α, β, α, β, γ, δ	type variables
n, m, i, j	index variables
x, y, z	term variables

$n,\ m$	$::=$			cohort index
	$\mid$	0		
	$\mid$	$n+1$		
$\alpha^{+},\ \beta^{+},\ \gamma^{+},\ \delta^{+}$	$::=$			positive variable
	$\mid$	α^{+}		
	$\mid$	α^{+n}		
	$\mid$	$\beta^{\pm}$		
$\alpha^{-},\ \beta^{-},\ \gamma^{-},\ \delta^{-}$	$::=$			negative variable
	$\mid$	α^{-}		
	$\mid$	α^{-n}		
	$\mid$	$\alpha^{\pm}$		
$\alpha^{\pm},\ \beta^{\pm}$	$::=$			positive or negative variable
	$\mid$	$\alpha^{\pm}$		
	$\mid$	$\alpha^{\pm n}$		
σ	$::=$			substitution
	$\mid$	$\cdot$		
	$\mid$	id		
	$\mid$	$\mathbf{P}\,/\alpha^{+}$		
	$\mid$	$\mathbf{N}\,/\alpha^{-}$		
	$\mid$	$\overrightarrow{P}/\overrightarrow{\alpha^{+}}$		
	$\mid$	$\overrightarrow{N}/\overrightarrow{\alpha^{-}}$		
	$\mid$	$\overrightarrow{\alpha^{-}}/\overrightarrow{\widehat{\alpha}^{-}}$		
	$\mid$	μ		
	$\mid$	$\sigma_1 \circ \sigma_2$		
	$\mid$	$\sigma_1 \lll \sigma_2$		
	$\mid$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\mid$	(σ)	S	
	$\mid$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\mid$	$\mathbf{nf}\,(\sigma')$	M	
	$\mid$	$\sigma'\vert_{vars}$	M	
e	$::=$			entry of a subtyping constraint
	$\mid$	$\widehat{\alpha}^{+} :\simeq P$		
	$\mid$	$\widehat{\alpha}^{-} :\simeq N$		
	$\mid$	$\widehat{\alpha}^{+} :\geqslant P$		
	$\mid$	(e)	S	
	$\mid$	$SC(\widehat{\alpha}^{-})$	M	
	$\mid$	$e_1\ \&\ e_2$	M	
e	$::=$			entry of a unification constraint
	$\mid$	$\widehat{\alpha}^{+} :\simeq P$		
	$\mid$	$\widehat{\alpha}^{-} :\simeq N$		
	$\mid$	(e)	S	
	$\mid$	$e_1\ \&\ e_2$	M	

UC	$::=$			unification constraint
	$\mid$	$\cdot$		
	$\mid$	e		
	$\mid$	$UC \backslash vars$		
	$\mid$	$UC \vert vars$		
	$\mid$	$UC_1 \cup UC_2$		
	$\mid$	$\overline{UC_i}^{\,i}$		concatenate
	$\mid$	(UC)	S	
	$\mid$	$UC' \vert_{vars}$	M	
	$\mid$	$UC_1 \ \& \ UC_2$	M	
	$\mid$	$UC_1 \cup UC_2$	M	
	$\mid$	$\vert SC \vert$	M	
SC	$::=$			subtyping constraint
	$\mid$	$\cdot$		
	$\mid$	e		
	$\mid$	$SC \backslash vars$		
	$\mid$	$SC \vert vars$		
	$\mid$	$SC_1 \cup SC_2$		
	$\mid$	UC		
	$\mid$	$\overline{SC_i}^{\,i}$		concatenate
	$\mid$	(SC)	S	
	$\mid$	$SC' \vert_{vars}$	M	
	$\mid$	$SC_1 \ \& \ SC_2$	M	
$\widehat{\sigma}$	$::=$			unification substitution
	$\mid$	$\cdot$		
	$\mid$	$P / \widehat{\alpha}^{+}$		
	$\mid$	$N / \widehat{\alpha}^{-}$		
	$\mid$	$\overrightarrow{P} / \overrightarrow{\widehat{\alpha}^{+}}$		
	$\mid$	$\overrightarrow{N} / \overrightarrow{\widehat{\alpha}^{-}}$		
	$\mid$	$(\widehat{\sigma})$	S	
	$\mid$	$\widehat{\sigma}_1 \circ \widehat{\sigma}_2$		
	$\mid$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\mid$	$\mathbf{nf}\,(\widehat{\sigma}')$	M	
	$\mid$	$\widehat{\sigma}' \vert_{vars}$	M	
$\widehat{\tau},\ \widehat{\rho}$	$::=$			anti-unification substitution
	$\mid$	$\cdot$		
	$\mid$	$\widehat{\alpha}^{-} :\approx N$		
	$\mid$	$\widehat{\alpha}^{-} :\approx N$		
	$\mid$	$\overrightarrow{\alpha^{-}} / \overrightarrow{\widehat{\alpha}^{-}}$		
	$\mid$	$\overrightarrow{N} / \overrightarrow{\widehat{\alpha}^{-}}$		
	$\mid$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
	$\mid$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	$\mid$	$(\widehat{\tau})$	S	
	$\mid$	$\widehat{\tau}' \vert_{vars}$	M	
	$\mid$	$\widehat{\tau}_1 \ \& \ \widehat{\tau}_2$	M	
$\overrightarrow{\alpha^{+}},\ \overrightarrow{\beta^{+}},\ \overrightarrow{\gamma^{+}},\ \overrightarrow{\delta^{+}}$	$::=$			positive variable list

	$\mid$	$\cdot$		empty list
	$\mid$	α^+		a variable
	$\mid$	$\overrightarrow{\alpha^+}$		a variable
	$\mid$	$\overrightarrow{\overrightarrow{\alpha^+}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$	$::=$			negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	α^-		a variable
	$\mid$	$\overrightarrow{\alpha^-}$		variables
	$\mid$	$\overrightarrow{\overrightarrow{\alpha^-}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$	$::=$			positive or negative variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\alpha^\pm$		a variable
	$\mid$	$\overrightarrow{\alpha^\pm}$		variables
	$\mid$	$\overrightarrow{\overrightarrow{\alpha^\pm}_i}^{\,i}$		concatenate lists
$P,\ Q,\ R$	$::=$			positive declarative types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^-}.P$		
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\widehat{\tau}]\,P$	M	
	$\mid$	$[\widehat{\sigma}]\,P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	(P)	S	
	$\mid$	$P_1 \vee P_2$	M	
	$\mid$	$\mathbf{nf}\,(P')$	M	
$N,\ M,\ K$	$::=$			negative declarative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$P \to N$		
	$\mid$	$\forall\overrightarrow{\alpha^+}.N$		
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\widehat{\tau}]\,N$	M	
	$\mid$	$[\mu]N$	M	
	$\mid$	$[\widehat{\sigma}]\,N$	M	
	$\mid$	(N)	S	
	$\mid$	$\mathbf{nf}\,(N')$	M	
$\vec{P},\ \vec{Q}$	$::=$			list of positive types
	$\mid$	$\cdot$		empty list
	$\mid$	P		a singel type
	$\mid$	$[\sigma]\vec{P}$	M	
	$\mid$	$\overrightarrow{\vec{P}_i}^{\,i}$		concatenate lists
	$\mid$	$(\vec{P})$	S	
	$\mid$	$\mathbf{nf}\,(\vec{P}')$	M	

$\overrightarrow{N},\ \overrightarrow{M}$	::=			list of negative types
	|	$\cdot$		empty list
	|	N		a singel type
	|	$[\sigma]\overrightarrow{N}$	M	
	|	$\overline{\overrightarrow{N}_i}^{\,i}$		concatenate lists
	|	$(\overrightarrow{N})$	S	
	|	$\mathbf{nf}\,(\overrightarrow{N'})$	M	
$\Delta,\ \Gamma$	::=			declarative type context
	|	$\cdot$		empty context
	|	$\overrightarrow{\alpha^+}$		list of variables
	|	$\overrightarrow{\alpha^-}$		list of variables
	|	$\{\overrightarrow{\alpha^\pm}\}$		list of variables
	|	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	|	(Γ)	S	
	|	$\Gamma, \overrightarrow{\alpha^+}$	M	append a list of variables
	|	$\Gamma, \overrightarrow{\alpha^-}$	M	append a list of variables
	|	$\Gamma, \overrightarrow{\alpha^\pm}$	M	append a list of variables
	|	$\Theta(\widehat{\alpha}^+)$	M	
	|	$\Theta(\widehat{\alpha}^-)$	M	
	|	$\Gamma_1 \cup \Gamma_2$		
	|	$\Gamma_1 \cap \mathit{vars}$		
	|	$\Gamma_1 \cup \Gamma_2$	M	
	|	$\mathbf{fv}\ N$	M	
	|	$\mathbf{fv}\ P$	M	
	|	$\mathbf{fv}\,P$	M	
	|	$\mathbf{fv}\,N$	M	
Θ	::=			algorithmic variable context
	|	$\cdot$		empty context
	|	$\vec{\alpha}\{\Delta\}$		from an ordered list of variables
	|	$\overrightarrow{\widehat{\alpha}^+\{\Delta\}}$		from a variable to a list
	|	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	|	(Θ)	S	
	|	$\Theta\|_{\mathit{vars}}$		leave only those variables that are in the set
	|	$\Theta_1 \cup \Theta_2$		
Ξ	::=			anti-unification type variable context
	|	$\cdot$		empty context
	|	$\overrightarrow{\widehat{\alpha}^+}$		list of positive variables
	|	$\overrightarrow{\widehat{\alpha}^-}$		list of negative variables
	|	$\Xi, \overrightarrow{\widehat{\alpha}^+}$	M	append a list of variables
	|	$\Xi, \overrightarrow{\widehat{\alpha}^-}$	M	append a list of variables
	|	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	|	(Ξ)	S	
	|	$\Xi_1 \cup \Xi_2$		
	|	$\Xi_1 \cap \mathit{vars}$		
	|	$\Xi'\|_{\mathit{vars}}$	M	
	|	$\mathbf{dom}\,(UC)$	M	
	|	$\mathbf{dom}\,(SC)$	M	

	$\mid$	$\mathbf{dom}\,(\widehat{\sigma})$	M	
	$\mid$	$\mathbf{dom}\,(\widehat{\tau})$	M	
	$\mid$	$\mathbf{dom}\,(\Theta)$	M	
	$\mid$	$\mathbf{uv}\; N$	M	
	$\mid$	$\mathbf{uv}\; P$	M	
$\vec{\alpha},\ \vec{\beta}$	$::=$			ordered positive or negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	$\overrightarrow{\alpha^{+}}$		list of variables
	$\mid$	$\overrightarrow{\alpha^{-}}$		list of variables
	$\mid$	$\overrightarrow{\alpha^{\pm}}$		list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^{+}}$		list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}}$		list of variables
	$\mid$	$\vec{\alpha}_1 \backslash \mathit{vars}$		setminus
	$\mid$	vars		
	$\mid$	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	$\mid$	$(\vec{\alpha})$	S	parenthesis
	$\mid$	$[\mu]\vec{\alpha}$	M	apply moving to list
	$\mid$	$[\vec{\mu}]\vec{\alpha}$	M	apply umoving to list
	$\mid$	$\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P$	M	
	$\mid$	$\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N$	M	
	$\mid$	$\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P$	M	
	$\mid$	$\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N$	M	
vars	$::=$			set of variables
	$\mid$	$\varnothing$		empty set
	$\mid$	$\mathit{vars}_1 \cap \mathit{vars}_2$		set intersection
	$\mid$	$\mathit{vars}_1 \cup \mathit{vars}_2$		set union
	$\mid$	$\mathit{vars}_1 \backslash \mathit{vars}_2$		set complement
	$\mid$	(vars)	S	parenthesis
	$\mid$	$\vec{\alpha}$		ordered list of variables
	$\mid$	$[\mu]\mathit{vars}$	M	apply moving to varset
	$\mid$	Ξ		algorithmic type context
	$\mid$	Γ		declarative type context
μ	$::=$			
	$\mid$	$\cdot$		empty moving
	$\mid$	$\mathit{pma}1 \mapsto \mathit{pma}2$		Positive unit substitution
	$\mid$	$\mathit{nma}1 \mapsto \mathit{nma}2$		Positive unit substitution
	$\mid$	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	$\mid$	$\mu_1 \circ \mu_2$	M	Composition
	$\mid$	$\overline{\mu_i}^{\,i}$		concatenate movings
	$\mid$	$\mu\vert_{\mathit{vars}}$	M	restriction on a set
	$\mid$	μ^{-1}	M	inversion
	$\mid$	$\mathbf{nf}\,(\mu')$	M	
$\vec{\mu}$	$::=$			
	$\mid$	$\cdot$		empty moving
	$\mid$	$\overrightarrow{\widehat{\alpha}^{+}}/\overrightarrow{\alpha^{+}}$		
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}}/\overrightarrow{\alpha^{-}}$		

$\widehat{\alpha}^{\pm}$	$::=$			positive/negative unification variable
	$\mid$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	$::=$			positive unification variable
	$\mid$	$\widehat{\alpha}^{+}$		
	$\mid$	$\widehat{\alpha}^{+}\{\Delta\}$		
	$\mid$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-}$, $\widehat{\beta}^{-}$	$::=$			negative unification variable
	$\mid$	$\widehat{\alpha}^{-}$		
	$\mid$	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	$\mid$	$\widehat{\alpha}^{-}\{\Delta\}$		
	$\mid$	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widehat{\alpha}^{+}}$, $\overrightarrow{\widehat{\beta}^{+}}$	$::=$			positive unification variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\widehat{\alpha}^{+}$		a variable
	$\mid$	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	$\mid$	$\overrightarrow{\overrightarrow{\widehat{\alpha}^{+}}_{i}}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}}$, $\overrightarrow{\widehat{\beta}^{-}}$	$::=$			negative unification variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\widehat{\alpha}^{-}$		a variable
	$\mid$	Ξ		from an antiunification context
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	$\mid$	$\overrightarrow{\overrightarrow{\widehat{\alpha}^{-}}_{i}}^{\,i}$		concatenate lists
P, Q	$::=$			a positive algorithmic type (potentially with algorithmic variables)
	$\mid$	$\widehat{\alpha}^{+}$		
	$\mid$	α^{+}		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^{-}}.P$		
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\widehat{\tau}]P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	$[\widehat{\sigma}]P$	M	
	$\mid$	$[\overrightarrow{\mu}]P$	M	
	$\mid$	(P)	S	
	$\mid$	$\mathbf{nf}\,(P')$	M	
N, M	$::=$			a negative algorithmic type (potentially with algorithmic variables)
	$\mid$	$\widehat{\alpha}^{-}$		
	$\mid$	α^{-}		
	$\mid$	$\uparrow P$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$\forall\overrightarrow{\alpha^{+}}.N$		
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\widehat{\tau}]N$	M	

	|	$[\mu] N$	M
	|	$[\widehat{\sigma}] N$	M
	|	$[\vec{\mu}] N$	M
	|	(N)	S
	|	$\mathbf{nf}\ (N')$	M
$auSol$	::=		
	|	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	|	$(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$	
$terminals$	::=		
	|	$\exists$	
	|	$\forall$	
	|	$\uparrow$	
	|	$\downarrow$	
	|	$\rightarrow$	
	|	$\leftrightarrow$	
	|	$\in$	
	|	$\notin$	
	|	$\cdot$	
	|	$\vdash$	
	|	$\leqslant$	
	|	$\geqslant$	
	|	$\simeq$	
	|	$\cup$	
	|	$\cap$	
	|	$\setminus$	
	|	$\subseteq$	
	|	$\mapsto$	
	|	$\stackrel{u}{\simeq}$	
	|	$\stackrel{a}{\simeq}$	
	|	$\varnothing$	
	|	$\circ$	
	|	$\Rightarrow$	
	|	$\vDash$	
	|	$\Dashv$	
	|	$\neq$	
	|	$\equiv_n$	
	|	$\vee$	
	|	$\Downarrow$	
	|	$:\geqslant$	
	|	$:\simeq$	
	|	Λ	
	|	λ	
	|	$\mathbf{let}^{\exists}$	
	|	$\bullet$	
	|	$\Rrightarrow$	
	|	$\Lleftarrow$	
$v,\ w$	::=		value terms

	$\vert$	x	
	$\vert$	$\{c\}$	
	$\vert$	$(v : P)$	
	$\vert$	(v)	M
$\vec{v}$	$::=$		list of arguments
	$\vert$	$\cdot$	
	$\vert$	v	
	$\vert$	$\overline{\vec{v}_i}^{\,i}$	concatenate
$c,\ d$	$::=$		computation terms
	$\vert$	$(c : N)$	
	$\vert$	$\lambda x : P.c$	
	$\vert$	$\Lambda \alpha^+.c$	
	$\vert$	$\textbf{return}\, v$	
	$\vert$	$\textbf{let}\, x = v; c$	
	$\vert$	$\textbf{let}\, x : P = v(\vec{v}); c$	
	$\vert$	$\textbf{let}\, x = v(\vec{v}); c$	
	$\vert$	$\textbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c$	
$vctx,\ \Phi$	$::=$		variable context
	$\vert$	$\cdot$	
	$\vert$	$x : P$	
	$\vert$	$\overline{\Phi_i}^{\,i}$	concatenate contexts
$formula$	$::=$		
	$\vert$	$judgement$	
	$\vert$	$judgement$ unique	
	$\vert$	$formula_1 \quad .. \quad formula_n$	
	$\vert$	$\mu : vars_1 \leftrightarrow vars_2$	
	$\vert$	$\mu\, \textbf{is bijective}$	
	$\vert$	$x : P \in \Phi$	
	$\vert$	$UC_1 \subseteq UC_2$	
	$\vert$	$UC_1 = UC_2$	
	$\vert$	$SC_1 \subseteq SC_2$	
	$\vert$	$e \in SC$	
	$\vert$	$e \in UC$	
	$\vert$	$vars_1 \subseteq vars_2$	
	$\vert$	$vars_1 \subseteq vars_2 \subseteq vars_3$	
	$\vert$	$vars_1 = vars_2$	
	$\vert$	$vars\, \textbf{are fresh}$	
	$\vert$	$\alpha^- \notin vars$	
	$\vert$	$\alpha^+ \notin vars$	
	$\vert$	$\alpha^- \in vars$	
	$\vert$	$\alpha^+ \in vars$	
	$\vert$	$\widehat{\alpha}^+ \in vars$	
	$\vert$	$\widehat{\alpha}^- \in vars$	
	$\vert$	$\widehat{\alpha}^- \in \Theta$	
	$\vert$	$\widehat{\alpha}^+ \in \Theta$	
	$\vert$	$\widehat{\alpha}^- \notin vars$	

	$\mid$	$\widehat{\alpha}^{+} \notin vars$	
	$\mid$	$\widehat{\alpha}^{-} \notin \Theta$	
	$\mid$	$\widehat{\alpha}^{+} \notin \Theta$	
	$\mid$	$\widehat{\alpha}^{-} \in \Xi$	
	$\mid$	$\widehat{\alpha}^{-} \notin \Xi$	
	$\mid$	$\widehat{\alpha}^{+} \in \Xi$	
	$\mid$	$\widehat{\alpha}^{+} \notin \Xi$	
	$\mid$	if any other rule is not applicable	
	$\mid$	$\overrightarrow{\alpha}_1 = \overrightarrow{\alpha}_2$	
	$\mid$	$e_1 = e_2$	
	$\mid$	$e_1 = e_2$	
	$\mid$	$\widehat{\sigma}_1 = \widehat{\sigma}_2$	
	$\mid$	$N = M$	
	$\mid$	$\Theta \subseteq \Theta'$	
	$\mid$	$\overrightarrow{v}_1 = \overrightarrow{v}_2$	
	$\mid$	$\mathbf{N} \neq \mathbf{M}$	
	$\mid$	$\mathbf{P} \neq \mathbf{Q}$	
	$\mid$	$N \neq M$	
	$\mid$	$P \neq Q$	
	$\mid$	$P \neq Q$	
	$\mid$	$N \neq M$	
	$\mid$	$\overrightarrow{v}_1 \neq \overrightarrow{v}_2$	
	$\mid$	$\overrightarrow{\alpha^{+}}_1 \neq \overrightarrow{\alpha^{+}}_2$	
	$\mid$	$\lvert\overrightarrow{\alpha^{-}}\rvert + \lvert\overrightarrow{\beta^{-}}\rvert > 0$	
	$\mid$	$\lvert\overrightarrow{\alpha^{+}}\rvert + \lvert\overrightarrow{\beta^{+}}\rvert > 0$	
A	$::=$		
	$\mid$	$\Gamma; \Theta \models N \leqslant M \dashv SC$	Negative subtyping
	$\mid$	$\Gamma; \Theta \models P \geqslant Q \dashv SC$	Positive supertyping
AT	$::=$		
	$\mid$	$\Gamma; \Phi \models v : P$	Positive type inference
	$\mid$	$\Gamma; \Phi \models c : N$	Negative type inference
	$\mid$	$\Gamma; \Phi; \Theta_1 \models N \bullet \overrightarrow{v} \Rrightarrow M \dashv \Theta_2; SC$	Application type inference
AU	$::=$		
	$\mid$	$\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	$\mid$	$\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$	
SCM	$::=$		
	$\mid$	$\Gamma \vdash e_1 \mathbin{\&} e_2 = e_3$	Subtyping Constraint Entry Merge
	$\mid$	$\Theta \vdash SC_1 \& SC_2 = SC_3$	Merge of subtyping constraints
UCM	$::=$		
	$\mid$	$\Gamma \vdash e_1 \& e_2 = e_3$	
	$\mid$	$\Theta \vdash UC_1 \& UC_2 = UC_3$	Merge of unification constraints
$SATSCE$	$::=$		
	$\mid$	$\Gamma \vdash P : e$	Positive type satisfies with the subtyping constr

	$\vert$	$\Gamma \vdash N : e$	Negative type satisfies with the subtyping constraint entry
SING	$::=$		
	$\vert$	e_1 **singular with** P	Positive Subtyping Constraint Entry Is Singular
	$\vert$	e_1 **singular with** N	Negative Subtyping Constraint Entry Is Singular
	$\vert$	SC **singular with** $\widehat{\sigma}$	Subtyping Constraint Is Singular
E1	$::=$		
	$\vert$	$N \simeq^{D} M$	Negative type equivalence
	$\vert$	$P \simeq^{D} Q$	Positive type equivalence
	$\vert$	$P \simeq^{D} Q$	Positive unification type equivalence
	$\vert$	$N \simeq^{D} M$	Positive unification type equivalence
D1	$::=$		
	$\vert$	$\Gamma \vdash N \simeq^{\leqslant} M$	Negative subtyping-induced equivalence
	$\vert$	$\Gamma \vdash P \simeq^{\leqslant} Q$	Positive subtyping-induced equivalence
	$\vert$	$\Gamma \vdash N \leqslant M$	Negative subtyping
	$\vert$	$\Gamma \vdash P \geqslant Q$	Positive supertyping
D1S	$::=$		
	$\vert$	$\Gamma_2 \vdash \sigma_1 \simeq^{\leqslant} \sigma_2 : \Gamma_1$	Equivalence of substitutions
	$\vert$	$\Gamma \vdash \sigma_1 \simeq^{\leqslant} \sigma_2 : vars$	Equivalence of substitutions
	$\vert$	$\Theta \vdash \widehat{\sigma}_1 \simeq^{\leqslant} \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions
	$\vert$	$\Gamma \vdash \widehat{\sigma}_1 \simeq^{\leqslant} \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions
D1C	$::=$		
	$\vert$	$\Gamma \vdash \Phi_1 \simeq^{\leqslant} \Phi_2$	Equivalence of contexts
DT	$::=$		
	$\vert$	$\Gamma; \Phi \vdash v : P$	Positive type inference
	$\vert$	$\Gamma; \Phi \vdash c : N$	Negative type inference
	$\vert$	$\Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!\!> M$	Application type inference
EQ	$::=$		
	$\vert$	$N = M$	Negative type equality (alpha-equivalence)
	$\vert$	$P = Q$	Positive type equuality (alphha-equivalence)
	$\vert$	$P = Q$	
LUBF	$::=$		
	$\vert$	$P_1 \vee P_2 === Q$	
	$\vert$	**ord** $vars$ **in** $P === \vec{\alpha}$	
	$\vert$	**ord** $vars$ **in** $N === \vec{\alpha}$	
	$\vert$	**ord** $vars$ **in** $P === \vec{\alpha}$	
	$\vert$	**ord** $vars$ **in** $N === \vec{\alpha}$	
	$\vert$	**nf** $(N') === N$	
	$\vert$	**nf** $(P') === P$	
	$\vert$	**nf** $(N') === N$	
	$\vert$	**nf** $(P') === P$	
	$\vert$	**nf** $(\vec{N}') === \vec{N}$	

$|\quad \mathbf{nf}\,(\vec{P}') === \vec{P}$
$|\quad \mathbf{nf}\,(\sigma') === \sigma$
$|\quad \mathbf{nf}\,(\widehat{\sigma}') === \widehat{\sigma}$
$|\quad \mathbf{nf}\,(\mu') === \mu$
$|\quad \sigma'|_{vars}$
$|\quad \widehat{\sigma}'|_{vars}$
$|\quad \widehat{\tau}'|_{vars}$
$|\quad \Xi'|_{vars}$
$|\quad SC'|_{vars}$
$|\quad UC'|_{vars}$
$|\quad e_1 \,\&\, e_2$
$|\quad e_1 \,\&\, e_2$
$|\quad UC_1 \,\&\, UC_2$
$|\quad UC_1 \cup UC_2$
$|\quad \Gamma_1 \cup \Gamma_2$
$|\quad SC_1 \,\&\, SC_2$
$|\quad \widehat{\tau}_1 \,\&\, \widehat{\tau}_2$
$|\quad \mathbf{dom}\,(UC) === \Xi$
$|\quad \mathbf{dom}\,(SC) === \Xi$
$|\quad \mathbf{dom}\,(\widehat{\sigma}) === \Xi$
$|\quad \mathbf{dom}\,(\widehat{\tau}) === \Xi$
$|\quad \mathbf{dom}\,(\Theta) === \Xi$
$|\quad |SC| === UC$
$|\quad \mathbf{fv}\, N === \Gamma$
$|\quad \mathbf{fv}\, P === \Gamma$
$|\quad \mathbf{fv}\, P === \Gamma$
$|\quad \mathbf{fv}\, N === \Gamma$
$|\quad \mathbf{uv}\, N === \Xi$
$|\quad \mathbf{uv}\, P === \Xi$

$LUB \quad ::=$
$|\quad \Gamma \models P_1 \vee P_2 = Q$ — Least Upper Bound (Least Common Supertype)
$|\quad \mathbf{upgrade}\, \Gamma \vdash P\, \mathbf{to}\, \Delta = Q$

$Nrm \quad ::=$
$|\quad \mathbf{nf}\,(N) = M$
$|\quad \mathbf{nf}\,(P) = Q$
$|\quad \mathbf{nf}\,(N) = M$
$|\quad \mathbf{nf}\,(P) = Q$

$Order \quad ::=$
$|\quad \mathbf{ord}\, vars\, \mathbf{in}\, N = \vec{\alpha}$
$|\quad \mathbf{ord}\, vars\, \mathbf{in}\, P = \vec{\alpha}$
$|\quad \mathbf{ord}\, vars\, \mathbf{in}\, N = \vec{\alpha}$
$|\quad \mathbf{ord}\, vars\, \mathbf{in}\, P = \vec{\alpha}$

$U \quad ::=$
$|\quad \Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC$ — Negative unification
$|\quad \Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC$ — Positive unification

WFT	::=		
	|	$\Gamma \vdash N$	Negative type well-formedness
	|	$\Gamma \vdash P$	Positive type well-formedness
WFAT	::=		
	|	$\Gamma ; \Xi \vdash N$	Negative algorithmic type well-formedness
	|	$\Gamma ; \Xi \vdash P$	Positive algorithmic type well-formedness
WFALL	::=		
	|	$\Gamma \vdash \overrightarrow{N}$	Negative type list well-formedness
	|	$\Gamma \vdash \overrightarrow{P}$	Positive type list well-formedness
	|	$\Gamma ; \Xi_2 \vdash \widehat{\tau} : \Xi_1$	Antiunification substitution well-formedness
	|	$\Gamma \vdash^{\supseteq} \Theta$	Unification context well-formedness
	|	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution signature
	|	$\Theta \vdash \widehat{\sigma} : \Xi$	Unification substitution signature
	|	$\Gamma \vdash \widehat{\sigma} : \Xi$	Unification substitution general signature
	|	$\Theta \vdash \widehat{\sigma} : UC$	Unification substitution satisfies unification constraint
	|	$\Theta \vdash \widehat{\sigma} : SC$	Unification substitution satisfies subtyping constraint
	|	$\Gamma \vdash e$	Unification constraint entry well-formedness
	|	$\Gamma \vdash e$	Subtyping constraint entry well-formedness
	|	$\Gamma \vdash P : e$	Positive type satisfies unification constraint
	|	$\Gamma \vdash N : e$	Negative type satisfies unification constraint
	|	$\Gamma \vdash P : e$	Positive type satisfies subtyping constraint
	|	$\Gamma \vdash N : e$	Negative type satisfies subtyping constraint
	|	$\Theta \vdash UC : \Xi$	Unification constraint well-formedness with specified domain
	|	$\Theta \vdash SC : \Xi$	Subtyping constraint well-formedness with specified domain
	|	$\Theta \vdash UC$	Unification constraint well-formedness
	|	$\Theta \vdash SC$	Subtyping constraint well-formedness
	|	$\Gamma \vdash \overrightarrow{v}$	Argument List well-formedness
	|	$\Gamma \vdash \Phi$	Context well-formedness
	|	$\Gamma \vdash v$	Value well-formedness
	|	$\Gamma \vdash c$	Computation well-formedness
judgement	::=		
	|	*A*	
	|	*AT*	
	|	*AU*	
	|	*SCM*	
	|	*UCM*	
	|	*SATSCE*	
	|	*SING*	
	|	*E1*	
	|	*D1*	
	|	*D1S*	
	|	*D1C*	
	|	*DT*	
	|	*EQ*	
	|	*LUB*	
	|	*Nrm*	
	|	*Order*	

$$
\begin{array}{lcl}
 & | & U \\
 & | & \mathit{WFT} \\
 & | & \mathit{WFAT} \\
 & | & \mathit{WFALL} \\
 & & \\
\mathit{user\_syntax} & ::= & \\
 & | & \alpha \\
 & | & n \\
 & | & x \\
 & | & n \\
 & | & \alpha^{+} \\
 & | & \alpha^{-} \\
 & | & \alpha^{\pm} \\
 & | & \sigma \\
 & | & e \\
 & | & e \\
 & | & \mathit{UC} \\
 & | & \mathit{SC} \\
 & | & \widehat{\sigma} \\
 & | & \widehat{\tau} \\
 & | & \overrightarrow{\alpha^{+}} \\
 & | & \overrightarrow{\alpha^{-}} \\
 & | & \overrightarrow{\alpha^{\pm}} \\
 & | & P \\
 & | & N \\
 & | & \overrightarrow{P} \\
 & | & \overrightarrow{N} \\
 & | & \Gamma \\
 & | & \Theta \\
 & | & \Xi \\
 & | & \overrightarrow{\alpha} \\
 & | & \mathit{vars} \\
 & | & \mu \\
 & | & \overrightarrow{\mu} \\
 & | & \widehat{\alpha}^{\pm} \\
 & | & \widehat{\alpha}^{+} \\
 & | & \widehat{\alpha}^{-} \\
 & | & \overrightarrow{\widehat{\alpha}^{+}} \\
 & | & \overrightarrow{\widehat{\alpha}^{-}} \\
 & | & P \\
 & | & N \\
 & | & \mathit{auSol} \\
 & | & \mathit{terminals} \\
 & | & v \\
 & | & \overrightarrow{v} \\
 & | & c \\
 & | & \mathit{vctx} \\
 & | & \mathit{formula}
\end{array}
$$

$\boxed{\Gamma; \Theta \models N \leqslant M \dashv \mathit{SC}}$ Negative subtyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^- \leqslant \alpha^- \dashv \cdot}\ \textsc{ANVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(P) \overset{u}{\simeq} \mathbf{nf}\,(Q) \dashv UC}{\Gamma;\ \Theta \models \uparrow P \leqslant \uparrow Q \dashv UC}\ \textsc{AShiftU}$$

$$\frac{\Gamma;\ \Theta \models P \geqslant Q \dashv SC_1 \quad \Gamma;\ \Theta \models N \leqslant M \dashv SC_2 \quad \Theta \vdash SC_1 \& SC_2 = SC}{\Gamma;\ \Theta \models P \to N \leqslant Q \to M \dashv SC}\ \textsc{AArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma;\ \Theta \models \forall \overrightarrow{\alpha^+}.N \leqslant \forall \overrightarrow{\beta^+}.M \dashv SC \backslash \overrightarrow{\widehat{\alpha}^+}}\ \textsc{AForall}$$

$\boxed{\Gamma;\ \Theta \models P \geqslant Q \dashv SC}$ Positive supertyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^+ \geqslant \alpha^+ \dashv \cdot}\ \textsc{APVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(N) \overset{u}{\simeq} \mathbf{nf}\,(M) \dashv UC}{\Gamma;\ \Theta \models \downarrow N \geqslant \downarrow M \dashv UC}\ \textsc{AShiftD}$$

$$\frac{\overrightarrow{\widehat{\alpha}^-}\ \textbf{are fresh} \quad \Gamma, \overrightarrow{\beta^-};\ \Theta, \overrightarrow{\widehat{\alpha}^-}\{\Gamma, \overrightarrow{\beta^-}\} \models [\overrightarrow{\widehat{\alpha}^-}/\overrightarrow{\alpha^-}]P \geqslant Q \dashv SC}{\Gamma;\ \Theta \models \exists \overrightarrow{\alpha^-}.P \geqslant \exists \overrightarrow{\beta^-}.Q \dashv SC \backslash \overrightarrow{\widehat{\alpha}^-}}\ \textsc{AExists}$$

$$\frac{\textbf{upgrade}\,\Gamma \vdash P\,\textbf{to}\,\Theta(\widehat{\alpha}^+) = Q}{\Gamma;\ \Theta \models \widehat{\alpha}^+ \geqslant P \dashv (\widehat{\alpha}^+ :\geqslant Q)}\ \textsc{APUVar}$$

$\boxed{\Gamma; \Phi \models v \colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \models x \colon \mathbf{nf}\,(P)}\ \textsc{ATVar}$$

$$\frac{\Gamma; \Phi \models c \colon N}{\Gamma; \Phi \models \{c\} \colon \downarrow N}\ \textsc{ATThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma; \Phi \models v \colon P \quad \Gamma; \cdot \models Q \geqslant P \dashv \cdot}{\Gamma; \Phi \models (v : Q) \colon \mathbf{nf}\,(Q)}\ \textsc{ATPAnnot}$$

$\boxed{\Gamma; \Phi \models c \colon N}$ Negative type inference

$$\frac{\Gamma \vdash M \quad \Gamma; \Phi \models c \colon N \quad \Gamma; \cdot \models N \leqslant M \dashv \cdot}{\Gamma; \Phi \models (c : M) \colon \mathbf{nf}\,(M)}\ \textsc{ATNAnnot}$$

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi, x : P \models c \colon N}{\Gamma; \Phi \models \lambda x : P.c \colon \mathbf{nf}\,(P \to N)}\ \textsc{ATtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \models c \colon N}{\Gamma; \Phi \models \Lambda \alpha^+.c \colon \mathbf{nf}\,(\forall \alpha^+.N)}\ \textsc{ATTLam}$$

$$\frac{\Gamma; \Phi \models v \colon P}{\Gamma; \Phi \models \mathbf{return}\,v \colon \uparrow P}\ \textsc{ATReturn}$$

$$\frac{\Gamma; \Phi \models v \colon P \quad \Gamma; \Phi, x : P \models c \colon N}{\Gamma; \Phi \models \mathbf{let}\,x = v; c \colon N}\ \textsc{ATVarLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma; \Phi \models v \colon \downarrow M \\ \Gamma; \Phi; \cdot \models M \bullet \vec{v} \Rrightarrow \uparrow Q \dashv \Theta; SC_1 \quad \Gamma;\ \Theta \models \uparrow Q \leqslant \uparrow P \dashv SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC \quad \Gamma; \Phi, x : P \models c \colon N\end{array}}{\Gamma; \Phi \models \mathbf{let}\,x : P = v(\vec{v}); c \colon N}\ \textsc{ATAppLetAnn}$$

$$\frac{\begin{array}{c}\Gamma; \Phi \vDash v \colon \downarrow M \quad \Gamma; \Phi; \cdot \vDash M \bullet \vec{v} \Rrightarrow \uparrow Q \dashv \Theta; SC \\ \texttt{<<multiple parses>>} \\ \Gamma; \Phi, x : [\widehat{\sigma}]\, Q \vDash c \colon N\end{array}}{\Gamma; \Phi \vDash \mathbf{let}\, x = v(\vec{v}); c \colon N}\ \textsc{ATAppLet}$$

$$\frac{\Gamma; \Phi \vDash v \colon \exists \overrightarrow{\alpha^-}.P \quad \Gamma, \overrightarrow{\alpha^-}; \Phi, x : P \vDash c \colon N \quad \Gamma \vdash N}{\Gamma; \Phi \vDash \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c \colon N}\ \textsc{ATUnpack}$$

$\boxed{\Gamma; \Phi; \Theta_1 \vDash N \bullet \vec{v} \Rrightarrow M \dashv \Theta_2; SC}$ Application type inference

$$\frac{}{\Gamma; \Phi; \Theta \vDash N \bullet \cdot \Rrightarrow \mathbf{nf}\,(N) \dashv \Theta; \cdot}\ \textsc{ATEmptyApp}$$

$$\frac{\begin{array}{l}\Gamma; \Phi \vDash v \colon P \quad \Gamma;\ \Theta \vDash Q \geqslant P \dashv SC_1 \quad \Gamma; \Phi; \Theta \vDash N \bullet \vec{v} \Rrightarrow M \dashv \Theta'; SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC\end{array}}{\Gamma; \Phi; \Theta \vDash Q \to N \bullet v, \vec{v} \Rrightarrow M \dashv \Theta'; SC}\ \textsc{ATArrowApp}$$

$$\frac{\begin{array}{c}\texttt{<<multiple parses>>} \\ \overrightarrow{\widehat{\alpha}^+}\ \textbf{are fresh} \quad \vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot\end{array}}{\texttt{<<multiple parses>>}}\ \textsc{ATForallApp}$$

$\boxed{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \vDash \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\cdot, \alpha^+, \cdot, \cdot)}\ \textsc{AUPVar}$$

$$\frac{\Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \downarrow N_1 \overset{a}{\simeq} \downarrow N_2 \dashv (\Xi, \downarrow M, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P_1 \overset{a}{\simeq} \exists \overrightarrow{\alpha^-}.P_2 \dashv (\Xi, \exists \overrightarrow{\alpha^-}.Q, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUExists}$$

$\boxed{\Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \vDash \alpha^- \overset{a}{\simeq} \alpha^- \dashv (\cdot, \alpha^-, \cdot, \cdot)}\ \textsc{AUNVar}$$

$$\frac{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \uparrow P_1 \overset{a}{\simeq} \uparrow P_2 \dashv (\Xi, \uparrow Q, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vDash \forall \overrightarrow{\alpha^+}.N_1 \overset{a}{\simeq} \forall \overrightarrow{\alpha^+}.N_2 \dashv (\Xi, \forall \overrightarrow{\alpha^+}.M, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUForall}$$

$$\frac{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \widehat{\tau}_1, \widehat{\tau}_2) \quad \Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \widehat{\tau}_1', \widehat{\tau}_2')}{\Gamma \vDash P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \widehat{\tau}_1 \cup \widehat{\tau}_1', \widehat{\tau}_2 \cup \widehat{\tau}_2')}\ \textsc{AUArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \vDash N \overset{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, \widehat{\alpha}^-_{\{N,M\}}, (\widehat{\alpha}^-_{\{N,M\}} :\approx N), (\widehat{\alpha}^-_{\{N,M\}} :\approx M))}\ \textsc{AUAU}$$

$\boxed{\Gamma \vdash e_1\ \&\ e_2 = e_3}$ Subtyping Constraint Entry Merge

$$\frac{\Gamma \vDash P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1)\ \&\ (\widehat{\alpha}^+ :\geqslant P_2) = (\widehat{\alpha}^+ :\geqslant Q)}\ \textsc{SCMESupSup}$$

$$\frac{\Gamma; \cdot \vDash P \geqslant Q \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\simeq P)\ \&\ (\widehat{\alpha}^+ :\geqslant Q) = (\widehat{\alpha}^+ :\simeq P)}\ \textsc{SCMEEqSup}$$

$$\frac{\Gamma;\cdot\vDash Q \geqslant P \dashv \cdot}{\Gamma\vdash(\widehat{\alpha}^+ :\geqslant P)\ \&\ (\widehat{\alpha}^+ :\simeq Q) = (\widehat{\alpha}^+ :\simeq Q)}\ \text{SCMESupEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma\vdash(\widehat{\alpha}^+ :\simeq P)\ \&\ (\widehat{\alpha}^+ :\simeq P') = (\widehat{\alpha}^+ :\simeq P)}\ \text{SCMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma\vdash(\widehat{\alpha}^- :\simeq N)\ \&\ (\widehat{\alpha}^- :\simeq N') = (\widehat{\alpha}^- :\simeq N)}\ \text{SCMENEqEq}$$

$\boxed{\Theta\vdash SC_1\&SC_2 = SC_3}$ Merge of subtyping constraints

$\boxed{\Gamma\vdash e_1\&e_2 = e_3}$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma\vdash(\widehat{\alpha}^+ :\simeq P)\&(\widehat{\alpha}^+ :\simeq P') = (\widehat{\alpha}^+ :\simeq P)}\ \text{UCMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma\vdash(\widehat{\alpha}^- :\simeq N)\&(\widehat{\alpha}^- :\simeq N') = (\widehat{\alpha}^- :\simeq N)}\ \text{UCMENEqEq}$$

$\boxed{\Theta\vdash UC_1\&UC_2 = UC_3}$ Merge of unification constraints

$\boxed{\Gamma\vdash P : e}$ Positive type satisfies with the subtyping constraint entry

$$\frac{\Gamma\vdash P \geqslant Q}{\Gamma\vdash P : (\widehat{\alpha}^+ :\geqslant Q)}\ \text{SATSCESup}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma\vdash P : (\widehat{\alpha}^+ :\simeq Q)}\ \text{SATSCEPEq}$$

$\boxed{\Gamma\vdash N : e}$ Negative type satisfies with the subtyping constraint entry

$$\frac{\texttt{<<multiple parses>>}}{\Gamma\vdash N : (\widehat{\alpha}^- :\simeq M)}\ \text{SATSCENEq}$$

$\boxed{e_1\ \textbf{singular with}\ P}$ Positive Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^+ :\simeq P\ \textbf{singular with}\ \textbf{nf}\ (P)}\ \text{SINGPEq}$$

$$\frac{}{\widehat{\alpha}^+ :\geqslant \exists\overrightarrow{\alpha^-}.\alpha^+\ \textbf{singular with}\ \alpha^+}\ \text{SINGSupVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\widehat{\alpha}^+ :\geqslant \exists\overrightarrow{\alpha^-}.\downarrow N\ \textbf{singular with}\ \exists\alpha^-.\downarrow\alpha^-}\ \text{SINGSupShift}$$

$\boxed{e_1\ \textbf{singular with}\ N}$ Negative Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^- :\simeq N\ \textbf{singular with}\ \textbf{nf}\ (N)}\ \text{SINGNEq}$$

$\boxed{SC\ \textbf{singular with}\ \widehat{\sigma}}$ Subtyping Constraint Is Singular

$\boxed{N \simeq^D M}$ Negative type equivalence

$$\frac{}{\alpha^- \simeq^D \alpha^-}\ \text{E1NVar}$$

$$\frac{P \simeq^D Q}{\uparrow P \simeq^D \uparrow Q}\ \text{E1ShiftU}$$

$$\frac{P \simeq^D Q \quad N \simeq^D M}{P\to N \simeq^D Q\to M}\ \text{E1Arrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\forall\overrightarrow{\alpha^+}.N \simeq^D \forall\overrightarrow{\beta^+}.M}\ \textsc{E1Forall}$$

$\boxed{P \simeq^D Q}$ Positive type equivalence

$$\frac{}{\alpha^+ \simeq^D \alpha^+}\ \textsc{E1PVar}$$

$$\frac{N \simeq^D M}{\downarrow N \simeq^D \downarrow M}\ \textsc{E1ShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\exists\overrightarrow{\alpha^-}.P \simeq^D \exists\overrightarrow{\beta^-}.Q}\ \textsc{E1Exists}$$

$\boxed{P \simeq^D Q}$ Positive unification type equivalence

$\boxed{N \simeq^D M}$ Positive unification type equivalence

$\boxed{\Gamma \vdash N \simeq^{\leqslant} M}$ Negative subtyping-induced equivalence

$$\frac{\Gamma \vdash N \leqslant M \quad \Gamma \vdash M \leqslant N}{\Gamma \vdash N \simeq^{\leqslant} M}\ \textsc{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq^{\leqslant} Q}$ Positive subtyping-induced equivalence

$$\frac{\Gamma \vdash P \geqslant Q \quad \Gamma \vdash Q \geqslant P}{\Gamma \vdash P \simeq^{\leqslant} Q}\ \textsc{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant \alpha^-}\ \textsc{D1NVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant \uparrow Q}\ \textsc{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant Q \quad \Gamma \vdash N \leqslant M}{\Gamma \vdash P \to N \leqslant Q \to M}\ \textsc{D1Arrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma, \overrightarrow{\beta^+} \vdash [\sigma]N \leqslant M}{\Gamma \vdash \forall\overrightarrow{\alpha^+}.N \leqslant \forall\overrightarrow{\beta^+}.M}\ \textsc{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant \alpha^+}\ \textsc{D1PVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant \downarrow M}\ \textsc{D1ShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vdash \sigma : \overrightarrow{\alpha^-} \quad \Gamma, \overrightarrow{\beta^-} \vdash [\sigma]P \geqslant Q}{\Gamma \vdash \exists\overrightarrow{\alpha^-}.P \geqslant \exists\overrightarrow{\beta^-}.Q}\ \textsc{D1Exists}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq^{\leqslant} \sigma_2 : \Gamma_1}$ Equivalence of substitutions

$\boxed{\Gamma \vdash \sigma_1 \simeq^{\leqslant} \sigma_2 : vars}$ Equivalence of substitutions

$\boxed{\Theta \vdash \widehat{\sigma}_1 \simeq^{\leqslant} \widehat{\sigma}_2 : vars}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash \widehat{\sigma}_1 \simeq^{\leqslant} \widehat{\sigma}_2 : vars}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash \Phi_1 \simeq^{\leqslant} \Phi_2}$ Equivalence of contexts

$\boxed{\Gamma; \Phi \vdash v : P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \vdash x : P}\ \textsc{DTVar}$$

$$\frac{\Gamma; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \{c\} \colon \downarrow N} \text{ DTThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant P}{\Gamma; \Phi \vdash (v : Q) \colon Q} \text{ DTPAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash v \colon P'} \text{ DTPEquiv}$$

$\boxed{\Gamma; \Phi \vdash c \colon N}$ Negative type inference

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \lambda x : P.c \colon P \to N} \text{ DTtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \Lambda \alpha^+.c \colon \forall \alpha^+.N} \text{ DTTLam}$$

$$\frac{\Gamma; \Phi \vdash v \colon P}{\Gamma; \Phi \vdash \mathbf{return}\, v \colon \uparrow P} \text{ DTReturn}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v; c \colon N} \text{ DTVarLet}$$

$$\frac{\Gamma; \Phi \vdash v \colon \downarrow M \quad \Gamma; \Phi \vdash M \bullet \vec{v} \Rightarrow\!\!\!> \uparrow Q \text{ unique} \quad \Gamma; \Phi, x : Q \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v(\vec{v}); c \colon N} \text{ DTAppLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma; \Phi \vdash v \colon \downarrow M \\ \Gamma; \Phi \vdash M \bullet \vec{v} \Rightarrow\!\!\!> \uparrow Q \quad \Gamma \vdash \uparrow Q \leqslant \uparrow P \quad \Gamma; \Phi, x : P \vdash c \colon N\end{array}}{\Gamma; \Phi \vdash \mathbf{let}\, x : P = v(\vec{v}); c \colon N} \text{ DTAppLetAnn}$$

$$\frac{\begin{array}{l}\texttt{<<multiple parses>>} \\ \Gamma, \overrightarrow{\alpha^-}; \Phi, x : P \vdash c \colon N \quad \Gamma \vdash N\end{array}}{\Gamma; \Phi \vdash \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c \colon N} \text{ DTUnpack}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash (c : M) \colon M} \text{ DTNAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash c \colon N'} \text{ DTNEquiv}$$

$\boxed{\Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!\!> M}$ Application type inference

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash N \bullet \cdot \Rightarrow\!\!\!> N'} \text{ DTEmptyApp}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant P \quad \Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!\!> M}{\Gamma; \Phi \vdash Q \to N \bullet v, \vec{v} \Rightarrow\!\!\!> M} \text{ DTArrowApp}$$

$$\frac{\begin{array}{l}\Gamma \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma; \Phi \vdash [\sigma]N \bullet \vec{v} \Rightarrow\!\!\!> M \\ \vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot\end{array}}{\Gamma; \Phi \vdash \forall \overrightarrow{\alpha^+}.N \bullet \vec{v} \Rightarrow\!\!\!> M} \text{ DTForallApp}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$

$\boxed{P_1 \vee P_2}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ N}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ N}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{N}')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{P}')}$

$\boxed{\mathbf{nf}\,(\sigma')}$

$\boxed{\mathbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\mathbf{nf}\,(\mu')}$

$\boxed{\sigma'|_{vars}}$

$\boxed{\widehat{\sigma}'|_{vars}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{SC'|_{vars}}$

$\boxed{UC'|_{vars}}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{UC_1 \ \& \ UC_2}$

$\boxed{UC_1 \cup UC_2}$

$\boxed{\Gamma_1 \cup \Gamma_2}$

$\boxed{SC_1 \ \& \ SC_2}$

$\boxed{\widehat{\tau}_1 \ \& \ \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(UC)}$

$\boxed{\mathbf{dom}\,(SC)}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{|SC|}$

$\boxed{\mathbf{fv}\ N}$

$\boxed{\mathbf{fv}\ P}$

$\boxed{\mathbf{fv}\,P}$

$\boxed{\mathbf{fv}\,N}$

$\boxed{\mathbf{uv}\ N}$

$\boxed{\mathbf{uv}\ P}$

$\boxed{\Gamma \vDash P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \vDash \alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBVar}$$

$$\frac{\Gamma, \cdot \vDash \mathbf{nf}\,(\downarrow N) \overset{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi]\,P} \quad \text{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \vDash P_1 \vee P_2 = Q}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P_1 \vee \exists \overrightarrow{\beta^-}.P_2 = Q} \quad \text{LUBExists}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}$

$$\frac{\texttt{<<multiple parses>>} \quad \Delta, \overrightarrow{\beta^\pm}, \overrightarrow{\gamma^\pm} \vDash [\overrightarrow{\beta^\pm}/\overrightarrow{\alpha^\pm}]P \vee [\overrightarrow{\gamma^\pm}/\overrightarrow{\alpha^\pm}]P = Q}{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q} \quad \text{LUBUpgrade}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^-) = \alpha^-} \quad \text{NrmNVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\uparrow P) = \uparrow Q} \quad \text{NrmShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(P \to N) = Q \to M} \quad \text{NrmArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\forall\overrightarrow{\alpha^{+}}.N) = \forall\overrightarrow{\alpha^{+}}'.N'}\ \textsc{NrmForall}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\alpha^{+}) = \alpha^{+}}\ \textsc{NrmPVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\downarrow N) = \downarrow M}\ \textsc{NrmShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\exists\overrightarrow{\alpha^{-}}.P) = \exists\overrightarrow{\alpha^{-}}'.P'}\ \textsc{NrmExists}$$

$\boxed{\mathbf{nf}\,(\,N\,) = \,M}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^{-}) = \widehat{\alpha}^{-}}\ \textsc{NrmNUVar}$$

$\boxed{\mathbf{nf}\,(\,P\,) = \,Q}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^{+}) = \widehat{\alpha}^{+}}\ \textsc{NrmPUVar}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}$

$$\frac{\alpha^{-} \in \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^{-} = \alpha^{-}}\ \textsc{ONVarIn}$$

$$\frac{\alpha^{-} \notin \mathit{vars}}{\texttt{<<multiple parses>>}}\ \textsc{ONVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\uparrow P = \vec{\alpha}}\ \textsc{OShiftU}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}_1 \quad \mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}_2}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)}\ \textsc{OArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\forall\overrightarrow{\alpha^{+}}.N = \vec{\alpha}}\ \textsc{OForall}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}$

$$\frac{\alpha^{+} \in \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^{+} = \alpha^{+}}\ \textsc{OPVarIn}$$

$$\frac{\alpha^{+} \notin \mathit{vars}}{\texttt{<<multiple parses>>}}\ \textsc{OPVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\downarrow N = \vec{\alpha}}\ \textsc{OShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\exists\overrightarrow{\alpha^{-}}.P = \vec{\alpha}}\ \textsc{OExists}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\,N\, = \vec{\alpha}}$

$$\frac{}{\texttt{<<multiple parses>>}}\ \textsc{ONUVar}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\,P\, = \vec{\alpha}}$

$$\frac{}{\text{<<multiple parses>>}}\ \text{OPUVar}$$

$\boxed{\Gamma;\Theta \models N \overset{u}{\simeq} M \dashv UC}$ Negative unification

$$\frac{}{\Gamma;\Theta \models \alpha^- \overset{u}{\simeq} \alpha^- \dashv \cdot}\ \text{UNVar}$$

$$\frac{\Gamma;\Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma;\Theta \models \uparrow P \overset{u}{\simeq} \uparrow Q \dashv UC}\ \text{UShiftU}$$

$$\frac{\Gamma;\Theta \models P \overset{u}{\simeq} Q \dashv UC_1 \quad \Gamma;\Theta \models N \overset{u}{\simeq} M \dashv UC_2}{\Gamma;\Theta \models P \to N \overset{u}{\simeq} Q \to M \dashv UC_1 \ \&\ UC_2}\ \text{UArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+};\Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma;\Theta \models \forall\overrightarrow{\alpha^+}.N \overset{u}{\simeq} \forall\overrightarrow{\alpha^+}.M \dashv UC}\ \text{UForall}$$

$$\frac{\Theta(\widehat{\alpha}^-) \vdash N}{\Gamma;\Theta \models \widehat{\alpha}^- \overset{u}{\simeq} N \dashv (\widehat{\alpha}^- :\simeq N)}\ \text{UNUVar}$$

$\boxed{\Gamma;\Theta \models P \overset{u}{\simeq} Q \dashv UC}$ Positive unification

$$\frac{}{\Gamma;\Theta \models \alpha^+ \overset{u}{\simeq} \alpha^+ \dashv \cdot}\ \text{UPVar}$$

$$\frac{\Gamma;\Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma;\Theta \models \downarrow N \overset{u}{\simeq} \downarrow M \dashv UC}\ \text{UShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-};\Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma;\Theta \models \exists\overrightarrow{\alpha^-}.P \overset{u}{\simeq} \exists\overrightarrow{\alpha^-}.Q \dashv UC}\ \text{UExists}$$

$$\frac{\Theta(\widehat{\alpha}^+) \vdash P}{\Gamma;\Theta \models \widehat{\alpha}^+ \overset{u}{\simeq} P \dashv (\widehat{\alpha}^+ :\simeq P)}\ \text{UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma \vdash \alpha^-}\ \text{WFTNVar}$$

$$\frac{\Gamma \vdash P}{\Gamma \vdash \uparrow P}\ \text{WFTShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash N}{\Gamma \vdash P \to N}\ \text{WFTArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+} \vdash N}{\Gamma \vdash \forall\overrightarrow{\alpha^+}.N}\ \text{WFTForall}$$

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma \vdash \alpha^+}\ \text{WFTPVar}$$

$$\frac{\Gamma \vdash N}{\Gamma \vdash \downarrow N}\ \text{WFTShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-} \vdash P}{\Gamma \vdash \exists \overrightarrow{\alpha^-}. P} \quad \text{WFTExists}$$

$\boxed{\Gamma; \Xi \vdash N}$ Negative algorithmic type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma; \Xi \vdash \alpha^-} \quad \text{WFATNVar}$$

$$\frac{\widehat{\alpha}^- \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^-} \quad \text{WFATNUVar}$$

$$\frac{\Gamma; \Xi \vdash P}{\Gamma; \Xi \vdash \uparrow P} \quad \text{WFATShiftU}$$

$$\frac{\Gamma; \Xi \vdash P \quad \Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash P \to N} \quad \text{WFATArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Xi \vdash N}{\Gamma; \Xi \vdash \forall \overrightarrow{\alpha^+}. N} \quad \text{WFATForall}$$

$\boxed{\Gamma; \Xi \vdash P}$ Positive algorithmic type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma; \Xi \vdash \alpha^+} \quad \text{WFATPVar}$$

$$\frac{\widehat{\alpha}^+ \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^+} \quad \text{WFATPUVar}$$

$$\frac{\Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash \downarrow N} \quad \text{WFATShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Xi \vdash P}{\Gamma; \Xi \vdash \exists \overrightarrow{\alpha^-}. P} \quad \text{WFATExists}$$

$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness

$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness

$\boxed{\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness

$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness

$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution signature

$\boxed{\Theta \vdash \widehat{\sigma} : \Xi}$ Unification substitution signature

$\boxed{\Gamma \vdash \widehat{\sigma} : \Xi}$ Unification substitution general signature

$\boxed{\Theta \vdash \widehat{\sigma} : UC}$ Unification substitution satisfies unification constraint

$\boxed{\Theta \vdash \widehat{\sigma} : SC}$ Unification substitution satisfies subtyping constraint

$\boxed{\Gamma \vdash e}$ Unification constraint entry well-formedness

$\boxed{\Gamma \vdash e}$ Subtyping constraint entry well-formedness

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies unification constraint

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies unification constraint

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies subtyping constraint

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies subtyping constraint

$\boxed{\Theta \vdash UC : \Xi}$ Unification constraint well-formedness with specified domain

$\boxed{\Theta \vdash SC : \Xi}$ Subtyping constraint well-formedness with specified domain

$\boxed{\Theta \vdash UC}$ Unification constraint well-formedness

$\boxed{\Theta \vdash SC}$ Subtyping constraint well-formedness

$\boxed{\Gamma \vdash \overrightarrow{v}}$ Argument List well-formedness

$\boxed{\Gamma \vdash \Phi}$ Context well-formedness

$\boxed{\Gamma \vdash v}$ Value well-formedness

$$\frac{}{\Gamma \vdash x} \quad \text{WFALLV{\scriptsize AR}}$$

$\boxed{\Gamma \vdash c}$ Computation well-formedness

$$\frac{\Gamma \vdash v \quad \Gamma \vdash c \quad \Gamma \vdash \vec{v}}{\Gamma \vdash \mathbf{let}\, x = v(\vec{v});\, c} \quad \text{WFALLA{\scriptsize PP}L{\scriptsize ET}}$$

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Definition rules:         94 good    33 bad
Definition rule clauses: 208 good    34 bad
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