

1 Syntax

The difference between the surface and core syntax is that the surface spines can have underscores. Notice that all the types are *core* types.

(Core) Types

$$A, B, C, D ::= 1 \mid \Pi x : A. B \mid \forall x : A. B \mid e_1 = e_2 : A$$

(Core) Contexts

$$\Gamma ::= \cdot \mid x : A \mid \Gamma_1, \Gamma_2$$

Core Terms

$$e, p ::= x \mid <> \mid \lambda^\Pi x : A. e \mid \lambda^\forall x : A. e \mid e(\vec{s}) \mid \mathbf{refl} \, e \mid \mathbf{subst} \, (p : e_1 = e_2 : A, x. B, e)$$

Core Spines

$$\vec{s} ::= \cdot \mid e \mid \{e\} \mid \vec{s}_1, \vec{s}_2$$

Surface Terms

$$\underline{e}, \underline{p} ::= x \mid <> \mid \lambda^\Pi x : A. \underline{e} \mid \lambda^\forall x : A. \underline{e} \mid \underline{e}(\overline{\underline{ss}}) \mid \mathbf{refl} \, \underline{e} \mid \mathbf{subst} \, (\underline{p} : \underline{e}_1 = \underline{e}_2 : A, x. B, \underline{e})$$

Surface Spines

$$\overline{\underline{ss}} ::= \cdot \mid \underline{e} \mid \{\underline{e}\} \mid \{_ \} \mid \overline{\underline{ss}}_1, \overline{\underline{ss}}_2$$

2 Core

$$\boxed{\Gamma \vdash e \Leftarrow A}$$

$$\frac{\Gamma \vdash e \Rightarrow A' \quad \Gamma \vdash A' \equiv A}{\Gamma \vdash e \Leftarrow A} \text{CORECSWITCH}$$

$$\boxed{\Gamma \vdash e \Rightarrow A}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A} \text{COREIVAR}$$

$$\frac{}{\Gamma \vdash <> \Rightarrow 1} \text{COREIUNIT}$$

$$\frac{\Gamma, x : A \vdash e \Rightarrow B}{\Gamma \vdash \lambda^{\Pi} x : A. e \Rightarrow \Pi x : A. B} \text{COREILAM}$$

$$\frac{\Gamma, x : A \vdash e \Rightarrow B}{\Gamma \vdash \lambda^{\forall} x : A. e \Rightarrow \forall x : A'. B} \text{COREIPLAM}$$

$$\frac{\Gamma \vdash e \Rightarrow A \quad \Gamma \vdash \vec{s} : A \gg B}{\Gamma \vdash e(\vec{s}) \Rightarrow B} \text{COREIAPP}$$

$$\frac{\Gamma \vdash e \Rightarrow A}{\Gamma \vdash \text{refl } e \Rightarrow e = e : A} \text{COREIREFL}$$

$$\frac{\begin{array}{l} \Gamma \vdash A \quad \Gamma, x : A \vdash B \\ \Gamma \vdash e_1 \Leftarrow A \quad \Gamma \vdash e_2 \Leftarrow A \\ \Gamma \vdash p \Leftarrow e_1 = e_2 : A \\ \Gamma \vdash e \Leftarrow [e_1/x]B \end{array}}{\Gamma \vdash \text{subst } (p : e_1 = e_2 : A, x. B, e) \Rightarrow [e_2/x]B} \text{COREISUBST}$$

$$\boxed{\Gamma \vdash \vec{s} : A \gg B}$$

$$\frac{}{\Gamma \vdash \cdot : A \gg A} \text{CORESAEMPTY}$$

$$\frac{\begin{array}{l} \Gamma \vdash \Pi x : B. A \\ \Gamma \vdash e \Leftarrow B \quad \Gamma \vdash \vec{s} : [e/x]A \gg C \end{array}}{\Gamma \vdash e, \vec{s} : \Pi x : B. A \gg C} \text{CORESAARG}$$

$$\frac{\begin{array}{l} \Gamma \vdash \forall x : B. A \\ \Gamma \vdash e \Leftarrow B \quad \Gamma \vdash \vec{s} : [e/x]A \gg C \end{array}}{\Gamma \vdash \{e\}, \vec{s} : \forall x : B. A \gg C} \text{CORESAIMPARG}$$

$$\boxed{\Gamma \vdash A \equiv B} \quad \text{Type Conversion}$$

$$\frac{\Gamma \vdash A \equiv A' \quad \Gamma, x : A \vdash B \equiv B'}{\Gamma \vdash \Pi x : A. B \equiv \Pi x : A'. B'} \text{CORETCPi}$$

$$\frac{\Gamma \vdash A \equiv A' \quad \Gamma, x : A \vdash B \equiv B'}{\Gamma \vdash \forall x : A. B \equiv \forall x : A'. B'} \text{CORETCforall}$$

$$\begin{array}{c}
\frac{\Gamma \vdash A \equiv B \quad \Gamma \vdash e_1 \equiv p_1 \Leftarrow A \quad \Gamma \vdash e_2 \equiv p_2 \Leftarrow A}{\Gamma \vdash e_1 = e_2 : A \equiv p_1 = p_2 : B} \quad \text{CORETCEQ} \\
\\
\frac{}{\Gamma \vdash A \equiv A} \quad \text{CORETCREFLC} \\
\\
\frac{\Gamma \vdash A \equiv B}{\Gamma \vdash B \equiv A} \quad \text{CORETCSYMC} \\
\\
\frac{\Gamma \vdash A \equiv B \quad \Gamma \vdash B \equiv C}{\Gamma \vdash A \equiv C} \quad \text{CORETCTRASC} \\
\\
\boxed{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A} \quad \text{Checking Conversion} \\
\\
\frac{\Gamma \vdash e_1 \equiv e_2 \Rightarrow A \quad \Gamma \vdash A \equiv B}{\Gamma \vdash e_1 \equiv e_2 \Leftarrow B} \quad \text{CORECCSWITCH} \\
\\
\frac{\Gamma \vdash A_1 \equiv A \quad \Gamma \vdash A_2 \equiv A \quad \Gamma, x : A \vdash e_1 \equiv e_2 \Leftarrow B}{\Gamma \vdash \lambda^\Pi x : A_1. e_1 \equiv \lambda^\Pi x : A_2. e_2 \Leftarrow \Pi x : A. B} \quad \text{CORECCLAM} \\
\\
\frac{\Gamma \vdash A_1 \equiv A \quad \Gamma \vdash A_2 \equiv A \quad \Gamma, x : A \vdash e_1 \equiv e_2 \Leftarrow B}{\Gamma \vdash \lambda^\forall x : A_1. e_1 \equiv \lambda^\forall x : A_2. e_2 \Leftarrow \forall x : A. B} \quad \text{CORECCPLAM} \\
\\
\frac{\Gamma \vdash p \equiv p' \Leftarrow A \quad \Gamma \vdash e_1 \equiv p \Leftarrow A \quad \Gamma \vdash e_2 \equiv p \Leftarrow A}{\Gamma \vdash \text{refl } e_1 \equiv \text{refl } e_2 \Leftarrow (p = p' : A)} \quad \text{CORECCREFL} \\
\\
\frac{}{\Gamma \vdash e \equiv e \Leftarrow A} \quad \text{CORECCREFLC} \\
\\
\frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A}{\Gamma \vdash e_2 \equiv e_1 \Leftarrow A} \quad \text{CORECCSYMC} \\
\\
\frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A \quad \Gamma \vdash e_2 \equiv e_3 \Leftarrow A}{\Gamma \vdash e_1 \equiv e_3 \Leftarrow A} \quad \text{CORECCTRASC} \\
\\
\frac{\Gamma \vdash e_2 \Leftarrow A \quad \Gamma, x : A \vdash e_1(\vec{s}) \Leftarrow B}{\Gamma \vdash (\lambda^\Pi x : A. e_1)(e_2, \vec{s}) \equiv [e_2/x]e_1(\vec{s}) \Leftarrow B} \quad \text{CORECCBETAR} \\
\\
\frac{\Gamma \vdash e_2 \Leftarrow A \quad \Gamma, x : A \vdash e_1(\vec{s}) \Leftarrow B}{\Gamma \vdash (\lambda^\forall x : A. e_1)(\{e_2\}, \vec{s}) \equiv [e_2/x]e_1(\vec{s}) \Leftarrow B} \quad \text{CORECCBETARIMP} \\
\\
\frac{\Gamma \vdash e \Leftarrow \Pi x : A. B}{\Gamma \vdash \lambda^\Pi x : A. e(x) \equiv e \Leftarrow \Pi x : A. B} \quad \text{CORECCETAR} \\
\\
\frac{\Gamma \vdash e \Leftarrow \forall x : A. B}{\Gamma \vdash \lambda^\Pi x : A. e(\{x\}) \equiv e \Leftarrow \forall x : A. B} \quad \text{CORECCETARIMP} \\
\\
\boxed{\Gamma \vdash \vec{s}_1 \equiv \vec{s}_2 : A \gg B} \quad \text{Spine Conversion} \\
\\
\frac{}{\Gamma \vdash \cdot \equiv \cdot : A \gg A} \quad \text{CORESCEMPTY}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A \quad \Gamma \vdash \vec{s}_1 \equiv \vec{s}_2 : [e_1/x]B \gg C}{\Gamma \vdash e_1, \vec{s}_1 \equiv e_2, \vec{s}_2 : \Pi x : A. B \gg C} \text{CORESCARG} \\
\\
\frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A \quad \Gamma \vdash \vec{s}_1 \equiv \vec{s}_2 : [e_1/x]B \gg C}{\Gamma \vdash \{e_1\}, \vec{s}_1 \equiv \{e_2\}, \vec{s}_2 : \forall x : A. B \gg C} \text{CORESCIMPIMPARG} \\
\\
\boxed{\Gamma \vdash A} \\
\\
\frac{}{\Gamma \vdash 1} \text{COREWFUNIT} \\
\\
\frac{\Gamma \vdash A \quad \Gamma, x : A \vdash B}{\Gamma \vdash \Pi x : A. B} \text{COREWFPI} \\
\\
\frac{\Gamma \vdash A \quad \Gamma, x : A \vdash B}{\Gamma \vdash \forall x : A. B} \text{COREWFFORALL} \\
\\
\frac{\Gamma \vdash A \quad \Gamma \vdash e_1 \Leftarrow A \quad \Gamma \vdash e_2 \Leftarrow A}{\Gamma \vdash e_1 = e_2 : A} \text{COREWFEQ}
\end{array}$$

3 Surface to Core

$$\boxed{\Gamma \vdash \underline{e} \Leftarrow A \rightsquigarrow e}$$

$$\frac{\Gamma \vdash \underline{e} \Rightarrow A \rightsquigarrow e \quad \Gamma \vdash A \equiv B}{\Gamma \vdash \underline{e} \Leftarrow B \rightsquigarrow e} \text{TypCSWITCH}$$

$$\boxed{\Gamma \vdash \underline{e} \Rightarrow A \rightsquigarrow e}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow x} \text{TypIVAR}$$

$$\frac{}{\Gamma \vdash \langle \rangle \Rightarrow 1 \rightsquigarrow \langle \rangle} \text{TypIUNIT}$$

$$\frac{\Gamma, x : A \vdash \underline{e} \Rightarrow B \rightsquigarrow e'}{\Gamma \vdash \lambda^{\Pi} x : A. \underline{e} \Rightarrow \Pi x : A. B \rightsquigarrow \lambda^{\Pi} x : A. e'} \text{TypILAM}$$

$$\frac{\Gamma, x : A \vdash \underline{e} \Rightarrow B \rightsquigarrow e'}{\Gamma \vdash \lambda^{\forall} x : A. \underline{e} \Rightarrow \forall x : A. B \rightsquigarrow \lambda^{\forall} x : A. e'} \text{TypIPLAM}$$

$$\frac{\Gamma \vdash \underline{e} \Rightarrow A \rightsquigarrow e \quad \Gamma \vdash \underline{\vec{s}s} : A \gg B \rightsquigarrow \vec{s}}{\Gamma \vdash \underline{e}(\underline{\vec{s}s}) \Rightarrow B \rightsquigarrow e(\vec{s})} \text{TypIAPP}$$

$$\frac{\Gamma \vdash \underline{e} \Rightarrow A \rightsquigarrow e}{\Gamma \vdash \mathbf{refl} \underline{e} \Rightarrow e = e : A \rightsquigarrow \mathbf{refl} e} \text{TypIREFL}$$

$$\frac{\begin{array}{l} \Gamma \vdash A \quad \Gamma, x : A \vdash B \\ \Gamma \vdash \underline{e}_1 \Leftarrow A \rightsquigarrow e_1 \quad \Gamma \vdash \underline{e}_2 \Leftarrow A \rightsquigarrow e_2 \\ \Gamma \vdash \underline{p} \Leftarrow e_1 = e_2 : A \rightsquigarrow p \\ \Gamma \vdash \underline{e} \Leftarrow [e_1/x]B \rightsquigarrow e \end{array}}{\Gamma \vdash \mathbf{subst} (\underline{p} : \underline{e}_1 = \underline{e}_2 : A, x. B, \underline{e}) \Rightarrow [e_2/x]B \rightsquigarrow \mathbf{subst} (p : e_1 = e_2 : A, x. B, e)} \text{TypISUBST}$$

$$\boxed{\Gamma \vdash \underline{\vec{s}s} : A \gg B \rightsquigarrow \vec{s}}$$

$$\frac{}{\Gamma \vdash \cdot : A \gg A \rightsquigarrow \cdot} \text{TypSAEMPTY}$$

$$\frac{\begin{array}{l} \Gamma \vdash \Pi x : B. A \\ \Gamma \vdash \underline{e} \Leftarrow B \rightsquigarrow e \quad \Gamma \vdash \underline{\vec{s}s} : [e/x]A \gg C \rightsquigarrow \vec{s} \end{array}}{\Gamma \vdash \underline{e}, \underline{\vec{s}s} : \Pi x : B. A \gg C \rightsquigarrow e, \vec{s}} \text{TypSAARG}$$

$$\frac{\begin{array}{l} \Gamma \vdash \forall x : B. A \\ \Gamma \vdash \underline{e} \Leftarrow B \rightsquigarrow e \quad \Gamma \vdash \underline{\vec{s}s} : [e/x]A \gg C \rightsquigarrow \vec{s} \end{array}}{\Gamma \vdash \{e\}, \underline{\vec{s}s} : \forall x : B. A \gg C \rightsquigarrow \{e\}, \vec{s}} \text{TypSAIMPARG}$$

$$\frac{\begin{array}{l} \Gamma \vdash \forall x : B. A \\ \Gamma \vdash e \Leftarrow B \quad \Gamma \vdash \underline{\vec{s}s} : [e/x]A \gg C \rightsquigarrow \vec{s} \end{array}}{\Gamma \vdash \{_, \underline{\vec{s}s} : \forall x : B. A \gg C \rightsquigarrow \{e\}, \vec{s}} \text{TypSAHOLEARG}$$

4 Algorithmic Syntax

Alg Types

$$\widehat{A}, \widehat{B}, \widehat{C}, \widehat{D} ::= 1 \mid \Pi x : \widehat{A}. \widehat{B} \mid \forall x : \widehat{A}. \widehat{B} \mid \widehat{e}_1 = \widehat{e}_2 : \widehat{A}$$

Alg Contexts

$$\widehat{\Gamma} ::= \cdot \mid x : \widehat{A} \mid \widehat{\Gamma}_1, \widehat{\Gamma}_2$$

Alg Core Terms

$$\widehat{e}, \widehat{p} ::= x \mid \widehat{x} \mid \langle \rangle \mid \lambda^\Pi x : \widehat{A}. \widehat{e} \mid \lambda^\forall x : \widehat{A}. \widehat{e} \mid \widehat{e}(\widehat{s}) \mid \mathbf{refl} \widehat{e} \mid \mathbf{subst} (p : \widehat{e}_1 = \widehat{e}_2 : \widehat{A}, x. \widehat{B}, \widehat{e})$$

Alg Core Spines

$$\widehat{s} ::= \cdot \mid \widehat{e} \mid \{\widehat{e}\} \mid \widehat{s}_1, \widehat{s}_2$$

Alg Surface Terms

$$\underline{\widehat{e}}, \underline{\widehat{p}} ::= x \mid \widehat{x} \mid \langle \rangle \mid \lambda^\Pi x : \widehat{A}. \underline{\widehat{e}} \mid \lambda^\forall x : \widehat{A}. \underline{\widehat{e}} \mid \underline{\widehat{e}}(\underline{\widehat{s}}) \mid \mathbf{refl} \underline{\widehat{e}} \mid \mathbf{subst} (p : \underline{\widehat{e}}_1 = \underline{\widehat{e}}_2 : \widehat{A}, x. \widehat{B}, \underline{\widehat{e}})$$

Alg Surface Spines

$$\underline{\widehat{s}} ::= \cdot \mid \underline{\widehat{e}} \mid \{\underline{\widehat{e}}\} \mid \{_\} \mid \underline{\widehat{s}}_1, \underline{\widehat{s}}_2$$

5 Surface to Algo

$$\boxed{M\Gamma ; \widehat{\Gamma} \models \underline{\widehat{e}} \Leftarrow \widehat{A} \rightsquigarrow \widehat{e} \Vdash M\Gamma'}$$

$$\frac{M\Gamma ; \widehat{\Gamma} \models \underline{\widehat{e}} \Rightarrow \widehat{A} \rightsquigarrow \widehat{e} \Vdash M\Gamma_1 \quad M\Gamma_1 ; \widehat{\Gamma} \models \widehat{A} \equiv \widehat{B} \Vdash M\Gamma_2}{M\Gamma ; \widehat{\Gamma} \models \underline{\widehat{e}} \Leftarrow \widehat{B} \rightsquigarrow \widehat{e} \Vdash M\Gamma_2} \text{ALGCSWITCH}$$

$$\boxed{M\Gamma ; \widehat{\Gamma} \models \underline{\widehat{e}} \Rightarrow \widehat{A} \rightsquigarrow \widehat{e} \Vdash M\Gamma'}$$

$$\frac{x : \widehat{A} \in \widehat{\Gamma}}{M\Gamma ; \widehat{\Gamma} \models x \Rightarrow \widehat{A} \rightsquigarrow x \Vdash M\Gamma} \text{ALGIVAR}$$

$$\frac{}{M\Gamma ; \widehat{\Gamma} \models \langle \rangle \Rightarrow 1 \rightsquigarrow \langle \rangle \Vdash M\Gamma} \text{ALGIUNIT}$$

$$\frac{M\Gamma ; \widehat{\Gamma}, x : \widehat{A} \models \underline{\widehat{e}} \Rightarrow \widehat{B} \rightsquigarrow \widehat{e} \Vdash M\Gamma'}{M\Gamma ; \widehat{\Gamma} \models \lambda^\Pi x : \widehat{A}. \underline{\widehat{e}} \Rightarrow \Pi x : \widehat{A}. \widehat{B} \rightsquigarrow \lambda^\Pi x : \widehat{A}. \widehat{e} \Vdash M\Gamma'} \text{ALGILAM}$$

$$\frac{M\Gamma ; \widehat{\Gamma}, x : \widehat{A} \models \underline{\widehat{e}} \Rightarrow \widehat{B} \rightsquigarrow \widehat{e} \Vdash M\Gamma'}{M\Gamma ; \widehat{\Gamma} \models \lambda^\forall x : \widehat{A}. \underline{\widehat{e}} \Rightarrow \forall x : \widehat{A}. \widehat{B} \rightsquigarrow \lambda^\forall x : \widehat{A}. \widehat{e} \Vdash M\Gamma'} \text{ALGIPLAM}$$

$$\frac{M\Gamma ; \widehat{\Gamma} \models \underline{\widehat{e}} \Rightarrow \widehat{A} \rightsquigarrow \widehat{e} \Vdash M\Gamma_1 \quad M\Gamma_1 ; \widehat{\Gamma} \models \widehat{s} : \widehat{A} \gg \widehat{B} \rightsquigarrow \widehat{s} \Vdash M\Gamma_2}{M\Gamma ; \widehat{\Gamma} \models \underline{\widehat{e}}(\widehat{s}) \Rightarrow \widehat{B} \rightsquigarrow \widehat{e}(\widehat{s}) \Vdash M\Gamma_2} \text{ALGIAPP}$$

$$\frac{M\Gamma ; \widehat{\Gamma} \models \underline{\widehat{e}} \Rightarrow \widehat{A} \rightsquigarrow \widehat{e} \Vdash M\Gamma'}{M\Gamma ; \widehat{\Gamma} \models \mathbf{refl} \underline{\widehat{e}} \Rightarrow \widehat{e} = \widehat{e} : \widehat{A} \rightsquigarrow \mathbf{refl} \widehat{e} \Vdash M\Gamma'} \text{ALGIREFL}$$

$$\boxed{M\Gamma ; \widehat{\Gamma} \models \widehat{s} : \widehat{A} \gg \widehat{B} \rightsquigarrow \widehat{s} \Vdash M\Gamma'}$$

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$$\begin{array}{c}
\frac{}{M\Gamma; \widehat{\Gamma} \models \cdot : \widehat{A} \gg \widehat{A} \rightsquigarrow \cdot \models M\Gamma} \text{ALGSAEMPTY} \\
\\
\frac{
\begin{array}{l}
M\Gamma; \widehat{\Gamma} \models \Pi x : \widehat{B}. \widehat{A} \models M\Gamma_1 \\
M\Gamma_1; \widehat{\Gamma} \models \widehat{e} \Leftarrow \widehat{B} \rightsquigarrow \widehat{e} \models M\Gamma_2 \\
M\Gamma_2; \widehat{\Gamma} \models \widehat{s} : [\widehat{e}/x] \widehat{A} \gg \widehat{C} \rightsquigarrow \widehat{s} \models M\Gamma_3
\end{array}
}{M\Gamma; \widehat{\Gamma} \models \widehat{e}, \widehat{s} : (\Pi x : \widehat{B}. \widehat{A}) \gg \widehat{C} \rightsquigarrow \widehat{e}, \widehat{s} \models M\Gamma_3} \text{ALGSAARG} \\
\\
\frac{
\begin{array}{l}
M\Gamma; \widehat{\Gamma} \models \forall x : \widehat{B}. \widehat{A} \models M\Gamma_1 \\
M\Gamma_1; \widehat{\Gamma} \models \widehat{e} \Leftarrow \widehat{B} \rightsquigarrow \widehat{e} \models M\Gamma_2 \\
M\Gamma_2; \widehat{\Gamma} \models \widehat{s} : [\widehat{e}/x] \widehat{A} \gg \widehat{C} \rightsquigarrow \widehat{s} \models M\Gamma_3
\end{array}
}{M\Gamma; \widehat{\Gamma} \models \{\widehat{e}\}, \widehat{s} : (\forall x : \widehat{B}. \widehat{A}) \gg \widehat{C} \rightsquigarrow \{\widehat{e}\}, \widehat{s} \models M\Gamma_3} \text{ALGSAIMPARG} \\
\\
\frac{
\begin{array}{l}
M\Gamma; \widehat{\Gamma} \models \forall x : \widehat{B}. \widehat{A} \models M\Gamma_1 \quad M\Gamma_1; \widehat{\Gamma} \models \widehat{s} : [\widehat{x}/x] \widehat{A} \gg \widehat{C} \rightsquigarrow \widehat{s} \models M\Gamma_2 \\
M\Gamma; \widehat{\Gamma} \models \{_\}, \widehat{s} : (\forall x : \widehat{B}. \widehat{A}) \gg \widehat{C} \rightsquigarrow \{\widehat{x}\}, \widehat{s} \models M\Gamma_2, (\widehat{\Gamma} \vdash ax \Leftarrow \widehat{B})
\end{array}
}{M\Gamma; \widehat{\Gamma} \models \{_\}, \widehat{s} : (\forall x : \widehat{B}. \widehat{A}) \gg \widehat{C} \rightsquigarrow \{\widehat{x}\}, \widehat{s} \models M\Gamma_2, (\widehat{\Gamma} \vdash ax \Leftarrow \widehat{B})} \text{ALGSAHOLEARG} \\
\\
\boxed{M\Gamma; \widehat{\Gamma} \models \widehat{A} \models M\Gamma'} \\
\\
\frac{}{M\Gamma; \widehat{\Gamma} \models 1 \models M\Gamma} \text{ALGWFUNIT} \\
\\
\frac{
\begin{array}{l}
M\Gamma; \widehat{\Gamma} \models \widehat{A} \models M\Gamma_1 \quad M\Gamma_1; \widehat{\Gamma}, x : \widehat{A} \models \widehat{B} \models M\Gamma_2
\end{array}
}{M\Gamma; \widehat{\Gamma} \models \Pi x : \widehat{A}. \widehat{B} \models M\Gamma_2} \text{ALGWFPi} \\
\\
\frac{
\begin{array}{l}
M\Gamma; \widehat{\Gamma} \models \widehat{A} \models M\Gamma_1 \quad M\Gamma_1; \widehat{\Gamma}, x : \widehat{A} \models \widehat{B} \models M\Gamma_2
\end{array}
}{M\Gamma; \widehat{\Gamma} \models \forall x : \widehat{A}. \widehat{B} \models M\Gamma_2} \text{ALGWFFORALL} \\
\\
\frac{
\begin{array}{l}
M\Gamma; \widehat{\Gamma} \models \widehat{A} \models M\Gamma_1 \\
M\Gamma_1; \widehat{\Gamma} \models \widehat{e}_1 \Leftarrow \widehat{A} \rightsquigarrow \widehat{e}_3 \models M\Gamma_2 \quad M\Gamma_2; \widehat{\Gamma} \models \widehat{e}_2 \Leftarrow \widehat{A} \rightsquigarrow \widehat{e}_3 \models M\Gamma_3
\end{array}
}{M\Gamma; \widehat{\Gamma} \models \widehat{e}_1 = \widehat{e}_2 : \widehat{A} \models M\Gamma_3} \text{ALGWFEQ}
\end{array}$$

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