α, β, α, β, γ, δ	type variables
n, m, i, j	index variables
x, y, z	term variables

$n,\ m$	::=			cohort index
	|	0		
	|	$n+1$		
$\alpha^+,\ \beta^+,\ \gamma^+,\ \delta^+$	::=			positive variable
	|	α^+		
	|	α^{+n}		
	|	$\beta^\pm$		
$\alpha^-,\ \beta^-,\ \gamma^-,\ \delta^-$	::=			negative variable
	|	α^-		
	|	α^{-n}		
	|	$\alpha^\pm$		
$\alpha^\pm,\ \beta^\pm$	::=			positive or negative variable
	|	$\alpha^\pm$		
	|	$\alpha^{\pm n}$		
σ	::=			substitution
	|	$\cdot$		
	|	id		
	|	$\mathbf{P}\,/\alpha^+$		
	|	$\mathbf{N}\,/\alpha^-$		
	|	$\vec{P}/\overrightarrow{\alpha^+}$		
	|	$\vec{N}/\overrightarrow{\alpha^-}$		
	|	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha}^-}$		
	|	μ		
	|	$\sigma_1 \circ \sigma_2$		
	|	$\sigma_1 \lll \sigma_2$		
	|	$\vec{\alpha}_1/\vec{\alpha}_2$		
	|	(σ)	S	
	|	$\overline{\sigma_i}^{\,i}$		concatenate
	|	$\mathbf{nf}\,(\sigma')$	M	
	|	$\sigma'\|_{vars}$	M	
e	::=			entry of a subtyping constraint
	|	$\widehat{\alpha}^+ :\simeq P$		
	|	$\widehat{\alpha}^- :\simeq N$		
	|	$\widehat{\alpha}^+ :\geqslant P$		
	|	(e)	S	
	|	$SC(\widehat{\alpha}^-)$	M	
	|	$e_1\ \&\ e_2$	M	
e	::=			entry of a unification constraint
	|	$\widehat{\alpha}^+ :\simeq P$		
	|	$\widehat{\alpha}^- :\simeq N$		
	|	(e)	S	
	|	$e_1\ \&\ e_2$	M	

UC	$::=$			unification constraint
	$\mid$	$\cdot$		
	$\mid$	e		
	$\mid$	$UC \backslash vars$		
	$\mid$	$UC \mid vars$		
	$\mid$	$UC_1 \cup UC_2$		
	$\mid$	$\overline{UC_i}^{\,i}$		concatenate
	$\mid$	(UC)	S	
	$\mid$	$UC'\vert_{vars}$	M	
	$\mid$	$UC_1 \mathbin{\&} UC_2$	M	
	$\mid$	$UC_1 \cup UC_2$	M	
	$\mid$	$\vert SC \vert$	M	
SC	$::=$			subtyping constraint
	$\mid$	$\cdot$		
	$\mid$	e		
	$\mid$	$SC \backslash vars$		
	$\mid$	$SC \mid vars$		
	$\mid$	$SC_1 \cup SC_2$		
	$\mid$	UC		
	$\mid$	$\overline{SC_i}^{\,i}$		concatenate
	$\mid$	(SC)	S	
	$\mid$	$SC'\vert_{vars}$	M	
	$\mid$	$SC_1 \mathbin{\&} SC_2$	M	
$\widehat{\sigma}$	$::=$			unification substitution
	$\mid$	$\cdot$		
	$\mid$	$P/\widehat{\alpha}^+$		
	$\mid$	$N/\widehat{\alpha}^-$		
	$\mid$	$\overrightarrow{P}/\overrightarrow{\widehat{\alpha}^+}$		
	$\mid$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha}^-}$		
	$\mid$	$(\widehat{\sigma})$	S	
	$\mid$	$\widehat{\sigma}_1 \circ \widehat{\sigma}_2$		
	$\mid$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\mid$	$\mathbf{nf}\,(\widehat{\sigma}')$	M	
	$\mid$	$\widehat{\sigma}'\vert_{vars}$	M	
$\widehat{\tau},\ \widehat{\rho}$	$::=$			anti-unification substitution
	$\mid$	$\cdot$		
	$\mid$	$\widehat{\alpha}^- \mapsto N$		
	$\mid$	$\widehat{\alpha}^- \mapsto N$		
	$\mid$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha}^-}$		
	$\mid$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha}^-}$		
	$\mid$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
	$\mid$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	$\mid$	$(\widehat{\tau})$	S	
	$\mid$	$\widehat{\tau}'\vert_{vars}$	M	
	$\mid$	$\widehat{\tau}_1 \mathbin{\&} \widehat{\tau}_2$	M	
$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+},\ \overrightarrow{\gamma^+},\ \overrightarrow{\delta^+}$	$::=$			positive variable list

	$\mid$	$\cdot$		empty list
	$\mid$	α^+		a variable
	$\mid$	$\overrightarrow{\alpha^+}$		a variable
	$\mid$	$\overrightarrow{\overrightarrow{\alpha^+}_i}^{i}$		concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$	$::=$			negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	α^-		a variable
	$\mid$	$\overrightarrow{\alpha^-}$		variables
	$\mid$	$\overrightarrow{\overrightarrow{\alpha^-}_i}^{i}$		concatenate lists
$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$	$::=$			positive or negative variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\alpha^\pm$		a variable
	$\mid$	$\overrightarrow{\alpha^\pm}$		variables
	$\mid$	$\overrightarrow{\overrightarrow{\alpha^\pm}_i}^{i}$		concatenate lists
$P,\ Q,\ R$	$::=$			positive declarative types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^-}.P$		
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\hat{\tau}]\,P$	M	
	$\mid$	$[\hat{\sigma}]\,P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	(P)	S	
	$\mid$	$P_1 \vee P_2$	M	
	$\mid$	$\mathbf{nf}\,(P')$	M	
$N,\ M,\ K$	$::=$			negative declarative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$\forall\overrightarrow{\alpha^+}.N$		
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\hat{\tau}]\,N$	M	
	$\mid$	$[\mu]N$	M	
	$\mid$	$[\hat{\sigma}]\,N$	M	
	$\mid$	(N)	S	
	$\mid$	$\mathbf{nf}\,(N')$	M	
$\vec{P},\ \vec{Q}$	$::=$			list of positive types
	$\mid$	$\cdot$		empty list
	$\mid$	P		a singel type
	$\mid$	$[\sigma]\vec{P}$	M	
	$\mid$	$\overrightarrow{\vec{P}_i}^{i}$		concatenate lists
	$\mid$	$(\vec{P})$	S	
	$\mid$	$\mathbf{nf}\,(\vec{P}')$	M	

$\overrightarrow{N},\ \overrightarrow{M}$	$::=$			list of negative types
	$\mid$	$\cdot$		empty list
	$\mid$	N		a singel type
	$\mid$	$[\sigma]\overrightarrow{N}$	M	
	$\mid$	$\overline{\overrightarrow{N}_i}^{\,i}$		concatenate lists
	$\mid$	$(\overrightarrow{N})$	S	
	$\mid$	$\mathbf{nf}\,(\overrightarrow{N'})$	M	
$\Delta,\ \Gamma$	$::=$			declarative type context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\alpha^{+}}$		list of variables
	$\mid$	$\overrightarrow{\alpha^{-}}$		list of variables
	$\mid$	$\{\overrightarrow{\alpha^{\pm}}\}$		list of variables
	$\mid$	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	$\mid$	(Γ)	S	
	$\mid$	$\Gamma, \overrightarrow{\alpha^{+}}$	M	append a list of variables
	$\mid$	$\Gamma, \overrightarrow{\alpha^{-}}$	M	append a list of variables
	$\mid$	$\Gamma, \overrightarrow{\alpha^{\pm}}$	M	append a list of variables
	$\mid$	$\Theta(\widehat{\alpha}^{+})$	M	
	$\mid$	$\Theta(\widehat{\alpha}^{-})$	M	
	$\mid$	$\Gamma_1 \cup \Gamma_2$		
	$\mid$	$\Gamma_1 \cap \mathit{vars}$		
	$\mid$	$\Gamma_1 \cup \Gamma_2$	M	
	$\mid$	$\mathbf{fv}\ N$	M	
	$\mid$	$\mathbf{fv}\ P$	M	
	$\mid$	$\mathbf{fv}\,P$	M	
	$\mid$	$\mathbf{fv}\,N$	M	
Θ	$::=$			algorithmic variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\vec{\alpha}\{\Delta\}$		from an ordered list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^{+}\{\Delta\}}$		from a variable to a list
	$\mid$	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	$\mid$	(Θ)	S	
	$\mid$	$\Theta\vert_{\mathit{vars}}$		leave only those variables that are in the set
	$\mid$	$\Theta_1 \cup \Theta_2$		
Ξ	$::=$			anti-unification type variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\widehat{\alpha}^{+}}$		list of positive variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}}$		list of negative variables
	$\mid$	$\Xi, \overrightarrow{\widehat{\alpha}^{+}}$	M	append a list of variables
	$\mid$	$\Xi, \overrightarrow{\widehat{\alpha}^{-}}$	M	append a list of variables
	$\mid$	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	$\mid$	(Ξ)	S	
	$\mid$	$\Xi_1 \cup \Xi_2$		
	$\mid$	$\Xi_1 \cap \mathit{vars}$		
	$\mid$	$\Xi'\vert_{\mathit{vars}}$	M	
	$\mid$	$\mathbf{dom}\,(UC)$	M	
	$\mid$	$\mathbf{dom}\,(SC)$	M	

	|	$\mathbf{dom}\,(\widehat{\sigma})$	M	
	|	$\mathbf{dom}\,(\widehat{\tau})$	M	
	|	$\mathbf{dom}\,(\Theta)$	M	
	|	$\mathbf{uv}\ N$	M	
	|	$\mathbf{uv}\ P$	M	
$\vec{\alpha},\ \vec{\beta}$	::=			ordered positive or negative variables
	|	$\cdot$		empty list
	|	$\overrightarrow{\alpha^{+}}$		list of variables
	|	$\overrightarrow{\alpha^{-}}$		list of variables
	|	$\overrightarrow{\alpha^{\pm}}$		list of variables
	|	$\overrightarrow{\widehat{\alpha}^{+}}$		list of variables
	|	$\overrightarrow{\widehat{\alpha}^{-}}$		list of variables
	|	$\vec{\alpha}_1 \backslash vars$		setminus
	|	$vars$		
	|	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	|	$(\vec{\alpha})$	S	parenthesis
	|	$[\mu]\vec{\alpha}$	M	apply moving to list
	|	$[\vec{\mu}]\vec{\alpha}$	M	apply umoving to list
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ P$	M	
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ N$	M	
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ P$	M	
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ N$	M	
$vars$	::=			set of variables
	|	$\varnothing$		empty set
	|	$vars_1 \cap vars_2$		set intersection
	|	$vars_1 \cup vars_2$		set union
	|	$vars_1 \backslash vars_2$		set complement
	|	$(vars)$	S	parenthesis
	|	$\vec{\alpha}$		ordered list of variables
	|	$[\mu]vars$	M	apply moving to varset
	|	Ξ		algorithmic type context
	|	Γ		declarative type context
μ	::=			
	|	$\cdot$		empty moving
	|	$pma1 \mapsto pma2$		Positive unit substitution
	|	$nma1 \mapsto nma2$		Positive unit substitution
	|	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	|	$\mu_1 \circ \mu_2$	M	Composition
	|	$\overline{\mu_i}^{\,i}$		concatenate movings
	|	$\mu\vert_{vars}$	M	restriction on a set
	|	μ^{-1}	M	inversion
	|	$\mathbf{nf}\,(\mu')$	M	
$\vec{\mu}$	::=			
	|	$\cdot$		empty moving
	|	$\overrightarrow{\widehat{\alpha}^{+}}/\overrightarrow{\alpha^{+}}$		
	|	$\overrightarrow{\widehat{\alpha}^{-}}/\overrightarrow{\alpha^{-}}$		

$\widehat{\alpha}^{\pm}$	$::=$			positive/negative unification variable
	$\mid$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	$::=$			positive unification variable
	$\mid$	$\widehat{\alpha}^{+}$		
	$\mid$	$\widehat{\alpha}^{+}\{\Delta\}$		
	$\mid$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-}$, $\widehat{\beta}^{-}$	$::=$			negative unification variable
	$\mid$	$\widehat{\alpha}^{-}$		
	$\mid$	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	$\mid$	$\widehat{\alpha}^{-}\{\Delta\}$		
	$\mid$	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widehat{\alpha}^{+}}$, $\overrightarrow{\widehat{\beta}^{+}}$	$::=$			positive unification variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\widehat{\alpha}^{+}$		a variable
	$\mid$	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	$\mid$	$\overline{\overrightarrow{\widehat{\alpha}^{+}}_{i}}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}}$, $\overrightarrow{\widehat{\beta}^{-}}$	$::=$			negative unification variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\widehat{\alpha}^{-}$		a variable
	$\mid$	Ξ		from an antiunification context
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	$\mid$	$\overline{\overrightarrow{\widehat{\alpha}^{-}}_{i}}^{\,i}$		concatenate lists
P, Q	$::=$			a positive algorithmic type
	$\mid$	$\widehat{\alpha}^{+}$		
	$\mid$	α^{+}		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^{-}}.P$		
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\widehat{\tau}]P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	$[\widehat{\sigma}]P$	M	
	$\mid$	$[\overrightarrow{\mu}]P$	M	
	$\mid$	(P)	S	
	$\mid$	$\mathbf{nf}\,(P')$	M	
N, M	$::=$			a negative algorithmic type
	$\mid$	$\widehat{\alpha}^{-}$		
	$\mid$	α^{-}		
	$\mid$	$\uparrow P$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$\forall\overrightarrow{\alpha^{+}}.N$		
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\widehat{\tau}]N$	M	

	$\mid$	$[\mu] N$	M
	$\mid$	$[\widehat{\sigma}] N$	M
	$\mid$	$[\vec{\mu}] N$	M
	$\mid$	(N)	S
	$\mid$	$\mathbf{nf}\ (N')$	M
$auSol$	$::=$		
	$\mid$	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	$\mid$	$(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$	
$terminals$	$::=$		
	$\mid$	$\exists$	
	$\mid$	$\forall$	
	$\mid$	$\uparrow$	
	$\mid$	$\downarrow$	
	$\mid$	$\rightarrow$	
	$\mid$	$\leftrightarrow$	
	$\mid$	$\in$	
	$\mid$	$\notin$	
	$\mid$	$\cdot$	
	$\mid$	$\vdash$	
	$\mid$	$\leqslant$	
	$\mid$	$\geqslant$	
	$\mid$	$\simeq$	
	$\mid$	$\cup$	
	$\mid$	$\cap$	
	$\mid$	$\setminus$	
	$\mid$	$\subseteq$	
	$\mid$	$\mapsto$	
	$\mid$	$\stackrel{u}{\simeq}$	
	$\mid$	$\stackrel{a}{\simeq}$	
	$\mid$	$\varnothing$	
	$\mid$	$\circ$	
	$\mid$	$\Rightarrow$	
	$\mid$	$\models$	
	$\mid$	$\dashv$	
	$\mid$	$\neq$	
	$\mid$	$\equiv_n$	
	$\mid$	$\vee$	
	$\mid$	$\Downarrow$	
	$\mid$	$:\geqslant$	
	$\mid$	$:\simeq$	
	$\mid$	Λ	
	$\mid$	λ	
	$\mid$	$\mathbf{let}^{\exists}$	
	$\mid$	$\bullet$	
	$\mid$	$\Rrightarrow$	
	$\mid$	$\lll$	
$v,\ w$	$::=$		value terms

	$\mid$	x	
	$\mid$	$\{c\}$	
	$\mid$	$(v : P)$	
	$\mid$	(v)	M
$\vec{v}$	$::=$		list of arguments
	$\mid$	$\cdot$	
	$\mid$	v	
	$\mid$	$\overline{\vec{v}_i}^{\,i}$	concatenate
$c,\ d$	$::=$		computation terms
	$\mid$	$(c : N)$	
	$\mid$	$\lambda x : P.c$	
	$\mid$	$\Lambda\alpha^{+}.c$	
	$\mid$	$\mathbf{return}\ v$	
	$\mid$	$\mathbf{let}\ x = v; c$	
	$\mid$	$\mathbf{let}\ x : P = v(\vec{v}); c$	
	$\mid$	$\mathbf{let}\ x = v(\vec{v}); c$	
	$\mid$	$\mathbf{let}^{\exists}(\overrightarrow{\alpha^{-}}, x) = v; c$	
$vctx,\ \Phi$	$::=$		variable context
	$\mid$	$\cdot$	
	$\mid$	$x : P$	
	$\mid$	$\overline{\Phi_i}^{\,i}$	concatenate contexts
$formula$	$::=$		
	$\mid$	$judgement$	
	$\mid$	$judgement$ unique	
	$\mid$	$formula_1 \quad .. \quad formula_n$	
	$\mid$	$\mu : vars_1 \leftrightarrow vars_2$	
	$\mid$	$\mu\ \mathbf{is\ bijective}$	
	$\mid$	$x : P \in \Phi$	
	$\mid$	$UC_1 \subseteq UC_2$	
	$\mid$	$UC_1 = UC_2$	
	$\mid$	$SC_1 \subseteq SC_2$	
	$\mid$	$e \in SC$	
	$\mid$	$e \in UC$	
	$\mid$	$vars_1 \subseteq vars_2$	
	$\mid$	$vars_1 \subseteq vars_2 \subseteq vars_3$	
	$\mid$	$vars_1 = vars_2$	
	$\mid$	$vars\ \mathbf{are\ fresh}$	
	$\mid$	$\alpha^{-} \notin vars$	
	$\mid$	$\alpha^{+} \notin vars$	
	$\mid$	$\alpha^{-} \in vars$	
	$\mid$	$\alpha^{+} \in vars$	
	$\mid$	$\widehat{\alpha}^{+} \in vars$	
	$\mid$	$\widehat{\alpha}^{-} \in vars$	
	$\mid$	$\widehat{\alpha}^{-} \in \Theta$	
	$\mid$	$\widehat{\alpha}^{+} \in \Theta$	
	$\mid$	$\widehat{\alpha}^{-} \notin vars$	

	|	$\widehat{\alpha}^+ \notin \mathit{vars}$	
	|	$\widehat{\alpha}^- \notin \Theta$	
	|	$\widehat{\alpha}^+ \notin \Theta$	
	|	$\widehat{\alpha}^- \in \Xi$	
	|	$\widehat{\alpha}^- \notin \Xi$	
	|	$\widehat{\alpha}^+ \in \Xi$	
	|	$\widehat{\alpha}^+ \notin \Xi$	
	|	if any other rule is not applicable	
	|	$\overrightarrow{\alpha}_1 = \overrightarrow{\alpha}_2$	
	|	$e_1 = e_2$	
	|	$e_1 = e_2$	
	|	$\widehat{\sigma}_1 = \widehat{\sigma}_2$	
	|	$N = M$	
	|	$\Theta \subseteq \Theta'$	
	|	$\overrightarrow{v}_1 = \overrightarrow{v}_2$	
	|	$\mathbf{N} \neq \mathbf{M}$	
	|	$\mathbf{P} \neq \mathbf{Q}$	
	|	$N \neq M$	
	|	$P \neq Q$	
	|	$P \neq Q$	
	|	$N \neq M$	
	|	$\overrightarrow{v}_1 \neq \overrightarrow{v}_2$	
	|	$\overrightarrow{\alpha^+}_1 \neq \overrightarrow{\alpha^+}_2$	
	|	$\lvert\overrightarrow{\alpha^-}\rvert + \lvert\overrightarrow{\beta^-}\rvert > 0$	
	|	$\lvert\overrightarrow{\alpha^+}\rvert + \lvert\overrightarrow{\beta^+}\rvert > 0$	
A	::=		
	|	$\Gamma; \Theta \models N \leqslant M \dashv SC$	Negative subtyping
	|	$\Gamma; \Theta \models P \geqslant Q \dashv SC$	Positive supertyping
AT	::=		
	|	$\Gamma; \Phi \models v : P$	Positive type inference
	|	$\Gamma; \Phi \models c : N$	Negative type inference
	|	$\Gamma; \Phi; \Theta_1 \models N \bullet \overrightarrow{v} \mathrel{\Rightarrow\!\!>} M \dashv \Theta_2; SC$	Application type inference
AU	::=		
	|	$\Gamma \models P_1 \stackrel{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	|	$\Gamma \models N_1 \stackrel{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$	
SCM	::=		
	|	$\Gamma \vdash e_1 \;\&\; e_2 = e_3$	Subtyping Constraint Entry Merge
	|	$\Theta \vdash SC_1 \& SC_2 = SC_3$	Merge of subtyping constraints
UCM	::=		
	|	$\Gamma \vdash e_1 \& e_2 = e_3$	
	|	$\Theta \vdash UC_1 \& UC_2 = UC_3$	Merge of unification constraints
$SATSCE$	::=		
	|	$\Gamma \vdash P : e$	Positive constraint entry satisfaction

	$\vert$	$\Gamma \vdash N : e$	Negative constraint entry satisfaction
SING	::=		
	$\vert$	e_1 **singular with** P	Positive Subtyping Constraint Entry Is Singular
	$\vert$	e_1 **singular with** N	Negative Subtyping Constraint Entry Is Singular
	$\vert$	SC **singular with** $\widehat{\sigma}$	Subtyping Constraint Is Singular
E1	::=		
	$\vert$	$N \simeq^D M$	Negative type equivalence
	$\vert$	$P \simeq^D Q$	Positive type equivalence
	$\vert$	$P \simeq^D Q$	Positive unification type equivalence
	$\vert$	$N \simeq^D M$	Positive unification type equivalence
D1	::=		
	$\vert$	$\Gamma \vdash N \simeq^{\leqslant} M$	Negative subtyping-induced equivalence
	$\vert$	$\Gamma \vdash P \simeq^{\leqslant} Q$	Positive subtyping-induced equivalence
	$\vert$	$\Gamma \vdash N \leqslant M$	Negative subtyping
	$\vert$	$\Gamma \vdash P \geqslant Q$	Positive supertyping
D1S	::=		
	$\vert$	$\Gamma_2 \vdash \sigma_1 \simeq^{\leqslant} \sigma_2 : \Gamma_1$	Equivalence of substitutions
	$\vert$	$\Gamma \vdash \sigma_1 \simeq^{\leqslant} \sigma_2 : vars$	Equivalence of substitutions
	$\vert$	$\Theta \vdash \widehat{\sigma}_1 \simeq^{\leqslant} \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions
	$\vert$	$\Gamma \vdash \widehat{\sigma}_1 \simeq^{\leqslant} \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions
D1C	::=		
	$\vert$	$\Gamma \vdash \Phi_1 \simeq^{\leqslant} \Phi_2$	Equivalence of contexts
DT	::=		
	$\vert$	$\Gamma; \Phi \vdash v : P$	Positive type inference
	$\vert$	$\Gamma; \Phi \vdash c : N$	Negative type inference
	$\vert$	$\Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!\!> M$	Application type inference
EQ	::=		
	$\vert$	$N = M$	Negative type equality (alpha-equivalence)
	$\vert$	$P = Q$	Positive type equuality (alphha-equivalence)
	$\vert$	$P = Q$	
LUBF	::=		
	$\vert$	$P_1 \vee P_2 === Q$	
	$\vert$	**ord** $vars$ **in** $P === \vec{\alpha}$	
	$\vert$	**ord** $vars$ **in** $N === \vec{\alpha}$	
	$\vert$	**ord** $vars$ **in** $P === \vec{\alpha}$	
	$\vert$	**ord** $vars$ **in** $N === \vec{\alpha}$	
	$\vert$	**nf** $(N') === N$	
	$\vert$	**nf** $(P') === P$	
	$\vert$	**nf** $(N') === N$	
	$\vert$	**nf** $(P') === P$	
	$\vert$	**nf** $(\vec{N}') === \vec{N}$	

	$\mid$	$\mathbf{nf}\,(\vec{P'}) === \vec{P}$	
	$\mid$	$\mathbf{nf}\,(\sigma') === \sigma$	
	$\mid$	$\mathbf{nf}\,(\widehat{\sigma}') === \widehat{\sigma}$	
	$\mid$	$\mathbf{nf}\,(\mu') === \mu$	
	$\mid$	$\sigma'\vert_{vars}$	
	$\mid$	$\widehat{\sigma}'\vert_{vars}$	
	$\mid$	$\widehat{\tau}'\vert_{vars}$	
	$\mid$	$\Xi'\vert_{vars}$	
	$\mid$	$SC'\vert_{vars}$	
	$\mid$	$UC'\vert_{vars}$	
	$\mid$	$e_1 \ \& \ e_2$	
	$\mid$	$e_1 \ \& \ e_2$	
	$\mid$	$UC_1 \ \& \ UC_2$	
	$\mid$	$UC_1 \cup UC_2$	
	$\mid$	$\Gamma_1 \cup \Gamma_2$	
	$\mid$	$SC_1 \ \& \ SC_2$	
	$\mid$	$\widehat{\tau}_1 \ \& \ \widehat{\tau}_2$	
	$\mid$	$\mathbf{dom}\,(UC) === \Xi$	
	$\mid$	$\mathbf{dom}\,(SC) === \Xi$	
	$\mid$	$\mathbf{dom}\,(\widehat{\sigma}) === \Xi$	
	$\mid$	$\mathbf{dom}\,(\widehat{\tau}) === \Xi$	
	$\mid$	$\mathbf{dom}\,(\Theta) === \Xi$	
	$\mid$	$\vert SC\vert === UC$	
	$\mid$	$\mathbf{fv}\ N === \Gamma$	
	$\mid$	$\mathbf{fv}\ P === \Gamma$	
	$\mid$	$\mathbf{fv}\, P === \Gamma$	
	$\mid$	$\mathbf{fv}\, N === \Gamma$	
	$\mid$	$\mathbf{uv}\ N === \Xi$	
	$\mid$	$\mathbf{uv}\ P === \Xi$	
LUB	$::=$		
	$\mid$	$\Gamma \models P_1 \vee P_2 = Q$	Least Upper Bound (Least Common Supertype)
	$\mid$	$\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q$	
Nrm	$::=$		
	$\mid$	$\mathbf{nf}\,(N) = M$	
	$\mid$	$\mathbf{nf}\,(P) = Q$	
	$\mid$	$\mathbf{nf}\,(N) = M$	
	$\mid$	$\mathbf{nf}\,(P) = Q$	
$Order$	$::=$		
	$\mid$	$\mathbf{ord}\ vars\,\mathbf{in}\,N = \vec{\alpha}$	variable ordering in a negative type
	$\mid$	$\mathbf{ord}\ vars\,\mathbf{in}\,P = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\,\mathbf{in}\ N = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\,\mathbf{in}\ P = \vec{\alpha}$	
U	$::=$		
	$\mid$	$\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC$	Negative unification
	$\mid$	$\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC$	Positive unification

WFT	::=		
	|	$\Gamma \vdash N$	Negative type well-formedness
	|	$\Gamma \vdash P$	Positive type well-formedness
WFAT	::=		
	|	$\Gamma; \Xi \vdash N$	Negative algorithmic type well-formedness
	|	$\Gamma; \Xi \vdash P$	Positive algorithmic type well-formedness
WFALL	::=		
	|	$\Gamma \vdash \overrightarrow{N}$	Negative type list well-formedness
	|	$\Gamma \vdash \overrightarrow{P}$	Positive type list well-formedness
	|	$\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1$	Antiunification substitution well-formedness
	|	$\Gamma \vdash^{\supseteq} \Theta$	Unification context well-formedness
	|	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution signature
	|	$\Theta \vdash \widehat{\sigma} : \Xi$	Unification substitution signature
	|	$\Gamma \vdash \widehat{\sigma} : \Xi$	Unification substitution general signature
	|	$\Theta \vdash \widehat{\sigma} : UC$	Unification substitution satisfies unification constraint
	|	$\Theta \vdash \widehat{\sigma} : SC$	Unification substitution satisfies subtyping constraint
	|	$\Gamma \vdash e$	Unification constraint entry well-formedness
	|	$\Gamma \vdash e$	Subtyping constraint entry well-formedness
	|	$\Gamma \vdash P : e$	Positive type satisfies unification constraint
	|	$\Gamma \vdash N : e$	Negative type satisfies unification constraint
	|	$\Gamma \vdash P : e$	Positive type satisfies subtyping constraint
	|	$\Gamma \vdash N : e$	Negative type satisfies subtyping constraint
	|	$\Theta \vdash UC : \Xi$	Unification constraint well-formedness with specified domain
	|	$\Theta \vdash SC : \Xi$	Subtyping constraint well-formedness with specified domain
	|	$\Theta \vdash UC$	Unification constraint well-formedness
	|	$\Theta \vdash SC$	Subtyping constraint well-formedness
	|	$\Gamma \vdash \overrightarrow{v}$	Argument List well-formedness
	|	$\Gamma \vdash \Phi$	Context well-formedness
	|	$\Gamma \vdash v$	Value well-formedness
	|	$\Gamma \vdash c$	Computation well-formedness
judgement	::=		
	|	*A*	
	|	*AT*	
	|	*AU*	
	|	*SCM*	
	|	*UCM*	
	|	*SATSCE*	
	|	*SING*	
	|	*E1*	
	|	*D1*	
	|	*D1S*	
	|	*D1C*	
	|	*DT*	
	|	*EQ*	
	|	*LUB*	
	|	*Nrm*	
	|	*Order*	

	$\|$	U
	$\|$	WFT
	$\|$	$WFAT$
	$\|$	$WFALL$
$user\_syntax$	$::=$	
	$\|$	α
	$\|$	n
	$\|$	x
	$\|$	n
	$\|$	α^+
	$\|$	α^-
	$\|$	$\alpha^\pm$
	$\|$	σ
	$\|$	e
	$\|$	e
	$\|$	UC
	$\|$	SC
	$\|$	$\widehat{\sigma}$
	$\|$	$\widehat{\tau}$
	$\|$	$\overrightarrow{\alpha^+}$
	$\|$	$\overrightarrow{\alpha^-}$
	$\|$	$\overrightarrow{\alpha^\pm}$
	$\|$	P
	$\|$	N
	$\|$	$\overrightarrow{P}$
	$\|$	$\overrightarrow{N}$
	$\|$	Γ
	$\|$	Θ
	$\|$	Ξ
	$\|$	$\overrightarrow{\alpha}$
	$\|$	$vars$
	$\|$	μ
	$\|$	$\overrightarrow{\mu}$
	$\|$	$\widehat{\alpha}^\pm$
	$\|$	$\widehat{\alpha}^+$
	$\|$	$\widehat{\alpha}^-$
	$\|$	$\overrightarrow{\widehat{\alpha}^+}$
	$\|$	$\overrightarrow{\widehat{\alpha}^-}$
	$\|$	P
	$\|$	N
	$\|$	$auSol$
	$\|$	$terminals$
	$\|$	v
	$\|$	$\overrightarrow{v}$
	$\|$	c
	$\|$	$vctx$
	$\|$	$formula$

$\boxed{\Gamma;\, \Theta \models N \leqslant M \dashv SC}$ Negative subtyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^- \leqslant \alpha^- \dashv \cdot}\ \text{ANVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(P) \overset{u}{\simeq} \mathbf{nf}\,(Q) \dashv UC}{\Gamma;\ \Theta \models \uparrow P \leqslant \uparrow Q \dashv UC}\ \text{AShiftU}$$

$$\frac{\begin{array}{l}\overrightarrow{\widehat{\alpha}^+} \textbf{ are fresh} \\ \texttt{<<multiple parses>>}\end{array}}{\Gamma;\ \Theta \models \forall \overrightarrow{\alpha^+}.N \leqslant \forall \overrightarrow{\beta^+}.M \dashv SC \backslash \overrightarrow{\widehat{\alpha}^+}}\ \text{AForall}$$

$$\frac{\begin{array}{l}\Gamma;\ \Theta \models P \geqslant Q \dashv SC_1 \quad \Gamma;\ \Theta \models N \leqslant M \dashv SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC\end{array}}{\Gamma;\ \Theta \models P \to N \leqslant Q \to M \dashv SC}\ \text{AArrow}$$

$\boxed{\Gamma;\ \Theta \models P \geqslant Q \dashv SC}$ Positive supertyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^+ \geqslant \alpha^+ \dashv \cdot}\ \text{APVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(N) \overset{u}{\simeq} \mathbf{nf}\,(M) \dashv UC}{\Gamma;\ \Theta \models \downarrow N \geqslant \downarrow M \dashv UC}\ \text{AShiftD}$$

$$\frac{\begin{array}{l}\overrightarrow{\widehat{\alpha}^-} \textbf{ are fresh} \\ \Gamma, \overrightarrow{\beta^-};\ \Theta, \overrightarrow{\widehat{\alpha}^-}\{\Gamma, \overrightarrow{\beta^-}\} \models [\overrightarrow{\widehat{\alpha}^-}/\overrightarrow{\alpha^-}]P \geqslant Q \dashv SC\end{array}}{\Gamma;\ \Theta \models \exists \overrightarrow{\alpha^-}.P \geqslant \exists \overrightarrow{\beta^-}.Q \dashv SC \backslash \overrightarrow{\widehat{\alpha}^-}}\ \text{AExists}$$

$$\frac{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Theta(\widehat{\alpha}^+) = Q}{\Gamma;\ \Theta \models \widehat{\alpha}^+ \geqslant P \dashv (\widehat{\alpha}^+ :\geqslant Q)}\ \text{APUVar}$$

$\boxed{\Gamma; \Phi \models v \colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \models x \colon \mathbf{nf}\,(P)}\ \text{ATVar}$$

$$\frac{\Gamma; \Phi \models c \colon N}{\Gamma; \Phi \models \{c\} \colon \downarrow N}\ \text{ATThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma; \Phi \models v \colon P \quad \Gamma;\ \cdot \models Q \geqslant P \dashv \cdot}{\Gamma; \Phi \models (v : Q) \colon \mathbf{nf}\,(Q)}\ \text{ATPAnnot}$$

$\boxed{\Gamma; \Phi \models c \colon N}$ Negative type inference

$$\frac{\Gamma \vdash M \quad \Gamma; \Phi \models c \colon N \quad \Gamma;\ \cdot \models N \leqslant M \dashv \cdot}{\Gamma; \Phi \models (c : M) \colon \mathbf{nf}\,(M)}\ \text{ATNAnnot}$$

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi, x : P \models c \colon N}{\Gamma; \Phi \models \lambda x : P.c \colon \mathbf{nf}\,(P \to N)}\ \text{ATtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \models c \colon N}{\Gamma; \Phi \models \Lambda \alpha^+.c \colon \mathbf{nf}\,(\forall \alpha^+.N)}\ \text{ATTLam}$$

$$\frac{\Gamma; \Phi \models v \colon P}{\Gamma; \Phi \models \mathbf{return}\, v \colon \uparrow P}\ \text{ATReturn}$$

$$\frac{\Gamma; \Phi \models v \colon P \quad \Gamma; \Phi, x : P \models c \colon N}{\Gamma; \Phi \models \mathbf{let}\, x = v; c \colon N}\ \text{ATVarLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma;\Phi \models v\colon {\downarrow}M \\ \Gamma;\Phi;\cdot \models M \bullet \vec{v} \Rightarrow\!\!\!> {\uparrow}Q \dashv \Theta; SC_1 \quad \Gamma;\Theta \models {\uparrow}Q \leqslant {\uparrow}P \dashv SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC \quad \Gamma;\Phi, x : P \models c\colon N\end{array}}{\Gamma;\Phi \models \mathbf{let}\, x : P = v(\vec{v}); c\colon N}\ \textsc{ATAppLetAnn}$$

$$\frac{\begin{array}{l}\Gamma;\Phi \models v\colon {\downarrow}M \quad \Gamma;\Phi;\cdot \models M \bullet \vec{v} \Rightarrow\!\!\!> {\uparrow}Q \dashv \Theta; SC \\ \texttt{<<multiple parses>>} \\ \Gamma;\Phi, x : [\hat{\sigma}]Q \models c\colon N\end{array}}{\Gamma;\Phi \models \mathbf{let}\, x = v(\vec{v}); c\colon N}\ \textsc{ATAppLet}$$

$$\frac{\Gamma;\Phi \models v\colon \exists\overrightarrow{\alpha^-}.P \quad \Gamma,\overrightarrow{\alpha^-};\Phi, x : P \models c\colon N \quad \Gamma \vdash N}{\Gamma;\Phi \models \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c\colon N}\ \textsc{ATUnpack}$$

$\boxed{\Gamma;\Phi;\Theta_1 \models N \bullet \vec{v} \Rightarrow\!\!\!> M \dashv \Theta_2; SC}$ Application type inference

$$\frac{}{\Gamma;\Phi;\Theta \models N \bullet \cdot \Rightarrow\!\!\!> \mathbf{nf}\,(N) \dashv \Theta; \cdot}\ \textsc{ATEmptyApp}$$

$$\frac{\begin{array}{l}\Gamma;\Phi \models v\colon P \quad \Gamma;\Theta \models Q \geqslant P \dashv SC_1 \quad \Gamma;\Phi;\Theta \models N \bullet \vec{v} \Rightarrow\!\!\!> M \dashv \Theta'; SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC\end{array}}{\Gamma;\Phi;\Theta \models Q \rightarrow N \bullet v, \vec{v} \Rightarrow\!\!\!> M \dashv \Theta'; SC}\ \textsc{ATArrowApp}$$

$$\frac{\begin{array}{l}\texttt{<<multiple parses>>} \\ \overrightarrow{\hat{\alpha}^+}\ \textbf{are fresh} \quad \vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot\end{array}}{\texttt{<<multiple parses>>}}\ \textsc{ATForallApp}$$

$\boxed{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \hat{\tau}_1, \hat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\cdot, \alpha^+, \cdot, \cdot)}\ \textsc{AUPVar}$$

$$\frac{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \hat{\tau}_1, \hat{\tau}_2)}{\Gamma \models {\downarrow}N_1 \overset{a}{\simeq} {\downarrow}N_2 \dashv (\Xi, {\downarrow}M, \hat{\tau}_1, \hat{\tau}_2)}\ \textsc{AUShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \models \exists\overrightarrow{\alpha^-}.P_1 \overset{a}{\simeq} \exists\overrightarrow{\alpha^-}.P_2 \dashv (\Xi, \exists\overrightarrow{\alpha^-}.Q, \hat{\tau}_1, \hat{\tau}_2)}\ \textsc{AUExists}$$

$\boxed{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \hat{\tau}_1, \hat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^- \overset{a}{\simeq} \alpha^- \dashv (\cdot, \alpha^-, \cdot, \cdot)}\ \textsc{AUNVar}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \hat{\tau}_1, \hat{\tau}_2)}{\Gamma \models {\uparrow}P_1 \overset{a}{\simeq} {\uparrow}P_2 \dashv (\Xi, {\uparrow}Q, \hat{\tau}_1, \hat{\tau}_2)}\ \textsc{AUShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \models \forall\overrightarrow{\alpha^+}.N_1 \overset{a}{\simeq} \forall\overrightarrow{\alpha^+}.N_2 \dashv (\Xi, \forall\overrightarrow{\alpha^+}.M, \hat{\tau}_1, \hat{\tau}_2)}\ \textsc{AUForall}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \hat{\tau}_1, \hat{\tau}_2) \quad \Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \hat{\tau}_1', \hat{\tau}_2')}{\Gamma \models P_1 \rightarrow N_1 \overset{a}{\simeq} P_2 \rightarrow N_2 \dashv (\Xi_1 \cup \Xi_2, Q \rightarrow M, \hat{\tau}_1 \cup \hat{\tau}_1', \hat{\tau}_2 \cup \hat{\tau}_2')}\ \textsc{AUArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \models N \overset{a}{\simeq} M \dashv (\hat{\alpha}^-_{\{N,M\}}, \hat{\alpha}^-_{\{N,M\}}, (\hat{\alpha}^-_{\{N,M\}} \mapsto N), (\hat{\alpha}^-_{\{N,M\}} \mapsto M))}\ \textsc{AUAU}$$

$\boxed{\Gamma \vdash e_1 \ \&\ e_2 = e_3}$ Subtyping Constraint Entry Merge

$$\frac{\Gamma \vDash P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1) \mathbin{\&} (\widehat{\alpha}^+ :\geqslant P_2) = (\widehat{\alpha}^+ :\geqslant Q)} \quad \text{SCMESupSup}$$

$$\frac{\Gamma; \cdot \vDash P \geqslant Q \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\simeq P) \mathbin{\&} (\widehat{\alpha}^+ :\geqslant Q) = (\widehat{\alpha}^+ :\simeq P)} \quad \text{SCMEEqSup}$$

$$\frac{\Gamma; \cdot \vDash Q \geqslant P \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P) \mathbin{\&} (\widehat{\alpha}^+ :\simeq Q) = (\widehat{\alpha}^+ :\simeq Q)} \quad \text{SCMESupEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\simeq P) \mathbin{\&} (\widehat{\alpha}^+ :\simeq P') = (\widehat{\alpha}^+ :\simeq P)} \quad \text{SCMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\simeq N) \mathbin{\&} (\widehat{\alpha}^- :\simeq N') = (\widehat{\alpha}^- :\simeq N)} \quad \text{SCMENEqEq}$$

$\boxed{\Theta \vdash SC_1 \& SC_2 = SC_3}$ Merge of subtyping constraints

$\boxed{\Gamma \vdash e_1 \& e_2 = e_3}$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\simeq P) \& (\widehat{\alpha}^+ :\simeq P') = (\widehat{\alpha}^+ :\simeq P)} \quad \text{UCMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\simeq N) \& (\widehat{\alpha}^- :\simeq N') = (\widehat{\alpha}^- :\simeq N)} \quad \text{UCMENEqEq}$$

$\boxed{\Theta \vdash UC_1 \& UC_2 = UC_3}$ Merge of unification constraints

$\boxed{\Gamma \vdash P : e}$ Positive constraint entry satisfaction

$$\frac{\Gamma \vdash P \geqslant Q}{\Gamma \vdash P : (\widehat{\alpha}^+ :\geqslant Q)} \quad \text{SATSCESup}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash P : (\widehat{\alpha}^+ :\simeq Q)} \quad \text{SATSCEPEq}$$

$\boxed{\Gamma \vdash N : e}$ Negative constraint entry satisfaction

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash N : (\widehat{\alpha}^- :\simeq M)} \quad \text{SATSCENEq}$$

$\boxed{e_1 \textbf{ singular with } P}$ Positive Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^+ :\simeq P \textbf{ singular with } \mathbf{nf}\,(P)} \quad \text{SINGPEq}$$

$$\frac{}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.\alpha^+ \textbf{ singular with } \alpha^+} \quad \text{SINGSupVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.{\downarrow} N \textbf{ singular with } \exists \alpha^-.{\downarrow}\alpha^-} \quad \text{SINGSupShift}$$

$\boxed{e_1 \textbf{ singular with } N}$ Negative Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^- :\simeq N \textbf{ singular with } \mathbf{nf}\,(N)} \quad \text{SINGNEq}$$

$\boxed{SC \textbf{ singular with } \widehat{\sigma}}$ Subtyping Constraint Is Singular

$\boxed{N \simeq^D M}$ Negative type equivalence

$$\frac{}{\alpha^- \simeq^D \alpha^-} \quad \text{E1NVar}$$

$$\frac{P \simeq^D Q}{\uparrow P \simeq^D \uparrow Q}\ \textsc{E1ShiftU}$$

$$\frac{P \simeq^D Q \quad N \simeq^D M}{P \to N \simeq^D Q \to M}\ \textsc{E1Arrow}$$

$$\frac{\begin{array}{l}\mu : (\overrightarrow{\beta^+} \cap \mathbf{fv}\, M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \mathbf{fv}\, N) \\ \texttt{<<multiple parses>>}\end{array}}{\forall \overrightarrow{\alpha^+}.N \simeq^D \forall \overrightarrow{\beta^+}.M}\ \textsc{E1Forall}$$

$\boxed{P \simeq^D Q}$ Positive type equivalence

$$\frac{}{\alpha^+ \simeq^D \alpha^+}\ \textsc{E1PVar}$$

$$\frac{N \simeq^D M}{\downarrow N \simeq^D \downarrow M}\ \textsc{E1ShiftD}$$

$$\frac{\begin{array}{l}\mu : (\overrightarrow{\beta^-} \cap \mathbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \mathbf{fv}\, P) \\ \texttt{<<multiple parses>>}\end{array}}{\exists \overrightarrow{\alpha^-}.P \simeq^D \exists \overrightarrow{\beta^-}.Q}\ \textsc{E1Exists}$$

$\boxed{P \simeq^D Q}$ Positive unification type equivalence

$\boxed{N \simeq^D M}$ Positive unification type equivalence

$\boxed{\Gamma \vdash N \simeq^{\leqslant} M}$ Negative subtyping-induced equivalence

$$\frac{\Gamma \vdash N \leqslant M \quad \Gamma \vdash M \leqslant N}{\Gamma \vdash N \simeq^{\leqslant} M}\ \textsc{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq^{\leqslant} Q}$ Positive subtyping-induced equivalence

$$\frac{\Gamma \vdash P \geqslant Q \quad \Gamma \vdash Q \geqslant P}{\Gamma \vdash P \simeq^{\leqslant} Q}\ \textsc{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant \alpha^-}\ \textsc{D1NVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant \uparrow Q}\ \textsc{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant Q \quad \Gamma \vdash N \leqslant M}{\Gamma \vdash P \to N \leqslant Q \to M}\ \textsc{D1Arrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma, \overrightarrow{\beta^+} \vdash [\sigma]N \leqslant M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant \forall \overrightarrow{\beta^+}.M}\ \textsc{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant \alpha^+}\ \textsc{D1PVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant \downarrow M}\ \textsc{D1ShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vdash \sigma : \overrightarrow{\alpha^-} \quad \Gamma, \overrightarrow{\beta^-} \vdash [\sigma]P \geqslant Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P \geqslant \exists \overrightarrow{\beta^-}.Q}\ \textsc{D1Exists}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq^{\leqslant} \sigma_2 : \Gamma_1}$ Equivalence of substitutions

$\boxed{\Gamma \vdash \sigma_1 \simeq^{\leqslant} \sigma_2 : vars}$ Equivalence of substitutions

$\boxed{\Theta \vdash \widehat{\sigma}_1 \simeq^{\leqslant} \widehat{\sigma}_2 : vars}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash \widehat{\sigma}_1 \simeq^{\leqslant} \widehat{\sigma}_2 : vars}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash \Phi_1 \simeq^{\leqslant} \Phi_2}$ Equivalence of contexts

$\boxed{\Gamma; \Phi \vdash v \colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \vdash x \colon P}\ \textsc{DTVar}$$

$$\frac{\Gamma; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \{c\} \colon {\downarrow} N}\ \textsc{DTThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant P}{\Gamma; \Phi \vdash (v : Q) \colon Q}\ \textsc{DTPAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash v \colon P'}\ \textsc{DTPEquiv}$$

$\boxed{\Gamma; \Phi \vdash c \colon N}$ Negative type inference

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \lambda x : P.c \colon P \to N}\ \textsc{DTtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \Lambda \alpha^+.c \colon \forall \alpha^+.N}\ \textsc{DTTLam}$$

$$\frac{\Gamma; \Phi \vdash v \colon P}{\Gamma; \Phi \vdash \textbf{return}\, v \colon {\uparrow} P}\ \textsc{DTReturn}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \textbf{let}\, x = v; c \colon N}\ \textsc{DTVarLet}$$

$$\frac{\Gamma; \Phi \vdash v \colon {\downarrow} M \quad \Gamma; \Phi \vdash M \bullet \vec{v} \Longrightarrow {\uparrow} Q \text{ unique} \quad \Gamma; \Phi, x : Q \vdash c \colon N}{\Gamma; \Phi \vdash \textbf{let}\, x = v(\vec{v}); c \colon N}\ \textsc{DTAppLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma; \Phi \vdash v \colon {\downarrow} M \\ \Gamma; \Phi \vdash M \bullet \vec{v} \Longrightarrow {\uparrow} Q \quad \Gamma \vdash {\uparrow} Q \leqslant {\uparrow} P \quad \Gamma; \Phi, x : P \vdash c \colon N\end{array}}{\Gamma; \Phi \vdash \textbf{let}\, x : P = v(\vec{v}); c \colon N}\ \textsc{DTAppLetAnn}$$

$$\frac{\begin{array}{l}\texttt{<<multiple parses>>} \\ \Gamma, \overrightarrow{\alpha^-}; \Phi, x : P \vdash c \colon N \quad \Gamma \vdash N\end{array}}{\Gamma; \Phi \vdash \textbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c \colon N}\ \textsc{DTUnpack}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash (c : M) \colon M}\ \textsc{DTNAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash c \colon N'}\ \textsc{DTNEquiv}$$

$\boxed{\Gamma; \Phi \vdash N \bullet \vec{v} \Longrightarrow M}$ Application type inference

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash N \bullet \cdot \Longrightarrow N'}\ \textsc{DTEmptyApp}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant P \quad \Gamma; \Phi \vdash N \bullet \vec{v} \Longrightarrow M}{\Gamma; \Phi \vdash Q \to N \bullet v, \vec{v} \Longrightarrow M}\ \textsc{DTArrowApp}$$

$$\frac{\Gamma \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma; \Phi \vdash [\sigma]N \bullet \vec{v} \Rrightarrow M \\ \vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot}{\Gamma; \Phi \vdash \forall \overrightarrow{\alpha^+}.N \bullet \vec{v} \Rrightarrow M} \quad \textsc{DTForallApp}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$

$\boxed{P_1 \vee P_2}$

$\boxed{\textbf{ord}\, vars\, \textbf{in}\, P}$

$\boxed{\textbf{ord}\, vars\, \textbf{in}\, N}$

$\boxed{\textbf{ord}\, vars\, \textbf{in}\, P}$

$\boxed{\textbf{ord}\, vars\, \textbf{in}\, N}$

$\boxed{\textbf{nf}\,(N')}$

$\boxed{\textbf{nf}\,(P')}$

$\boxed{\textbf{nf}\,(N')}$

$\boxed{\textbf{nf}\,(P')}$

$\boxed{\textbf{nf}\,(\vec{N}')}$

$\boxed{\textbf{nf}\,(\vec{P}')}$

$\boxed{\textbf{nf}\,(\sigma')}$

$\boxed{\textbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\mathbf{nf}\,(\mu')}$

$\boxed{\sigma'|_{vars}}$

$\boxed{\widehat{\sigma}'|_{vars}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{SC'|_{vars}}$

$\boxed{UC'|_{vars}}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{UC_1 \ \& \ UC_2}$

$\boxed{UC_1 \cup UC_2}$

$\boxed{\Gamma_1 \cup \Gamma_2}$

$\boxed{SC_1 \ \& \ SC_2}$

$\boxed{\widehat{\tau}_1 \ \& \ \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(UC)}$

$\boxed{\mathbf{dom}\,(SC)}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{|SC|}$

$\boxed{\mathbf{fv}\ N}$

$\boxed{\mathbf{fv}\ P}$

$\boxed{\mathbf{fv}\,P}$

$\boxed{\mathbf{fv}\,N}$

$\boxed{\mathbf{uv}\ N}$

$\boxed{\mathbf{uv}\ P}$

$\boxed{\Gamma \models P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \models \alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBVAR}$$

$$\frac{\Gamma, \cdot \models \mathbf{nf}\,(\downarrow N) \overset{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi]\,P} \quad \text{LUBSHIFT}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \models P_1 \vee P_2 = Q}{\Gamma \models \exists \overrightarrow{\alpha^-}.P_1 \vee \exists \overrightarrow{\beta^-}.P_2 = Q} \quad \text{LUBEXISTS}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}$

<<multiple parses>>

$$\frac{\Delta, \overrightarrow{\beta^\pm}, \overrightarrow{\gamma^\pm} \models [\overrightarrow{\beta^\pm}/\overrightarrow{\alpha^\pm}]P \vee [\overrightarrow{\gamma^\pm}/\overrightarrow{\alpha^\pm}]P = Q}{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q} \quad \text{LUBUPGRADE}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^-) = \alpha^-}\ \textsc{NrmNVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\uparrow P) = \uparrow Q}\ \textsc{NrmShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(P \to N) = Q \to M}\ \textsc{NrmArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\forall \overrightarrow{\alpha^+}.N) = \forall \overrightarrow{\alpha^{+\prime}}.N'}\ \textsc{NrmForall}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\alpha^+) = \alpha^+}\ \textsc{NrmPVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\downarrow N) = \downarrow M}\ \textsc{NrmShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\exists \overrightarrow{\alpha^-}.P) = \exists \overrightarrow{\alpha^{-\prime}}.P'}\ \textsc{NrmExists}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^-) = \widehat{\alpha}^-}\ \textsc{NrmNUVar}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^+) = \widehat{\alpha}^+}\ \textsc{NrmPUVar}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}$ variable ordering in a negative type

$$\frac{\alpha^- \in \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^- = \alpha^-}\ \textsc{ONVarIn}$$

$$\frac{\alpha^- \notin \mathit{vars}}{\texttt{<<multiple parses>>}}\ \textsc{ONVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\uparrow P = \vec{\alpha}}\ \textsc{OShiftU}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}_1 \quad \mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}_2}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)}\ \textsc{OArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\forall \overrightarrow{\alpha^+}.N = \vec{\alpha}}\ \textsc{OForall}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}$
variable ordering in a positive type

$$\frac{\alpha^+ \in \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^+ = \alpha^+}\ \textsc{OPVarIn}$$

$$\frac{\alpha^+ \notin \mathit{vars}}{\texttt{<<multiple parses>>}}\ \textsc{OPVarNIn}$$

$$\frac{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N = \vec{\alpha}}{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ {\downarrow}N = \vec{\alpha}}\ \textsc{OShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ \exists\overrightarrow{\alpha^-}.P = \vec{\alpha}}\ \textsc{OExists}$$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N = \vec{\alpha}}$

$$\frac{}{\texttt{<<multiple parses>>}}\ \textsc{ONUVar}$$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P = \vec{\alpha}}$

$$\frac{}{\texttt{<<multiple parses>>}}\ \textsc{OPUVar}$$

$\boxed{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC}$ Negative unification

$$\frac{}{\Gamma; \Theta \models \alpha^- \overset{u}{\simeq} \alpha^- \dashv \cdot}\ \textsc{UNVar}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models {\uparrow}P \overset{u}{\simeq} {\uparrow}Q \dashv UC}\ \textsc{UShiftU}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC_1 \quad \Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC_2}{\Gamma; \Theta \models P \to N \overset{u}{\simeq} Q \to M \dashv UC_1\ \&\ UC_2}\ \textsc{UArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \forall\overrightarrow{\alpha^+}.N \overset{u}{\simeq} \forall\overrightarrow{\alpha^+}.M \dashv UC}\ \textsc{UForall}$$

$$\frac{\Theta(\widehat{\alpha}^-) \vdash N}{\Gamma; \Theta \models \widehat{\alpha}^- \overset{u}{\simeq} N \dashv (\widehat{\alpha}^- :\simeq N)}\ \textsc{UNUVar}$$

$\boxed{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC}$ Positive unification

$$\frac{}{\Gamma; \Theta \models \alpha^+ \overset{u}{\simeq} \alpha^+ \dashv \cdot}\ \textsc{UPVar}$$

$$\frac{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models {\downarrow}N \overset{u}{\simeq} {\downarrow}M \dashv UC}\ \textsc{UShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \exists\overrightarrow{\alpha^-}.P \overset{u}{\simeq} \exists\overrightarrow{\alpha^-}.Q \dashv UC}\ \textsc{UExists}$$

$$\frac{\Theta(\widehat{\alpha}^+) \vdash P}{\Gamma; \Theta \models \widehat{\alpha}^+ \overset{u}{\simeq} P \dashv (\widehat{\alpha}^+ :\simeq P)}\ \textsc{UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma \vdash \alpha^-}\ \textsc{WFTNVar}$$

$$\frac{\Gamma \vdash P}{\Gamma \vdash {\uparrow}P}\ \textsc{WFTShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash N}{\Gamma \vdash P \to N}\ \textsc{WFTArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+} \vdash N}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N} \quad \text{WFTForall}$$

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma \vdash \alpha^+} \quad \text{WFTPVar}$$

$$\frac{\Gamma \vdash N}{\Gamma \vdash \downarrow N} \quad \text{WFTShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-} \vdash P}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P} \quad \text{WFTExists}$$

$\boxed{\Gamma; \Xi \vdash N}$ Negative algorithmic type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma; \Xi \vdash \alpha^-} \quad \text{WFATNVar}$$

$$\frac{\widehat{\alpha}^- \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^-} \quad \text{WFATNUVar}$$

$$\frac{\Gamma; \Xi \vdash P}{\Gamma; \Xi \vdash \uparrow P} \quad \text{WFATShiftU}$$

$$\frac{\Gamma; \Xi \vdash P \quad \Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash P \rightarrow N} \quad \text{WFATArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Xi \vdash N}{\Gamma; \Xi \vdash \forall \overrightarrow{\alpha^+}.N} \quad \text{WFATForall}$$

$\boxed{\Gamma; \Xi \vdash P}$ Positive algorithmic type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma; \Xi \vdash \alpha^+} \quad \text{WFATPVar}$$

$$\frac{\widehat{\alpha}^+ \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^+} \quad \text{WFATPUVar}$$

$$\frac{\Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash \downarrow N} \quad \text{WFATShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Xi \vdash P}{\Gamma; \Xi \vdash \exists \overrightarrow{\alpha^-}.P} \quad \text{WFATExists}$$

$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness

$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness

$\boxed{\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness

$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness

$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution signature

$\boxed{\Theta \vdash \widehat{\sigma} : \Xi}$ Unification substitution signature

$\boxed{\Gamma \vdash \widehat{\sigma} : \Xi}$ Unification substitution general signature

$\boxed{\Theta \vdash \widehat{\sigma} : UC}$ Unification substitution satisfies unification constraint

$\boxed{\Theta \vdash \widehat{\sigma} : SC}$ Unification substitution satisfies subtyping constraint

$\boxed{\Gamma \vdash e}$ Unification constraint entry well-formedness

$\boxed{\Gamma \vdash e}$ Subtyping constraint entry well-formedness

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies unification constraint

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies unification constraint
$\boxed{\Gamma \vdash P : e}$ Positive type satisfies subtyping constraint
$\boxed{\Gamma \vdash N : e}$ Negative type satisfies subtyping constraint
$\boxed{\Theta \vdash UC : \Xi}$ Unification constraint well-formedness with specified domain
$\boxed{\Theta \vdash SC : \Xi}$ Subtyping constraint well-formedness with specified domain
$\boxed{\Theta \vdash UC}$ Unification constraint well-formedness
$\boxed{\Theta \vdash SC}$ Subtyping constraint well-formedness
$\boxed{\Gamma \vdash \overrightarrow{v}}$ Argument List well-formedness
$\boxed{\Gamma \vdash \Phi}$ Context well-formedness
$\boxed{\Gamma \vdash v}$ Value well-formedness

$$\frac{}{\Gamma \vdash x} \quad \text{WFALLV{\scriptsize AR}}$$

$\boxed{\Gamma \vdash c}$ Computation well-formedness

$$\frac{\Gamma \vdash v \quad \Gamma \vdash c \quad \Gamma \vdash \overrightarrow{v}}{\Gamma \vdash \mathbf{let}\, x = v(\overrightarrow{v}); c} \quad \text{WFALLA{\scriptsize PP}L{\scriptsize ET}}$$

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Definition rules:        94 good    33 bad
Definition rule clauses: 213 good    34 bad
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