Math 1271 - Lecture 050
Spring 2018
Quiz IV
2/22/18
Time Limit: 20 Minutes

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Section ____________________

You may *not* use your books, notes, graphing calculator, phones or any other internet devices on this exam. Please show all work clearly and legibly.

Problem	Points	Score
1	10	
2	10	
Total:	20	

1. (10 points) Find an equation for a tangent line to the curve defined by $y^2 - x^2 - 3x + 6 = 0$ at the point $(x_0, y_0) = (2, -2)$.

Response. The tangent line to the curve is given by

$$T(x) = m(x - x_0) + y_0 \text{ where } m = \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\Big|_{(x_0,y_0)}$$

First we differentiate with respect to both sides:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}(y^2 - x^2 - 3x + 6) &= \frac{\mathrm{d}}{\mathrm{d}x}(0)\\ 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 2x - 3 &= 0\end{aligned}$$

Now we solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$$\begin{aligned}2y\frac{\mathrm{d}y}{\mathrm{d}x} - 2x - 3 &= 0\\ \implies 2y\frac{\mathrm{d}y}{\mathrm{d}x} &= 2x + 3\\ \implies \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{2x+3}{2y}\end{aligned}$$

So to find $m = \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\Big|_{(x_0,y_0)}$, we just need to plug in $(x_0, y_0) = (2, -2)$:

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\Big|_{(x_0,y_0)} = \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\Big|_{(2,-2)} = \frac{2(2)+3}{2(-2)} = \frac{-7}{4}$$

From this we get that

$$T(x) = \frac{-7}{4}(x-2) - 2 = \frac{-7}{4}x + \frac{3}{2}$$

$\square$

2. Find y' (in terms of x and y) for:

(a) (5 points) $y = x^{\sqrt{x}}$

Response. We first rewrite (note that this step can be skipped) then apply natural log to both sides.

$$\begin{aligned} y &= \left(e^{\ln(x)}\right)^{\sqrt{x}} \\ &= e^{\ln(x)\sqrt{x}} \\ \implies \ln(y) &= \ln(x)\sqrt{x} \end{aligned}$$

We now differentiate each side w/r/t x:

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln(y) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(x)\sqrt{x}\right)$$

To compute $\frac{\mathrm{d}}{\mathrm{d}x}\ln(y)$, we use the chain rule. We let $f(x) = \ln(x)$ and $g(x) = y$. Then, $f'(x) = \frac{1}{x}$ and $g'(x) = y'$, so

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln(y) = \frac{\mathrm{d}}{\mathrm{d}x}f(g(x)) = g'(x)f'(g(x)) = \frac{y'}{y}.$$

We compute the derivative of the other side using the product rule to yield

$$\begin{aligned} \frac{y'}{y} &= \frac{\sqrt{x}}{x} + \frac{\ln(x)}{2\sqrt{x}} \\ &= \frac{2+\ln(x)}{2\sqrt{x}} \\ \implies y' &= \left(\frac{2+\ln(x)}{2\sqrt{x}}\right) y \end{aligned}$$

Finally, to put it in terms of x, we substitute $y = x^{\sqrt{x}}$ and yield

$$y' = \left(\frac{2+\ln(x)}{2\sqrt{x}}\right) x^{\sqrt{x}}$$

$\square$

(b) (5 points) $x = \cos(y^2)$

Response. Don't overthink it! We can put this one in terms of x and y, so lets just differentiate each side:

$$\frac{\mathrm{d}}{\mathrm{d}x}x = \frac{\mathrm{d}}{\mathrm{d}x}\cos(y^2)$$

We use the chain rule on the right: we let $f(x) = \cos(x)$ and $g(x) = y^2$. Then, $f'(x) = -\sin(x)$ and $g'(x) = 2yy'$. Thus,

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos(y^2) = \frac{\mathrm{d}}{\mathrm{d}x}f(g(x)) = g'(x)f'(g(x)) = 2yy'(-\sin(y^2)).$$

Now we have from differentiating both sides above:

$$\begin{aligned} 1 &= -2yy'\sin(y^2) \\ \implies \frac{-1}{2y\sin(y^2)} &= y' \end{aligned}$$

(Depending on what you did, there are several possible correct answers to this in mixed terms of x and y which are not obviously the same.) $\square$