

**MAKE SURE EACH MEMBER OF YOUR GROUP UNDERSTANDS  
YOUR RESPONSE TO EACH PROBLEM BEFORE MOVING ON**

**1.)**

Show that the function  $f(x) = e^x - 2$  has a root in the interval  $(0, 1)$ . What theorem are you using?

**2.)**

**a.)**

Show that  $\frac{d}{dx}x^2 = 2x$  using the limit definition of derivative.

**b.)**

The binomial theorem states that

$$(a + b)^n = a^n + na^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + \binom{n}{n-2}a^2b^{n-2} + nab^{n-1} + b^n \quad (1)$$

$$= \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k \quad (2)$$

where  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ .

The power law states that  $\frac{d}{dx}x^n = nx^{n-1}$ . Use the binomial theorem in conjunction with the limit definition of derivative to show this.

(HINT: stuck on those pesky binomial coefficients? It turns out they're mostly a red herring here. Rephrase equation (1) to say  $(a + b)^n = a^n + na^{n-1}b + b^2g(a, b)$  where  $g(a, b)$  is some polynomial in  $a$  and  $b$  that we can pretty much ignore for the purposes of this problem. Why does that work? Why can we ignore it here??) **IF**

**YOUR GROUP HAS COMPLETED BOTH PROBLEMS LET DAVID  
KNOW!**