$\alpha,\ \beta,\ \alpha,\ \beta,\ \gamma,\ \delta$ type variables
$n,\ m,\ i,\ j$ index variables
$x,\ y,\ z$ term variables

$n,\ m$	$::=$		cohort index
	$\mid$	0	
	$\mid$	$n+1$	

$\alpha^+,\ \beta^+,\ \gamma^+,\ \delta^+$	$::=$		positive variable
	$\mid$	α^+	
	$\mid$	α^{+n}	
	$\mid$	$\beta^{\pm}$	

$\alpha^-,\ \beta^-,\ \gamma^-,\ \delta^-$	$::=$		negative variable
	$\mid$	α^-	
	$\mid$	α^{-n}	
	$\mid$	$\alpha^{\pm}$	

$\alpha^{\pm},\ \beta^{\pm}$	$::=$		positive or negative variable
	$\mid$	$\alpha^{\pm}$	
	$\mid$	$\alpha^{\pm n}$	

σ	$::=$		substitution
	$\mid$	$\cdot$	
	$\mid$	id	
	$\mid$	$\mathbf{P}/\alpha^+$	
	$\mid$	$\mathbf{N}/\alpha^-$	
	$\mid$	$\overrightarrow{P}/\overrightarrow{\alpha^+}$	
	$\mid$	$\overrightarrow{N}/\overrightarrow{\alpha^-}$	
	$\mid$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha}^-}$	
	$\mid$	μ	
	$\mid$	$\sigma_1 \circ \sigma_2$	
	$\mid$	$\sigma_1 \lll \sigma_2$	
	$\mid$	$\overrightarrow{\alpha}_1/\overrightarrow{\alpha}_2$	
	$\mid$	(σ) S	
	$\mid$	$\overline{\sigma_i}^{\,i}$	concatenate
	$\mid$	$\mathbf{nf}\,(\sigma')$ M	
	$\mid$	$\sigma'|_{vars}$ M	

e	$::=$		entry of a subtyping constraint
	$\mid$	$\widehat{\alpha}^+ :\approx P$	
	$\mid$	$\widehat{\alpha}^- :\approx N$	
	$\mid$	$\widehat{\alpha}^+ :\geqslant P$	
	$\mid$	(e) S	
	$\mid$	$SC(\widehat{\alpha}^-)$ M	
	$\mid$	$e_1\ \&\ e_2$ M	

e	$::=$		entry of a unification constraint
	$\mid$	$\widehat{\alpha}^+ :\approx P$	
	$\mid$	$\widehat{\alpha}^- :\approx N$	
	$\mid$	(e) S	
	$\mid$	$e_1\ \&\ e_2$ M	

UC		$::=$			unification constraint
	$\mid$	$\cdot$			
	$\mid$	e			
	$\mid$	$UC \backslash vars$			
	$\mid$	$UC\mid vars$			
	$\mid$	$UC_1 \cup UC_2$			
	$\mid$	$\overrightarrow{UC_i}^{\,i}$			concatenate
	$\mid$	(UC)		S	
	$\mid$	$UC'\mid_{vars}$		M	
	$\mid$	$UC_1 \ \& \ UC_2$		M	
	$\mid$	$UC_1 \cup UC_2$		M	
	$\mid$	$\mid SC \mid$		M	

SC		$::=$			subtyping constraint
	$\mid$	$\cdot$			
	$\mid$	e			
	$\mid$	$SC \backslash vars$			
	$\mid$	$SC\mid vars$			
	$\mid$	$SC_1 \cup SC_2$			
	$\mid$	UC			
	$\mid$	$\overrightarrow{SC_i}^{\,i}$			concatenate
	$\mid$	(SC)		S	
	$\mid$	$SC'\mid_{vars}$		M	
	$\mid$	$SC_1 \ \& \ SC_2$		M	

$\widehat{\sigma}$		$::=$			unification substitution
	$\mid$	$\cdot$			
	$\mid$	$P/\widehat{\alpha}^+$			
	$\mid$	$N/\widehat{\alpha}^-$			
	$\mid$	$\overrightarrow{P}/\overrightarrow{\widehat{\alpha}^+}$			
	$\mid$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha}^-}$			
	$\mid$	$(\widehat{\sigma})$		S	
	$\mid$	$\widehat{\sigma}_1 \circ \widehat{\sigma}_2$			
	$\mid$	$\overrightarrow{\widehat{\sigma}_i}^{\,i}$			concatenate
	$\mid$	$\mathbf{nf}\,(\widehat{\sigma}')$		M	
	$\mid$	$\widehat{\sigma}'\mid_{vars}$		M	

$\widehat{\tau},\ \widehat{\rho}$		$::=$			anti-unification substitution
	$\mid$	$\cdot$			
	$\mid$	$\widehat{\alpha}^- :\approx N$			
	$\mid$	$\widehat{\alpha}^- :\approx N$			
	$\mid$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha}^-}$			
	$\mid$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha}^-}$			
	$\mid$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$			
	$\mid$	$\overrightarrow{\widehat{\tau}_i}^{\,i}$			concatenate
	$\mid$	$(\widehat{\tau})$		S	
	$\mid$	$\widehat{\tau}'\mid_{vars}$		M	
	$\mid$	$\widehat{\tau}_1 \ \& \ \widehat{\tau}_2$		M	

$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+},\ \overrightarrow{\gamma^+},\ \overrightarrow{\delta^+}$		$::=$			positive variable list

	.		empty list
	α^+		a variable
	$\overrightarrow{\alpha^+}$		a variable
	$\overrightarrow{\alpha^+}_i^{\ i}$		concatenate lists

$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$ ::= negative variables

	.		empty list
	α^-		a variable
	$\overrightarrow{\alpha^-}$		variables
	$\overrightarrow{\alpha^-}_i^{\ i}$		concatenate lists

$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$ ::= positive or negative variable list

	.		empty list
	$\alpha^\pm$		a variable
	$\overrightarrow{\mathbf{pa}}$		variables
	$\overrightarrow{\alpha^\pm}_i^{\ i}$		concatenate lists

$P,\ Q,\ R$ ::= positive declarative types

	α^+	
	$\downarrow N$	
	$\exists \overrightarrow{\alpha^-}.P$	
	$[\sigma]P$	M
	$[\widehat{\tau}]P$	M
	$[\widehat{\sigma}]P$	M
	$[\mu]P$	M
	(P)	S
	$P_1 \vee P_2$	M
	$\mathbf{nf}\,(P')$	M

$N,\ M,\ K$ ::= negative declarative types

	α^-	
	$\uparrow P$	
	$P \to N$	
	$\forall \overrightarrow{\alpha^+}.N$	
	$[\sigma]N$	M
	$[\widehat{\tau}]N$	M
	$[\mu]N$	M
	$[\widehat{\sigma}]N$	M
	(N)	S
	$\mathbf{nf}\,(N')$	M

$\overrightarrow{P},\ \overrightarrow{Q}$ ::= list of positive types

	.		empty list
	P		a singel type
	$[\sigma]\overrightarrow{P}$	M	
	$\overrightarrow{\overrightarrow{P}}_i^{\ i}$		concatenate lists
	$(\overrightarrow{P})$	S	
	$\mathbf{nf}\,(\overrightarrow{P'})$	M	

$\vec{N}$, $\vec{M}$	::=			list of negative types
	|	$\cdot$		empty list
	|	N		a singel type
	|	$[\sigma]\vec{N}$	M	
	|	$\overrightarrow{\vec{N_i}}^{\,i}$		concatenate lists
	|	$(\vec{N})$	S	
	|	$\mathbf{nf}\,(\vec{N'})$	M	

Δ, Γ	::=			declarative type context
	|	$\cdot$		empty context
	|	$\overrightarrow{\alpha^+}$		list of variables
	|	$\overrightarrow{\alpha^-}$		list of variables
	|	$\overrightarrow{\{\alpha^{\pm}\}}$		list of variables
	|	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	|	(Γ)	S	
	|	$\Gamma, \overrightarrow{\alpha^+}$	M	append a list of variables
	|	$\Gamma, \overrightarrow{\alpha^-}$	M	append a list of variables
	|	$\Gamma, \overrightarrow{\alpha^{\pm}}$	M	append a list of variables
	|	$\Theta(\widehat{\alpha}^+)$	M	
	|	$\Theta(\widehat{\alpha}^-)$	M	
	|	$\Gamma_1 \cup \Gamma_2$		
	|	$\Gamma_1 \cap vars$		
	|	$\Gamma_1 \cup \Gamma_2$	M	
	|	$\mathbf{fv}\,N$	M	
	|	$\mathbf{fv}\,P$	M	
	|	$\mathbf{fv}\,P$	M	
	|	$\mathbf{fv}\,N$	M	

Θ	::=			algorithmic variable context
	|	$\cdot$		empty context
	|	$\vec{\alpha}\{\Delta\}$		from an ordered list of variables
	|	$\widehat{\alpha}^+\{\Delta\}$		from a variable to a list
	|	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	|	(Θ)	S	
	|	$\Theta|_{vars}$		leave only those variables that are in the set
	|	$\Theta_1 \cup \Theta_2$		

Ξ	::=			anti-unification type variable context
	|	$\cdot$		empty context
	|	$\overrightarrow{\widehat{\alpha}^+}$		list of positive variables
	|	$\overrightarrow{\widehat{\alpha}^-}$		list of negative variables
	|	$\Xi, \overrightarrow{\widehat{\alpha}^+}$	M	append a list of variables
	|	$\Xi, \overrightarrow{\widehat{\alpha}^-}$	M	append a list of variables
	|	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	|	(Ξ)	S	
	|	$\Xi_1 \cup \Xi_2$		
	|	$\Xi_1 \cap vars$		
	|	$\Xi'|_{vars}$	M	
	|	$\mathbf{dom}\,(UC)$	M	
	|	$\mathbf{dom}\,(SC)$	M	

| $\textbf{dom}\,(\hat{\sigma})$ M

| $\textbf{dom}\,(\hat{\tau})$ M

| $\textbf{dom}\,(\Theta)$ M

| $\textbf{uv}\,N$ M

| $\textbf{uv}\,P$ M

$\vec{\alpha},\ \vec{\beta}$	::=			ordered positive or negative variables
	|	$\cdot$		empty list
	|	$\overrightarrow{\alpha^+}$		list of variables
	|	$\overrightarrow{\alpha^-}$		list of variables
	|	$\overrightarrow{\alpha^{\pm}}$		list of variables
	|	$\overrightarrow{\hat{\alpha}^+}$		list of variables
	|	$\overrightarrow{\hat{\alpha}^-}$		list of variables
	|	$\vec{\alpha}_1 \backslash vars$		setminus
	|	$vars$		
	|	$\overrightarrow{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	|	$(\vec{\alpha})$	S	parenthesis
	|	$[\mu]\vec{\alpha}$	M	apply moving to list
	|	$[\vec{\mu}]\vec{\alpha}$	M	apply umoving to list
	|	$\textbf{ord}\,vars\,\textbf{in}\,P$	M	
	|	$\textbf{ord}\,vars\,\textbf{in}\,N$	M	
	|	$\textbf{ord}\,vars\,\textbf{in}\,P$	M	
	|	$\textbf{ord}\,vars\,\textbf{in}\,N$	M	

$vars$	::=			set of variables
	|	$\varnothing$		empty set
	|	$vars_1 \cap vars_2$		set intersection
	|	$vars_1 \cup vars_2$		set union
	|	$vars_1 \backslash vars_2$		set complement
	|	$(vars)$	S	parenthesis
	|	$\vec{\alpha}$		ordered list of variables
	|	$[\mu]vars$	M	apply moving to varset
	|	Ξ		algorithmic type context
	|	Γ		declarative type context

μ	::=			
	|	$\cdot$		empty moving
	|	$pma1 \mapsto pma2$		Positive unit substitution
	|	$nma1 \mapsto nma2$		Positive unit substitution
	|	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	|	$\mu_1 \circ \mu_2$	M	Composition
	|	$\overline{\mu_i}^{\,i}$		concatenate movings
	|	$\mu|_{vars}$	M	restriction on a set
	|	μ^{-1}	M	inversion
	|	$\textbf{nf}\,(\mu')$	M	

$\vec{\mu}$	::=			
	|	$\cdot$		empty moving
	|	$\overrightarrow{\hat{\alpha}^+}/\overrightarrow{\alpha^+}$		
	|	$\overrightarrow{\hat{\alpha}^-}/\overrightarrow{\alpha^-}$		

$\widehat{\alpha}^{\pm}$	::=		positive/negative unification variable
	|	$\widehat{\alpha}^{\pm}$	

$\widehat{\alpha}^{+}$	::=		positive unification variable
	|	$\widehat{\alpha}^{+}$	
	|	$\widehat{\alpha}^{+}\{\Delta\}$	
	|	$\widehat{\alpha}^{\pm}$	

$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	::=		negative unification variable
	|	$\widehat{\alpha}^{-}$	
	|	$\widehat{\alpha}^{-}_{\{N,M\}}$	
	|	$\widehat{\alpha}^{-}\{\Delta\}$	
	|	$\widehat{\alpha}^{\pm}$	

$\overrightarrow{\widehat{\alpha}^{+}},\ \overrightarrow{\widehat{\beta}^{+}}$	::=		positive unification variable list
	|	$\cdot$	empty list
	|	$\widehat{\alpha}^{+}$	a variable
	|	$\overrightarrow{\widehat{\alpha}^{+}}$	from a normal variable, context unspecified
	|	$\overrightarrow{\overrightarrow{\widehat{\alpha}^{+}}}^{i}_{i}$	concatenate lists

$\overrightarrow{\widehat{\alpha}^{-}},\ \overrightarrow{\widehat{\beta}^{-}}$	::=		negative unification variable list
	|	$\cdot$	empty list
	|	$\widehat{\alpha}^{-}$	a variable
	|	Ξ	from an antiunification context
	|	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$	from a normal variable
	|	$\overrightarrow{\widehat{\alpha}^{-}}$	from a normal variable, context unspecified
	|	$\overrightarrow{\overrightarrow{\widehat{\alpha}^{-}}}^{i}_{i}$	concatenate lists

$P,\ Q$	::=		a positive algorithmic type (potentially with algorithmic variables)
	|	$\widehat{\alpha}^{+}$	
	|	α^{+}	
	|	$\downarrow N$	
	|	$\exists\overrightarrow{\widehat{\alpha}^{-}}.P$	
	|	$[\sigma]P$	M
	|	$[\widehat{\tau}]P$	M
	|	$[\mu]P$	M
	|	$[\widehat{\sigma}]P$	M
	|	$[\overrightarrow{\mu}]P$	M
	|	(P)	S
	|	$\mathbf{nf}\,(P')$	M

$N,\ M$	::=		a negative algorithmic type (potentially with algorithmic variables)
	|	$\widehat{\alpha}^{-}$	
	|	α^{-}	
	|	$\uparrow P$	
	|	$P \to N$	
	|	$\forall\overrightarrow{\alpha^{+}}.N$	
	|	$[\sigma]N$	M
	|	$[\widehat{\tau}]N$	M

	$[\mu]\,N$	M
	$[\widehat{\sigma}]\,N$	M
	$[\overrightarrow{\mu}]N$	M
	(N)	S
	$\mathbf{nf}\,(N')$	M

$auSol$ ::=

| | $(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$ |
| | $(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$ |

$terminals$ ::=

	$\exists$
	$\forall$
	$\uparrow$
	$\downarrow$
	$\rightarrow$
	$\leftrightarrow$
	$\in$
	$\notin$
	$\cdot$
	$\vdash$
	$\leqslant$
	$\geqslant$
	$\simeq$
	$\cup$
	$\cap$
	$\setminus$
	$\sqsubseteq$
	$\mapsto$
	$\overset{u}{\simeq}$
	$\overset{a}{\simeq}$
	$\varnothing$
	$\circ$
	$\Rightarrow$
	$\vDash$
	$\dashv$
	$\neq$
	$\equiv_n$
	$\vee$
	$\Downarrow$
	$:\geqslant$
	$:\simeq$
	Λ
	λ
	$\mathbf{let}^{\exists}$
	$\bullet$
	$\Rrightarrow$
	$\Lleftarrow$

$v,\ w$ ::=   value terms

| x
| $\{c\}$
| $(v : P)$
| (v) M

$\vec{v}$ ::=   list of arguments
| .
| v
| $\overrightarrow{\vec{v}_i}^{\,i}$ concatenate

$c,\ d$ ::=   computation terms
| $(c : N)$
| $\lambda x : P.c$
| $\Lambda \alpha^+.c$
| $\mathbf{return}\ v$
| $\mathbf{let}\ x = v; c$
| $\mathbf{let}\ x : P = v(\vec{v}); c$
| $\mathbf{let}\ x = v(\vec{v}); c$
| $\mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c$

$vctx,\ \Phi$ ::=   variable context
| .
| $x : P$
| $\overline{\Phi_i}^{\,i}$ concatenate contexts

$formula$ ::=
| $judgement$
| $judgement$ unique
| $formula_1\quad .. \quad formula_n$
| $\mu : vars_1 \leftrightarrow vars_2$
| $\mu\ \mathbf{is\ bijective}$
| $x : P \in \Phi$
| $UC_1 \subseteq UC_2$
| $UC_1 = UC_2$
| $SC_1 \subseteq SC_2$
| $e \in SC$
| $e \in UC$
| $vars_1 \subseteq vars_2$
| $vars_1 \subseteq vars_2 \subseteq vars_3$
| $vars_1 = vars_2$
| $vars\ \mathbf{are\ fresh}$
| $\alpha^- \notin vars$
| $\alpha^+ \notin vars$
| $\alpha^- \in vars$
| $\alpha^+ \in vars$
| $\widehat{\alpha}^+ \in vars$
| $\widehat{\alpha}^- \in vars$
| $\widehat{\alpha}^- \in \Theta$
| $\widehat{\alpha}^+ \in \Theta$
| $\widehat{\alpha}^- \notin vars$

| | $\widehat{\alpha}^+ \notin vars$
| | $\widehat{\alpha}^- \notin \Theta$
| | $\widehat{\alpha}^+ \notin \Theta$
| | $\widehat{\alpha}^- \in \Xi$
| | $\widehat{\alpha}^- \notin \Xi$
| | $\widehat{\alpha}^+ \in \Xi$
| | $\widehat{\alpha}^+ \notin \Xi$
| | if any other rule is not applicable
| | $\vec{\alpha}_1 = \vec{\alpha}_2$
| | $e_1 = e_2$
| | $e_1 = e_2$
| | $\widehat{\sigma}_1 = \widehat{\sigma}_2$
| | $N = M$
| | $\Theta \subseteq \Theta'$
| | $\vec{v}_1 = \vec{v}_2$
| | $\mathbf{N} \neq \mathbf{M}$
| | $\mathbf{P} \neq \mathbf{Q}$
| | $N \neq M$
| | $P \neq Q$
| | $P \neq Q$
| | $N \neq M$
| | $\vec{v}_1 \neq \vec{v}_2$
| | $\overrightarrow{\alpha^+}_1 \neq \overrightarrow{\alpha^+}_2$
| | $|\overrightarrow{\alpha^-}| + |\overrightarrow{\beta^-}| > 0$
| | $|\overrightarrow{\alpha^+}| + |\overrightarrow{\beta^+}| > 0$

A ::=

| | $\Gamma; \Theta \vDash N \leqslant M \dashv SC$ Negative subtyping
| | $\Gamma; \Theta \vDash P \geqslant Q \dashv SC$ Positive supertyping

AT ::=

| | $\Gamma; \Phi \vDash v : P$ Positive type inference
| | $\Gamma; \Phi \vDash c : N$ Negative type inference
| | $\Gamma; \Phi; \Theta_1 \vDash N \bullet \vec{v} \Rrightarrow M \dashv \Theta_2; SC$ Application type inference

AU ::=

| | $\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$
| | $\Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$

SCM ::=

| | $\Gamma \vdash e_1 \ \& \ e_2 = e_3$ Subtyping Constraint Entry Merge
| | $\Theta \vdash SC_1 \& SC_2 = SC_3$ Merge of subtyping constraints

UCM ::=

| | $\Gamma \vdash e_1 \& e_2 = e_3$
| | $\Theta \vdash UC_1 \& UC_2 = UC_3$ Merge of unification constraints

$SATSCE$ ::=

| | $\Gamma \vdash P : e$ Positive type satisfies with the subtyping constr

| | $\Gamma \vdash N : e$ | Negative type satisfies with the subtyping constraint entry |

$SING$ ::=

	e_1 **singular with** P	Positive Subtyping Constraint Entry Is Singular
	e_1 **singular with** N	Negative Subtyping Constraint Entry Is Singular
	SC **singular with** $\widehat{\sigma}$	Subtyping Constraint Is Singular

$E1$ ::=

	$N \simeq^D M$	Negative type equivalence
	$P \simeq^D Q$	Positive type equivalence
	$P \simeq^D Q$	Positive unification type equivalence
	$N \simeq^D M$	Positive unification type equivalence

$D1$ ::=

	$\Gamma \vdash N \simeq^{\leqslant} M$	Negative subtyping-induced equivalence
	$\Gamma \vdash P \simeq^{\leqslant} Q$	Positive subtyping-induced equivalence
	$\Gamma \vdash N \leqslant M$	Negative subtyping
	$\Gamma \vdash P \geqslant Q$	Positive supertyping

$D1S$ ::=

	$\Gamma_2 \vdash \sigma_1 \simeq^{\leqslant} \sigma_2 : \Gamma_1$	Equivalence of substitutions
	$\Gamma \vdash \sigma_1 \simeq^{\leqslant} \sigma_2 : vars$	Equivalence of substitutions
	$\Theta \vdash \widehat{\sigma}_1 \simeq^{\leqslant} \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions
	$\Gamma \vdash \widehat{\sigma}_1 \simeq^{\leqslant} \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions

$D1C$ ::=

| | $\Gamma \vdash \Phi_1 \simeq^{\leqslant} \Phi_2$ | Equivalence of contexts |

DT ::=

	$\Gamma; \Phi \vdash v : P$	Positive type inference
	$\Gamma; \Phi \vdash c : N$	Negative type inference
	$\Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M$	Application type inference

EQ ::=

	$N = M$	Negative type equality (alpha-equivalence)
	$P = Q$	Positive type equuality (alphha-equivalence)
	$P = Q$	

$LUBF$ ::=

	$P_1 \vee P_2 === Q$	
	ord $vars$ **in** $P === \vec{\alpha}$	
	ord $vars$ **in** $N === \vec{\alpha}$	
	ord $vars$ **in** $P === \vec{\alpha}$	
	ord $vars$ **in** $N === \vec{\alpha}$	
	nf $(N') === N$	
	nf $(P') === P$	
	nf $(N') === N$	
	nf $(P') === P$	
	nf $(\vec{N'}) === \vec{N}$	

| $\mathbf{nf}\,(\vec{P'}) === \vec{P}$
| $\mathbf{nf}\,(\sigma') === \sigma$
| $\mathbf{nf}\,(\widehat{\sigma}') === \widehat{\sigma}$
| $\mathbf{nf}\,(\mu') === \mu$
| $\sigma'|_{vars}$
| $\widehat{\sigma}'|_{vars}$
| $\widehat{\tau}'|_{vars}$
| $\Xi'|_{vars}$
| $SC'|_{vars}$
| $UC'|_{vars}$
| $e_1 \ \& \ e_2$
| $e_1 \ \& \ e_2$
| $UC_1 \ \& \ UC_2$
| $UC_1 \cup UC_2$
| $\Gamma_1 \cup \Gamma_2$
| $SC_1 \ \& \ SC_2$
| $\widehat{\tau}_1 \ \& \ \widehat{\tau}_2$
| $\mathbf{dom}\,(UC) === \Xi$
| $\mathbf{dom}\,(SC) === \Xi$
| $\mathbf{dom}\,(\widehat{\sigma}) === \Xi$
| $\mathbf{dom}\,(\widehat{\tau}) === \Xi$
| $\mathbf{dom}\,(\Theta) === \Xi$
| $|SC| === UC$
| $\mathbf{fv}\ N === \Gamma$
| $\mathbf{fv}\ P === \Gamma$
| $\mathbf{fv}\ P === \Gamma$
| $\mathbf{fv}\ N === \Gamma$
| $\mathbf{uv}\ N === \Xi$
| $\mathbf{uv}\ P === \Xi$

LUB ::=
| $\Gamma \vDash P_1 \vee P_2 = Q$ Least Upper Bound (Least Common Supertype)
| $\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q$

Nrm ::=
| $\mathbf{nf}\,(N) = M$
| $\mathbf{nf}\,(P) = Q$
| $\mathbf{nf}\,(N) = M$
| $\mathbf{nf}\,(P) = Q$

$Order$ ::=
| $\mathbf{ord}\,vars\,\mathbf{in}\,N = \vec{\alpha}$
| $\mathbf{ord}\,vars\,\mathbf{in}\,P = \vec{\alpha}$
| $\mathbf{ord}\,vars\,\mathbf{in}\,N = \vec{\alpha}$
| $\mathbf{ord}\,vars\,\mathbf{in}\,P = \vec{\alpha}$

U ::=
| $\Gamma; \Theta \vDash N \stackrel{u}{\simeq} M \dashv UC$ Negative unification
| $\Gamma; \Theta \vDash P \stackrel{u}{\simeq} Q \dashv UC$ Positive unification

WFT	::=		
	|	$\Gamma \vdash N$	Negative type well-formedness
	|	$\Gamma \vdash P$	Positive type well-formedness

$WFAT$	::=		
	|	$\Gamma; \Xi \vdash N$	Negative algorithmic type well-formedness
	|	$\Gamma; \Xi \vdash P$	Positive algorithmic type well-formedness

$WFALL$	::=		
	|	$\Gamma \vdash \vec{N}$	Negative type list well-formedness
	|	$\Gamma \vdash \vec{P}$	Positive type list well-formedness
	|	$\Gamma; \Xi_2 \vdash \hat{\tau} : \Xi_1$	Antiunification substitution well-formedness
	|	$\Gamma \vdash^{\supseteq} \Theta$	Unification context well-formedness
	|	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution signature
	|	$\Theta \vdash \hat{\sigma} : \Xi$	Unification substitution signature
	|	$\Gamma \vdash \hat{\sigma} : \Xi$	Unification substitution general signature
	|	$\Theta \vdash \hat{\sigma} : UC$	Unification substitution satisfies unification constraint
	|	$\Theta \vdash \hat{\sigma} : SC$	Unification substitution satisfies subtyping constraint
	|	$\Gamma \vdash e$	Unification constraint entry well-formedness
	|	$\Gamma \vdash e$	Subtyping constraint entry well-formedness
	|	$\Gamma \vdash P : e$	Positive type satisfies unification constraint
	|	$\Gamma \vdash N : e$	Negative type satisfies unification constraint
	|	$\Gamma \vdash P : e$	Positive type satisfies subtyping constraint
	|	$\Gamma \vdash N : e$	Negative type satisfies subtyping constraint
	|	$\Theta \vdash UC : \Xi$	Unification constraint well-formedness with specified domain
	|	$\Theta \vdash SC : \Xi$	Subtyping constraint well-formedness with specified domain
	|	$\Theta \vdash UC$	Unification constraint well-formedness
	|	$\Theta \vdash SC$	Subtyping constraint well-formedness
	|	$\Gamma \vdash \vec{v}$	Argument List well-formedness
	|	$\Gamma \vdash \Phi$	Context well-formedness
	|	$\Gamma \vdash v$	Value well-formedness
	|	$\Gamma \vdash c$	Computation well-formedness

$judgement$	::=	
	|	A
	|	AT
	|	AU
	|	SCM
	|	UCM
	|	$SATSCE$
	|	$SING$
	|	$E1$
	|	$D1$
	|	$D1S$
	|	$D1C$
	|	DT
	|	EQ
	|	LUB
	|	Nrm
	|	$Order$

	U
$\vert$	WFT
$\vert$	$WFAT$
$\vert$	$WFALL$

$user\_syntax$ $::=$

$\vert$	α
$\vert$	n
$\vert$	x
$\vert$	n
$\vert$	α^+
$\vert$	α^-
$\vert$	$\alpha^\pm$
$\vert$	σ
$\vert$	e
$\vert$	e
$\vert$	UC
$\vert$	SC
$\vert$	$\hat{\sigma}$
$\vert$	$\hat{\tau}$
$\vert$	$\overrightarrow{\alpha^+}$
$\vert$	$\overrightarrow{\alpha^-}$
$\vert$	$\overrightarrow{\alpha^\pm}$
$\vert$	P
$\vert$	N
$\vert$	$\overrightarrow{P}$
$\vert$	$\overrightarrow{N}$
$\vert$	Γ
$\vert$	Θ
$\vert$	Ξ
$\vert$	$\overrightarrow{\alpha}$
$\vert$	$vars$
$\vert$	μ
$\vert$	$\overrightarrow{\mu}$
$\vert$	$\hat{\alpha}^\pm$
$\vert$	$\hat{\alpha}^+$
$\vert$	$\hat{\alpha}^-$
$\vert$	$\overrightarrow{\hat{\alpha}^+}$
$\vert$	$\overrightarrow{\hat{\alpha}^-}$
$\vert$	P
$\vert$	N
$\vert$	$auSol$
$\vert$	$terminals$
$\vert$	v
$\vert$	$\overrightarrow{v}$
$\vert$	c
$\vert$	$vctx$
$\vert$	$formula$

$\boxed{\Gamma;\ \Theta \models N \leqslant M \dashv SC}$ Negative subtyping

$$\frac{}{\Gamma; \Theta \models \alpha^- \leqslant \alpha^- \dashv \cdot} \quad \text{ANVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(P) \overset{u}{\simeq} \mathbf{nf}\,(Q) \dashv UC}{\Gamma; \Theta \models \uparrow P \leqslant \uparrow Q \dashv UC} \quad \text{AShiftU}$$

$$\frac{\Gamma; \Theta \models P \geqslant Q \dashv SC_1 \quad \Gamma; \Theta \models N \leqslant M \dashv SC_2 \quad \Theta \vdash SC_1 \& SC_2 = SC}{\Gamma; \Theta \models P \to N \leqslant Q \to M \dashv SC} \quad \text{AArrow}$$

$$\frac{\textcolor{red}{\texttt{<<multiple parses>>}}}{\Gamma; \Theta \models \overrightarrow{\forall \alpha^+}.N \leqslant \overrightarrow{\forall \beta^+}.M \dashv \overrightarrow{SC \backslash \widehat{\alpha^+}}} \quad \text{AForall}$$

$\boxed{\Gamma; \Theta \models P \geqslant Q \dashv SC}$ Positive supertyping

$$\frac{}{\Gamma; \Theta \models \alpha^+ \geqslant \alpha^+ \dashv \cdot} \quad \text{APVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(N) \overset{u}{\simeq} \mathbf{nf}\,(M) \dashv UC}{\Gamma; \Theta \models \downarrow N \geqslant \downarrow M \dashv UC} \quad \text{AShiftD}$$

$$\frac{\overrightarrow{\widehat{\alpha^-}}\,\textbf{are fresh} \quad \Gamma, \overrightarrow{\beta^-}; \Theta, \overrightarrow{\widehat{\alpha^-}}\{\Gamma, \overrightarrow{\beta^-}\} \models [\overrightarrow{\widehat{\alpha^-}/\alpha^-}]P \geqslant Q \dashv SC}{\Gamma; \Theta \models \overrightarrow{\exists \alpha^-}.P \geqslant \overrightarrow{\exists \beta^-}.Q \dashv \overrightarrow{SC \backslash \widehat{\alpha^-}}} \quad \text{AExists}$$

$$\frac{\textbf{upgrade}\,\Gamma \vdash P\,\textbf{to}\,\Theta(\widehat{\alpha^+}) = Q}{\Gamma; \Theta \models \widehat{\alpha^+} \geqslant P \dashv (\widehat{\alpha^+} :\geqslant Q)} \quad \text{APUVar}$$

$\boxed{\Gamma; \Phi \models v : P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \models x : \mathbf{nf}\,(P)} \quad \text{ATVar}$$

$$\frac{\Gamma; \Phi \models c : N}{\Gamma; \Phi \models \{c\} : \downarrow N} \quad \text{ATThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma; \Phi \models v : P \quad \Gamma; \cdot \models Q \geqslant P \dashv \cdot}{\Gamma; \Phi \models (v : Q) : \mathbf{nf}\,(Q)} \quad \text{ATPAnnot}$$

$\boxed{\Gamma; \Phi \models c : N}$ Negative type inference

$$\frac{\Gamma \vdash M \quad \Gamma; \Phi \models c : N \quad \Gamma; \cdot \models N \leqslant M \dashv \cdot}{\Gamma; \Phi \models (c : M) : \mathbf{nf}\,(M)} \quad \text{ATNAnnot}$$

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi, x : P \models c : N}{\Gamma; \Phi \models \lambda x : P.c : \mathbf{nf}\,(P \to N)} \quad \text{ATtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \models c : N}{\Gamma; \Phi \models \Lambda \alpha^+.c : \mathbf{nf}\,(\forall \alpha^+.N)} \quad \text{ATTLam}$$

$$\frac{\Gamma; \Phi \models v : P}{\Gamma; \Phi \models \mathbf{return}\,v : \uparrow P} \quad \text{ATReturn}$$

$$\frac{\Gamma; \Phi \models v : P \quad \Gamma; \Phi, x : P \models c : N}{\Gamma; \Phi \models \mathbf{let}\,x = v; c : N} \quad \text{ATVarLet}$$

$$\frac{\begin{array}{c}\Gamma \vdash P \quad \Gamma; \Phi \models v : \downarrow M \\ \Gamma; \Phi; \cdot \models M \bullet \overrightarrow{v} \Rrightarrow \uparrow Q \dashv \Theta; SC_1 \quad \Gamma; \Theta \models \uparrow Q \leqslant \uparrow P \dashv SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC \quad \Gamma; \Phi, x : P \models c : N\end{array}}{\Gamma; \Phi \models \mathbf{let}\,x : P = v(\overrightarrow{v}); c : N} \quad \text{ATAppLetAnn}$$

$$\Gamma; \Phi \models v \colon \downarrow M \quad \Gamma; \Phi; \cdot \models M \bullet \vec{v} \Rrightarrow \uparrow Q \dashv \Theta; SC$$

<<multiple parses>>

$$\frac{\Gamma; \Phi, x \colon [\hat{\sigma}] \, Q \models c \colon N}{\Gamma; \Phi \models \mathbf{let}\ x = v(\vec{v}); c \colon N} \quad \text{ATAppLet}$$

$$\frac{\Gamma; \Phi \models v \colon \exists \overrightarrow{\alpha^{-}}.P \quad \Gamma, \overrightarrow{\alpha^{-}}; \Phi, x \colon P \models c \colon N \quad \Gamma \vdash N}{\Gamma; \Phi \models \mathbf{let}^{\exists}(\overrightarrow{\alpha^{-}}, x) = v; c \colon N} \quad \text{ATUnpack}$$

$\boxed{\Gamma; \Phi; \Theta_1 \models N \bullet \vec{v} \Rrightarrow M \dashv \Theta_2; SC}$ Application type inference

$$\frac{}{\Gamma; \Phi; \Theta \models N \bullet \cdot \Rrightarrow \mathbf{nf}\,(N) \dashv \Theta; \cdot} \quad \text{ATEmptyApp}$$

$$\frac{\begin{array}{c} \Gamma; \Phi \models v \colon P \quad \Gamma; \Theta \models Q \geqslant P \dashv SC_1 \quad \Gamma; \Phi; \Theta \models N \bullet \vec{v} \Rrightarrow M \dashv \Theta'; SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC \end{array}}{\Gamma; \Phi; \Theta \models Q \to N \bullet v, \vec{v} \Rrightarrow M \dashv \Theta'; SC} \quad \text{ATArrowApp}$$

<<multiple parses>>

$$\frac{\overrightarrow{\widehat{\alpha^{+}}}\ \mathbf{are\ fresh} \quad \vec{v} \neq \cdot \quad \overrightarrow{\alpha^{+}} \neq \cdot}{\text{<<multiple parses>>}} \quad \text{ATForallApp}$$

$\boxed{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \hat{\tau}_1, \hat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^{+} \overset{a}{\simeq} \alpha^{+} \dashv (\cdot, \alpha^{+}, \cdot, \cdot)} \quad \text{AUPVar}$$

$$\frac{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \hat{\tau}_1, \hat{\tau}_2)}{\Gamma \models \downarrow N_1 \overset{a}{\simeq} \downarrow N_2 \dashv (\Xi, \downarrow M, \hat{\tau}_1, \hat{\tau}_2)} \quad \text{AUShiftD}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma \models \exists \overrightarrow{\alpha^{-}}.P_1 \overset{a}{\simeq} \exists \overrightarrow{\alpha^{-}}.P_2 \dashv (\Xi, \exists \overrightarrow{\alpha^{-}}.Q, \hat{\tau}_1, \hat{\tau}_2)} \quad \text{AUExists}$$

$\boxed{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \hat{\tau}_1, \hat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^{-} \overset{a}{\simeq} \alpha^{-} \dashv (\cdot, \alpha^{-}, \cdot, \cdot)} \quad \text{AUNVar}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \hat{\tau}_1, \hat{\tau}_2)}{\Gamma \models \uparrow P_1 \overset{a}{\simeq} \uparrow P_2 \dashv (\Xi, \uparrow Q, \hat{\tau}_1, \hat{\tau}_2)} \quad \text{AUShiftU}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma \models \forall \overrightarrow{\alpha^{+}}.N_1 \overset{a}{\simeq} \forall \overrightarrow{\alpha^{+}}.N_2 \dashv (\Xi, \forall \overrightarrow{\alpha^{+}}.M, \hat{\tau}_1, \hat{\tau}_2)} \quad \text{AUForall}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \hat{\tau}_1, \hat{\tau}_2) \quad \Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \hat{\tau}_1', \hat{\tau}_2')}{\Gamma \models P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \hat{\tau}_1 \cup \hat{\tau}_1', \hat{\tau}_2 \cup \hat{\tau}_2')} \quad \text{AUArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \models N \overset{a}{\simeq} M \dashv (\widehat{\alpha}^{-}_{\{N,M\}}, \widehat{\alpha}^{-}_{\{N,M\}}, (\widehat{\alpha}^{-}_{\{N,M\}} :\approx N), (\widehat{\alpha}^{-}_{\{N,M\}} :\approx M))} \quad \text{AUAU}$$

$\boxed{\Gamma \vdash e_1 \ \& \ e_2 = e_3}$ Subtyping Constraint Entry Merge

$$\frac{\Gamma \models P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^{+} :\geqslant P_1) \ \& \ (\widehat{\alpha}^{+} :\geqslant P_2) = (\widehat{\alpha}^{+} :\geqslant Q)} \quad \text{SCMESupSup}$$

$$\frac{\Gamma; \cdot \models P \geqslant Q \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^{+} :\approx P) \ \& \ (\widehat{\alpha}^{+} :\geqslant Q) = (\widehat{\alpha}^{+} :\approx P)} \quad \text{SCMEEqSup}$$

$$\frac{\Gamma; \cdot \vDash \boxed{Q} \geqslant P \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P) \,\&\, (\widehat{\alpha}^+ :\approx Q) = (\widehat{\alpha}^+ :\approx Q)} \quad \text{SCMESUPEQ}$$

$$\frac{\text{\color{red}<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \,\&\, (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \text{SCMEPEQEQ}$$

$$\frac{\text{\color{red}<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N) \,\&\, (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \text{SCMENEQEQ}$$

$\boxed{\Theta \vdash SC_1 \& SC_2 = SC_3}$ Merge of subtyping constraints

$\boxed{\Gamma \vdash e_1 \& e_2 = e_3}$

$$\frac{\text{\color{red}<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P)\&(\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \text{UCMEPEQEQ}$$

$$\frac{\text{\color{red}<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N)\&(\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \text{UCMENEQEQ}$$

$\boxed{\Theta \vdash UC_1 \& UC_2 = UC_3}$ Merge of unification constraints

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies with the subtyping constraint entry

$$\frac{\Gamma \vdash P \geqslant Q}{\Gamma \vdash P : (\widehat{\alpha}^+ :\geqslant Q)} \quad \text{SATSCESUP}$$

$$\frac{\text{\color{red}<<multiple parses>>}}{\Gamma \vdash P : (\widehat{\alpha}^+ :\approx Q)} \quad \text{SATSCEPEQ}$$

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies with the subtyping constraint entry

$$\frac{\text{\color{red}<<multiple parses>>}}{\Gamma \vdash N : (\widehat{\alpha}^- :\approx M)} \quad \text{SATSCENEQ}$$

$\boxed{e_1 \textbf{ singular with } P}$ Positive Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^+ :\approx P \textbf{ singular with nf } (P)} \quad \text{SINGPEQ}$$

$$\frac{}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.\alpha^+ \textbf{ singular with } \alpha^+} \quad \text{SINGSUPVAR}$$

$$\frac{\text{\color{red}<<multiple parses>>}}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.{\downarrow}N \textbf{ singular with } \exists \alpha^-.{\downarrow}\alpha^-} \quad \text{SINGSUPSHIFT}$$

$\boxed{e_1 \textbf{ singular with } N}$ Negative Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^- :\approx N \textbf{ singular with nf } (N)} \quad \text{SINGNEQ}$$

$\boxed{SC \textbf{ singular with } \widehat{\sigma}}$ Subtyping Constraint Is Singular

$\boxed{N \simeq^D M}$ Negative type equivalence

$$\frac{}{\alpha^- \simeq^D \alpha^-} \quad \text{E1NVAR}$$

$$\frac{P \simeq^D Q}{{\uparrow}P \simeq^D {\uparrow}Q} \quad \text{E1SHIFTU}$$

$$\frac{P \simeq^D Q \quad N \simeq^D M}{P \to N \simeq^D Q \to M} \quad \text{E1ARROW}$$

$$\frac{\texttt{<<multiple parses>>}}{\overrightarrow{\forall \alpha^+}.N \simeq^D \overrightarrow{\forall \beta^+}.M} \quad \text{E1FORALL}$$

$\boxed{P \simeq^D Q}$ Positive type equivalence

$$\frac{}{\alpha^+ \simeq^D \alpha^+} \quad \text{E1PVAR}$$

$$\frac{N \simeq^D M}{\downarrow N \simeq^D \downarrow M} \quad \text{E1SHIFTD}$$

$$\frac{\texttt{<<multiple parses>>}}{\overrightarrow{\exists \alpha^-}.P \simeq^D \overrightarrow{\exists \beta^-}.Q} \quad \text{E1EXISTS}$$

$\boxed{P \simeq^D Q}$ Positive unification type equivalence
$\boxed{N \simeq^D M}$ Positive unification type equivalence
$\boxed{\Gamma \vdash N \simeq^\leqslant M}$ Negative subtyping-induced equivalence

$$\frac{\Gamma \vdash N \leqslant M \quad \Gamma \vdash M \leqslant N}{\Gamma \vdash N \simeq^\leqslant M} \quad \text{D1NDEF}$$

$\boxed{\Gamma \vdash P \simeq^\leqslant Q}$ Positive subtyping-induced equivalence

$$\frac{\Gamma \vdash P \geqslant Q \quad \Gamma \vdash Q \geqslant P}{\Gamma \vdash P \simeq^\leqslant Q} \quad \text{D1PDEF}$$

$\boxed{\Gamma \vdash N \leqslant M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant \alpha^-} \quad \text{D1NVAR}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant \uparrow Q} \quad \text{D1SHIFTU}$$

$$\frac{\Gamma \vdash P \geqslant Q \quad \Gamma \vdash N \leqslant M}{\Gamma \vdash P \rightarrow N \leqslant Q \rightarrow M} \quad \text{D1ARROW}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma, \overrightarrow{\beta^+} \vdash [\sigma]N \leqslant M}{\Gamma \vdash \overrightarrow{\forall \alpha^+}.N \leqslant \overrightarrow{\forall \beta^+}.M} \quad \text{D1FORALL}$$

$\boxed{\Gamma \vdash P \geqslant Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant \alpha^+} \quad \text{D1PVAR}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant \downarrow M} \quad \text{D1SHIFTD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vdash \sigma : \overrightarrow{\alpha^-} \quad \Gamma, \overrightarrow{\beta^-} \vdash [\sigma]P \geqslant Q}{\Gamma \vdash \overrightarrow{\exists \alpha^-}.P \geqslant \overrightarrow{\exists \beta^-}.Q} \quad \text{D1EXISTS}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq^\leqslant \sigma_2 : \Gamma_1}$ Equivalence of substitutions
$\boxed{\Gamma \vdash \sigma_1 \simeq^\leqslant \sigma_2 : vars}$ Equivalence of substitutions
$\boxed{\Theta \vdash \hat{\sigma}_1 \simeq^\leqslant \hat{\sigma}_2 : vars}$ Equivalence of unification substitutions
$\boxed{\Gamma \vdash \hat{\sigma}_1 \simeq^\leqslant \hat{\sigma}_2 : vars}$ Equivalence of unification substitutions
$\boxed{\Gamma \vdash \Phi_1 \simeq^\leqslant \Phi_2}$ Equivalence of contexts
$\boxed{\Gamma; \Phi \vdash v : P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \vdash x : P} \quad \text{DTVAR}$$

$$\frac{\Gamma;\Phi \vdash c : N}{\Gamma;\Phi \vdash \{c\} : \downarrow N} \quad \text{DTThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma;\Phi \vdash v : P \quad \Gamma \vdash Q \geqslant P}{\Gamma;\Phi \vdash (v : Q) : Q} \quad \text{DTPAnnot}$$

$$\frac{\text{\color{red}{<<multiple parses>>}}}{\Gamma;\Phi \vdash v : P'} \quad \text{DTPEquiv}$$

$\boxed{\Gamma;\Phi \vdash c : N}$ Negative type inference

$$\frac{\Gamma \vdash P \quad \Gamma;\Phi, x : P \vdash c : N}{\Gamma;\Phi \vdash \lambda x : P.c : P \to N} \quad \text{DTtLam}$$

$$\frac{\Gamma, \alpha^+;\Phi \vdash c : N}{\Gamma;\Phi \vdash \Lambda\alpha^+.c : \forall\alpha^+.N} \quad \text{DTTLam}$$

$$\frac{\Gamma;\Phi \vdash v : P}{\Gamma;\Phi \vdash \mathbf{return}\, v : \uparrow P} \quad \text{DTReturn}$$

$$\frac{\Gamma;\Phi \vdash v : P \quad \Gamma;\Phi, x : P \vdash c : N}{\Gamma;\Phi \vdash \mathbf{let}\, x = v; c : N} \quad \text{DTVarLet}$$

$$\frac{\Gamma;\Phi \vdash v : \downarrow M \quad \Gamma;\Phi \vdash M \bullet \vec{v} \Rrightarrow \uparrow Q \text{ unique} \quad \Gamma;\Phi, x : Q \vdash c : N}{\Gamma;\Phi \vdash \mathbf{let}\, x = v(\vec{v}); c : N} \quad \text{DTAppLet}$$

$$\frac{\begin{array}{c}\Gamma \vdash P \quad \Gamma;\Phi \vdash v : \downarrow M \\ \Gamma;\Phi \vdash M \bullet \vec{v} \Rrightarrow \uparrow Q \quad \Gamma \vdash \uparrow Q \leqslant \uparrow P \quad \Gamma;\Phi, x : P \vdash c : N\end{array}}{\Gamma;\Phi \vdash \mathbf{let}\, x : P = v(\vec{v}); c : N} \quad \text{DTAppLetAnn}$$

$$\frac{\begin{array}{c}\text{\color{red}{<<multiple parses>>}} \\ \Gamma, \overrightarrow{\alpha^-};\Phi, x : P \vdash c : N \quad \Gamma \vdash N\end{array}}{\Gamma;\Phi \vdash \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c : N} \quad \text{DTUnpack}$$

$$\frac{\text{\color{red}{<<multiple parses>>}}}{\Gamma;\Phi \vdash (c : M) : M} \quad \text{DTNAnnot}$$

$$\frac{\text{\color{red}{<<multiple parses>>}}}{\Gamma;\Phi \vdash c : N'} \quad \text{DTNEquiv}$$

$\boxed{\Gamma;\Phi \vdash N \bullet \vec{v} \Rrightarrow M}$ Application type inference

$$\frac{\text{\color{red}{<<multiple parses>>}}}{\Gamma;\Phi \vdash N \bullet \cdot \Rrightarrow N'} \quad \text{DTEmptyApp}$$

$$\frac{\Gamma;\Phi \vdash v : P \quad \Gamma \vdash Q \geqslant P \quad \Gamma;\Phi \vdash N \bullet \vec{v} \Rrightarrow M}{\Gamma;\Phi \vdash Q \to N \bullet v, \vec{v} \Rrightarrow M} \quad \text{DTArrowApp}$$

$$\frac{\begin{array}{c}\Gamma \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma;\Phi \vdash [\sigma]N \bullet \vec{v} \Rrightarrow M \\ \vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot\end{array}}{\Gamma;\Phi \vdash \forall\overrightarrow{\alpha^+}.N \bullet \vec{v} \Rrightarrow M} \quad \text{DTForallApp}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$

$\boxed{P_1 \vee P_2}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N}$

$\boxed{\textbf{nf}\,(N')}$

$\boxed{\textbf{nf}\,(P')}$

$\boxed{\textbf{nf}\,(\ N')}$

$\boxed{\textbf{nf}\,(\ P')}$

$\boxed{\textbf{nf}\,(\overrightarrow{N'})}$

$\boxed{\textbf{nf}\,(\overrightarrow{P'})}$

$\boxed{\textbf{nf}\,(\sigma')}$

$\boxed{\textbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\textbf{nf}\,(\mu')}$

$\boxed{\sigma'|_{\mathit{vars}}}$

$\boxed{\widehat{\sigma}'|_{\mathit{vars}}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{SC'|_{vars}}$

$\boxed{UC'|_{vars}}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{UC_1 \ \& \ UC_2}$

$\boxed{UC_1 \cup UC_2}$

$\boxed{\Gamma_1 \cup \Gamma_2}$

$\boxed{SC_1 \ \& \ SC_2}$

$\boxed{\widehat{\tau}_1 \ \& \ \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(UC)}$

$\boxed{\mathbf{dom}\,(SC)}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{\lVert SC \rVert}$

$\boxed{\mathbf{fv}\,N}$

$\boxed{\mathbf{fv}\,P}$

$\boxed{\mathbf{fv}\,P}$

$\boxed{\mathbf{fv}\,N}$

$\boxed{\mathbf{uv}\,N}$

$\boxed{\mathbf{uv}\,P}$

$\boxed{\Gamma \vDash P_1 \lor P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \vDash \alpha^+ \lor \alpha^+ = \alpha^+}\ \text{LUBVar}$$

$$\frac{\Gamma, \cdot \vDash \mathbf{nf}\,(\downarrow N) \overset{a}{\simeq} \mathbf{nf}\,(\downarrow M) \Dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \downarrow N \lor \downarrow M = \exists \overrightarrow{\alpha^-}.\overrightarrow{[\alpha^-/\Xi]}P}\ \text{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \vDash P_1 \lor P_2 = Q}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P_1 \lor \exists \overrightarrow{\beta^-}.P_2 = Q}\ \text{LUBExists}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P \,\mathbf{to}\, \Delta = Q}$

$$\frac{\color{red}\text{<<multiple parses>>} \\ \Delta, \overrightarrow{\beta^{\pm}}, \overrightarrow{\gamma^{\pm}} \vDash \overrightarrow{[\beta^{\pm}/\alpha^{\pm}]}P \lor \overrightarrow{[\gamma^{\pm}/\alpha^{\pm}]}P = Q}{\mathbf{upgrade}\,\Gamma \vdash P \,\mathbf{to}\, \Delta = Q}\ \text{LUBUpgrade}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^-) = \alpha^-}\ \text{NrmNVar}$$

$$\frac{\color{red}\text{<<multiple parses>>}}{\mathbf{nf}\,(\uparrow P) = \uparrow Q}\ \text{NrmShiftU}$$

$$\frac{\color{red}\text{<<multiple parses>>}}{\mathbf{nf}\,(P \to N) = Q \to M}\ \text{NrmArrow}$$

$$\boxed{\mathbf{nf}\,(P) = Q}$$

$$\frac{\color{red}\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\forall\overrightarrow{\alpha^+}.N) = \forall\overrightarrow{\alpha^+}'.N'}\ \text{NrmForall}$$

$$\frac{}{\mathbf{nf}\,(\alpha^+) = \alpha^+}\ \text{NrmPVar}$$

$$\frac{\color{red}\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\downarrow N) = \downarrow M}\ \text{NrmShiftD}$$

$$\frac{\color{red}\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\exists\overrightarrow{\alpha^-}.P) = \exists\overrightarrow{\alpha^-}'.P'}\ \text{NrmExists}$$

$$\boxed{\mathbf{nf}\,(N) = M}$$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^-) = \widehat{\alpha}^-}\ \text{NrmNUVar}$$

$$\boxed{\mathbf{nf}\,(P) = Q}$$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^+) = \widehat{\alpha}^+}\ \text{NrmPUVar}$$

$$\boxed{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,N = \vec{\alpha}}$$

$$\frac{\alpha^- \in \textit{vars}}{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,\alpha^- = \alpha^-}\ \text{ONVarIn}$$

$$\frac{\alpha^- \notin \textit{vars}}{\color{red}\texttt{<<multiple parses>>}}\ \text{ONVarNIn}$$

$$\frac{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,P = \vec{\alpha}}{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,\uparrow P = \vec{\alpha}}\ \text{OShiftU}$$

$$\frac{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,P = \vec{\alpha}_1 \quad \mathbf{ord}\,\textit{vars}\,\mathbf{in}\,N = \vec{\alpha}_2}{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,P \to N = \vec{\alpha}_1,(\vec{\alpha}_2\backslash\vec{\alpha}_1)}\ \text{OArrow}$$

$$\frac{\color{red}\texttt{<<multiple parses>>}}{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,\forall\overrightarrow{\alpha^+}.N = \vec{\alpha}}\ \text{OForall}$$

$$\boxed{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,P = \vec{\alpha}}$$

$$\frac{\alpha^+ \in \textit{vars}}{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,\alpha^+ = \alpha^+}\ \text{OPVarIn}$$

$$\frac{\alpha^+ \notin \textit{vars}}{\color{red}\texttt{<<multiple parses>>}}\ \text{OPVarNIn}$$

$$\frac{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,N = \vec{\alpha}}{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,\downarrow N = \vec{\alpha}}\ \text{OShiftD}$$

$$\frac{\color{red}\texttt{<<multiple parses>>}}{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,\exists\overrightarrow{\alpha^-}.P = \vec{\alpha}}\ \text{OExists}$$

$$\boxed{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,N = \vec{\alpha}}$$

$$\frac{}{\color{red}\texttt{<<multiple parses>>}}\ \text{ONUVar}$$

$$\boxed{\mathbf{ord}\,\textit{vars}\,\mathbf{in}\,P = \vec{\alpha}}$$

$$\frac{}{\text{\textcolor{red}{<<multiple parses>>}}} \text{ OPUVar}$$

$\boxed{\Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv UC}$ Negative unification

$$\frac{}{\Gamma; \Theta \models \alpha^- \stackrel{u}{\simeq} \alpha^- \dashv \cdot} \text{ UNVar}$$

$$\frac{\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \uparrow P \stackrel{u}{\simeq} \uparrow Q \dashv UC} \text{ UShiftU}$$

$$\frac{\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC_1 \quad \Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv UC_2}{\Gamma; \Theta \models P \to N \stackrel{u}{\simeq} Q \to M \dashv UC_1 \,\&\, UC_2} \text{ UArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Theta \models N \stackrel{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \forall \overrightarrow{\alpha^+}.N \stackrel{u}{\simeq} \forall \overrightarrow{\alpha^+}.M \dashv UC} \text{ UForall}$$

$$\frac{\Theta(\widehat{\alpha}^-) \vdash N}{\Gamma; \Theta \models \widehat{\alpha}^- \stackrel{u}{\simeq} N \dashv (\widehat{\alpha}^- :\approx N)} \text{ UNUVar}$$

$\boxed{\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC}$ Positive unification

$$\frac{}{\Gamma; \Theta \models \alpha^+ \stackrel{u}{\simeq} \alpha^+ \dashv \cdot} \text{ UPVar}$$

$$\frac{\Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \downarrow N \stackrel{u}{\simeq} \downarrow M \dashv UC} \text{ UShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \exists \overrightarrow{\alpha^-}.P \stackrel{u}{\simeq} \exists \overrightarrow{\alpha^-}.Q \dashv UC} \text{ UExists}$$

$$\frac{\Theta(\widehat{\alpha}^+) \vdash P}{\Gamma; \Theta \models \widehat{\alpha}^+ \stackrel{u}{\simeq} P \dashv (\widehat{\alpha}^+ :\approx P)} \text{ UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma \vdash \alpha^-} \text{ WFTNVar}$$

$$\frac{\Gamma \vdash P}{\Gamma \vdash \uparrow P} \text{ WFTShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash N}{\Gamma \vdash P \to N} \text{ WFTArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+} \vdash N}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N} \text{ WFTForall}$$

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma \vdash \alpha^+} \text{ WFTPVar}$$

$$\frac{\Gamma \vdash N}{\Gamma \vdash \downarrow N} \text{ WFTShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-} \vdash P}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P} \quad \text{WFTEXISTS}$$

$\boxed{\Gamma; \Xi \vdash N}$ Negative algorithmic type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma; \Xi \vdash \alpha^-} \quad \text{WFATNVAR}$$

$$\frac{\widehat{\alpha}^- \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^-} \quad \text{WFATNUVAR}$$

$$\frac{\Gamma; \Xi \vdash P}{\Gamma; \Xi \vdash {\uparrow} P} \quad \text{WFATSHIFTU}$$

$$\frac{\Gamma; \Xi \vdash P \quad \Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash P \to N} \quad \text{WFATARROW}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Xi \vdash N}{\Gamma; \Xi \vdash \forall \overrightarrow{\alpha^+}.N} \quad \text{WFATFORALL}$$

$\boxed{\Gamma; \Xi \vdash P}$ Positive algorithmic type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma; \Xi \vdash \alpha^+} \quad \text{WFATPVAR}$$

$$\frac{\widehat{\alpha}^+ \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^+} \quad \text{WFATPUVAR}$$

$$\frac{\Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash {\downarrow} N} \quad \text{WFATSHIFTD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Xi \vdash P}{\Gamma; \Xi \vdash \exists \overrightarrow{\alpha^-}.P} \quad \text{WFATEXISTS}$$

$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness

$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness

$\boxed{\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness

$\boxed{\Gamma \vdash^{\sqsupseteq} \Theta}$ Unification context well-formedness

$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution signature

$\boxed{\Theta \vdash \widehat{\sigma} : \Xi}$ Unification substitution signature

$\boxed{\Gamma \vdash \widehat{\sigma} : \Xi}$ Unification substitution general signature

$\boxed{\Theta \vdash \widehat{\sigma} : UC}$ Unification substitution satisfies unification constraint

$\boxed{\Theta \vdash \widehat{\sigma} : SC}$ Unification substitution satisfies subtyping constraint

$\boxed{\Gamma \vdash e}$ Unification constraint entry well-formedness

$\boxed{\Gamma \vdash e}$ Subtyping constraint entry well-formedness

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies unification constraint

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies unification constraint

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies subtyping constraint

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies subtyping constraint

$\boxed{\Theta \vdash UC : \Xi}$ Unification constraint well-formedness with specified domain

$\boxed{\Theta \vdash SC : \Xi}$ Subtyping constraint well-formedness with specified domain

$\boxed{\Theta \vdash UC}$ Unification constraint well-formedness

$\boxed{\Theta \vdash SC}$ Subtyping constraint well-formedness

$\boxed{\Gamma \vdash \overrightarrow{v}}$ Argument List well-formedness

$\boxed{\Gamma \vdash \Phi}$ Context well-formedness

$\boxed{\Gamma \vdash v}$ Value well-formedness

$$\frac{}{\Gamma \vdash x} \quad \text{WFALLVAR}$$

$\boxed{\Gamma \vdash c}$ Computation well-formedness

$$\frac{\Gamma \vdash v \quad \Gamma \vdash c \quad \Gamma \vdash \vec{v}}{\Gamma \vdash \mathbf{let}\ x = v(\vec{v}); c} \quad \text{WFALLAPPLET}$$

```
Definition rules:        94 good     33 bad
Definition rule clauses: 208 good     34 bad
```