α, β, α, β, γ, δ	type variables
n, m, i, j	index variables
x, y, z	term variables

n, m	$::=$			cohort index
	$\mid$	0		
	$\mid$	$n+1$		
$\alpha^+, \beta^+, \gamma^+, \delta^+$	$::=$			positive variable
	$\mid$	α^+		
	$\mid$	α^{+n}		
	$\mid$	$\beta^{\pm}$		
$\alpha^-, \beta^-, \gamma^-, \delta^-$	$::=$			negative variable
	$\mid$	α^-		
	$\mid$	α^{-n}		
	$\mid$	$\alpha^{\pm}$		
$\alpha^{\pm}, \beta^{\pm}$	$::=$			positive or negative variable
	$\mid$	$\alpha^{\pm}$		
	$\mid$	$\alpha^{\pm n}$		
σ	$::=$			substitution
	$\mid$	$\cdot$		
	$\mid$	id		
	$\mid$	P/α^+		
	$\mid$	N/α^-		
	$\mid$	$\overrightarrow{P}/\overrightarrow{\alpha^+}$		
	$\mid$	$\overrightarrow{N}/\overrightarrow{\alpha^-}$		
	$\mid$	$\mathbf{pmas}\,/\overrightarrow{\alpha^+}$		
	$\mid$	$\mathbf{nmas}\,/\overrightarrow{\alpha^-}$		
	$\mid$	$\overrightarrow{\widetilde{\alpha^+}}/\overrightarrow{\alpha^+}$		
	$\mid$	$\overrightarrow{\widehat{\alpha^-}}/\overrightarrow{\alpha^-}$		
	$\mid$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\mid$	μ		
	$\mid$	$\widehat{\sigma}$		
	$\mid$	$\sigma_1 \circ \sigma_2$		
	$\mid$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\mid$	(σ)	S	
	$\mid$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\mid$	$\mathbf{nf}\,(\sigma')$	M	
	$\mid$	$\sigma'\vert_{vars}$	M	
e	$::=$			entry of a unification solution
	$\mid$	$\widehat{\alpha}^+ :\approx P$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		
	$\mid$	$\widehat{\alpha}^+ :\geqslant P$		
	$\mid$	(e)	S	
	$\mid$	$\widehat{\sigma}(\widehat{\alpha}^+)$	M	
	$\mid$	$\widehat{\sigma}(\widehat{\alpha}^-)$	M	
	$\mid$	$e_1 \,\&\, e_2$	M	
$\widehat{\sigma}$	$::=$			unification solution (substitution)

	$\vert$	$\cdot$		
	$\vert$	e		
	$\vert$	$\widehat{\sigma}\backslash vars$		
	$\vert$	$\widehat{\sigma}\vert vars$		
	$\vert$	$\widehat{\sigma}_1 \cup \widehat{\sigma}_2$		
	$\vert$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\vert$	$(\widehat{\sigma})$	S	
	$\vert$	$\mathbf{nf}\,(\widehat{\sigma}')$	M	
	$\vert$	$\widehat{\sigma}'\vert_{vars}$	M	
	$\vert$	$\widehat{\sigma}_1\ \&\ \widehat{\sigma}_2$	M	
$\widehat{\tau},\ \widehat{\rho}$	$::=$			anti-unification substitution
	$\vert$	$\cdot$		
	$\vert$	$\widehat{\alpha}^- :\approx N$		
	$\vert$	$\widehat{\alpha}^- :\approx N$		
	$\vert$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha}^-}$		
	$\vert$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha}^-}$		
	$\vert$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
	$\vert$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	$\vert$	$(\widehat{\tau})$	S	
	$\vert$	$\widehat{\tau}'\vert_{vars}$	M	
	$\vert$	$\widehat{\tau}_1\ \&\ \widehat{\tau}_2$	M	
$P,\ Q,\ R$	$::=$			positive types
	$\vert$	α^+		
	$\vert$	$\downarrow N$		
	$\vert$	$\exists\alpha^-.P$		
	$\vert$	$[\sigma]P$	M	
$N,\ M,\ K$	$::=$			negative types
	$\vert$	α^-		
	$\vert$	$\uparrow P$		
	$\vert$	$\forall\alpha^+.N$		
	$\vert$	$P \to N$		
	$\vert$	$[\sigma]N$	M	
$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+},\ \overrightarrow{\gamma^+},\ \overrightarrow{\delta^+}$	$::=$			positive variable list
	$\vert$	$\cdot$		empty list
	$\vert$	α^+		a variable
	$\vert$	$\overrightarrow{\alpha^+}$		a variable
	$\vert$	$\overline{\overrightarrow{\alpha^+}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$	$::=$			negative variables
	$\vert$	$\cdot$		empty list
	$\vert$	α^-		a variable
	$\vert$	$\overrightarrow{\alpha^-}$		variables
	$\vert$	$\overline{\overrightarrow{\alpha^-}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$	$::=$			positive or negative variable list

	$\|$	$\cdot$		empty list
	$\|$	$\alpha^{\pm}$		a variable
	$\|$	$\overrightarrow{\mathbf{pa}}$		variables
	$\|$	$\overline{\overrightarrow{\alpha^{\pm}}_i}^{\,i}$		concatenate lists
$P,\ Q$	$::=$			multi-quantified positive types
	$\|$	α^{+}		
	$\|$	$\downarrow N$		
	$\|$	$\exists\overrightarrow{\alpha^{-}}.P$		$P \neq \exists\ldots$
	$\|$	$[\sigma]P$	M	
	$\|$	$[\widehat{\tau}]P$	M	
	$\|$	$[\widehat{\sigma}]P$	M	
	$\|$	$[\mu]P$	M	
	$\|$	(P)	S	
	$\|$	$P_1 \vee P_2$	M	
	$\|$	$\mathbf{nf}\,(P')$	M	
$N,\ M$	$::=$			multi-quantified negative types
	$\|$	α^{-}		
	$\|$	$\uparrow P$		
	$\|$	$P \to N$		
	$\|$	$\forall\overrightarrow{\alpha^{+}}.N$		$N \neq \forall\ldots$
	$\|$	$[\sigma]N$	M	
	$\|$	$[\widehat{\tau}]N$	M	
	$\|$	$[\mu]N$	M	
	$\|$	$[\widehat{\sigma}]N$	M	
	$\|$	(N)	S	
	$\|$	$\mathbf{nf}\,(N')$	M	
$\vec{P},\ \vec{Q}$	$::=$			list of positive types
	$\|$	$\cdot$		empty list
	$\|$	P		a singel type
	$\|$	$\overline{\vec{P}_i}^{\,i}$		concatenate lists
	$\|$	$\mathbf{nf}\,(\vec{P}')$	M	
$\vec{N},\ \vec{M}$	$::=$			list of negative types
	$\|$	$\cdot$		empty list
	$\|$	N		a singel type
	$\|$	$\overline{\vec{N}_i}^{\,i}$		concatenate lists
	$\|$	$\mathbf{nf}\,(\vec{N}')$	M	
$\Delta,\ \Gamma$	$::=$			declarative type context
	$\|$	$\cdot$		empty context
	$\|$	$\overrightarrow{\alpha^{+}}$		list of variables
	$\|$	$\overrightarrow{\alpha^{-}}$		list of variables
	$\|$	$\overrightarrow{\alpha^{\pm}}$		list of variables
	$\|$	vars		
	$\|$	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	$\|$	(Γ)	S	
	$\|$	$\Theta(\widehat{\alpha}^{+})$	M	

	$\mid$	$\Theta(\widehat{\alpha}^-)$	M	
Θ	$::=$			unification type variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\vec{\alpha}[\Delta]$		from an ordered list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^+\{\Delta\}}$		from a variable to a list
	$\mid$	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	$\mid$	(Θ)	S	
	$\mid$	$\Theta\vert_{vars}$		leave only those variables that are in the set
	$\mid$	$\Theta_1 \cup \Theta_2$		
Ξ	$::=$			anti-unification type variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\widehat{\alpha}^-}$		list of variables
	$\mid$	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	$\mid$	(Ξ)	S	
	$\mid$	$\Xi_1 \cup \Xi_2$		
	$\mid$	$\Xi_1 \cap \Xi_2$		
	$\mid$	$\Xi'\vert_{vars}$	M	
$\vec{\alpha},\ \vec{\beta}$	$::=$			ordered positive or negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	$\overrightarrow{\alpha^+}$		list of variables
	$\mid$	$\overrightarrow{\alpha^-}$		list of variables
	$\mid$	$\overrightarrow{\alpha^\pm}$		list of variables
	$\mid$	$\overrightarrow{\widetilde{\alpha}^+}$		list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^-}$		list of variables
	$\mid$	$\vec{\alpha}_1 \backslash vars$		setminus
	$\mid$	Γ		context
	$\mid$	$vars$		
	$\mid$	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	$\mid$	$(\vec{\alpha})$	S	parenthesis
	$\mid$	$[\mu]\vec{\alpha}$	M	apply moving to list
	$\mid$	$\mathbf{ord}\, vars\, \mathbf{in}\, P$	M	
	$\mid$	$\mathbf{ord}\, vars\, \mathbf{in}\, N$	M	
	$\mid$	$\mathbf{ord}\, vars\, \mathbf{in}\, P$	M	
	$\mid$	$\mathbf{ord}\, vars\, \mathbf{in}\, N$	M	
$vars$	$::=$			set of variables
	$\mid$	$\varnothing$		empty set
	$\mid$	$\mathbf{fv}\, P$		free variables
	$\mid$	$\mathbf{fv}\, N$		free variables
	$\mid$	$\mathbf{fv}\, \mathbf{imP}$		free variables
	$\mid$	$\mathbf{fv}\, \mathbf{imN}$		free variables
	$\mid$	$vars_1 \cap vars_2$		set intersection
	$\mid$	$vars_1 \cup vars_2$		set union
	$\mid$	$vars_1 \backslash vars_2$		set complement
	$\mid$	$\mathbf{mv}\, \mathbf{imP}$		movable variables
	$\mid$	$\mathbf{mv}\, \mathbf{imN}$		movable variables
	$\mid$	$\mathbf{uv}\, N$		unification variables

	|	$\mathbf{uv}\ P$		unification variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathbf{fv}\ P$		free variables
	|	$(vars)$	S	parenthesis
	|	$\overrightarrow{\alpha}$		ordered list of variables
	|	$[\mu]vars$	M	apply moving to varset
	|	$\mathbf{dom}\,(\widehat{\sigma})$	M	
	|	$\mathbf{dom}\,(\widehat{\tau})$	M	
	|	$\mathbf{dom}\,(\Theta)$	M	
μ	::=			
	|	$\cdot$		empty moving
	|	$pma1 \mapsto pma2$		Positive unit substitution
	|	$nma1 \mapsto nma2$		Positive unit substitution
	|	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	|	$\mu_1 \circ \mu_2$	M	Composition
	|	$\overline{\mu_i}^{\,i}$		concatenate movings
	|	$\mu\|_{vars}$	M	restriction on a set
	|	μ^{-1}	M	inversion
	|	$\mathbf{nf}\,(\mu')$	M	
$\widehat{\alpha}^{\pm}$	::=			positive/negative unification variable
	|	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	::=			positive unification variable
	|	$\widehat{\alpha}^{+}$		
	|	$\widehat{\alpha}^{+}\{\Delta\}$		
	|	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	::=			negative unification variable
	|	$\widehat{\alpha}^{-}$		
	|	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	|	$\widehat{\alpha}^{-}\{\Delta\}$		
	|	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widetilde{\alpha}^{+}},\ \overrightarrow{\widetilde{\beta}^{+}}$	::=			positive unification variable list
	|	$\cdot$		empty list
	|	$\widehat{\alpha}^{+}$		a variable
	|	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	|	$\overline{\overrightarrow{\widetilde{\alpha}^{+}}_i}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}},\ \overrightarrow{\widehat{\beta}^{-}}$	::=			negative unification variable list
	|	$\cdot$		empty list
	|	$\widehat{\alpha}^{-}$		a variable
	|	Ξ		from an antiunification context
	|	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	|	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	|	$\overline{\overrightarrow{\widehat{\alpha}^{-}}_i}^{\,i}$		concatenate lists

P, Q	$::=$			a positive algorithmic type (potentially with metavariables)
	$\vert$	α^{+}		
	$\vert$	$\mathbf{pma}$		
	$\vert$	$\widehat{\alpha}^{+}$		
	$\vert$	$\downarrow N$		
	$\vert$	$\exists\overrightarrow{\alpha^{-}}.P$		
	$\vert$	$[\sigma]P$	M	
	$\vert$	$[\widehat{\tau}]P$	M	
	$\vert$	$[\mu]P$	M	
	$\vert$	(P)	S	
	$\vert$	$\mathbf{nf}\,(P')$	M	
N, M	$::=$			a negative algorithmic type (potentially with metavariables)
	$\vert$	α^{-}		
	$\vert$	$\widehat{\alpha}^{-}$		
	$\vert$	$\uparrow P$		
	$\vert$	$P \to N$		
	$\vert$	$\forall\overrightarrow{\alpha^{+}}.N$		
	$\vert$	$[\sigma]N$	M	
	$\vert$	$[\widehat{\tau}]N$	M	
	$\vert$	$[\mu]N$	M	
	$\vert$	(N)	S	
	$\vert$	$\mathbf{nf}\,(N')$	M	
auSol	$::=$			
	$\vert$	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$		
	$\vert$	$(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$		
terminals	$::=$			
	$\vert$	$\exists$		
	$\vert$	$\forall$		
	$\vert$	$\uparrow$		
	$\vert$	$\downarrow$		
	$\vert$	$\to$		
	$\vert$	$\leftrightarrow$		
	$\vert$	$\in$		
	$\vert$	$\notin$		
	$\vert$	$\cdot$		
	$\vert$	$\vdash$		
	$\vert$	$\leqslant$		
	$\vert$	$\geqslant$		
	$\vert$	$\simeq$		
	$\vert$	$\cup$		
	$\vert$	$\cap$		
	$\vert$	$\setminus$		
	$\vert$	$\subseteq$		
	$\vert$	$\mapsto$		
	$\vert$	$\overset{u}{\simeq}$		
	$\vert$	$\overset{a}{\simeq}$		
	$\vert$	$\varnothing$		

	$\mid$	$\circ$	
	$\mid$	$\Rightarrow$	
	$\mid$	$\vDash$	
	$\mid$	$\Dashv$	
	$\mid$	$\neq$	
	$\mid$	$\equiv_n$	
	$\mid$	$\vee$	
	$\mid$	$\Downarrow$	
	$\mid$	$:\geqslant$	
	$\mid$	$:\simeq$	
	$\mid$	Λ	
	$\mid$	λ	
	$\mid$	$\mathbf{let}^{\exists}$	
	$\mid$	$\bullet$	
	$\mid$	$\Rightarrow\!\!>$	
$v,\ w$	$::=$		value terms
	$\mid$	x	
	$\mid$	$\{c\}$	
	$\mid$	$(v : P)$	
	$\mid$	(v)	M
$\vec{v}$	$::=$		list of arguments
	$\mid$	$\cdot$	
	$\mid$	v	
	$\mid$	$\overline{\vec{v}_i}^{\,i}$	concatenate
$c,\ d$	$::=$		computation terms
	$\mid$	$(c : N)$	
	$\mid$	$\lambda x : P.c$	
	$\mid$	$\Lambda\alpha^{+}.c$	
	$\mid$	$\mathbf{return}\, v$	
	$\mid$	$\mathbf{let}\, x : P = v(\vec{v}); c$	
	$\mid$	$\mathbf{let}\, x = v(\vec{v}); c$	
	$\mid$	$\mathbf{let}^{\exists}(\alpha^{-}, x) = v; c$	
$vctx,\ \Phi$	$::=$		variable context
	$\mid$	$\cdot$	
	$\mid$	$x : P$	
	$\mid$	$\overline{\Phi_i}^{\,i}$	concatenate contexts
formula	$::=$		
	$\mid$	*judgement*	
	$\mid$	judgement *uniquely*	
	$\mid$	$formula_1 \quad .. \quad formula_n$	
	$\mid$	$\mu : vars_1 \leftrightarrow vars_2$	
	$\mid$	$\mu\, \mathbf{is\, bijective}$	
	$\mid$	$\widehat{\sigma}\, \mathbf{is\, functional}$	
	$\mid$	$\widehat{\sigma}_1 \in \widehat{\sigma}_2$	
	$\mid$	$v : P \in \Phi$	

	|	$\widehat{\sigma}_1 \subseteq \widehat{\sigma}_2$	
	|	$vars_1 \subseteq vars_2$	
	|	$vars_1 = vars_2$	
	|	$vars$ **is fresh**	
	|	$\alpha^- \notin vars$	
	|	$\alpha^+ \notin vars$	
	|	$\alpha^- \in vars$	
	|	$\alpha^+ \in vars$	
	|	$\widehat{\alpha}^+ \in vars$	
	|	$\widehat{\alpha}^- \in vars$	
	|	$\widehat{\alpha}^- \in \Theta$	
	|	$\widehat{\alpha}^+ \in \Theta$	
	|	if any other rule is not applicable	
	|	$\vec{\alpha}_1 = \vec{\alpha}_2$	
	|	$e_1 = e_2$	
	|	$N = M$	
	|	$N \neq M$	
	|	$P \neq Q$	
	|	$N \neq M$	
	|	$P \neq Q$	
	|	$P \neq Q$	
	|	$N \neq M$	
A	::=		
	|	$\Gamma;\ \Theta \models N \leqslant M \dashv \widehat{\sigma}$	Negative subtyping
	|	$\Gamma;\ \Theta \models P \geqslant Q \dashv \widehat{\sigma}$	Positive supertyping
AT	::=		
	|	$\Gamma; \Phi \models v \colon P$	Positive type inference
	|	$\Gamma; \Phi \models c \colon N$	Negative type inference
	|	$\Gamma; \Phi; \Theta \models N \bullet \vec{v} \Longrightarrow\!\!> M \dashv \widehat{\sigma}$	Application type inference
AU	::=		
	|	$\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	|	$\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$	
$E1$	::=		
	|	$N \simeq_1^D M$	Negative multi-quantified type equivalence
	|	$P \simeq_1^D Q$	Positive multi-quantified type equivalence
	|	$P \simeq Q$	
$D1$	::=		
	|	$\Gamma \vdash N \simeq_1^{\leqslant} M$	Negative equivalence on MQ types
	|	$\Gamma \vdash P \simeq_1^{\leqslant} Q$	Positive equivalence on MQ types
	|	$\Gamma \vdash N \leqslant_1 M$	Negative subtyping
	|	$\Gamma \vdash P \geqslant_1 Q$	Positive supertyping
	|	$\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1$	Equivalence of substitutions
$D0$	::=		

	$\mid$	$\Gamma \vdash N \simeq_0^{\leqslant} M$	Negative equivalence
	$\mid$	$\Gamma \vdash P \simeq_0^{\leqslant} Q$	Positive equivalence
	$\mid$	$\Gamma \vdash N \leqslant_0 M$	Negative subtyping
	$\mid$	$\Gamma \vdash P \geqslant_0 Q$	Positive supertyping
DT	$::=$		
	$\mid$	$\Gamma; \Phi \vdash v \colon P$	Positive type inference
	$\mid$	$\Gamma; \Phi \vdash c \colon N$	Negative type inference
	$\mid$	$\Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M$	Spin Application type inference
EQ	$::=$		
	$\mid$	$N = M$	Negative type equality (alpha-equivalence)
	$\mid$	$P = Q$	Positive type equuality (alphha-equivalence)
	$\mid$	$P = Q$	
$LUBF$	$::=$		
	$\mid$	$P_1 \vee P_2 === Q$	
	$\mid$	$\textbf{ord}\, vars\, \textbf{in}\, P === \vec{\alpha}$	
	$\mid$	$\textbf{ord}\, vars\, \textbf{in}\, N === \vec{\alpha}$	
	$\mid$	$\textbf{ord}\, vars\, \textbf{in}\, P === \vec{\alpha}$	
	$\mid$	$\textbf{ord}\, vars\, \textbf{in}\, N === \vec{\alpha}$	
	$\mid$	$\textbf{nf}\,(N') === N$	
	$\mid$	$\textbf{nf}\,(P') === P$	
	$\mid$	$\textbf{nf}\,(N') === N$	
	$\mid$	$\textbf{nf}\,(P') === P$	
	$\mid$	$\textbf{nf}\,(\vec{N}') === \vec{N}$	
	$\mid$	$\textbf{nf}\,(\vec{P}') === \vec{P}$	
	$\mid$	$\textbf{nf}\,(\sigma') === \sigma$	
	$\mid$	$\textbf{nf}\,(\mu') === \mu$	
	$\mid$	$\textbf{nf}\,(\widehat{\sigma}') === \widehat{\sigma}$	
	$\mid$	$\sigma'\vert_{vars}$	
	$\mid$	$\widehat{\sigma}'\vert_{vars}$	
	$\mid$	$\widehat{\tau}'\vert_{vars}$	
	$\mid$	$\Xi'\vert_{vars}$	
	$\mid$	$e_1 \mathbin{\&} e_2$	
	$\mid$	$\widehat{\sigma}_1 \mathbin{\&} \widehat{\sigma}_2$	
	$\mid$	$\widehat{\tau}_1 \mathbin{\&} \widehat{\tau}_2$	
	$\mid$	$\textbf{dom}\,(\widehat{\sigma}) === vars$	
	$\mid$	$\textbf{dom}\,(\widehat{\tau}) === vars$	
	$\mid$	$\textbf{dom}\,(\Theta) === vars$	
LUB	$::=$		
	$\mid$	$\Gamma \vDash P_1 \vee P_2 = Q$	Least Upper Bound (Least Common Supertype)
	$\mid$	$\textbf{upgrade}\, \Gamma \vdash P\, \textbf{to}\, \Delta = Q$	
Nrm	$::=$		
	$\mid$	$\textbf{nf}\,(N) = M$	
	$\mid$	$\textbf{nf}\,(P) = Q$	
	$\mid$	$\textbf{nf}\,(N) = M$	
	$\mid$	$\textbf{nf}\,(P) = Q$	

Order	::=		
	|	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}$	
	|	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}$	
	|	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}$	
	|	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}$	
SM	::=		
	|	$\Gamma \vdash e_1 \ \&\ e_2 = e_3$	Unification Solution Entry Merge
	|	$\Theta \vdash \widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3$	Merge unification solutions
SImp	::=		
	|	$\Gamma \vdash e_1 \Rightarrow e_2$	Weakening of unification solution entries
	|	$\Theta \vdash \widehat{\sigma}_1 \Rightarrow \widehat{\sigma}_2$	Weakening of unification solutions
	|	$\Gamma \vdash e_1 \simeq e_2$	
	|	$\Theta \vdash \widehat{\sigma}_1 \simeq \widehat{\sigma}_2$	
U	::=		
	|	$\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv \widehat{\sigma}$	Negative unification
	|	$\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv \widehat{\sigma}$	Positive unification
WF	::=		
	|	$\Gamma \vdash N$	Negative type well-formedness
	|	$\Gamma \vdash P$	Positive type well-formedness
	|	$\Gamma \vdash N$	Negative type well-formedness
	|	$\Gamma \vdash P$	Positive type well-formedness
	|	$\Gamma \vdash \vec{N}$	Negative type list well-formedness
	|	$\Gamma \vdash \vec{P}$	Positive type list well-formedness
	|	$\Gamma; \Theta \vdash N$	Negative unification type well-formedness
	|	$\Gamma; \Theta \vdash P$	Positive unifiication type well-formedness
	|	$\Gamma; \Xi \vdash N$	Negative anti-unification type well-formedness
	|	$\Gamma; \Xi \vdash P$	Positive anti-unification type well-formedness
	|	$\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1$	Antiunification substitution well-formedness
	|	$\widehat{\sigma} : \Theta$	Unification substitution well-formedness
	|	$\Gamma \vdash^{\supseteq} \Theta$	Unification context well-formedness
	|	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution well-formedness
	|	$\Gamma \vdash e$	Unification solution entry well-formedness
judgement	::=		
	|	*A*	
	|	*AT*	
	|	*AU*	
	|	*E1*	
	|	*D1*	
	|	*D0*	
	|	*DT*	
	|	*EQ*	
	|	*LUB*	
	|	*Nrm*	
	|	*Order*	
	|	*SM*	

	$SImp$
	U
	WF

$user\_syntax$ ::=

	α
	n
	x
	n
	α^{+}
	α^{-}
	$\alpha^{\pm}$
	σ
	e
	$\widehat{\sigma}$
	$\widehat{\tau}$
	P
	N
	$\overrightarrow{\alpha^{+}}$
	$\overrightarrow{\alpha^{-}}$
	$\overrightarrow{\alpha^{\pm}}$
	P
	N
	$\overrightarrow{P}$
	$\overrightarrow{N}$
	Γ
	Θ
	Ξ
	$\overrightarrow{\alpha}$
	$vars$
	μ
	$\widehat{\alpha}^{\pm}$
	$\widehat{\alpha}^{+}$
	$\widehat{\alpha}^{-}$
	$\overrightarrow{\widetilde{\alpha}^{+}}$
	$\overrightarrow{\widehat{\alpha}^{-}}$
	P
	N
	$auSol$
	$terminals$
	v
	$\overrightarrow{v}$
	c
	$vctx$
	$formula$

$\boxed{\Gamma;\,\Theta \models N \leqslant M \dashv \widehat{\sigma}}$ Negative subtyping

$$\frac{}{\Gamma;\,\Theta \models \alpha^{-} \leqslant \alpha^{-} \dashv \cdot}\quad \textsc{ANVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(P) \stackrel{u}{\simeq} \mathbf{nf}\,(Q) \dashv \widehat{\sigma}}{\Gamma;\, \Theta \models {\uparrow}P \leqslant {\uparrow}Q \dashv \widehat{\sigma}} \quad \textsc{AShiftU}$$

$$\frac{\Gamma;\, \Theta \models P \geqslant Q \dashv \widehat{\sigma}_1 \quad \Gamma;\, \Theta \models N \leqslant M \dashv \widehat{\sigma}_2}{\Gamma;\, \Theta \models P \to N \leqslant Q \to M \dashv \widehat{\sigma}_1 \;\&\; \widehat{\sigma}_2} \quad \textsc{AArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma;\, \Theta \models \forall \overrightarrow{\alpha^+}.N \leqslant \forall \overrightarrow{\beta^+}.M \dashv \widehat{\sigma} \backslash \overrightarrow{\widehat{\alpha}^+}} \quad \textsc{AForall}$$

$\boxed{\Gamma;\, \Theta \models P \geqslant Q \dashv \widehat{\sigma}}$ Positive supertyping

$$\frac{}{\Gamma;\, \Theta \models \alpha^+ \geqslant \alpha^+ \dashv \cdot} \quad \textsc{APVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(N) \stackrel{u}{\simeq} \mathbf{nf}\,(M) \dashv \widehat{\sigma}}{\Gamma;\, \Theta \models {\downarrow}N \geqslant {\downarrow}M \dashv \widehat{\sigma}} \quad \textsc{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-};\, \Theta, \overrightarrow{\widehat{\alpha}^-}[\Gamma, \overrightarrow{\beta^-}] \models [\overrightarrow{\widehat{\alpha}^-}/\overrightarrow{\alpha^-}]\,P \geqslant Q \dashv \widehat{\sigma}}{\Gamma;\, \Theta \models \exists \overrightarrow{\alpha^-}.P \geqslant \exists \overrightarrow{\beta^-}.Q \dashv \widehat{\sigma} \backslash \overrightarrow{\widehat{\alpha}^-}} \quad \textsc{AExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}{\Gamma;\, \Theta \models \widehat{\alpha}^+ \geqslant P \dashv (\widehat{\alpha}^+ :\geqslant Q)} \quad \textsc{APUVar}$$

$\boxed{\Gamma; \Phi \models v \colon P}$ Positive type inference

$$\frac{v : P \in \Phi}{\Gamma; \Phi \models v \colon P} \quad \textsc{ATVar}$$

$$\frac{\Gamma; \Phi \models c \colon N}{\Gamma; \Phi \models \{c\} \colon {\downarrow}N} \quad \textsc{ATThunk}$$

$$\frac{\Gamma; \Phi \models v \colon P \quad \Gamma; \cdot \models Q \geqslant P \dashv \cdot}{\Gamma; \Phi \models (v : Q) \colon Q} \quad \textsc{ATAnnot}$$

$\boxed{\Gamma; \Phi \models c \colon N}$ Negative type inference

$$\frac{\Gamma; \Phi \models c \colon N \quad \Gamma; \cdot \models N \leqslant M \dashv \cdot}{\Gamma; \Phi \models (c : M) \colon M} \quad \textsc{ATAnnotN}$$

$$\frac{\Gamma; \Phi, x : P \models c \colon N}{\Gamma; \Phi \models \lambda x : P.c \colon P \to N} \quad \textsc{ATtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \models c \colon N}{\Gamma; \Phi \models \Lambda \alpha^+.c \colon \forall \alpha^+.N} \quad \textsc{ATTLam}$$

$$\frac{\Gamma; \Phi \models v \colon P}{\Gamma; \Phi \models \mathbf{return}\, v \colon {\uparrow}P} \quad \textsc{ATReturn}$$

$$\frac{\begin{array}{c}\Gamma; \Phi \models v \colon {\downarrow}M \quad \Gamma; \Phi; \cdot \models M \bullet \overrightarrow{v} \Rightarrow\!\!\!> {\uparrow}Q \dashv \widehat{\sigma}_1 \quad \Gamma;\, \mathbf{uv}\; Q[\Gamma] \models {\uparrow}Q \leqslant {\uparrow}P \dashv \widehat{\sigma}_2 \\ \mathbf{uv}\; Q[\Gamma] \vdash \widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma} \quad \Gamma; \Phi, x : P \models c \colon N\end{array}}{\Gamma; \Phi \models \mathbf{let}\, x : P = v(\overrightarrow{v});\, c \colon N} \quad \textsc{ATLetAnn}$$

$$\frac{\Gamma; \Phi \models v \colon {\downarrow}M \quad \Gamma; \Phi; \cdot \models M \bullet \overrightarrow{v} \Rightarrow\!\!\!> {\uparrow}Q \dashv \widehat{\sigma} \quad \mathbf{uv}\,(Q) = \varnothing \quad \Gamma; \Phi, x : Q \models c \colon N}{\Gamma; \Phi \models \mathbf{let}\, x = v(\overrightarrow{v});\, c \colon N} \quad \textsc{ATLet}$$

$$\frac{\Gamma, \alpha^-; \Phi \models v \colon \exists \alpha^-.P \quad \Gamma, \alpha^-; \Phi, x : P \models c \colon N \quad \Gamma \vdash N}{\Gamma; \Phi \models \mathbf{let}^{\exists}(\alpha^-, x) = v;\, c \colon N} \quad \textsc{ATUnpack}$$

$\boxed{\Gamma; \Phi; \Theta \models N \bullet \overrightarrow{v} \Rightarrow\!\!\!> M \dashv \widehat{\sigma}}$ Application type inference

$$\frac{N \neq \forall\overrightarrow{\alpha^+}.M}{\Gamma;\Phi;\Theta \vDash N \bullet \cdot \Rrightarrow N \dashv \cdot}\ \textsc{ATEmtpty}$$

$$\frac{\Gamma;\Phi \vDash v\colon P \quad \Gamma;\Theta \vDash Q \geqslant P \dashv \widehat{\sigma}_1 \quad \Gamma;\Phi;\Theta \vDash N \bullet \vec{v} \Rrightarrow M \dashv \widehat{\sigma}_2}{\Gamma;\Phi;\Theta \vDash Q \to N \bullet v, \vec{v} \Rrightarrow M \dashv \widehat{\sigma}_1 \,\&\, \widehat{\sigma}_2}\ \textsc{ATArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma;\Phi;\Theta \vDash \forall\overrightarrow{\alpha^+}.N \bullet \vec{v} \Rrightarrow M \dashv \widehat{\sigma}}\ \textsc{ATForall}$$

$\boxed{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \vDash \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\cdot, \alpha^+, \cdot, \cdot)}\ \textsc{AUPVar}$$

$$\frac{\Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash {\downarrow}N_1 \overset{a}{\simeq} {\downarrow}N_2 \dashv (\Xi, {\downarrow}M, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \Gamma = \varnothing \quad \Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \exists\overrightarrow{\alpha^-}.P_1 \overset{a}{\simeq} \exists\overrightarrow{\alpha^-}.P_2 \dashv (\Xi, \exists\overrightarrow{\alpha^-}.Q, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUExists}$$

$\boxed{\Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \vDash \alpha^- \overset{a}{\simeq} \alpha^- \dashv (\cdot, \alpha^-, \cdot, \cdot)}\ \textsc{AUNVar}$$

$$\frac{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash {\uparrow}P_1 \overset{a}{\simeq} {\uparrow}P_2 \dashv (\Xi, {\uparrow}Q, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUShiftU}$$

$$\frac{\overrightarrow{\alpha^+} \cap \Gamma = \varnothing \quad \Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \forall\overrightarrow{\alpha^+}.N_1 \overset{a}{\simeq} \forall\overrightarrow{\alpha^+}.N_2 \dashv (\Xi, \forall\overrightarrow{\alpha^+}.M, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUForall}$$

$$\frac{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \widehat{\tau}_1, \widehat{\tau}_2) \quad \Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \widehat{\tau}'_1, \widehat{\tau}'_2)}{\Gamma \vDash P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \widehat{\tau}_1 \cup \widehat{\tau}'_1, \widehat{\tau}_2 \cup \widehat{\tau}'_2)}\ \textsc{AUArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \vDash N \overset{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, \widehat{\alpha}^-_{\{N,M\}}, (\widehat{\alpha}^-_{\{N,M\}} :\approx N), (\widehat{\alpha}^-_{\{N,M\}} :\approx M))}\ \textsc{AUAU}$$

$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-}\ \textsc{E1NVar}$$

$$\frac{P \simeq_1^D Q}{{\uparrow}P \simeq_1^D {\uparrow}Q}\ \textsc{E1ShiftU}$$

$$\frac{P \simeq_1^D Q \quad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M}\ \textsc{E1Arrow}$$

$$\frac{\overrightarrow{\alpha^+} \cap \mathbf{fv}\,M = \varnothing \quad \mu : (\overrightarrow{\beta^+} \cap \mathbf{fv}\,M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \mathbf{fv}\,N) \quad N \simeq_1^D [\mu]M}{\forall\overrightarrow{\alpha^+}.N \simeq_1^D \forall\overrightarrow{\beta^+}.M}\ \textsc{E1Forall}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+}\ \textsc{E1PVar}$$

$$\frac{N \simeq_1^D M}{\downarrow N \simeq_1^D \downarrow M} \quad \text{E1ShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \mathbf{fv}\, Q = \varnothing \quad \mu : (\overrightarrow{\beta^-} \cap \mathbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \mathbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists \overrightarrow{\alpha^-}.P \simeq_1^D \exists \overrightarrow{\beta^-}.Q} \quad \text{E1Exists}$$

$\boxed{P \simeq Q}$

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M} \quad \text{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q} \quad \text{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-} \quad \text{D1NVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant_1 \uparrow Q} \quad \text{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M} \quad \text{D1Arrow}$$

$$\frac{\mathbf{fv}\, N \cap \overrightarrow{\beta^+} = \varnothing \quad \Gamma, \overrightarrow{\beta^+} \vdash P_i \quad \Gamma, \overrightarrow{\beta^+} \vdash [\overrightarrow{P}/\overrightarrow{\alpha^+}]N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant_1 \forall \overrightarrow{\beta^+}.M} \quad \text{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+} \quad \text{D1PVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M} \quad \text{D1ShiftD}$$

$$\frac{\mathbf{fv}\, P \cap \overrightarrow{\beta^-} = \varnothing \quad \Gamma, \overrightarrow{\beta^-} \vdash N_i \quad \Gamma, \overrightarrow{\beta^-} \vdash [\overrightarrow{N}/\overrightarrow{\alpha^-}]P \geqslant_1 Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P \geqslant_1 \exists \overrightarrow{\beta^-}.Q} \quad \text{D1Exists}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1}$ Equivalence of substitutions

$\boxed{\Gamma \vdash N \simeq_0^{\leqslant} M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq_0^{\leqslant} M} \quad \text{D0NDef}$$

$\boxed{\Gamma \vdash P \simeq_0^{\leqslant} Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq_0^{\leqslant} Q} \quad \text{D0PDef}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_0 \alpha^-} \quad \text{D0NVar}$$

$$\frac{\Gamma \vdash P \simeq_0^{\leqslant} Q}{\Gamma \vdash \uparrow P \leqslant_0 \uparrow Q} \quad \text{D0ShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/\alpha^+]N \leqslant_0 M \quad M \neq \forall \beta^+.M'}{\Gamma \vdash \forall \alpha^+.N \leqslant_0 M} \text{ D0ForallL}$$

$$\frac{\Gamma, \alpha^+ \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall \alpha^+.M} \text{ D0ForallR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \to N \leqslant_0 Q \to M} \text{ D0Arrow}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_0 \alpha^+} \text{ D0PVar}$$

$$\frac{\Gamma \vdash N \simeq_0^{\leqslant} M}{\Gamma \vdash \downarrow N \geqslant_0 \downarrow M} \text{ D0ShiftD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/\alpha^-]P \geqslant_0 Q \quad Q \neq \exists \alpha^-.Q'}{\Gamma \vdash \exists \alpha^-.P \geqslant_0 Q} \text{ D0ExistsL}$$

$$\frac{\Gamma, \alpha^- \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists \alpha^-.Q} \text{ D0ExistsR}$$

$\boxed{\Gamma; \Phi \vdash v \colon P}$ Positive type inference

$$\frac{v : P \in \Phi}{\Gamma; \Phi \vdash v \colon P} \text{ DTVar}$$

$$\frac{\Gamma; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \{c\} \colon \downarrow N} \text{ DTThunk}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma; \Phi \vdash (v : Q) \colon Q} \text{ DTAnnotP}$$

$\boxed{\Gamma; \Phi \vdash c \colon N}$ Negative type inference

$$\frac{\Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \lambda x : P.c \colon P \to N} \text{ DTtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \Lambda \alpha^+.c \colon \forall \alpha^+.N} \text{ DTTLam}$$

$$\frac{\Gamma; \Phi \vdash v \colon P}{\Gamma; \Phi \vdash \mathbf{return}\, v \colon \uparrow P} \text{ DTReturn}$$

$$\frac{\Gamma; \Phi \vdash v \colon \downarrow M \quad \Gamma; \Phi \vdash M \bullet \overrightarrow{v} \Rightarrow\!\!\!> \uparrow Q \quad \Gamma \vdash \uparrow Q \leqslant_1 \uparrow P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x : P = v(\overrightarrow{v}); c \colon N} \text{ DTLetAnn}$$

$$\frac{\Gamma; \Phi \vdash v \colon \downarrow M \quad \Gamma; \Phi \vdash M \bullet \overrightarrow{v} \Rightarrow\!\!\!> \uparrow Q \text{ uniquely} \quad \Gamma; \Phi, x : Q \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v(\overrightarrow{v}); c \colon N} \text{ DTLet}$$

$$\frac{\Gamma, \alpha^-; \Phi \vdash v \colon \exists \alpha^-.P \quad \Gamma, \alpha^-; \Phi, x : P \vdash c \colon N \quad \Gamma \vdash N}{\Gamma; \Phi \vdash \mathbf{let}^{\exists}(\alpha^-, x) = v; c \colon N} \text{ DTUnpack}$$

$$\frac{\Gamma; \Phi \vdash c \colon N \quad \Gamma \vdash N \leqslant_1 M}{\Gamma; \Phi \vdash (c : M) \colon M} \text{ DTAnnotN}$$

$\boxed{\Gamma; \Phi \vdash N \bullet \overrightarrow{v} \Rightarrow\!\!\!> M}$ Spin Application type inference

$$\frac{N \neq \forall \overrightarrow{\alpha^+}.M}{\Gamma; \Phi \vdash N \bullet \cdot \Rightarrow\!\!\!> N} \text{ DTEmtpty}$$

$$\frac{\Gamma; \Phi \vdash v : P \quad \Gamma \vdash Q \geqslant_1 P \quad \Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M}{\Gamma; \Phi \vdash Q \to N \bullet v, \vec{v} \Rrightarrow M} \text{DTArrow}$$

$$\frac{\Gamma \vdash \vec{P} \quad \Gamma; \Phi \vdash [\vec{P}/\overrightarrow{\alpha^+}]N \bullet \vec{v} \Rrightarrow M}{\Gamma; \Phi \vdash \forall \overrightarrow{\alpha^+}.N \bullet \vec{v} \Rrightarrow M} \text{DTForall}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$

$\boxed{P_1 \vee P_2}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N}$

$\boxed{\textbf{nf}\ (N')}$

$\boxed{\textbf{nf}\ (P')}$

$\boxed{\textbf{nf}\ (N')}$

$\boxed{\textbf{nf}\ (P')}$

$\boxed{\textbf{nf}\ (\vec{N}')}$

$\boxed{\textbf{nf}\ (\vec{P}')}$

$\boxed{\textbf{nf}\ (\sigma')}$

$\boxed{\textbf{nf}\ (\mu')}$

$\boxed{\mathbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\sigma'|_{vars}}$

$\boxed{\widehat{\sigma}'|_{vars}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{\widehat{\sigma}_1 \ \& \ \widehat{\sigma}_2}$

$\boxed{\widehat{\tau}_1 \ \& \ \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{\Gamma \vDash P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \vDash \alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBVar}$$

$$\frac{\Gamma, \cdot \vDash \mathbf{nf}\,(\downarrow N) \stackrel{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi]P} \quad \text{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \vDash P_1 \vee P_2 = Q}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P_1 \vee \exists \overrightarrow{\beta^-}.P_2 = Q} \quad \text{LUBExists}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}$

$$\frac{\Gamma = \Delta, \overrightarrow{\alpha^{\pm}} \quad \overrightarrow{\beta^{\pm}}\ \textbf{is fresh} \quad \overrightarrow{\gamma^{\pm}}\ \textbf{is fresh} \\ \Delta, \overrightarrow{\beta^{\pm}}, \overrightarrow{\gamma^{\pm}} \models [\overrightarrow{\beta^{\pm}}/\overrightarrow{\alpha^{\pm}}]P \vee [\overrightarrow{\gamma^{\pm}}/\overrightarrow{\alpha^{\pm}}]P = Q}{\textbf{upgrade}\,\Gamma \vdash P\,\textbf{to}\,\Delta = Q}\ \textsc{LUBUpgrade}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^{-}) = \alpha^{-}}\ \textsc{NrmNVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\uparrow P) = \uparrow Q}\ \textsc{NrmShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(P \to N) = Q \to M}\ \textsc{NrmArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\forall \overrightarrow{\alpha^{+}}.N) = \forall \overrightarrow{\alpha^{+}}'.N'}\ \textsc{NrmForall}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\alpha^{+}) = \alpha^{+}}\ \textsc{NrmPVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\downarrow N) = \downarrow M}\ \textsc{NrmShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\exists \overrightarrow{\alpha^{-}}.P) = \exists \overrightarrow{\alpha^{-}}'.P'}\ \textsc{NrmExists}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^{-}) = \widehat{\alpha}^{-}}\ \textsc{NrmNUVar}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^{+}) = \widehat{\alpha}^{+}}\ \textsc{NrmPUVar}$$

$\boxed{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}$

$$\frac{\alpha^{-} \in \mathit{vars}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\alpha^{-} = \alpha^{-}}\ \textsc{ONVarIn}$$

$$\frac{\alpha^{-} \notin \mathit{vars}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\alpha^{-} = \cdot}\ \textsc{ONVarNIn}$$

$$\frac{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\uparrow P = \vec{\alpha}}\ \textsc{OShiftU}$$

$$\frac{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}_1 \quad \mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}_2}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)}\ \textsc{OArrow}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^{+}} = \varnothing \quad \mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\forall \overrightarrow{\alpha^{+}}.N = \vec{\alpha}}\ \textsc{OForall}$$

$\boxed{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}$

$$\frac{\alpha^{+} \in \mathit{vars}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\alpha^{+} = \alpha^{+}}\ \textsc{OPVarIn}$$

$$\frac{\alpha^+ \notin \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \alpha^+ = \cdot}\ \textsc{OPVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ {\downarrow}N = \vec{\alpha}}\ \textsc{OShiftD}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^-} = \varnothing \quad \mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \exists\overrightarrow{\alpha^-}.P = \vec{\alpha}}\ \textsc{OExists}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \widehat{\alpha}^- = \cdot}\ \textsc{ONUVar}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \widehat{\alpha}^+ = \cdot}\ \textsc{OPUVar}$$

$\boxed{\Gamma \vdash e_1\ \&\ e_2 = e_3}$ Unification Solution Entry Merge

$$\frac{\Gamma \vDash P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1)\ \&\ (\widehat{\alpha}^+ :\geqslant P_2) = (\widehat{\alpha}^+ :\geqslant Q)}\ \textsc{SMESupSup}$$

$$\frac{\Gamma; \cdot \vDash P \geqslant Q \dashv \widehat{\sigma}'}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P)\ \&\ (\widehat{\alpha}^+ :\geqslant Q) = (\widehat{\alpha}^+ :\approx P)}\ \textsc{SMEEqSup}$$

$$\frac{\Gamma; \cdot \vDash Q \geqslant P \dashv \widehat{\sigma}'}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P)\ \&\ (\widehat{\alpha}^+ :\approx Q) = (\widehat{\alpha}^+ :\approx Q)}\ \textsc{SMESupEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P)\ \&\ (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)}\ \textsc{SMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1)\ \&\ (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)}\ \textsc{SMENEqEq}$$

$\boxed{\Theta \vdash \widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3}$ Merge unification solutions

$\boxed{\Gamma \vdash e_1 \Rightarrow e_2}$ Weakening of unification solution entries

$$\frac{\Gamma \vdash P_1 \geqslant_1 P_2}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1) \Rightarrow (\widehat{\alpha}^+ :\geqslant P_2)}\ \textsc{SImpESupSup}$$

$$\frac{\Gamma \vdash P_1 \geqslant_1 P_2}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P_1) \Rightarrow (\widehat{\alpha}^+ :\geqslant P_2)}\ \textsc{SImpEEqSup}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P_1) \Rightarrow (\widehat{\alpha}^+ :\approx P_2)}\ \textsc{SImpEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \Rightarrow (\widehat{\alpha}^- :\approx N_2)}\ \textsc{SImpENEqEq}$$

$\boxed{\Theta \vdash \widehat{\sigma}_1 \Rightarrow \widehat{\sigma}_2}$ Weakening of unification solutions

$\boxed{\Gamma \vdash e_1 \simeq e_2}$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1) \simeq (\widehat{\alpha}^+ :\geqslant P_2)}\ \textsc{SImpEEqSupSup}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P_1) \simeq (\widehat{\alpha}^+ :\approx P_2)}\ \textsc{SImpEEqPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \simeq (\widehat{\alpha}^- :\approx N_2)} \quad \textsc{SImpEEqNEqEq}$$

$\boxed{\Theta \vdash \widehat{\sigma}_1 \simeq \widehat{\sigma}_2}$

$\boxed{\Gamma; \Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}}$ Negative unification

$$\frac{}{\Gamma; \Theta \vDash \alpha^- \overset{u}{\simeq} \alpha^- \dashv \cdot} \quad \textsc{UNVar}$$

$$\frac{\Gamma; \Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}{\Gamma; \Theta \vDash \uparrow P \overset{u}{\simeq} \uparrow Q \dashv \widehat{\sigma}} \quad \textsc{UShiftU}$$

$$\frac{\Gamma; \Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}_1 \quad \Gamma; \Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}_2}{\Gamma; \Theta \vDash P \to N \overset{u}{\simeq} Q \to M \dashv \widehat{\sigma}_1 \ \&\ \widehat{\sigma}_2} \quad \textsc{UArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}}{\Gamma; \Theta \vDash \forall \overrightarrow{\alpha^+}. N \overset{u}{\simeq} \forall \overrightarrow{\alpha^+}. M \dashv \widehat{\sigma}} \quad \textsc{UForall}$$

$$\frac{\widehat{\alpha}^- \{\Delta\} \in \Theta \quad \Delta \vdash N}{\Gamma; \Theta \vDash \widehat{\alpha}^- \overset{u}{\simeq} N \dashv (\widehat{\alpha}^- :\approx N)} \quad \textsc{UNUVar}$$

$\boxed{\Gamma; \Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}$ Positive unification

$$\frac{}{\Gamma; \Theta \vDash \alpha^+ \overset{u}{\simeq} \alpha^+ \dashv \cdot} \quad \textsc{UPVar}$$

$$\frac{\Gamma; \Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}}{\Gamma; \Theta \vDash \downarrow N \overset{u}{\simeq} \downarrow M \dashv \widehat{\sigma}} \quad \textsc{UShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}{\Gamma; \Theta \vDash \exists \overrightarrow{\alpha^-}. P \overset{u}{\simeq} \exists \overrightarrow{\alpha^-}. Q \dashv \widehat{\sigma}} \quad \textsc{UExists}$$

$$\frac{\widehat{\alpha}^+ \{\Delta\} \in \Theta \quad \Delta \vdash P}{\Gamma; \Theta \vDash \widehat{\alpha}^+ \overset{u}{\simeq} P \dashv (\widehat{\alpha}^+ :\approx P)} \quad \textsc{UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness
$\boxed{\Gamma \vdash P}$ Positive type well-formedness
$\boxed{\Gamma \vdash N}$ Negative type well-formedness
$\boxed{\Gamma \vdash P}$ Positive type well-formedness
$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness
$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness
$\boxed{\Gamma; \Theta \vdash N}$ Negative unification type well-formedness
$\boxed{\Gamma; \Theta \vdash P}$ Positive unifiication type well-formedness
$\boxed{\Gamma; \Xi \vdash N}$ Negative anti-unification type well-formedness
$\boxed{\Gamma; \Xi \vdash P}$ Positive anti-unification type well-formedness
$\boxed{\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness
$\boxed{\widehat{\sigma} : \Theta}$ Unification substitution well-formedness
$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness
$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution well-formedness
$\boxed{\Gamma \vdash e}$ Unification solution entry well-formedness

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Definition rules:        98 good    16 bad
Definition rule clauses: 195 good    16 bad
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