1.)

For each of the functions listed, **(i)** identify any points of discontinuity and **(ii)** say whether it is continuous from the left, right, or neither.

a.)

$$f(x) = \frac{x+4}{2x^2+7x-4}$$

Response. In Stewart, §2.5, theorem 4, it is stated that if $g(x)$ and $h(x)$ are continuous at a, then $\frac{g(x)}{h(x)}$ is continuous at a *unless* $h(a) = 0$. Polynomials are continuous everywhere, so we need only find the zeros of $2x^2 + 7x - 4$. By factoring $2x^2 + 7x - 4 = (2x - 1)(x + 4)$, we see that f is continuous everywhere except $x = \frac{1}{2}, -4$. As $f(\frac{1}{2})$ and $f(4)$ are undefined, f cannot possible be continuous (from the right or from the left, for that matter) at either point as the definition of continuity ($f(x)$ is continuous at $a \iff \lim_{x\to a} f(x) = f(a)$) requires $f(a)$ to be defined. □

b.)

$$f(x) = \frac{x^2-1}{x^2+1}$$

Response. $x^2 + 1$ is never 0 for any real x. Thus, $f(x)$ is continuous everywhere! □

c.)

$$f(x) = \begin{cases} \sqrt{1-x} & x \leq 1 \\ \frac{2x^3-2}{x-2} & 1 < x \leq 3 \\ 52\sin(\pi x) & 3 < x \end{cases}$$

Response. There are two steps to checking for discontinuities here: check for any discontinuities within each "leg" of the piecewise, then check for discontinuities at the "seams" (the points at which we change from one function to another). $\sqrt{1-x}$ is continuous everywhere it is defined except at $x = 1$ where it is continuous only from the left—however, we shouldn't take that to mean that f has a discontinuity at 1, as 1 is one of the "seams" of the piecewise, and we'll handle it later. $\frac{2x^3-2}{x-2}$ is undefined at $x = 2$, which is in the domain of the second leg of the piecewise, so we have a discontinuity (not continuous from right *or* left) at $x = 2$. Finally, $52\sin(\pi x)$ is continuous everywhere. What is left to check is the two "seams" at $x = 1$ and $x = 3$.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{1-x} = 0 = f(1),$$

so f is at least continuous from the left there.

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{2x^3-2}{x-2} = 0 = f(1)$$

so it is continuous from the right as well. Thus, f is continuous at 1. On the other hand,

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{2x^3-2}{x-2} = 52 = f(3)$$

so f is continuous from the left at 3, but

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 52\sin(\pi x) = 52\sin(3\pi) = 0 \neq f(3),$$

so f is *not* continuous from the right at 3.

To summarize, we found two discontinuities: f is not continuous from either direction at $x = 2$, and f is continuous only from the left at $x = 3$. □