Practice Worksheet for Midterm I

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Note: this is more problems than will be on the test, but it should give you a pretty good idea of what to expect in terms of difficulty.

Limits, finite and infinite

1.)

compute the following limits:

a.)

$$\lim_{x\to 2} \frac{x^2-4}{x^2+3x-10} =$$

b.)

$$\lim_{x\to 0} \frac{x^2-2x+1}{x^3-6} =$$

c.)

$$\lim_{x\to -4} \frac{|x^2+8x+12|}{x+2} =$$

2.)

Compute more limits

a.)

$\lim_{x \to 1^-} \frac{x^2 + 2}{x^2 - 1} =$

b.)

$\lim_{x \to \infty} \frac{cos^2(x)}{x + 3} =$

c.)

$\lim_{x \to -\infty} \frac{x^3 - 2x + 2}{4x^3 - 6} =$

Continuity

3.)

Identify the points x at which $f(x)$ is *not* continuous.

$$f(x) = \begin{cases} x^2 & x < -5 \\ \frac{1}{x^2-9} & -5 \leq x < 0 \\ \frac{x^2-1}{9e^x} & 0 \leq x \end{cases}$$

4.)

Find a and b such that $f(x)$ is continuous everywhere.

$$f(x) = \begin{cases} ax + b & x < 0 \\ x^2 - a & 0 \leq x < 2 \\ x^3 & 2 \leq x \end{cases}$$

5.)

Show that $f(x)$ achieves the value $f(c) = 1/2$ for some $0 \leq c \leq 5$. State which theorem you are using. Does $f(x)$ have a root anywhere?

$$f(x) = \frac{1}{x-2}$$

Definition of Derivative

6.)

Use the (limit) definition of the derivative to compute $f'(x)$.

a.)

$$f(x) = 5x - 3$$

b.)

$$f(x) = \sqrt{1 - 2x}$$

c.)

$$f(x) = \frac{3x + 1}{x - 1}$$

Rules of Differentiation

7.)

Compute the derivative f' of $f(x)$ (using the rules of differentiation, *not* the limit-definition). Use that to find the equation for $T(x)$, the tangent line to $y = f(x)$ at $x = x_0$

a.)

$$f(x) = x^3 + 10x \qquad x = 3$$

b.)

$$f(x) = (x^2 + 3x)e^x \qquad x_0 = 2$$

c.)

$$f(x) = \frac{x^2 \cos(x)}{x + 1} \qquad x_0 = \pi$$

8.)

Compute the derivative f' of $f(x)$.

a.)

$$f(x) = \sin(x^2)$$

b.)

$$f(x) = e^{\cos(x^3 - x)}$$

c.)

$$f(x) = \frac{1}{\sqrt{x^3 - 8}}$$

9.)

Let $f(x) = g(h(x))$. Compute the following using the below table of values or state that insufficient information is given with justification

x	g(x)	h(x)	g'(x)	h'(x)
1	7	3	8	2
2	6	1	-2	4
3	-1	4	-8	-2

a.)

$f'(2) =$

b.)

$f(2) =$

c.)

$f'(3) =$

d.)

$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right)\Big|_{x=1} =$