α, β	type variables
n, m, i, j	index variables

$\alpha^{+},\ \beta^{+}$	$::=$			positive variable
	$\mid$	α^{+}		
$\alpha^{-},\ \beta^{-}$	$::=$			negative variable
	$\mid$	α^{-}		
σ	$::=$			substitution
	$\mid$	$\cdot$		
	$\mid$	$P/a+$		
	$\mid$	$N/a-$		
	$\mid$	$\vec{P}/\overrightarrow{\alpha^{+}}$		
	$\mid$	$\vec{N}/\overrightarrow{\alpha^{-}}$		
	$\mid$	$\overrightarrow{\widetilde{\alpha^{+}}}/\overrightarrow{\alpha^{+}}$		
	$\mid$	$\overrightarrow{\widetilde{\alpha^{-}}}/\overrightarrow{\alpha^{-}}$		
	$\mid$	$\overrightarrow{\widetilde{\alpha^{+}}}/\overrightarrow{\alpha^{+}}$		
	$\mid$	$\overrightarrow{\widehat{\alpha^{-}}}/\overrightarrow{\alpha^{-}}$		
	$\mid$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\mid$	$\overline{\sigma_i}^{\,i}$		concatenate
e	$::=$			entry of a unification solution
	$\mid$	$\Gamma \vdash \widehat{\alpha}^{+} :\approx P$		
	$\mid$	$\Gamma \vdash \widehat{\alpha}^{-} :\approx N$		
	$\mid$	$\Gamma \vdash \widehat{\alpha}^{+} :\geqslant P$		
	$\mid$	(e)	S	
	$\mid$	$e_1\ \&\ e_2$	M	
$\widehat{\sigma}$	$::=$			unification solution (substitution)
	$\mid$	$\cdot$		
	$\mid$	e		
	$\mid$	$\widehat{\sigma}\backslash\overrightarrow{\widetilde{\alpha^{+}}}$		
	$\mid$	$\widehat{\sigma}\backslash\overrightarrow{\widehat{\alpha^{-}}}$		
	$\mid$	$\widehat{\sigma}\backslash\widehat{\alpha}^{+}$		
	$\mid$	$\widehat{\sigma}\backslash\widehat{\alpha}^{-}$		
	$\mid$	$\widehat{\sigma}_1 \cup \widehat{\sigma}_2$		
	$\mid$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\mid$	$(\widehat{\sigma})$	S	
	$\mid$	$\widehat{\sigma}_1\ \&\ \widehat{\sigma}_2$	M	
$P,\ Q$	$::=$			positive types
	$\mid$	$a+$		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\alpha^{-}.P$		
	$\mid$	$[\sigma]P$	M	
$N,\ M$	$::=$			negative types
	$\mid$	$a-$		
	$\mid$	$\uparrow P$		
	$\mid$	$\forall\alpha^{+}.N$		
	$\mid$	$P \to N$		

	$\mid$	$[\sigma]N$	M	
$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+}$	$::=$			positive variable list
	$\mid$	$\cdot$		empty list
	$\mid$	α^+		a variable
	$\mid$	$\overline{\overrightarrow{\alpha^+}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-}$	$::=$			negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	α^-		a variable
	$\mid$	$\overline{\overrightarrow{\alpha^-}_i}^{\,i}$		concatenate lists
$P,\ Q$	$::=$			multi-quantified positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^-}.P$		$P \neq \exists\ldots$
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	(P)	S	
	$\mid$	$\mathbf{nf}\,(P')$	M	
$N,\ M$	$::=$			multi-quantified negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$\forall\overrightarrow{\alpha^+}.N$		$N \neq \forall\ldots$
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\mu]N$	M	
	$\mid$	(N)	S	
	$\mid$	$\mathbf{nf}\,(N')$	M	
$\overrightarrow{P}$	$::=$			list of positive types
	$\mid$	$\cdot$		empty list
	$\mid$	P		a singel type
	$\mid$	$\overline{\overrightarrow{P}_i}^{\,i}$		concatenate lists
$\overrightarrow{N}$	$::=$			list of negative types
	$\mid$	$\cdot$		empty list
	$\mid$	N		a singel type
	$\mid$	$\overline{\overrightarrow{N}_i}^{\,i}$		concatenate lists
$\Delta,\ \Gamma$	$::=$			declarative type context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\alpha^+}$		list of variables
	$\mid$	$\overrightarrow{\alpha^-}$		list of variables
	$\mid$	$vars$		
	$\mid$	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	$\mid$	(Γ)	S	

$\vec{\alpha},\ \vec{\beta}$	::=			ordered positive or negative variables
	|	$\cdot$		empty list
	|	$\overrightarrow{\alpha^+}$		list of variables
	|	$\overrightarrow{\alpha^-}$		list of variables
	|	$\vec{\alpha}_1 \backslash \mathit{vars}$		setminus
	|	Γ		context
	|	vars		
	|	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	|	$(\vec{\alpha})$	S	parenthesis
	|	$[\mu]\vec{\alpha}$	M	apply moving to list
	|	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P$	M	
	|	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N$	M	
	|	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P$	M	
	|	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N$	M	
vars	::=			set of variables
	|	$\varnothing$		empty set
	|	$\mathbf{fv}\ P$		free variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathbf{fv}\ P$		free variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathit{vars}_1 \cap \mathit{vars}_2$		set intersection
	|	$\mathit{vars}_1 \cup \mathit{vars}_2$		set union
	|	$\mathit{vars}_1 \backslash \mathit{vars}_2$		set complement
	|	$\mathbf{mv}\ P$		movable variables
	|	$\mathbf{mv}\ N$		movable variables
	|	$\mathbf{uv}\ N$		unification variables
	|	$\mathbf{uv}\ P$		unification variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathbf{fv}\ P$		free variables
	|	(vars)	S	parenthesis
	|	$\{\vec{\alpha}\}$		ordered list of variables
	|	$[\mu]\mathit{vars}$	M	apply moving to varset
μ	::=			
	|	$\cdot$		empty moving
	|	$\widetilde{\alpha}^+_1 \mapsto \widetilde{\alpha}^+_2$		Positive unit substitution
	|	$\widetilde{\alpha}^-_1 \mapsto \widetilde{\alpha}^-_2$		Positive unit substitution
	|	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	|	$\overline{\mu_i}^{\,i}$		concatenate movings
	|	$\mu\|_{\mathit{vars}}$	M	restriction on a set
	|	μ^{-1}	M	inversion
n	::=			cohort index
	|	0		
	|	$n+1$		
$\widetilde{\alpha}^+$	::=			positive movable variable
	|	$\widetilde{\alpha}^{+n}$		

$\widetilde{\alpha}^{-}$	$::=$		negative movable variable
	$\mid$ $\widetilde{\alpha}^{-n}$		
$\overrightarrow{\widetilde{\alpha}^{+}}$, $\overrightarrow{\widetilde{\beta}^{+}}$	$::=$		positive movable variable list
	$\mid$ $\cdot$		empty list
	$\mid$ $\widetilde{\alpha}^{+}$		a variable
	$\mid$ $\overrightarrow{\widetilde{\alpha}^{+n}}$		from a non-movable variable
	$\mid$ $\overline{\overrightarrow{\widetilde{\alpha}^{+}}_{i}}^{i}$		concatenate lists
$\overrightarrow{\widetilde{\alpha}^{-}}$, $\overrightarrow{\widetilde{\beta}^{-}}$	$::=$		negatiive movable variable list
	$\mid$ $\cdot$		empty list
	$\mid$ $\widetilde{\alpha}^{-}$		a variable
	$\mid$ $\overrightarrow{\widetilde{\alpha}^{-n}}$		from a non-movable variable
	$\mid$ $\overline{\overrightarrow{\widetilde{\alpha}^{-}}_{i}}^{i}$		concatenate lists
$P,\ Q$	$::=$		multi-quantified positive types with movable variables
	$\mid$ α^{+}		
	$\mid$ $\widetilde{\alpha}^{+}$		
	$\mid$ $\downarrow N$		
	$\mid$ $\exists\overrightarrow{\alpha^{-}}.P$		
	$\mid$ $[\sigma]P$	M	
	$\mid$ $[\mu]P$	M	
$N,\ M$	$::=$		multi-quantified negative types with movable variables
	$\mid$ α^{-}		
	$\mid$ $\widetilde{\alpha}^{-}$		
	$\mid$ $\uparrow P$		
	$\mid$ $P \rightarrow N$		
	$\mid$ $\forall\overrightarrow{\alpha^{+}}.N$		
	$\mid$ $[\sigma]N$	M	
	$\mid$ $[\mu]N$	M	
$\widehat{\alpha}^{+}$	$::=$		positive unification variable
	$\mid$ $\widehat{\alpha}^{+}$		
$\widehat{\alpha}^{-}$	$::=$		negative unification variable
	$\mid$ $\widehat{\alpha}^{-}$		
	$\mid$ $\widehat{\alpha}^{-}_{\{N,M\}}$		
$\overrightarrow{\widehat{\alpha}^{+}}$, $\overrightarrow{\widehat{\beta}^{+}}$	$::=$		positive unification variable list
	$\mid$ $\cdot$		empty list
	$\mid$ $\widehat{\alpha}^{+}$		a variable
	$\mid$ $\overrightarrow{\widehat{\alpha}^{+}\{\Delta\}}$		from a normal variable
	$\mid$ $\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	$\mid$ $\overline{\overrightarrow{\widehat{\alpha}^{+}}_{i}}^{i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}}$, $\overrightarrow{\widehat{\beta}^{-}}$	$::=$		negative unification variable list
	$\mid$ $\cdot$		empty list
	$\mid$ $\widehat{\alpha}^{-}$		a variable

	$\mid$	$\overrightarrow{\widehat{\alpha}^-\{\Delta\}}$		from a normal variable
	$\mid$	$\overrightarrow{\widehat{\alpha}^-}$		from a normal variable, context unspecified
	$\mid$	$\overline{\overrightarrow{\widehat{\alpha}^-}_i}^{\,i}$		concatenate lists
P, Q	$::=$			a positive algorithmic type (potentially with metavariables)
	$\mid$	α^+		
	$\mid$	$\widetilde{\alpha}^+$		
	$\mid$	$\widehat{\alpha}^+\{\Delta\}$		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^-}.P$		
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	$\mathbf{nf}\,(P')$	M	
N, M	$::=$			a negative algorithmic type (potentially with metavariables)
	$\mid$	α^-		
	$\mid$	$\widetilde{\alpha}^-$		
	$\mid$	$\widehat{\alpha}^-\{\Delta\}$		
	$\mid$	$\widehat{\alpha}^-$		
	$\mid$	$\uparrow P$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$\forall\overrightarrow{\alpha^+}.N$		
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\mu]N$	M	
	$\mid$	$\mathbf{nf}\,(N')$	M	
terminals	$::=$			
	$\mid$	$\exists$		
	$\mid$	$\forall$		
	$\mid$	$\uparrow$		
	$\mid$	$\downarrow$		
	$\mid$	$\rightarrow$		
	$\mid$	$\leftrightarrow$		
	$\mid$	$\in$		
	$\mid$	$\notin$		
	$\mid$	$\cdot$		
	$\mid$	$\vdash$		
	$\mid$	$\leqslant$		
	$\mid$	$\geqslant$		
	$\mid$	$\simeq$		
	$\mid$	$\cup$		
	$\mid$	$\cap$		
	$\mid$	$\setminus$		
	$\mid$	$\subseteq$		
	$\mid$	$\mapsto$		
	$\mid$	$\stackrel{u}{\simeq}$		
	$\mid$	$\stackrel{a}{\simeq}$		
	$\mid$	$\varnothing$		
	$\mid$	$\vDash$		
	$\mid$	$\dashv$		

	$\mid$	$\neq$
	$\mid$	$\equiv_n$
	$\mid$	$\vee$
	$\mid$	$\Downarrow$
	$\mid$	$:\geqslant$
	$\mid$	$:\simeq$

formula	$::=$	
	$\mid$	*judgement*
	$\mid$	$\mathit{formula}_1 \quad .. \quad \mathit{formula}_n$
	$\mid$	$\mu : \mathit{vars}_1 \leftrightarrow \mathit{vars}_2$
	$\mid$	μ **is bijective**
	$\mid$	$\widehat{\sigma}$ **is functional**
	$\mid$	$\widehat{\sigma}_1 \in \widehat{\sigma}_2$
	$\mid$	$\mathit{vars}_1 \subseteq \mathit{vars}_2$
	$\mid$	$\mathit{vars}_1 = \mathit{vars}_2$
	$\mid$	vars **is fresh**
	$\mid$	$\alpha^- \notin \mathit{vars}$
	$\mid$	$\alpha^+ \notin \mathit{vars}$
	$\mid$	$\alpha^- \in \mathit{vars}$
	$\mid$	$\alpha^+ \in \mathit{vars}$
	$\mid$	if any other rule is not applicable
	$\mid$	$N \neq M$
	$\mid$	$P \neq Q$

E1A	$::=$		
	$\mid$	$n \models N \simeq_1^A M \dashv \mu$	Negative multi-quantified type equivalence (algorit
	$\mid$	$n \models P \simeq_1^A Q \dashv \mu$	Positive multi-quantified type equivalence (algorith
A	$::=$		
	$\mid$	$\Gamma \models N \leqslant M \dashv \widehat{\sigma}$	Negative subtyping
	$\mid$	$\Gamma \models P \geqslant Q \dashv \widehat{\sigma}$	Positive supertyping
E1	$::=$		
	$\mid$	$N \simeq_1^D M$	Negative multi-quantified type equivalence
	$\mid$	$P \simeq_1^D Q$	Positive multi-quantified type equivalence
D1	$::=$		
	$\mid$	$\Gamma \vdash N \simeq_1^{\leqslant} M$	Negative equivalence on MQ types
	$\mid$	$\Gamma \vdash P \simeq_1^{\leqslant} Q$	Positive equivalence on MQ types
	$\mid$	$\Gamma \vdash N \leqslant_1 M$	Negative subtyping
	$\mid$	$\Gamma \vdash P \geqslant_1 Q$	Positive supertyping
D0	$::=$		
	$\mid$	$\Gamma \vdash N \simeq_0^{\leqslant} M$	Negative equivalence
	$\mid$	$\Gamma \vdash P \simeq_0^{\leqslant} Q$	Positive equivalence
	$\mid$	$\Gamma \vdash N \leqslant_0 M$	Negative subtyping
	$\mid$	$\Gamma \vdash P \geqslant_0 Q$	Positive supertyping

$LUBF$	$::=$		
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, P \; === \; \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, N \; === \; \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, P \; === \; \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, N \; === \; \vec{\alpha}$	
	$\vert$	$\mathbf{nf}\,(N') === N$	
	$\vert$	$\mathbf{nf}\,(P') === P$	
	$\vert$	$\mathbf{nf}\,(N') === N$	
	$\vert$	$\mathbf{nf}\,(P') === P$	
	$\vert$	$e_1 \;\&\; e_2$	
	$\vert$	$\widehat{\sigma}_1 \;\&\; \widehat{\sigma}_2$	
LUB	$::=$		
	$\vert$	$\Gamma \models P_1 \vee P_2 = Q$	Least Upper Bound (Least Common Supertype)
	$\vert$	$\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q$	
AU	$::=$		
	$\vert$	$\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (Q, \widehat{\sigma}_1, \widehat{\sigma}_2)$	
	$\vert$	$\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (M, \widehat{\sigma}_1, \widehat{\sigma}_2)$	
Nrm	$::=$		
	$\vert$	$\mathbf{nf}\,(N) = M$	
	$\vert$	$\mathbf{nf}\,(P) = Q$	
	$\vert$	$\mathbf{nf}\,(N) = M$	
	$\vert$	$\mathbf{nf}\,(P) = Q$	
$Order$	$::=$		
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, N = \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, P = \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, N = \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, P = \vec{\alpha}$	
SM	$::=$		
	$\vert$	$e_1 \;\&\; e_2 = e_3$	Unification Solution Entry Merge
	$\vert$	$\widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3$	Merge unification solutions
U	$::=$		
	$\vert$	$N \overset{u}{\simeq} M \dashv \widehat{\sigma}$	Negative unification
	$\vert$	$P \overset{u}{\simeq} Q \dashv \widehat{\sigma}$	Positive unification
WF	$::=$		
	$\vert$	$\Gamma \vdash N$	Negative type well-formedness
	$\vert$	$\Gamma \vdash P$	Positive type well-formedness
	$\vert$	$\Gamma \vdash N$	Negative type well-formedness
	$\vert$	$\Gamma \vdash P$	Positive type well-formedness
$judgement$	$::=$		
	$\vert$	$E1A$	
	$\vert$	A	

$$
\begin{array}{lll}
 & | & \mathit{E1} \\
 & | & \mathit{D1} \\
 & | & \mathit{D0} \\
 & | & \mathit{LUB} \\
 & | & \mathit{AU} \\
 & | & \mathit{Nrm} \\
 & | & \mathit{Order} \\
 & | & \mathit{SM} \\
 & | & \mathit{U} \\
 & | & \mathit{WF} \\
 & & \\
\mathit{user\_syntax} & ::= & \\
 & | & \alpha \\
 & | & n \\
 & | & \alpha^{+} \\
 & | & \alpha^{-} \\
 & | & \sigma \\
 & | & e \\
 & | & \widehat{\sigma} \\
 & | & P \\
 & | & N \\
 & | & \overrightarrow{\alpha^{+}} \\
 & | & \overrightarrow{\alpha^{-}} \\
 & | & P \\
 & | & N \\
 & | & \overrightarrow{P} \\
 & | & \overrightarrow{N} \\
 & | & \Gamma \\
 & | & \overrightarrow{\alpha} \\
 & | & \mathit{vars} \\
 & | & \mu \\
 & | & n \\
 & | & \widetilde{\alpha}^{+} \\
 & | & \widetilde{\alpha}^{-} \\
 & | & \overrightarrow{\widetilde{\alpha}^{+}} \\
 & | & \overrightarrow{\widetilde{\alpha}^{-}} \\
 & | & P \\
 & | & N \\
 & | & \widehat{\alpha}^{+} \\
 & | & \widehat{\alpha}^{-} \\
 & | & \overrightarrow{\widetilde{\alpha}^{+}} \\
 & | & \overrightarrow{\widehat{\alpha}^{-}} \\
 & | & P \\
 & | & N \\
 & | & \mathit{terminals} \\
 & | & \mathit{formula}
\end{array}
$$

$\boxed{n \vDash N \simeq_1^{A} M \dashv \mu}$ Negative multi-quantified type equivalence (algorithmic)

$$\frac{}{n \vDash \alpha^- \simeq_1^A \alpha^- \Dashv \cdot}\ \text{E1ANVar}$$

$$\frac{n \vDash P \simeq_1^A Q \Dashv \mu}{n \vDash {\uparrow} P \simeq_1^A {\uparrow} Q \Dashv \mu}\ \text{E1AShiftU}$$

$$\frac{n \vDash P \simeq_1^A Q \Dashv \mu_1 \quad n \vDash N \simeq_1^A M \Dashv \mu_2 \quad \mu_1 \cup \mu_2 \textbf{ is bijective}}{n \vDash P \to N \simeq_1^A Q \to M \Dashv \mu_1 \cup \mu_2}\ \text{E1AArrow}$$

$$\frac{n+1 \vDash [\overrightarrow{\widetilde{\alpha}^{+n}}/\overrightarrow{\alpha^+}]N \simeq_1^A [\overrightarrow{\widetilde{\beta}^{+n}}/\overrightarrow{\beta^+}]M \Dashv \mu}{n \vDash \forall \overrightarrow{\alpha^+}.N \simeq_1^A \forall \overrightarrow{\beta^+}.M \Dashv \mu|_{\mathbf{mv}\,M}}\ \text{E1AForall}$$

$$\frac{}{n \vDash \widetilde{\alpha}^{-n} \simeq_1^A \widetilde{\beta}^{-n} \Dashv \widetilde{\beta}^{-n} \mapsto \widetilde{\alpha}^{-n}}\ \text{E1ANMVar}$$

$\boxed{n \vDash P \simeq_1^A Q \Dashv \mu}$ Positive multi-quantified type equivalence (algorithmic)

$$\frac{}{n \vDash \alpha^+ \simeq_1^A \alpha^+ \Dashv \cdot}\ \text{E1APVar}$$

$$\frac{n \vDash N \simeq_1^A M \Dashv \mu}{n \vDash {\downarrow} N \simeq_1^A {\downarrow} M \Dashv \mu}\ \text{E1AShiftD}$$

$$\frac{n+1 \vDash [\overrightarrow{\widetilde{\alpha}^{-n}}/\overrightarrow{\alpha^-}]P \simeq_1^A [\overrightarrow{\widetilde{\beta}^{-n}}/\overrightarrow{\beta^-}]Q \Dashv \mu}{n \vDash \exists \overrightarrow{\alpha^-}.P \simeq_1^A \exists \overrightarrow{\beta^-}.Q \Dashv \mu|_{\mathbf{mv}\,Q}}\ \text{E1AExists}$$

$$\frac{}{n \vDash \widetilde{\alpha}^{+n} \simeq_1^A \widetilde{\beta}^{+n} \Dashv \widetilde{\beta}^{+n} \mapsto \widetilde{\alpha}^{+n}}\ \text{E1APMVar}$$

$\boxed{\Gamma \vDash N \leqslant M \Dashv \widehat{\sigma}}$ Negative subtyping

$$\frac{}{\Gamma \vDash \alpha^- \leqslant \alpha^- \Dashv \cdot}\ \text{ANVar}$$

$$\frac{\mathbf{nf}\,(P) \stackrel{u}{\simeq} \mathbf{nf}\,(Q) \Dashv \widehat{\sigma}}{\Gamma \vDash {\uparrow} P \leqslant {\uparrow} Q \Dashv \widehat{\sigma}}\ \text{AShiftU}$$

$$\frac{\Gamma \vDash P \geqslant Q \Dashv \widehat{\sigma}_1 \quad \Gamma \vDash N \leqslant M \Dashv \widehat{\sigma}_2}{\Gamma \vDash P \to N \leqslant Q \to M \Dashv \widehat{\sigma}_1 \mathbin{\&} \widehat{\sigma}_2}\ \text{AArrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vDash [\overrightarrow{\widehat{\alpha}^+\{\Gamma, \overrightarrow{\beta^+}\}}/\overrightarrow{\alpha^+}]N \leqslant M \Dashv \widehat{\sigma}}{\Gamma \vDash \forall \overrightarrow{\alpha^+}.N \leqslant \forall \overrightarrow{\beta^+}.M \Dashv \widehat{\sigma} \backslash \overrightarrow{\widehat{\alpha}^+}}\ \text{AForall}$$

$\boxed{\Gamma \vDash P \geqslant Q \Dashv \widehat{\sigma}}$ Positive supertyping

$$\frac{}{\Gamma \vDash \alpha^+ \geqslant \alpha^+ \Dashv \cdot}\ \text{APVar}$$

$$\frac{\mathbf{nf}\,(N) \stackrel{u}{\simeq} \mathbf{nf}\,(M) \Dashv \widehat{\sigma}}{\Gamma \vDash {\downarrow} N \geqslant {\downarrow} M \Dashv \widehat{\sigma}}\ \text{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vDash [\overrightarrow{\widehat{\alpha}^-\{\Gamma, \overrightarrow{\beta^-}\}}/\overrightarrow{\alpha^-}]P \geqslant Q \Dashv \widehat{\sigma}}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P \geqslant \exists \overrightarrow{\beta^-}.Q \Dashv \widehat{\sigma}}\ \text{AExists}$$

$$\frac{\mathbf{upgrade}\,\Gamma \vdash \mathbf{nf}\,(P)\,\mathbf{to}\,\Delta = Q}{\Gamma \vDash \widehat{\alpha}^+\{\Delta\} \geqslant P \Dashv (\Delta \vdash \widehat{\alpha}^+ :\geqslant Q)}\ \text{APUVar}$$

$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-}\ \textsc{E1NVar}$$

$$\frac{P \simeq_1^D Q}{{\uparrow}P \simeq_1^D {\uparrow}Q}\ \textsc{E1ShiftU}$$

$$\frac{P \simeq_1^D Q \quad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M}\ \textsc{E1Arrow}$$

$$\frac{\{\overrightarrow{\alpha^+}\} \cap \mathbf{fv}\, M = \varnothing \quad \mu : (\{\overrightarrow{\beta^+}\} \cap \mathbf{fv}\, M) \leftrightarrow (\{\overrightarrow{\alpha^+}\} \cap \mathbf{fv}\, N) \quad N \simeq_1^D [\mu]M}{\forall \overrightarrow{\alpha^+}.N \simeq_1^D \forall \overrightarrow{\beta^+}.M}\ \textsc{E1Forall}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+}\ \textsc{E1PVar}$$

$$\frac{N \simeq_1^D M}{{\downarrow}N \simeq_1^D {\downarrow}M}\ \textsc{E1ShiftD}$$

$$\frac{\{\overrightarrow{\alpha^-}\} \cap \mathbf{fv}\, Q = \varnothing \quad \mu : (\{\overrightarrow{\beta^-}\} \cap \mathbf{fv}\, Q) \leftrightarrow (\{\overrightarrow{\alpha^-}\} \cap \mathbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists \overrightarrow{\alpha^-}.P \simeq_1^D \exists \overrightarrow{\beta^-}.Q}\ \textsc{E1Exists}$$

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M}\ \textsc{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q}\ \textsc{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-}\ \textsc{D1NVar}$$

$$\frac{\Gamma \vdash P \simeq_1^{\leqslant} Q}{\Gamma \vdash {\uparrow}P \leqslant_1 {\uparrow}Q}\ \textsc{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M}\ \textsc{D1Arrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vdash P_i \quad \Gamma, \overrightarrow{\beta^+} \vdash [\overrightarrow{P}/\overrightarrow{\alpha^+}]N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant_1 \forall \overrightarrow{\beta^+}.M}\ \textsc{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+}\ \textsc{D1PVar}$$

$$\frac{\Gamma \vdash N \simeq_1^{\leqslant} M}{\Gamma \vdash {\downarrow}N \geqslant_1 {\downarrow}M}\ \textsc{D1ShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vdash N_i \quad \Gamma, \overrightarrow{\beta^-} \vdash [\overrightarrow{N}/\overrightarrow{\alpha^-}]P \geqslant_1 Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P \geqslant_1 \exists \overrightarrow{\beta^-}.Q}\ \textsc{D1ExistsL}$$

$\boxed{\Gamma \vdash N \simeq_0^{\leqslant} M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq_0^{\leqslant} M} \text{ D0NDEF}$$

$\boxed{\Gamma \vdash P \simeq_0^{\leqslant} Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq_0^{\leqslant} Q} \text{ D0PDEF}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash a- \leqslant_0 a-} \text{ D0NVAR}$$

$$\frac{\Gamma \vdash P \simeq_0^{\leqslant} Q}{\Gamma \vdash \uparrow P \leqslant_0 \uparrow Q} \text{ D0SHIFTU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/a+]N \leqslant_0 M \quad M \neq \forall \beta^+.M'}{\Gamma \vdash \forall \alpha^+.N \leqslant_0 M} \text{ D0FORALLL}$$

$$\frac{\Gamma, \alpha^+ \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall \alpha^+.M} \text{ D0FORALLR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \to N \leqslant_0 Q \to M} \text{ D0ARROW}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash a+ \geqslant_0 a+} \text{ D0PVAR}$$

$$\frac{\Gamma \vdash N \simeq_0^{\leqslant} M}{\Gamma \vdash \downarrow N \geqslant_0 \downarrow M} \text{ D0SHIFTD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/a-]P \geqslant_0 Q \quad Q \neq \exists \alpha^-.Q'}{\Gamma \vdash \exists \alpha^-.P \geqslant_0 Q} \text{ D0EXISTSL}$$

$$\frac{\Gamma, \alpha^- \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists \alpha^-.Q} \text{ D0EXISTSR}$$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N}$

$\boxed{\textbf{nf}\ (N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{e_1 \ \&\ e_2}$

$\boxed{\widehat{\sigma}_1 \ \&\ \widehat{\sigma}_2}$

$\boxed{\Gamma \models P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \models \alpha^+ \vee \alpha^+ = \alpha^+}\ \textsc{LUBVar}$$

$$\frac{\Gamma \models {\downarrow}N \overset{a}{\simeq} {\downarrow}M \dashv (P, \widehat{\sigma}_1, \widehat{\sigma}_2)}{\Gamma \models {\downarrow}N \vee {\downarrow}M = \exists\overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/(\mathbf{uv}\ P)]P}\ \textsc{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \models P_1 \vee P_2 = Q}{\Gamma \models \exists\overrightarrow{\alpha^-}.P_1 \vee \exists\overrightarrow{\beta^-}.P_2 = Q}\ \textsc{LUBExists}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}$

$\boxed{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (Q, \widehat{\sigma}_1, \widehat{\sigma}_2)}$

$$\frac{}{\Gamma \models \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\alpha^+, \cdot, \cdot)}\ \textsc{AUPVar}$$

$$\frac{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (M, \widehat{\sigma}_1, \widehat{\sigma}_2)}{\Gamma \models {\downarrow}N_1 \overset{a}{\simeq} {\downarrow}N_2 \dashv ({\downarrow}M, \widehat{\sigma}_1, \widehat{\sigma}_2)}\ \textsc{AUPShift}$$

$$\frac{\{\overrightarrow{\alpha^-}\} \cap \{\Gamma\} = \varnothing \quad \Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (Q, \widehat{\sigma}_1, \widehat{\sigma}_2)}{\Gamma \models \exists\overrightarrow{\alpha^-}.P_1 \overset{a}{\simeq} \exists\overrightarrow{\alpha^-}.P_2 \dashv (\exists\overrightarrow{\alpha^-}.Q, \widehat{\sigma}_1, \widehat{\sigma}_2)}\ \textsc{AUPExists}$$

$\boxed{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (M, \widehat{\sigma}_1, \widehat{\sigma}_2)}$

$$\frac{}{\Gamma \models \alpha^- \overset{a}{\simeq} \alpha^- \dashv (\alpha^-, \cdot, \cdot)}\ \textsc{AUNVar}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (Q, \widehat{\sigma}_1, \widehat{\sigma}_2)}{\Gamma \models {\uparrow}P_1 \overset{a}{\simeq} {\uparrow}P_2 \dashv ({\uparrow}Q, \widehat{\sigma}_1, \widehat{\sigma}_2)}\ \textsc{AUNShift}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (Q, \widehat{\sigma}_1, \widehat{\sigma}_2) \quad \Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (M, \widehat{\sigma}'_1, \widehat{\sigma}'_2)}{\Gamma \models P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (Q \to M, \widehat{\sigma}_1 \cup \widehat{\sigma}'_1, \widehat{\sigma}_2 \cup \widehat{\sigma}'_2)}\ \textsc{AUNArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \models N \overset{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, (\Gamma \vdash \widehat{\alpha}^-_{\{N,M\}} :\approx N), (\Gamma \vdash \widehat{\alpha}^-_{\{N,M\}} :\approx M))}\ \textsc{AUNAU}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^-) = \alpha^-}\quad \textsc{NrmNVar}$$

$$\frac{\mathbf{nf}\,(P) = Q}{\mathbf{nf}\,(\uparrow P) = \uparrow Q}\quad \textsc{NrmShiftU}$$

$$\frac{\mathbf{nf}\,(P) = Q \quad \mathbf{nf}\,(N) = M}{\mathbf{nf}\,(P \to N) = Q \to M}\quad \textsc{NrmArrow}$$

$$\frac{\mathbf{nf}\,(N) = N' \quad \mathbf{ord}\,\{\overrightarrow{\alpha^+}\}\,\mathbf{in}\,N' = \overrightarrow{\alpha^+}'}{\mathbf{nf}\,(\forall\overrightarrow{\alpha^+}.N) = \forall\overrightarrow{\alpha^+}'.N'}\quad \textsc{NrmForall}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\alpha^+) = \alpha^+}\quad \textsc{NrmPVar}$$

$$\frac{\mathbf{nf}\,(N) = M}{\mathbf{nf}\,(\downarrow N) = \downarrow M}\quad \textsc{NrmShiftD}$$

$$\frac{\mathbf{nf}\,(P) = P' \quad \mathbf{ord}\,\{\overrightarrow{\alpha^-}\}\,\mathbf{in}\,P' = \overrightarrow{\alpha^-}'}{\mathbf{nf}\,(\exists\overrightarrow{\alpha^-}.P) = \exists\overrightarrow{\alpha^-}'.P'}\quad \textsc{NrmExists}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^-\{\Delta\}) = \widehat{\alpha}^-\{\Delta\}}\quad \textsc{NrmNUVar}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^+\{\Delta\}) = \widehat{\alpha}^+\{\Delta\}}\quad \textsc{NrmPUVar}$$

$\boxed{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}$

$$\frac{\alpha^- \in \mathit{vars}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\alpha^- = \alpha^-}\quad \textsc{ONVarIn}$$

$$\frac{\alpha^- \notin \mathit{vars}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\alpha^- = \cdot}\quad \textsc{ONVarNIn}$$

$$\frac{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\uparrow P = \vec{\alpha}}\quad \textsc{OShiftU}$$

$$\frac{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}_1 \quad \mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}_2}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P \to N = \vec{\alpha}_1, (\vec{\alpha}_2\backslash\{\vec{\alpha}_1\})}\quad \textsc{OArrow}$$

$$\frac{\mathit{vars} \cap \{\overrightarrow{\alpha^+}\} = \varnothing \quad \mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\forall\overrightarrow{\alpha^+}.N = \vec{\alpha}}\quad \textsc{OForall}$$

$\boxed{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}$

$$\frac{\alpha^+ \in \mathit{vars}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\alpha^+ = \alpha^+}\quad \textsc{OPVarIn}$$

$$\frac{\alpha^+ \notin \mathit{vars}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\alpha^+ = \cdot}\quad \textsc{OPVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ {\downarrow}N = \vec{\alpha}}\ \textsc{OShiftD}$$

$$\frac{\mathit{vars} \cap \{\overrightarrow{\alpha^-}\} = \varnothing \quad \mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \exists\overrightarrow{\alpha^-}.P = \vec{\alpha}}\ \textsc{OExists}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \widehat{\alpha}^-\{\Delta\} = \cdot}\ \textsc{ONUVar}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \widehat{\alpha}^+\{\Delta\} = \cdot}\ \textsc{OPUVar}$$

$\boxed{e_1\ \&\ e_2 = e_3}$ Unification Solution Entry Merge

$$\frac{\Gamma \models P_1 \vee P_2 = Q}{(\Gamma \vdash \widehat{\alpha}^+ :\geqslant P_1)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\geqslant P_2) = (\Gamma \vdash \widehat{\alpha}^+ :\geqslant Q)}\ \textsc{SMESupSup}$$

$$\frac{\Gamma \models P \geqslant Q \dashv \widehat{\sigma}'}{(\Gamma \vdash \widehat{\alpha}^+ :\approx P)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\geqslant Q) = (\Gamma \vdash \widehat{\alpha}^+ :\approx P)}\ \textsc{SMEEqSup}$$

$$\frac{\Gamma \models Q \geqslant P \dashv \widehat{\sigma}'}{(\Gamma \vdash \widehat{\alpha}^+ :\geqslant P)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\approx Q) = (\Gamma \vdash \widehat{\alpha}^+ :\approx Q)}\ \textsc{SMESupEq}$$

$$\frac{}{(\Gamma \vdash \widehat{\alpha}^+ :\approx P)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\approx P) = (\Gamma \vdash \widehat{\alpha}^+ :\approx P)}\ \textsc{SMEPEqEq}$$

$$\frac{}{(\Gamma \vdash \widehat{\alpha}^- :\approx N)\ \&\ (\Gamma \vdash \widehat{\alpha}^- :\approx N) = (\Gamma \vdash \widehat{\alpha}^- :\approx N)}\ \textsc{SMENEqEq}$$

$\boxed{\widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3}$ Merge unification solutions

$\boxed{N \stackrel{u}{\simeq} M \dashv \widehat{\sigma}}$ Negative unification

$$\frac{}{\alpha^- \stackrel{u}{\simeq} \alpha^- \dashv \cdot}\ \textsc{UNVar}$$

$$\frac{P \stackrel{u}{\simeq} Q \dashv \widehat{\sigma}}{{\uparrow}P \stackrel{u}{\simeq} {\uparrow}Q \dashv \widehat{\sigma}}\ \textsc{UShiftU}$$

$$\frac{P \stackrel{u}{\simeq} Q \dashv \widehat{\sigma}_1 \quad N \stackrel{u}{\simeq} M \dashv \widehat{\sigma}_2}{P \to N \stackrel{u}{\simeq} Q \to M \dashv \widehat{\sigma}_1\ \&\ \widehat{\sigma}_2}\ \textsc{UArrow}$$

$$\frac{N \stackrel{u}{\simeq} M \dashv \widehat{\sigma}}{\forall\overrightarrow{\alpha^+}.N \stackrel{u}{\simeq} \forall\overrightarrow{\alpha^+}.M \dashv \widehat{\sigma}}\ \textsc{UForall}$$

$$\frac{\mathbf{fv}\ N \subseteq \{\Delta\}}{\widehat{\alpha}^-\{\Delta\} \stackrel{u}{\simeq} N \dashv (\Delta \vdash \widehat{\alpha}^- :\approx N)}\ \textsc{UNUVar}$$

$\boxed{P \stackrel{u}{\simeq} Q \dashv \widehat{\sigma}}$ Positive unification

$$\frac{}{\alpha^+ \stackrel{u}{\simeq} \alpha^+ \dashv \cdot}\ \textsc{UPVar}$$

$$\frac{N \stackrel{u}{\simeq} M \dashv \widehat{\sigma}}{{\downarrow}N \stackrel{u}{\simeq} {\downarrow}M \dashv \widehat{\sigma}}\ \textsc{UShiftD}$$

$$\frac{\boxed{P} \overset{u}{\simeq} Q \dashv \widehat{\sigma}}{\exists\overrightarrow{\alpha^-}.\boxed{P} \overset{u}{\simeq} \exists\overrightarrow{\alpha^-}.Q \dashv \widehat{\sigma}} \quad \textsc{UExists}$$

$$\frac{\mathbf{fv}\, P \subseteq \{\Delta\}}{\widehat{\alpha}^+\{\Delta\} \overset{u}{\simeq} P \dashv (\Delta \vdash \widehat{\alpha}^+ :\approx P)} \quad \textsc{UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

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Definition rules:        88 good    0 bad
Definition rule clauses: 151 good    0 bad
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