1 Syntax

The difference between the surface and core syntax is that the surface spines can have underscores. Notice that all the types are *core* types.

$$
\begin{array}{rcl}
\text{(Core) Types} & & \\
A,\ B,\ C,\ D & ::= & 1 \mid \Pi x : A.\, B \mid \forall x : A.\, B \mid e_1 = e_2 : A \\[1em]
\text{(Core) Contexts} & & \\
\Gamma & ::= & \cdot \mid x : A \mid \Gamma_1, \Gamma_2 \\
\text{Core Terms} & & \\
e,\ p & ::= & x \mid <> \mid \lambda^{\Pi} x : A.\, e \mid \lambda^{\forall} x : A.\, e \mid e(\vec{s}) \mid \textbf{refl}\; e \mid \textbf{subst}\,(p : e_1 = e_2 : A, x.\, B, e) \\[1em]
\text{Core Spines} & & \\
\vec{s} & ::= & \cdot \mid e \mid \{e\} \mid \vec{s}_1, \vec{s}_2 \\[1em]
\text{Surface Terms} & & \\
\underline{e},\ \underline{p} & ::= & x \mid <> \mid \lambda^{\Pi} x : A.\, \underline{e} \mid \lambda^{\forall} x : A.\, \underline{e} \mid \underline{e}(\underline{\overrightarrow{ss}}) \mid \textbf{refl}\; \underline{e} \mid \textbf{subst}\,(\underline{p} : \underline{e}_1 = \underline{e}_2 : A, x.\, B, \underline{e}) \\[1em]
\text{Surface Spines} & & \\
\underline{\overrightarrow{ss}} & ::= & \cdot \mid \underline{e} \mid \{\underline{e}\} \mid \{\_\} \mid \underline{\overrightarrow{ss}}_1, \underline{\overrightarrow{ss}}_2
\end{array}
$$

2 Core

$\boxed{\Gamma \vdash e \Leftarrow A}$

$$\frac{\Gamma \vdash e \Rightarrow A' \quad \Gamma \vdash A' \equiv A}{\Gamma \vdash e \Leftarrow A}\ \textsc{CoreCSwitch}$$

$\boxed{\Gamma \vdash e \Rightarrow A}$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A}\ \textsc{CoreIVar}$$

$$\frac{}{\Gamma \vdash \textless\textgreater \Rightarrow 1}\ \textsc{CoreIUnit}$$

$$\frac{\Gamma, x : A \vdash e \Rightarrow B}{\Gamma \vdash \lambda^{\Pi} x : A.\, e \Rightarrow \Pi x : A.\, B}\ \textsc{CoreILam}$$

$$\frac{\Gamma, x : A \vdash e \Rightarrow B}{\Gamma \vdash \lambda^{\forall} x : A.\, e \Rightarrow \forall x : A'.\, B}\ \textsc{CoreIPLam}$$

$$\frac{\Gamma \vdash e \Rightarrow A \quad \Gamma \vdash \vec{s} : A \gg B}{\Gamma \vdash e(\vec{s}) \Rightarrow B}\ \textsc{CoreIApp}$$

$$\frac{\Gamma \vdash e \Rightarrow A}{\Gamma \vdash \textbf{refl}\, e \Rightarrow e = e : A}\ \textsc{CoreIRefl}$$

$$\frac{\begin{array}{l}\Gamma \vdash A \quad \Gamma, x : A \vdash B \\ \Gamma \vdash e_1 \Leftarrow A \quad \Gamma \vdash e_2 \Leftarrow A \\ \Gamma \vdash p \Leftarrow e_1 = e_2 : A \\ \Gamma \vdash e \Leftarrow [e_1/x]B\end{array}}{\Gamma \vdash \textbf{subst}\,(p : e_1 = e_2 : A, x.\, B, e) \Rightarrow [e_2/x]B}\ \textsc{CoreISubst}$$

$\boxed{\Gamma \vdash \vec{s} : A \gg B}$

$$\frac{}{\Gamma \vdash \cdot : A \gg A}\ \textsc{CoreSAEmpty}$$

$$\frac{\begin{array}{l}\Gamma \vdash \Pi x : B.\, A \\ \Gamma \vdash e \Leftarrow B \quad \Gamma \vdash \vec{s} : [e/x]A \gg C\end{array}}{\Gamma \vdash e, \vec{s} : \Pi x : B.\, A \gg C}\ \textsc{CoreSAArg}$$

$$\frac{\begin{array}{l}\Gamma \vdash \forall x : B.\, A \\ \Gamma \vdash e \Leftarrow B \quad \Gamma \vdash \vec{s} : [e/x]A \gg C\end{array}}{\Gamma \vdash \{e\}, \vec{s} : \forall x : B.\, A \gg C}\ \textsc{CoreSAImpArg}$$

$\boxed{\Gamma \vdash A \equiv B}$ Type Conversion

$$\frac{\Gamma \vdash A \equiv A' \quad \Gamma, x : A \vdash B \equiv B'}{\Gamma \vdash \Pi x : A.\, B \equiv \Pi x : A'.\, B'}\ \textsc{CoreTCPi}$$

$$\frac{\Gamma \vdash A \equiv A' \quad \Gamma, x : A \vdash B \equiv B'}{\Gamma \vdash \forall x : A.\, B \equiv \forall x : A'.\, B'}\ \textsc{CoreTCForall}$$

$$\frac{\Gamma \vdash A \equiv B \quad \Gamma \vdash e_1 \equiv p_1 \Leftarrow A \quad \Gamma \vdash e_2 \equiv p_2 \Leftarrow A}{\Gamma \vdash e_1 = e_2 : A \equiv p_1 = p_2 : B} \ \textsc{CoreTCEq}$$

$$\frac{}{\Gamma \vdash A \equiv A} \ \textsc{CoreTCReflC}$$

$$\frac{\Gamma \vdash A \equiv B}{\Gamma \vdash B \equiv A} \ \textsc{CoreTCSymC}$$

$$\frac{\Gamma \vdash A \equiv B \quad \Gamma \vdash B \equiv C}{\Gamma \vdash A \equiv C} \ \textsc{CoreTCTransC}$$

$\boxed{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A}$ Checking Conversion

$$\frac{\Gamma \vdash e_1 \equiv e_2 \Rightarrow A \quad \Gamma \vdash A \equiv B}{\Gamma \vdash e_1 \equiv e_2 \Leftarrow B} \ \textsc{CoreCCSwitch}$$

$$\frac{\Gamma \vdash A_1 \equiv A \quad \Gamma \vdash A_2 \equiv A \quad \Gamma, x : A \vdash e_1 \equiv e_2 \Leftarrow B}{\Gamma \vdash \lambda^{\Pi} x : A_1 .\, e_1 \equiv \lambda^{\Pi} x : A_2 .\, e_2 \Leftarrow \Pi x : A.\, B} \ \textsc{CoreCCLam}$$

$$\frac{\Gamma \vdash A_1 \equiv A \quad \Gamma \vdash A_2 \equiv A \quad \Gamma, x : A \vdash e_1 \equiv e_2 \Leftarrow B}{\Gamma \vdash \lambda^{\forall} x : A_1 .\, e_1 \equiv \lambda^{\forall} x : A_2 .\, e_2 \Leftarrow \forall x : A.\, B} \ \textsc{CoreCCPLam}$$

$$\frac{\Gamma \vdash p \equiv p' \Leftarrow A \quad \Gamma \vdash e_1 \equiv p \Leftarrow A \quad \Gamma \vdash e_2 \equiv p \Leftarrow A}{\Gamma \vdash \mathbf{refl}\, e_1 \equiv \mathbf{refl}\, e_2 \Leftarrow (p = p' : A)} \ \textsc{CoreCCRefl}$$

$$\frac{}{\Gamma \vdash e \equiv e \Leftarrow A} \ \textsc{CoreCCReflC}$$

$$\frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A}{\Gamma \vdash e_2 \equiv e_1 \Leftarrow A} \ \textsc{CoreCCSymC}$$

$$\frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A \quad \Gamma \vdash e_2 \equiv e_3 \Leftarrow A}{\Gamma \vdash e_1 \equiv e_3 \Leftarrow A} \ \textsc{CoreCCTransC}$$

$$\frac{\Gamma \vdash e_2 \Leftarrow A \quad \Gamma, x : A \vdash e_1(\vec{s}) \Leftarrow B}{\Gamma \vdash (\lambda^{\Pi} x : A.\, e_1)(e_2, \vec{s}) \equiv [e_2/x] e_1(\vec{s}) \Leftarrow B} \ \textsc{CoreCCBetaR}$$

$$\frac{\Gamma \vdash e_2 \Leftarrow A \quad \Gamma, x : A \vdash e_1(\vec{s}) \Leftarrow B}{\Gamma \vdash (\lambda^{\forall} x : A.\, e_1)(\{e_2\}, \vec{s}) \equiv [e_2/x] e_1(\vec{s}) \Leftarrow B} \ \textsc{CoreCCBetaRImp}$$

$$\frac{\Gamma \vdash e \Leftarrow \Pi x : A.\, B}{\Gamma \vdash \lambda^{\Pi} x : A.\, e(x) \equiv e \Leftarrow \Pi x : A.\, B} \ \textsc{CoreCCEtaR}$$

$$\frac{\Gamma \vdash e \Leftarrow \forall x : A.\, B}{\Gamma \vdash \lambda^{\Pi} x : A.\, e(\{x\}) \equiv e \Leftarrow \forall x : A.\, B} \ \textsc{CoreCCEtaRImp}$$

$\boxed{\Gamma \vdash \vec{s}_1 \equiv \vec{s}_2 : A \gg B}$ Spine Conversion

$$\frac{}{\Gamma \vdash \cdot \equiv \cdot : A \gg A} \ \textsc{CoreSCEmpty}$$

$$\frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A \quad \Gamma \vdash \vec{s}_1 \equiv \vec{s}_2 : [e_1/x]B \gg C}{\Gamma \vdash e_1, \vec{s}_1 \equiv e_2, \vec{s}_2 : \Pi x : A.\, B \gg C} \textsc{CoreSCArg}$$

$$\frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A \quad \Gamma \vdash \vec{s}_1 \equiv \vec{s}_2 : [e_1/x]B \gg C}{\Gamma \vdash \{e_1\}, \vec{s}_1 \equiv \{e_2\}, \vec{s}_2 : \forall x : A.\, B \gg C} \textsc{CoreSCImpImpArg}$$

$\boxed{\Gamma \vdash A}$

$$\frac{}{\Gamma \vdash 1} \textsc{CoreWFUnit}$$

$$\frac{\Gamma \vdash A \quad \Gamma, x : A \vdash B}{\Gamma \vdash \Pi x : A.\, B} \textsc{CoreWFPi}$$

$$\frac{\Gamma \vdash A \quad \Gamma, x : A \vdash B}{\Gamma \vdash \forall x : A.\, B} \textsc{CoreWFForall}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash e_1 \Leftarrow A \quad \Gamma \vdash e_2 \Leftarrow A}{\Gamma \vdash e_1 = e_2 : A} \textsc{CoreWFEq}$$

3 Surface to Core

$$\boxed{\Gamma \vdash \underline{e} \Leftarrow A \rightsquigarrow e}$$

$$\frac{\Gamma \vdash \underline{e} \Rightarrow A \rightsquigarrow e \quad \Gamma \vdash A \equiv B}{\Gamma \vdash \underline{e} \Leftarrow B \rightsquigarrow e} \textsc{TypCSwitch}$$

$$\boxed{\Gamma \vdash \underline{e} \Rightarrow A \rightsquigarrow e}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow x} \textsc{TypIVar}$$

$$\frac{}{\Gamma \vdash \langle\rangle \Rightarrow 1 \rightsquigarrow \langle\rangle} \textsc{TypIUnit}$$

$$\frac{\Gamma, x : A \vdash \underline{e} \Rightarrow B \rightsquigarrow e'}{\Gamma \vdash \lambda^{\Pi} x : A.\, \underline{e} \Rightarrow \Pi x : A.\, B \rightsquigarrow \lambda^{\Pi} x : A.\, e'} \textsc{TypILam}$$

$$\frac{\Gamma, x : A \vdash \underline{e} \Rightarrow B \rightsquigarrow e'}{\Gamma \vdash \lambda^{\forall} x : A.\, \underline{e} \Rightarrow \forall x : A.\, B \rightsquigarrow \lambda^{\forall} x : A.\, e'} \textsc{TypIPLam}$$

$$\frac{\Gamma \vdash \underline{e} \Rightarrow A \rightsquigarrow e \quad \Gamma \vdash \underline{\vec{ss}} : A \gg B \rightsquigarrow \vec{s}}{\Gamma \vdash \underline{e}(\underline{\vec{ss}}) \Rightarrow B \rightsquigarrow e(\vec{s})} \textsc{TypIApp}$$

$$\frac{\Gamma \vdash \underline{e} \Rightarrow A \rightsquigarrow e}{\Gamma \vdash \mathbf{refl}\, \underline{e} \Rightarrow e = e : A \rightsquigarrow \mathbf{refl}\, e} \textsc{TypIRefl}$$

$$\frac{\begin{array}{l}\Gamma \vdash A \quad \Gamma, x : A \vdash B \\ \Gamma \vdash \underline{e}_1 \Leftarrow A \rightsquigarrow e_1 \quad \Gamma \vdash \underline{e}_2 \Leftarrow A \rightsquigarrow e_2 \\ \Gamma \vdash \underline{p} \Leftarrow e_1 = e_2 : A \rightsquigarrow p \\ \Gamma \vdash \underline{e} \Leftarrow [e_1/x]B \rightsquigarrow e\end{array}}{\Gamma \vdash \mathbf{subst}\,(\underline{p} : \underline{e}_1 = \underline{e}_2 : A, x.\, B, \underline{e}) \Rightarrow [e_2/x]B \rightsquigarrow \mathbf{subst}\,(p : e_1 = e_2 : A, x.\, B, e)} \textsc{TypISubst}$$

$$\boxed{\Gamma \vdash \underline{\vec{ss}} : A \gg B \rightsquigarrow \vec{s}}$$

$$\frac{}{\Gamma \vdash \cdot : A \gg A \rightsquigarrow \cdot} \textsc{TypSAEmpty}$$

$$\frac{\begin{array}{l}\Gamma \vdash \Pi x : B.\, A \\ \Gamma \vdash \underline{e} \Leftarrow B \rightsquigarrow e \quad \Gamma \vdash \underline{\vec{ss}} : [e/x]A \gg C \rightsquigarrow \vec{s}\end{array}}{\Gamma \vdash \underline{e}, \underline{\vec{ss}} : \Pi x : B.\, A \gg C \rightsquigarrow e, \vec{s}} \textsc{TypSAArg}$$

$$\frac{\begin{array}{l}\Gamma \vdash \forall x : B.\, A \\ \Gamma \vdash \underline{e} \Leftarrow B \rightsquigarrow e \quad \Gamma \vdash \underline{\vec{ss}} : [e/x]A \gg C \rightsquigarrow \vec{s}\end{array}}{\Gamma \vdash \{\underline{e}\}, \underline{\vec{ss}} : \forall x : B.\, A \gg C \rightsquigarrow \{e\}, \vec{s}} \textsc{TypSAImpArg}$$

$$\frac{\begin{array}{l}\Gamma \vdash \forall x : B.\, A \\ \Gamma \vdash e \Leftarrow B \quad \Gamma \vdash \underline{\vec{ss}} : [e/x]A \gg C \rightsquigarrow \vec{s}\end{array}}{\Gamma \vdash \{\_\}, \underline{\vec{ss}} : \forall x : B.\, A \gg C \rightsquigarrow \{e\}, \vec{s}} \textsc{TypSAHoleArg}$$

4 Algorithmic Syntax

$$
\begin{array}{rcl}
\text{Alg Types} & & \\
\widehat{A},\ \widehat{B},\ \widehat{C},\ \widehat{D} & ::= & 1 \mid \Pi x : \widehat{A}.\, \widehat{B} \mid \forall x : \widehat{A}.\, \widehat{B} \mid \widehat{e}_1 = \widehat{e}_2 : \widehat{A} \\[1em]
\text{Alg Contexts} & & \\
\widehat{\Gamma} & ::= & \cdot \mid x : \widehat{A} \mid \widehat{\Gamma}_1, \widehat{\Gamma}_2 \\
\text{Alg Core Terms} & & \\
\widehat{e},\ \widehat{p} & ::= & x \mid \widehat{x} \mid \texttt{<>} \mid \lambda^{\Pi} x : \widehat{A}.\, \widehat{e} \mid \lambda^{\forall} x : \widehat{A}.\, \widehat{e} \mid \widehat{e}(\widehat{\vec{s}}) \mid \mathbf{refl}\, \widehat{e} \mid \mathbf{subst}\,(p : \widehat{e}_1 = \widehat{e}_2 : \widehat{A}, x.\, \widehat{B}, \widehat{e}) \\[1em]
\text{Alg Core Spines} & & \\
\widehat{\vec{s}} & ::= & \cdot \mid \widehat{e} \mid \{\widehat{e}\} \mid \widehat{\vec{s}}_1, \widehat{\vec{s}}_2 \\[1em]
\text{Alg Surface Terms} & & \\
\underline{\widehat{e}},\ \underline{\widehat{p}} & ::= & x \mid \widehat{x} \mid \texttt{<>} \mid \lambda^{\Pi} x : \widehat{A}.\, \underline{\widehat{e}} \mid \lambda^{\forall} x : \widehat{A}.\, \underline{\widehat{e}} \mid \underline{\widehat{e}}(\underline{\widehat{\vec{s}}}) \mid \mathbf{refl}\, \underline{\widehat{e}} \mid \mathbf{subst}\,(p : \underline{\widehat{e}}_1 = \underline{\widehat{e}}_2 : \widehat{A}, x.\, \widehat{B}, \underline{\widehat{e}}) \\[1em]
\text{Alg Surface Spines} & & \\
\underline{\widehat{\vec{s}}} & ::= & \cdot \mid \underline{\widehat{e}} \mid \{\underline{\widehat{e}}\} \mid \{\_\} \mid \underline{\widehat{\vec{s}}}_1, \underline{\widehat{\vec{s}}}_2
\end{array}
$$

5 Surface to Algo

$$\boxed{M\Gamma \,;\, \widehat{\Gamma} \vDash \underline{\widehat{e}} \Leftarrow \widehat{A} \rightsquigarrow \widehat{e} \dashv M\Gamma'}$$

$$\frac{M\Gamma \,;\, \widehat{\Gamma} \vDash \underline{\widehat{e}} \Rightarrow \widehat{A} \rightsquigarrow \widehat{e} \dashv M\Gamma_1 \quad M\Gamma_1 \,;\, \widehat{\Gamma} \vDash \widehat{A} \equiv \widehat{B} \dashv M\Gamma_2}{M\Gamma \,;\, \widehat{\Gamma} \vDash \underline{\widehat{e}} \Leftarrow \widehat{B} \rightsquigarrow \widehat{e} \dashv M\Gamma_2} \ \textsc{AlgCSwitch}$$

$$\boxed{M\Gamma \,;\, \widehat{\Gamma} \vDash \underline{\widehat{e}} \Rightarrow \widehat{A} \rightsquigarrow \widehat{e} \dashv M\Gamma'}$$

$$\frac{x : \widehat{A} \in \widehat{\Gamma}}{M\Gamma \,;\, \widehat{\Gamma} \vDash x \Rightarrow \widehat{A} \rightsquigarrow x \dashv M\Gamma} \ \textsc{AlgIVar}$$

$$\frac{}{M\Gamma \,;\, \widehat{\Gamma} \vDash \texttt{<>} \Rightarrow 1 \rightsquigarrow \texttt{<>} \dashv M\Gamma} \ \textsc{AlgIUnit}$$

$$\frac{M\Gamma \,;\, \widehat{\Gamma}, x : \widehat{A} \vDash \underline{\widehat{e}} \Rightarrow \widehat{B} \rightsquigarrow \widehat{e} \dashv M\Gamma'}{M\Gamma \,;\, \widehat{\Gamma} \vDash \lambda^{\Pi} x : \widehat{A}.\, \underline{\widehat{e}} \Rightarrow \Pi x : \widehat{A}.\, \widehat{B} \rightsquigarrow \lambda^{\Pi} x : \widehat{A}.\, \widehat{e} \dashv M\Gamma'} \ \textsc{AlgILam}$$

$$\frac{M\Gamma \,;\, \widehat{\Gamma}, x : \widehat{A} \vDash \underline{\widehat{e}} \Rightarrow \widehat{B} \rightsquigarrow \widehat{e} \dashv M\Gamma'}{M\Gamma \,;\, \widehat{\Gamma} \vDash \lambda^{\forall} x : \widehat{A}.\, \underline{\widehat{e}} \Rightarrow \forall x : \widehat{A}.\, \widehat{B} \rightsquigarrow \lambda^{\forall} x : \widehat{A}.\, \widehat{e} \dashv M\Gamma'} \ \textsc{AlgIPLam}$$

$$\frac{M\Gamma \,;\, \widehat{\Gamma} \vDash \underline{\widehat{e}} \Rightarrow \widehat{A} \rightsquigarrow \widehat{e} \dashv M\Gamma_1 \quad M\Gamma_1 \,;\, \widehat{\Gamma} \vDash \underline{\widehat{\vec{s}}} : \widehat{A} \gg \widehat{B} \rightsquigarrow \widehat{\vec{s}} \dashv M\Gamma_2}{M\Gamma \,;\, \widehat{\Gamma} \vDash \underline{\widehat{e}}(\underline{\widehat{\vec{s}}}) \Rightarrow \widehat{B} \rightsquigarrow \widehat{e}(\widehat{\vec{s}}) \dashv M\Gamma_2} \ \textsc{AlgIApp}$$

$$\frac{M\Gamma \,;\, \widehat{\Gamma} \vDash \underline{\widehat{e}} \Rightarrow \widehat{A} \rightsquigarrow \widehat{e} \dashv M\Gamma'}{M\Gamma \,;\, \widehat{\Gamma} \vDash \mathbf{refl}\, \underline{\widehat{e}} \Rightarrow \widehat{e} = \widehat{e} : \widehat{A} \rightsquigarrow \mathbf{refl}\, \widehat{e} \dashv M\Gamma'} \ \textsc{AlgIRefl}$$

$$\boxed{M\Gamma \,;\, \widehat{\Gamma} \vDash \underline{\widehat{\vec{s}}} : \widehat{A} \gg \widehat{B} \rightsquigarrow \widehat{\vec{s}} \dashv M\Gamma'}$$

$$\frac{}{M\Gamma\ ;\widehat{\Gamma} \vDash \cdot : \widehat{A} \gg \widehat{A} \rightsquigarrow \cdot \dashv M\Gamma}\ \textsc{AlgSAEmpty}$$

$$\frac{\begin{array}{l} M\Gamma\ ;\widehat{\Gamma} \vDash \Pi x : \widehat{B}.\,\widehat{A} \dashv M\Gamma_1 \\ M\Gamma_1\ ;\widehat{\Gamma} \vDash \underline{\widehat{e}} \Leftarrow \widehat{B} \rightsquigarrow \widehat{e} \dashv M\Gamma_2 \\ M\Gamma_2\ ;\widehat{\Gamma} \vDash \underline{\widehat{\vec{s}}} : [\widehat{e}/x]\widehat{A} \gg \widehat{C} \rightsquigarrow \widehat{\vec{s}} \dashv M\Gamma_3 \end{array}}{M\Gamma\ ;\widehat{\Gamma} \vDash \underline{\widehat{e}}, \underline{\widehat{\vec{s}}} : (\Pi x : \widehat{B}.\,\widehat{A}) \gg \widehat{C} \rightsquigarrow \widehat{e}, \widehat{\vec{s}} \dashv M\Gamma_3}\ \textsc{AlgSAArg}$$

$$\frac{\begin{array}{l} M\Gamma\ ;\widehat{\Gamma} \vDash \forall x : \widehat{B}.\,\widehat{A} \dashv M\Gamma_1 \\ M\Gamma_1\ ;\widehat{\Gamma} \vDash \underline{\widehat{e}} \Leftarrow \widehat{B} \rightsquigarrow \widehat{e} \dashv M\Gamma_2 \\ M\Gamma_2\ ;\widehat{\Gamma} \vDash \underline{\widehat{\vec{s}}} : [\widehat{e}/x]\widehat{A} \gg \widehat{C} \rightsquigarrow \widehat{\vec{s}} \dashv M\Gamma_3 \end{array}}{M\Gamma\ ;\widehat{\Gamma} \vDash \{\underline{\widehat{e}}\}, \underline{\widehat{\vec{s}}} : (\forall x : \widehat{B}.\,\widehat{A}) \gg \widehat{C} \rightsquigarrow \{\widehat{e}\}, \widehat{\vec{s}} \dashv M\Gamma_3}\ \textsc{AlgSAImpArg}$$

$$\frac{M\Gamma\ ;\widehat{\Gamma} \vDash \forall x : \widehat{B}.\,\widehat{A} \dashv M\Gamma_1 \quad M\Gamma_1\ ;\widehat{\Gamma} \vDash \underline{\widehat{\vec{s}}} : [\widehat{x}/x]\widehat{A} \gg \widehat{C} \rightsquigarrow \widehat{\vec{s}} \dashv M\Gamma_2}{M\Gamma\ ;\widehat{\Gamma} \vDash \{\_\}, \underline{\widehat{\vec{s}}} : (\forall x : \widehat{B}.\,\widehat{A}) \gg \widehat{C} \rightsquigarrow \{\widehat{x}\}, \widehat{\vec{s}} \dashv M\Gamma_2, (\widehat{\Gamma} \vdash ax \Leftarrow \widehat{B})}\ \textsc{AlgSAHoleArg}$$

$$\boxed{M\Gamma\ ;\widehat{\Gamma} \vDash \widehat{A} \dashv M\Gamma'}$$

$$\frac{}{M\Gamma\ ;\widehat{\Gamma} \vDash 1 \dashv M\Gamma}\ \textsc{AlgWFUnit}$$

$$\frac{M\Gamma\ ;\widehat{\Gamma} \vDash \widehat{A} \dashv M\Gamma_1 \quad M\Gamma_1\ ;\widehat{\Gamma}, x : \widehat{A} \vDash \widehat{B} \dashv M\Gamma_2}{M\Gamma\ ;\widehat{\Gamma} \vDash \Pi x : \widehat{A}.\,\widehat{B} \dashv M\Gamma_2}\ \textsc{AlgWFPi}$$

$$\frac{M\Gamma\ ;\widehat{\Gamma} \vDash \widehat{A} \dashv M\Gamma_1 \quad M\Gamma_1\ ;\widehat{\Gamma}, x : \widehat{A} \vDash \widehat{B} \dashv M\Gamma_2}{M\Gamma\ ;\widehat{\Gamma} \vDash \forall x : \widehat{A}.\,\widehat{B} \dashv M\Gamma_2}\ \textsc{AlgWFForall}$$

$$\frac{\begin{array}{l} M\Gamma\ ;\widehat{\Gamma} \vDash \widehat{A} \dashv M\Gamma_1 \\ M\Gamma_1\ ;\widehat{\Gamma} \vDash \widehat{e_1} \Leftarrow \widehat{A} \rightsquigarrow \widehat{e_3} \dashv M\Gamma_2 \quad M\Gamma_2\ ;\widehat{\Gamma} \vDash \widehat{e_2} \Leftarrow \widehat{A} \rightsquigarrow \widehat{e_3} \dashv M\Gamma_3 \end{array}}{M\Gamma\ ;\widehat{\Gamma} \vDash \widehat{e_1} = \widehat{e_2} : \widehat{A} \dashv M\Gamma_3}\ \textsc{AlgWFEq}$$