □

c.)

$$\lim_{x\to\infty}\frac{2+5x^2}{1+x-x^2}$$

Response. Multiply through by $\dfrac{1/x^2}{1/x^2} = 1$ to get $\displaystyle\lim_{x\to\infty}\frac{\frac{2}{x^2}+5}{\frac{1}{x^2}+\frac{1}{x}-1} = -5$ as each c/x^r term goes to 0. □

d.)

$$\lim_{x\to -\infty}\frac{1-x^6}{1+x^5}$$

Response. Multiply through by $\dfrac{1/x^5}{1/x^5} = 1$ to get $\displaystyle\lim_{x\to-\infty}\frac{\frac{1}{x^5}-x}{\frac{1}{x^5}+1}$. Note $\displaystyle\lim_{x\to-\infty}(\frac{1}{x^5}-x) = \infty$, while $\displaystyle\lim_{x\to-\infty}\frac{1}{x^5}+1 = 1$. Thus, $\displaystyle\lim_{x\to-\infty}\frac{1-x^6}{1+x^5} = \infty$. □

e.)

$$\lim_{x\to\infty}\frac{\sqrt{1+4x^6}}{2-x^3}$$

Response. Multiply through by $1 = \frac{1/\sqrt{x^6}}{1/x^3}$ to get $\displaystyle\lim_{x\to\infty}\frac{\sqrt{\frac{1}{x^6}+4}}{\frac{2}{x^3}-1}$. Note that $\displaystyle\lim_{x\to\infty}\sqrt{\frac{1}{x^6}+4} = \sqrt{\lim_{x\to\infty}\frac{1}{x^6}+4} = \sqrt{4} = 2$, while $\displaystyle\lim_{x\to\infty}\frac{2}{x^3}-1 = -1$. Thus, $\displaystyle\lim_{x\to\infty}\frac{\sqrt{1+4x^6}}{2-x^3} = -2$ □

f.)

$$\lim_{x\to\infty} e^{-x}\sin^2(x^2)$$

Response. Squeeze theorem! For any x, it is the case that $0 \le \sin^2(x^2) \le 1$, and $e^{-x} > 0$ for any x, so we can multiply through and now now have that $0 = 0 * e^{-x} \le e^{-x}\sin^2(x^2) \le e^{-x}$. The squeeze theorem now says $\displaystyle\lim_{x\to\infty} 0 \le \lim_{x\to\infty} e^{-x}\sin^2(x^2) \le \lim_{x\to\infty} e^{-x}$. Those two outer limits are 0, so we must have $\displaystyle\lim_{x\to\infty} e^{-x}\sin^2(x^2) = 0$. □