α, β, α, β, γ, δ	type variables
n, m, i, j	index variables
x, y, z	term variables

n, m	$::=$			cohort index
	$\|$	0		
	$\|$	$n+1$		
$\alpha^+, \beta^+, \gamma^+, \delta^+$	$::=$			positive variable
	$\|$	α^+		
	$\|$	α^{+n}		
	$\|$	$\beta^\pm$		
$\alpha^-, \beta^-, \gamma^-, \delta^-$	$::=$			negative variable
	$\|$	α^-		
	$\|$	α^{-n}		
	$\|$	$\alpha^\pm$		
$\alpha^\pm, \beta^\pm$	$::=$			positive or negative variable
	$\|$	$\alpha^\pm$		
	$\|$	$\alpha^{\pm n}$		
σ	$::=$			substitution
	$\|$	$\cdot$		
	$\|$	id		
	$\|$	P/α^+		
	$\|$	N/α^-		
	$\|$	$\vec{P}/\overrightarrow{\alpha^+}$		
	$\|$	$\vec{N}/\overrightarrow{\alpha^-}$		
	$\|$	$\mathbf{pmas}/\overrightarrow{\alpha^+}$		
	$\|$	$\mathbf{nmas}/\overrightarrow{\alpha^-}$		
	$\|$	$\overrightarrow{\widetilde{\alpha^+}}/\overrightarrow{\alpha^+}$		
	$\|$	$\overrightarrow{\widehat{\alpha^-}}/\overrightarrow{\alpha^-}$		
	$\|$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\|$	μ		
	$\|$	$\sigma_1 \circ \sigma_2$		
	$\|$	$\sigma_1 \lll \sigma_2$		
	$\|$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\|$	(σ)	S	
	$\|$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\|$	$\mathbf{nf}\,(\sigma')$	M	
	$\|$	$\sigma'\|_{vars}$	M	
e	$::=$			entry of a subtyping constraint
	$\|$	$\widehat{\alpha}^+ :\approx P$		
	$\|$	$\widehat{\alpha}^- :\approx N$		
	$\|$	$\widehat{\alpha}^+ :\geqslant P$		
	$\|$	(e)	S	
	$\|$	$SC(\widehat{\alpha}^-)$	M	
	$\|$	$e_1 \ \&\ e_2$	M	
e	$::=$			entry of a unification constraint
	$\|$	$\widehat{\alpha}^+ :\approx P$		

		$\mid$	$\widehat{\alpha}^- :\approx N$		
		$\mid$	(e)	S	
		$\mid$	$e_1 \ \& \ e_2$	M	
UC	$::=$				unification constraint
		$\mid$	$\cdot$		
		$\mid$	e		
		$\mid$	$UC \backslash vars$		
		$\mid$	$UC \mid vars$		
		$\mid$	$UC_1 \cup UC_2$		
		$\mid$	$\overline{UC_i}^{\,i}$		concatenate
		$\mid$	(UC)	S	
		$\mid$	$\mathbf{UC}\mid_{vars}$	M	
		$\mid$	$UC_1 \ \& \ UC_2$	M	
		$\mid$	$UC_1 \cup UC_2$	M	
		$\mid$	$\mid SC \mid$	M	
SC	$::=$				subtyping constraint
		$\mid$	$\cdot$		
		$\mid$	e		
		$\mid$	$SC \backslash vars$		
		$\mid$	$SC \mid vars$		
		$\mid$	$SC_1 \cup SC_2$		
		$\mid$	UC		
		$\mid$	$\overline{SC_i}^{\,i}$		concatenate
		$\mid$	(SC)	S	
		$\mid$	$\mathbf{SC}\mid_{vars}$	M	
		$\mid$	$SC_1 \ \& \ SC_2$	M	
$\widehat{\sigma}$	$::=$				unification substitution
		$\mid$	$\cdot$		
		$\mid$	$P/\widehat{\alpha}^+$		
		$\mid$	$N/\widehat{\alpha}^-$		
		$\mid$	$\overrightarrow{P}/\overrightarrow{\widehat{\alpha^+}}$		
		$\mid$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha^-}}$		
		$\mid$	$(\widehat{\sigma})$	S	
		$\mid$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
		$\mid$	$\mathbf{nf}\ (\widehat{\sigma}')$	M	
		$\mid$	$\widehat{\sigma}'\mid_{vars}$	M	
$\widehat{\tau},\ \widehat{\rho}$	$::=$				anti-unification substitution
		$\mid$	$\cdot$		
		$\mid$	$\widehat{\alpha}^- :\approx N$		
		$\mid$	$\widehat{\alpha}^- :\approx N$		
		$\mid$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
		$\mid$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha^-}}$		
		$\mid$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
		$\mid$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
		$\mid$	$(\widehat{\tau})$	S	
		$\mid$	$\widehat{\tau}'\mid_{vars}$	M	

	$\mid$	$\widehat{\tau}_1 \mathbin{\&} \widehat{\tau}_2$	M
$P,\ Q,\ R$	$::=$		positive types
	$\mid$	α^+	
	$\mid$	$\downarrow N$	
	$\mid$	$\exists \alpha^-. P$	
	$\mid$	$[\sigma] P$	M
$N,\ M,\ K$	$::=$		negative types
	$\mid$	α^-	
	$\mid$	$\uparrow P$	
	$\mid$	$\forall \alpha^+. N$	
	$\mid$	$P \to N$	
	$\mid$	$[\sigma] N$	M
$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+},\ \overrightarrow{\gamma^+},\ \overrightarrow{\delta^+}$	$::=$		positive variable list
	$\mid$	$\cdot$	empty list
	$\mid$	α^+	a variable
	$\mid$	$\overrightarrow{\alpha^+}$	a variable
	$\mid$	$\overline{\overrightarrow{\alpha^+}_i}^{\,i}$	concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$	$::=$		negative variables
	$\mid$	$\cdot$	empty list
	$\mid$	α^-	a variable
	$\mid$	$\overrightarrow{\alpha^-}$	variables
	$\mid$	$\overline{\overrightarrow{\alpha^-}_i}^{\,i}$	concatenate lists
$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$	$::=$		positive or negative variable list
	$\mid$	$\cdot$	empty list
	$\mid$	$\alpha^\pm$	a variable
	$\mid$	$\overrightarrow{\mathbf{pa}}$	variables
	$\mid$	$\overline{\overrightarrow{\alpha^\pm}_i}^{\,i}$	concatenate lists
$P,\ Q,\ R$	$::=$		multi-quantified positive types
	$\mid$	α^+	
	$\mid$	$\downarrow N$	
	$\mid$	$\exists \overrightarrow{\alpha^-}. P$	
	$\mid$	$[\sigma] P$	M
	$\mid$	$[\widehat{\tau}] P$	M
	$\mid$	$[\widehat{\sigma}] P$	M
	$\mid$	$[\mu] P$	M
	$\mid$	(P)	S
	$\mid$	$P_1 \vee P_2$	M
	$\mid$	$\mathbf{nf}\,(P')$	M
$N,\ M,\ K$	$::=$		multi-quantified negative types
	$\mid$	α^-	
	$\mid$	$\uparrow P$	
	$\mid$	$P \to N$	

	|	$\forall\overrightarrow{\alpha^+}.N$		
	|	$[\sigma]N$	M	
	|	$[\widehat{\tau}]N$	M	
	|	$[\mu]N$	M	
	|	$[\widehat{\sigma}]N$	M	
	|	(N)	S	
	|	$\mathbf{nf}\,(N')$	M	
$\vec{P},\ \vec{Q}$	::=			list of positive types
	|	$\cdot$		empty list
	|	P		a singel type
	|	$[\sigma]\vec{P}$	M	
	|	$\overline{\vec{P}_i}^{\,i}$		concatenate lists
	|	$(\vec{P})$	S	
	|	$\mathbf{nf}\,(\vec{P}')$	M	
$\vec{N},\ \vec{M}$	::=			list of negative types
	|	$\cdot$		empty list
	|	N		a singel type
	|	$[\sigma]\vec{N}$	M	
	|	$\overline{\vec{N}_i}^{\,i}$		concatenate lists
	|	$(\vec{N})$	S	
	|	$\mathbf{nf}\,(\vec{N}')$	M	
$\Delta,\ \Gamma$	::=			declarative type context
	|	$\cdot$		empty context
	|	$\overrightarrow{\alpha^+}$		list of variables
	|	$\overrightarrow{\alpha^-}$		list of variables
	|	$\overrightarrow{\alpha^\pm}$		list of variables
	|	$vars$		
	|	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	|	(Γ)	S	
	|	$\Theta(\widehat{\alpha}^+)$	M	
	|	$\Theta(\widehat{\alpha}^-)$	M	
	|	$\Gamma_1 \cup \Gamma_2$	M	
Θ	::=			algorithmic variable context
	|	$\cdot$		empty context
	|	$\vec{\alpha}\{\Delta\}$		from an ordered list of variables
	|	$\overrightarrow{\widehat{\alpha}^+\{\Delta\}}$		from a variable to a list
	|	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	|	(Θ)	S	
	|	$\Theta\vert_{vars}$		leave only those variables that are in the set
	|	$\Theta_1 \cup \Theta_2$		
Ξ	::=			anti-unification type variable context
	|	$\cdot$		empty context
	|	$\overrightarrow{\widetilde{\alpha^+}}$		list of positive variables
	|	$\overrightarrow{\overset{\frown}{\alpha^-}}$		list of negative variables
	|	$\mathbf{uv}\ N$		unification variables

	|	$\mathbf{uv}\ P$		unification variables
	|	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	|	(Ξ)	S	
	|	$\Xi_1 \cup \Xi_2$		
	|	$\Xi_1 \cap \Xi_2$		
	|	$\Xi'\vert_{vars}$	M	
	|	$\mathbf{dom}\,(UC)$	M	
	|	$\mathbf{dom}\,(SC)$	M	
	|	$\mathbf{dom}\,(\widehat{\sigma})$	M	
	|	$\mathbf{dom}\,(\widehat{\tau})$	M	
	|	$\mathbf{dom}\,(\Theta)$	M	
$\vec{\alpha},\ \vec{\beta}$	::=			ordered positive or negative variables
	|	$\cdot$		empty list
	|	$\overrightarrow{\alpha^+}$		list of variables
	|	$\overrightarrow{\alpha^-}$		list of variables
	|	$\overrightarrow{\alpha^\pm}$		list of variables
	|	$\overrightarrow{\widetilde{\alpha}^+}$		list of variables
	|	$\overrightarrow{\widehat{\alpha}^-}$		list of variables
	|	$\vec{\alpha}_1 \backslash vars$		setminus
	|	Γ		context
	|	$vars$		
	|	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	|	$(\vec{\alpha})$	S	parenthesis
	|	$[\mu]\vec{\alpha}$	M	apply moving to list
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ P$	M	
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ N$	M	
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ P$	M	
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ N$	M	
$vars$	::=			set of variables
	|	$\varnothing$		empty set
	|	$\mathbf{fv}\ P$		free variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathbf{fv}\ \mathbf{imP}$		free variables
	|	$\mathbf{fv}\ \mathbf{imN}$		free variables
	|	$vars_1 \cap vars_2$		set intersection
	|	$vars_1 \cup vars_2$		set union
	|	$vars_1 \backslash vars_2$		set complement
	|	$\mathbf{mv}\ \mathbf{imP}$		movable variables
	|	$\mathbf{mv}\ \mathbf{imN}$		movable variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathbf{fv}\ P$		free variables
	|	$(vars)$	S	parenthesis
	|	$\vec{\alpha}$		ordered list of variables
	|	$[\mu]vars$	M	apply moving to varset
	|	Ξ		anti-unification context
μ	::=			
	|	$\cdot$		empty moving

	$\vert$	$pma1 \mapsto pma2$		Positive unit substitution
	$\vert$	$nma1 \mapsto nma2$		Positive unit substitution
	$\vert$	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	$\vert$	$\mu_1 \circ \mu_2$	M	Composition
	$\vert$	$\overline{\mu_i}^{\,i}$		concatenate movings
	$\vert$	$\mu\vert_{vars}$	M	restriction on a set
	$\vert$	μ^{-1}	M	inversion
	$\vert$	$\mathbf{nf}\,(\mu')$	M	
$\widehat{\alpha}^{\pm}$	$::=$			positive/negative unification variable
	$\vert$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	$::=$			positive unification variable
	$\vert$	$\widehat{\alpha}^{+}$		
	$\vert$	$\widehat{\alpha}^{+}\{\Delta\}$		
	$\vert$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	$::=$			negative unification variable
	$\vert$	$\widehat{\alpha}^{-}$		
	$\vert$	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	$\vert$	$\widehat{\alpha}^{-}\{\Delta\}$		
	$\vert$	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widetilde{\alpha}^{+}},\ \overrightarrow{\widetilde{\beta}^{+}}$	$::=$			positive unification variable list
	$\vert$	$\cdot$		empty list
	$\vert$	$\widehat{\alpha}^{+}$		a variable
	$\vert$	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	$\vert$	$\overline{\overrightarrow{\widetilde{\alpha}^{+}}_{i}}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}},\ \overrightarrow{\widehat{\beta}^{-}}$	$::=$			negative unification variable list
	$\vert$	$\cdot$		empty list
	$\vert$	$\widehat{\alpha}^{-}$		a variable
	$\vert$	Ξ		from an antiunification context
	$\vert$	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	$\vert$	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	$\vert$	$\overline{\overrightarrow{\widehat{\alpha}^{-}}_{i}}^{\,i}$		concatenate lists
$P,\ Q$	$::=$			a positive algorithmic type (potentially with metavariables)
	$\vert$	$\widehat{\alpha}^{+}$		
	$\vert$	α^{+}		
	$\vert$	$\downarrow N$		
	$\vert$	$\exists\overrightarrow{\alpha^{-}}.\,P$		
	$\vert$	$[\sigma]P$	M	
	$\vert$	$[\widehat{\tau}]P$	M	
	$\vert$	$[\mu]P$	M	
	$\vert$	$[\overrightarrow{\widehat{\alpha}^{-}}/\overrightarrow{\alpha^{-}}]P$	M	
	$\vert$	(P)	S	
	$\vert$	$\mathbf{nf}\,(P')$	M	

N, M	::=			a negative algorithmic type (potentially with metavariables)
	|	$\widehat{\alpha}^-$		
	|	α^-		
	|	$\uparrow P$		
	|	$P \to N$		
	|	$\forall \overrightarrow{\alpha^+}. N$		
	|	$[\sigma] N$	M	
	|	$[\widehat{\tau}] N$	M	
	|	$[\mu] N$	M	
	|	$[\overrightarrow{\widehat{\alpha}^-} / \overrightarrow{\alpha^-}] N$	M	
	|	(N)	S	
	|	$\mathbf{nf}\,(N')$	M	
auSol	::=			
	|	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$		
	|	$(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$		
terminals	::=			
	|	$\exists$		
	|	$\forall$		
	|	$\uparrow$		
	|	$\downarrow$		
	|	$\rightarrow$		
	|	$\leftrightarrow$		
	|	$\in$		
	|	$\notin$		
	|	$\cdot$		
	|	$\vdash$		
	|	$\leqslant$		
	|	$\geqslant$		
	|	$\simeq$		
	|	$\cup$		
	|	$\cap$		
	|	$\setminus$		
	|	$\subseteq$		
	|	$\mapsto$		
	|	$\stackrel{u}{\simeq}$		
	|	$\stackrel{a}{\simeq}$		
	|	$\varnothing$		
	|	$\circ$		
	|	$\Rightarrow$		
	|	$\models$		
	|	$\dashv$		
	|	$\neq$		
	|	$\equiv_n$		
	|	$\vee$		
	|	$\Downarrow$		
	|	$:\geqslant$		
	|	$:\simeq$		
	|	Λ		

	$\mid$	λ	
	$\mid$	$\mathbf{let}^{\exists}$	
	$\mid$	$\bullet$	
	$\mid$	$\Rrightarrow$	
	$\mid$	$\Lleftarrow$	
$v,\ w$	$::=$		value terms
	$\mid$	x	
	$\mid$	$\{c\}$	
	$\mid$	$(v : P)$	
	$\mid$	(v)	M
$\overrightarrow{v}$	$::=$		list of arguments
	$\mid$	$\cdot$	
	$\mid$	v	
	$\mid$	$\overline{\overrightarrow{v}_i}^{\,i}$	concatenate
$c,\ d$	$::=$		computation terms
	$\mid$	$(c : N)$	
	$\mid$	$\lambda x : P.c$	
	$\mid$	$\Lambda \alpha^+.c$	
	$\mid$	$\mathbf{return}\, v$	
	$\mid$	$\mathbf{let}\, x = v; c$	
	$\mid$	$\mathbf{let}\, x : P = v(\overrightarrow{v}); c$	
	$\mid$	$\mathbf{let}\, x = v(\overrightarrow{v}); c$	
	$\mid$	$\mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c$	
$vctx,\ \Phi$	$::=$		variable context
	$\mid$	$\cdot$	
	$\mid$	$x : P$	
	$\mid$	$\overline{\Phi_i}^{\,i}$	concatenate contexts
$formula$	$::=$		
	$\mid$	$judgement$	
	$\mid$	$judgement$ unique	
	$\mid$	$formula_1 \quad .. \quad formula_n$	
	$\mid$	$\mu : vars_1 \leftrightarrow vars_2$	
	$\mid$	$\mu\, \mathbf{is\ bijective}$	
	$\mid$	$x : P \in \Phi$	
	$\mid$	$UC_1 \subseteq UC_2$	
	$\mid$	$UC_1 = UC_2$	
	$\mid$	$SC_1 \subseteq SC_2$	
	$\mid$	$vars_1 \subseteq vars_2$	
	$\mid$	$vars_1 \subseteq vars_2 \subseteq vars_3$	
	$\mid$	$vars_1 = vars_2$	
	$\mid$	$vars\, \mathbf{is\ fresh}$	
	$\mid$	$\alpha^- \notin vars$	
	$\mid$	$\alpha^+ \notin vars$	
	$\mid$	$\alpha^- \in vars$	
	$\mid$	$\alpha^+ \in vars$	

| $\widehat{\alpha}^+ \in vars$
| $\widehat{\alpha}^- \in vars$
| $\widehat{\alpha}^- \in \Theta$
| $\widehat{\alpha}^+ \in \Theta$
| $\widehat{\alpha}^- \notin vars$
| $\widehat{\alpha}^+ \notin vars$
| $\widehat{\alpha}^- \notin \Theta$
| $\widehat{\alpha}^+ \notin \Theta$
| $\widehat{\alpha}^- \in \Xi$
| $\widehat{\alpha}^- \notin \Xi$
| $\widehat{\alpha}^+ \in \Xi$
| $\widehat{\alpha}^+ \notin \Xi$
| if any other rule is not applicable
| $\vec{\alpha}_1 = \vec{\alpha}_2$
| $e_1 = e_2$
| $e_1 = e_2$
| $\widehat{\sigma}_1 = \widehat{\sigma}_2$
| $N = M$
| $\Theta \subseteq \Theta'$
| $\vec{v}_1 = \vec{v}_2$
| $N \neq M$
| $P \neq Q$
| $N \neq M$
| $P \neq Q$
| $P \neq Q$
| $N \neq M$
| $\vec{v}_1 \neq \vec{v}_2$
| $\overrightarrow{\alpha^+}_1 \neq \overrightarrow{\alpha^+}_2$
| $|\overrightarrow{\alpha^-}| + |\overrightarrow{\beta^-}| > 0$
| $|\overrightarrow{\alpha^+}| + |\overrightarrow{\beta^+}| > 0$

A ::=
| $\Gamma; \Theta \models N \leqslant M \dashv SC$ — Negative subtyping
| $\Gamma; \Theta \models P \geqslant Q \dashv SC$ — Positive supertyping

AT ::=
| $\Gamma; \Phi \models v : P$ — Positive type inference
| $\Gamma; \Phi \models c : N$ — Negative type inference
| $\Gamma; \Phi; \Theta_1 \models N \bullet \vec{v} \Rrightarrow M \dashv \Theta_2; SC$ — Application type inference

AU ::=
| $\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$
| $\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$

SCM ::=
| $\Gamma \vdash e_1 \ \& \ e_2 = e_3$ — Subtyping Constraint Entry Merge
| $\Theta \vdash SC_1 \& SC_2 = SC_3$ — Merge of subtyping constraints

UCM ::=
| $\Gamma \vdash e_1 \& e_2 = e_3$

	$\mid$	$\Theta \vdash UC_1 \& UC_2 = UC_3$	

SATSCE	::=		
	$\mid$	$\Gamma \vdash P : e$	Positive type satisfies with the subtyping constraint entry
	$\mid$	$\Gamma \vdash N : e$	Negative type satisfies with the subtyping constraint entry
SING	::=		
	$\mid$	e_1 **singular with** P	Positive Subtyping Constraint Entry Is Singular
	$\mid$	e_1 **singular with** N	Negative Subtyping Constraint Entry Is Singular
	$\mid$	SC **singular with** $\hat{\sigma}$	Subtyping Constraint Is Singular
E1	::=		
	$\mid$	$N \simeq_1^D M$	Negative multi-quantified type equivalence
	$\mid$	$P \simeq_1^D Q$	Positive multi-quantified type equivalence
	$\mid$	$P \simeq_1^D Q$	Positive unification type equivalence
	$\mid$	$N \simeq_1^D M$	Positive unification type equivalence
D1	::=		
	$\mid$	$\Gamma \vdash N \simeq_1^{\leqslant} M$	Negative equivalence on MQ types
	$\mid$	$\Gamma \vdash P \simeq_1^{\leqslant} Q$	Positive equivalence on MQ types
	$\mid$	$\Gamma \vdash N \leqslant_1 M$	Negative subtyping
	$\mid$	$\Gamma \vdash P \geqslant_1 Q$	Positive supertyping
	$\mid$	$\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1$	Equivalence of substitutions
	$\mid$	$\Gamma \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : vars$	Equivalence of substitutions
	$\mid$	$\Theta \vdash \hat{\sigma}_1 \simeq_1^{\leqslant} \hat{\sigma}_2 : \mathbf{varset}$	Equivalence of unification substitutions
	$\mid$	$\Gamma \vdash \hat{\sigma}_1 \simeq_1^{\leqslant} \hat{\sigma}_2 : vars$	Equivalence of unification substitutions
	$\mid$	$\Gamma \vdash \Phi_1 \simeq_1^{\leqslant} \Phi_2$	Equivalence of contexts
D0	::=		
	$\mid$	$\Gamma \vdash N \simeq_0^{\leqslant} M$	Negative equivalence
	$\mid$	$\Gamma \vdash P \simeq_0^{\leqslant} Q$	Positive equivalence
	$\mid$	$\Gamma \vdash N \leqslant_0 M$	Negative subtyping
	$\mid$	$\Gamma \vdash P \geqslant_0 Q$	Positive supertyping
DT	::=		
	$\mid$	$\Gamma; \Phi \vdash v : P$	Positive type inference
	$\mid$	$\Gamma; \Phi \vdash c : N$	Negative type inference
	$\mid$	$\Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M$	Application type inference
EQ	::=		
	$\mid$	$N = M$	Negative type equality (alpha-equivalence)
	$\mid$	$P = Q$	Positive type equuality (alphha-equivalence)
	$\mid$	$P = Q$	
LUBF	::=		
	$\mid$	$P_1 \vee P_2 === Q$	
	$\mid$	**ord** *vars* **in** P $=== \vec{\alpha}$	
	$\mid$	**ord** *vars* **in** N $=== \vec{\alpha}$	
	$\mid$	**ord** *vars* **in** $P === \vec{\alpha}$	

| $\mathbf{ord}\, \mathit{vars}\, \mathbf{in}\, N === \vec{\alpha}$
| $\mathbf{nf}\, (N') === N$
| $\mathbf{nf}\, (P') === P$
| $\mathbf{nf}\, (N') === N$
| $\mathbf{nf}\, (P') === P$
| $\mathbf{nf}\, (\vec{N}') === \vec{N}$
| $\mathbf{nf}\, (\vec{P}') === \vec{P}$
| $\mathbf{nf}\, (\sigma') === \sigma$
| $\mathbf{nf}\, (\widehat{\sigma}') === \widehat{\sigma}$
| $\mathbf{nf}\, (\mu') === \mu$
| $\sigma'|_{\mathit{vars}}$
| $\widehat{\sigma}'|_{\mathit{vars}}$
| $\widehat{\tau}'|_{\mathit{vars}}$
| $\Xi'|_{\mathit{vars}}$
| $\mathit{SC}|_{\mathit{vars}}$
| $\mathit{UC}|_{\mathit{vars}}$
| $e_1 \ \&\ e_2$
| $e_1 \ \&\ e_2$
| $\mathit{UC}_1 \ \&\ \mathit{UC}_2$
| $\mathit{UC}_1 \cup \mathit{UC}_2$
| $\Gamma_1 \cup \Gamma_2$
| $\mathit{SC}_1 \ \&\ \mathit{SC}_2$
| $\widehat{\tau}_1 \ \&\ \widehat{\tau}_2$
| $\mathbf{dom}\, (\mathit{UC}) === \Xi$
| $\mathbf{dom}\, (\mathit{SC}) === \Xi$
| $\mathbf{dom}\, (\widehat{\sigma}) === \Xi$
| $\mathbf{dom}\, (\widehat{\tau}) === \Xi$
| $\mathbf{dom}\, (\Theta) === \Xi$
| $|\mathit{SC}| === \mathit{UC}$

LUB ::=
| $\Gamma \models P_1 \vee P_2 = Q$ — Least Upper Bound (Least Common Supertype)
| $\mathbf{upgrade}\, \Gamma \vdash P\, \mathbf{to}\, \Delta = Q$

Nrm ::=
| $\mathbf{nf}\, (N) = M$
| $\mathbf{nf}\, (P) = Q$
| $\mathbf{nf}\, (N) = M$
| $\mathbf{nf}\, (P) = Q$

Order ::=
| $\mathbf{ord}\, \mathit{vars}\, \mathbf{in}\, N = \vec{\alpha}$
| $\mathbf{ord}\, \mathit{vars}\, \mathbf{in}\, P = \vec{\alpha}$
| $\mathbf{ord}\, \mathit{vars}\, \mathbf{in}\, N = \vec{\alpha}$
| $\mathbf{ord}\, \mathit{vars}\, \mathbf{in}\, P = \vec{\alpha}$

U ::=
| $\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv \mathit{UC}$ — Negative unification
| $\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv \mathit{UC}$ — Positive unification

WFT	$::=$		
	$\mid$	$\Gamma \vdash N$	Negative type well-formedness
	$\mid$	$\Gamma \vdash P$	Positive type well-formedness
	$\mid$	$\Gamma \vdash N$	Negative type well-formedness
	$\mid$	$\Gamma \vdash P$	Positive type well-formedness
	$\mid$	$\Gamma \vdash \overrightarrow{N}$	Negative type list well-formedness
	$\mid$	$\Gamma \vdash \overrightarrow{P}$	Positive type list well-formedness
$WFAT$	$::=$		
	$\mid$	$\Gamma; \Xi \vdash N$	Negative algorithmic type well-formedness
	$\mid$	$\Gamma; \Xi \vdash P$	Positive algorithmic type well-formedness
	$\mid$	$\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1$	Antiunification substitution well-formedness
	$\mid$	$\Gamma \vdash^{\supseteq} \Theta$	Unification context well-formedness
	$\mid$	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution signature
	$\mid$	$\Theta \vdash \widehat{\sigma}$	Unification substitution signature
	$\mid$	$(\Theta \vdash UC) \vdash \widehat{\sigma}$	Unification substitution satisfies unification constraint
	$\mid$	$(\Theta \vdash SC) \vdash \widehat{\sigma}$	Unification substitution satisfies subtyping constraint
	$\mid$	$\Gamma \vdash \widehat{\sigma} : \Xi$	Unification substitution general signature
	$\mid$	$\Gamma \vdash e$	Unification constraint entry well-formedness
	$\mid$	$\Gamma \vdash e$	Subtyping constraint entry well-formedness
	$\mid$	$\Gamma \vdash P : e$	Positive type satisfies unification constraint
	$\mid$	$\Gamma \vdash N : e$	Negative type satisfies unification constraint
	$\mid$	$\Gamma \vdash P : e$	Positive type satisfies subtyping constraint
	$\mid$	$\Gamma \vdash N : e$	Negative type satisfies subtyping constraint
	$\mid$	$\Theta \vdash UC$	Unification constraint well-formedness
	$\mid$	$\Theta \vdash SC$	Subtyping constraint well-formedness
	$\mid$	$\Gamma \vdash \overrightarrow{v}$	Argument List well-formedness
	$\mid$	$\Gamma \vdash \Phi$	Context well-formedness
	$\mid$	$\Gamma \vdash v$	Value well-formedness
	$\mid$	$\Gamma \vdash c$	Computation well-formedness
$judgement$	$::=$		
	$\mid$	A	
	$\mid$	AT	
	$\mid$	AU	
	$\mid$	SCM	
	$\mid$	UCM	
	$\mid$	$SATSCE$	
	$\mid$	$SING$	
	$\mid$	$E1$	
	$\mid$	$D1$	
	$\mid$	$D0$	
	$\mid$	DT	
	$\mid$	EQ	
	$\mid$	LUB	
	$\mid$	Nrm	
	$\mid$	$Order$	
	$\mid$	U	
	$\mid$	WFT	
	$\mid$	$WFAT$	

$$
\begin{array}{lcl}
\mathit{user\_syntax} & ::= & \\
& | & \alpha \\
& | & n \\
& | & x \\
& | & n \\
& | & \alpha^{+} \\
& | & \alpha^{-} \\
& | & \alpha^{\pm} \\
& | & \sigma \\
& | & e \\
& | & e \\
& | & \mathit{UC} \\
& | & \mathit{SC} \\
& | & \widehat{\sigma} \\
& | & \widehat{\tau} \\
& | & P \\
& | & N \\
& | & \overrightarrow{\alpha^{+}} \\
& | & \overrightarrow{\alpha^{-}} \\
& | & \overrightarrow{\alpha^{\pm}} \\
& | & P \\
& | & N \\
& | & \overrightarrow{P} \\
& | & \overrightarrow{N} \\
& | & \Gamma \\
& | & \Theta \\
& | & \Xi \\
& | & \overrightarrow{\alpha} \\
& | & \mathit{vars} \\
& | & \mu \\
& | & \widehat{\alpha}^{\pm} \\
& | & \widehat{\alpha}^{+} \\
& | & \widehat{\alpha}^{-} \\
& | & \overrightarrow{\widetilde{\alpha}^{+}} \\
& | & \overrightarrow{\widehat{\alpha}^{-}} \\
& | & P \\
& | & N \\
& | & \mathit{auSol} \\
& | & \mathit{terminals} \\
& | & v \\
& | & \overrightarrow{v} \\
& | & c \\
& | & \mathit{vctx} \\
& | & \mathit{formula}
\end{array}
$$

$\boxed{\Gamma;\, \Theta \vDash N \leqslant M \dashv \mathit{SC}}$ Negative subtyping

$$
\frac{}{\Gamma;\, \Theta \vDash \alpha^{-} \leqslant \alpha^{-} \dashv \cdot} \quad \text{ANVar}
$$

$$\frac{\Gamma;\Theta \models \mathbf{nf}\,(P) \stackrel{u}{\simeq} \mathbf{nf}\,(Q) \dashv UC}{\Gamma;\,\Theta \models {\uparrow}P \leqslant {\uparrow}Q \dashv UC}\ \textsc{AShiftU}$$

$$\frac{\Gamma;\,\Theta \models P \geqslant Q \dashv SC_1 \quad \Gamma;\,\Theta \models N \leqslant M \dashv SC_2 \quad \Theta \vdash SC_1 \& SC_2 = SC}{\Gamma;\,\Theta \models P \to N \leqslant Q \to M \dashv SC}\ \textsc{AArrow}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma;\,\Theta \models \forall\overrightarrow{\alpha^+}.N \leqslant \forall\overrightarrow{\beta^+}.M \dashv SC\backslash\overrightarrow{\widehat{\alpha}^+}}\ \textsc{AForall}$$

$\boxed{\Gamma;\,\Theta \models P \geqslant Q \dashv SC}$ Positive supertyping

$$\frac{}{\Gamma;\,\Theta \models \alpha^+ \geqslant \alpha^+ \dashv \cdot}\ \textsc{APVar}$$

$$\frac{\Gamma;\Theta \models \mathbf{nf}\,(N) \stackrel{u}{\simeq} \mathbf{nf}\,(M) \dashv UC}{\Gamma;\,\Theta \models {\downarrow}N \geqslant {\downarrow}M \dashv UC}\ \textsc{AShiftD}$$

$$\frac{\Gamma,\overrightarrow{\beta^-};\,\Theta,\overrightarrow{\widehat{\alpha}^-}\{\Gamma,\overrightarrow{\beta^-}\} \models [\overrightarrow{\widehat{\alpha}^-}/\overrightarrow{\alpha^-}]P \geqslant Q \dashv SC}{\Gamma;\,\Theta \models \exists\overrightarrow{\alpha^-}.P \geqslant \exists\overrightarrow{\beta^-}.Q \dashv SC\backslash\overrightarrow{\widehat{\alpha}^-}}\ \textsc{AExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}{\Gamma;\,\Theta \models \widehat{\alpha}^+ \geqslant P \dashv (\widehat{\alpha}^+ :\geqslant Q)}\ \textsc{APUVar}$$

$\boxed{\Gamma;\Phi \models v\colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma;\Phi \models x\colon \mathbf{nf}\,(P)}\ \textsc{ATVar}$$

$$\frac{\Gamma;\Phi \models c\colon N}{\Gamma;\Phi \models \{c\}\colon {\downarrow}N}\ \textsc{ATThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma;\Phi \models v\colon P \quad \Gamma;\cdot \models Q \geqslant P \dashv \cdot}{\Gamma;\Phi \models (v : Q)\colon \mathbf{nf}\,(Q)}\ \textsc{ATPAnnot}$$

$\boxed{\Gamma;\Phi \models c\colon N}$ Negative type inference

$$\frac{\Gamma \vdash M \quad \Gamma;\Phi \models c\colon N \quad \Gamma;\cdot \models N \leqslant M \dashv \cdot}{\Gamma;\Phi \models (c : M)\colon \mathbf{nf}\,(M)}\ \textsc{ATNAnnot}$$

$$\frac{\Gamma \vdash P \quad \Gamma;\Phi,x : P \models c\colon N}{\Gamma;\Phi \models \lambda x : P.c\colon \mathbf{nf}\,(P \to N)}\ \textsc{ATtLam}$$

$$\frac{\Gamma,\alpha^+;\Phi \models c\colon N}{\Gamma;\Phi \models \Lambda\alpha^+.c\colon \mathbf{nf}\,(\forall\alpha^+.N)}\ \textsc{ATTLam}$$

$$\frac{\Gamma;\Phi \models v\colon P}{\Gamma;\Phi \models \mathbf{return}\,v\colon {\uparrow}P}\ \textsc{ATReturn}$$

$$\frac{\Gamma;\Phi \models v\colon P \quad \Gamma;\Phi,x : P \models c\colon N}{\Gamma;\Phi \models \mathbf{let}\,x = v;\,c\colon N}\ \textsc{ATVarLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma;\Phi \models v\colon {\downarrow}M \\ \Gamma;\Phi;\cdot \models M \bullet \vec{v} \Rrightarrow {\uparrow}Q \dashv \Theta;SC_1 \quad \Gamma;\,\Theta \models {\uparrow}Q \leqslant {\uparrow}P \dashv SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC \quad \Gamma;\Phi,x : P \models c\colon N\end{array}}{\Gamma;\Phi \models \mathbf{let}\,x : P = v(\vec{v});\,c\colon N}\ \textsc{ATAppLetAnn}$$

$$\frac{\begin{array}{l}\Gamma;\Phi \models v\colon {\downarrow}M \quad \Gamma;\Phi;\cdot \models M \bullet \vec{v} \Rrightarrow {\uparrow}Q \dashv \Theta;SC \\ \text{<<multiple parses>>} \\ \Gamma;\Phi,x : [\widehat{\sigma}]Q \models c\colon N\end{array}}{\Gamma;\Phi \models \mathbf{let}\,x = v(\vec{v});\,c\colon N}\ \textsc{ATAppLet}$$

$$\frac{\Gamma; \Phi \models v \colon \exists \overrightarrow{\alpha^-}.P \quad \Gamma, \overrightarrow{\alpha^-}; \Phi, x : P \models c \colon N \quad \Gamma \vdash N}{\Gamma; \Phi \models \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c \colon N} \ \textsc{ATUnpack}$$

$\boxed{\Gamma; \Phi; \Theta_1 \models N \bullet \vec{v} \mathrel{=\!\!\gg} M \dashv \Theta_2; SC}$ Application type inference

$$\frac{}{\Gamma; \Phi; \Theta \models N \bullet \cdot \mathrel{=\!\!\gg} \mathbf{nf}\,(N) \dashv \Theta; \cdot} \ \textsc{ATEmptyApp}$$

$$\frac{\begin{array}{l}\Gamma; \Phi \models v \colon P \quad \Gamma; \Theta \models Q \geqslant P \dashv SC_1 \quad \Gamma; \Phi; \Theta \models N \bullet \vec{v} \mathrel{=\!\!\gg} M \dashv \Theta'; SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC\end{array}}{\Gamma; \Phi; \Theta \models Q \to N \bullet v, \vec{v} \mathrel{=\!\!\gg} M \dashv \Theta'; SC} \ \textsc{ATArrowApp}$$

<<multiple parses>>

$$\frac{\vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot}{\Gamma; \Phi; \Theta \models \forall \overrightarrow{\alpha^+}.N \bullet \vec{v} \mathrel{=\!\!\gg} M \dashv \Theta'; SC} \ \textsc{ATForallApp}$$

$\boxed{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\cdot, \alpha^+, \cdot, \cdot)} \ \textsc{AUPVar}$$

$$\frac{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models {\downarrow}N_1 \overset{a}{\simeq} {\downarrow}N_2 \dashv (\Xi, {\downarrow}M, \widehat{\tau}_1, \widehat{\tau}_2)} \ \textsc{AUShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \Gamma = \varnothing \quad \Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \exists \overrightarrow{\alpha^-}.P_1 \overset{a}{\simeq} \exists \overrightarrow{\alpha^-}.P_2 \dashv (\Xi, \exists \overrightarrow{\alpha^-}.Q, \widehat{\tau}_1, \widehat{\tau}_2)} \ \textsc{AUExists}$$

$\boxed{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^- \overset{a}{\simeq} \alpha^- \dashv (\cdot, \alpha^-, \cdot, \cdot)} \ \textsc{AUNVar}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models {\uparrow}P_1 \overset{a}{\simeq} {\uparrow}P_2 \dashv (\Xi, {\uparrow}Q, \widehat{\tau}_1, \widehat{\tau}_2)} \ \textsc{AUShiftU}$$

$$\frac{\overrightarrow{\alpha^+} \cap \Gamma = \varnothing \quad \Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \forall \overrightarrow{\alpha^+}.N_1 \overset{a}{\simeq} \forall \overrightarrow{\alpha^+}.N_2 \dashv (\Xi, \forall \overrightarrow{\alpha^+}.M, \widehat{\tau}_1, \widehat{\tau}_2)} \ \textsc{AUForall}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \widehat{\tau}_1, \widehat{\tau}_2) \quad \Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \widehat{\tau}'_1, \widehat{\tau}'_2)}{\Gamma \models P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \widehat{\tau}_1 \cup \widehat{\tau}'_1, \widehat{\tau}_2 \cup \widehat{\tau}'_2)} \ \textsc{AUArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \models N \overset{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, \widehat{\alpha}^-_{\{N,M\}}, (\widehat{\alpha}^-_{\{N,M\}} :\approx N), (\widehat{\alpha}^-_{\{N,M\}} :\approx M))} \ \textsc{AUAU}$$

$\boxed{\Gamma \vdash e_1 \ \& \ e_2 = e_3}$ Subtyping Constraint Entry Merge

$$\frac{\Gamma \models P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1) \ \& \ (\widehat{\alpha}^+ :\geqslant P_2) = (\widehat{\alpha}^+ :\geqslant Q)} \ \textsc{SCMESupSup}$$

$$\frac{\Gamma; \cdot \models P \geqslant Q \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \ \& \ (\widehat{\alpha}^+ :\geqslant Q) = (\widehat{\alpha}^+ :\approx P)} \ \textsc{SCMEEqSup}$$

$$\frac{\Gamma; \cdot \models Q \geqslant P \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P) \ \& \ (\widehat{\alpha}^+ :\approx Q) = (\widehat{\alpha}^+ :\approx Q)} \ \textsc{SCMESupEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \ \&\ (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \text{SCMEPEQEQ}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \ \&\ (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \text{SCMENEQEQ}$$

$\boxed{\Theta \vdash SC_1 \& SC_2 = SC_3}$ Merge of subtyping constraints
$\boxed{\Gamma \vdash e_1 \& e_2 = e_3}$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P)\&(\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \text{UCMEPEQEQ}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1)\&(\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \text{UCMENEQEQ}$$

$\boxed{\Theta \vdash UC_1 \& UC_2 = UC_3}$
$\boxed{\Gamma \vdash P : e}$ Positive type satisfies with the subtyping constraint entry

$$\frac{\Gamma \vdash P \geqslant_1 Q}{\Gamma \vdash P : (\widehat{\alpha}^+ :\geqslant Q)} \quad \text{SATSCESUP}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash P : (\widehat{\alpha}^+ :\approx Q)} \quad \text{SATSCEPEQ}$$

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies with the subtyping constraint entry

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash N : (\widehat{\alpha}^- :\approx M)} \quad \text{SATSCENEQ}$$

$\boxed{e_1 \textbf{ singular with } P}$ Positive Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^+ :\approx P \textbf{ singular with } \textbf{nf}\,(P)} \quad \text{SINGPEQ}$$

$$\frac{}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.\alpha^+ \textbf{ singular with } \alpha^+} \quad \text{SINGSUPVAR}$$

$$\frac{N \simeq_1^D \alpha_i^-}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.{\downarrow}N \textbf{ singular with } \exists \alpha^-.{\downarrow}\alpha^-} \quad \text{SINGSUPSHIFT}$$

$\boxed{e_1 \textbf{ singular with } N}$ Negative Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^- :\approx N \textbf{ singular with } \textbf{nf}\,(N)} \quad \text{SINGNEQ}$$

$\boxed{SC \textbf{ singular with } \widehat{\sigma}}$ Subtyping Constraint Is Singular
$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-} \quad \text{E1NVAR}$$

$$\frac{P \simeq_1^D Q}{{\uparrow}P \simeq_1^D {\uparrow}Q} \quad \text{E1SHIFTU}$$

$$\frac{P \simeq_1^D Q \quad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M} \quad \text{E1ARROW}$$

$$\frac{\overrightarrow{\alpha^+} \cap \textbf{fv}\,M = \varnothing \quad \mu : (\overrightarrow{\beta^+} \cap \textbf{fv}\,M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \textbf{fv}\,N) \quad N \simeq_1^D [\mu]M}{\forall \overrightarrow{\alpha^+}.N \simeq_1^D \forall \overrightarrow{\beta^+}.M} \quad \text{E1FORALL}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+} \quad \text{E1PVar}$$

$$\frac{N \simeq_1^D M}{\downarrow N \simeq_1^D \downarrow M} \quad \text{E1ShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \mathbf{fv}\, Q = \varnothing \quad \mu : (\overrightarrow{\beta^-} \cap \mathbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \mathbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists \overrightarrow{\alpha^-}.P \simeq_1^D \exists \overrightarrow{\beta^-}.Q} \quad \text{E1Exists}$$

$\boxed{P \simeq_1^D Q}$ Positive unification type equivalence

$\boxed{N \simeq_1^D M}$ Positive unification type equivalence

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M} \quad \text{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q} \quad \text{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-} \quad \text{D1NVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant_1 \uparrow Q} \quad \text{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M} \quad \text{D1Arrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma, \overrightarrow{\beta^+} \vdash [\sigma]N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant_1 \forall \overrightarrow{\beta^+}.M} \quad \text{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+} \quad \text{D1PVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M} \quad \text{D1ShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vdash \sigma : \overrightarrow{\alpha^-} \quad \Gamma, \overrightarrow{\beta^-} \vdash [\sigma]P \geqslant_1 Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P \geqslant_1 \exists \overrightarrow{\beta^-}.Q} \quad \text{D1Exists}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1}$ Equivalence of substitutions

$\boxed{\Gamma \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : vars}$ Equivalence of substitutions

$\boxed{\Theta \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : \mathbf{varset}}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : vars}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash \Phi_1 \simeq_1^{\leqslant} \Phi_2}$ Equivalence of contexts

$\boxed{\Gamma \vdash N \simeq_0^{\leqslant} M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq_0^{\leqslant} M} \quad \text{D0NDef}$$

$\boxed{\Gamma \vdash P \simeq_0^{\leqslant} Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq_0^{\leqslant} Q} \quad \textsc{D0PDef}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_0 \alpha^-} \quad \textsc{D0NVar}$$

$$\frac{\Gamma \vdash P \simeq_0^{\leqslant} Q}{\Gamma \vdash \uparrow P \leqslant_0 \uparrow Q} \quad \textsc{D0ShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/\alpha^+]N \leqslant_0 M \quad M \neq \forall \beta^+.M'}{\Gamma \vdash \forall \alpha^+.N \leqslant_0 M} \quad \textsc{D0ForallL}$$

$$\frac{\Gamma, \alpha^+ \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall \alpha^+.M} \quad \textsc{D0ForallR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \to N \leqslant_0 Q \to M} \quad \textsc{D0Arrow}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_0 \alpha^+} \quad \textsc{D0PVar}$$

$$\frac{\Gamma \vdash N \simeq_0^{\leqslant} M}{\Gamma \vdash \downarrow N \geqslant_0 \downarrow M} \quad \textsc{D0ShiftD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/\alpha^-]P \geqslant_0 Q \quad Q \neq \exists \alpha^-.Q'}{\Gamma \vdash \exists \alpha^-.P \geqslant_0 Q} \quad \textsc{D0ExistsL}$$

$$\frac{\Gamma, \alpha^- \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists \alpha^-.Q} \quad \textsc{D0ExistsR}$$

$\boxed{\Gamma; \Phi \vdash v \colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \vdash x \colon P} \quad \textsc{DTVar}$$

$$\frac{\Gamma; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \{c\} \colon \downarrow N} \quad \textsc{DTThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma; \Phi \vdash (v : Q) \colon Q} \quad \textsc{DTPAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash v \colon P'} \quad \textsc{DTPEquiv}$$

$\boxed{\Gamma; \Phi \vdash c \colon N}$ Negative type inference

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \lambda x : P.c \colon P \to N} \quad \textsc{DTtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \Lambda \alpha^+.c \colon \forall \alpha^+.N} \quad \textsc{DTTLam}$$

$$\frac{\Gamma; \Phi \vdash v \colon P}{\Gamma; \Phi \vdash \mathbf{return}\, v \colon \uparrow P} \quad \textsc{DTReturn}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v; c \colon N} \quad \textsc{DTVarLet}$$

$$\frac{\Gamma; \Phi \vdash v \colon {\downarrow} M \quad \Gamma; \Phi \vdash M \bullet \vec{v} \Rightarrow\!\!> {\uparrow} Q \text{ unique} \quad \Gamma; \Phi, x : Q \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v(\vec{v}); c \colon N} \quad \textsc{DTAppLet}$$

$$\frac{\begin{array}{l} \Gamma \vdash P \quad \Gamma; \Phi \vdash v \colon {\downarrow} M \\ \Gamma; \Phi \vdash M \bullet \vec{v} \Rightarrow\!\!> {\uparrow} Q \quad \Gamma \vdash {\uparrow} Q \leqslant_1 {\uparrow} P \quad \Gamma; \Phi, x : P \vdash c \colon N \end{array}}{\Gamma; \Phi \vdash \mathbf{let}\, x : P = v(\vec{v}); c \colon N} \quad \textsc{DTAppLetAnn}$$

$$\frac{\begin{array}{c} \texttt{<<multiple parses>>} \\ \Gamma, \overrightarrow{\alpha^-}; \Phi, x : P \vdash c \colon N \quad \Gamma \vdash N \end{array}}{\Gamma; \Phi \vdash \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c \colon N} \quad \textsc{DTUnpack}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash (c : M) \colon M} \quad \textsc{DTNAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash c \colon N'} \quad \textsc{DTNEquiv}$$

$\boxed{\Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!> M}$ Application type inference

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash N \bullet \cdot \Rightarrow\!\!> N'} \quad \textsc{DTEmptyApp}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P \quad \Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!> M}{\Gamma; \Phi \vdash Q \to N \bullet v, \vec{v} \Rightarrow\!\!> M} \quad \textsc{DTArrowApp}$$

$$\frac{\begin{array}{l} \Gamma \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma; \Phi \vdash [\sigma] N \bullet \vec{v} \Rightarrow\!\!> M \\ \vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot \end{array}}{\Gamma; \Phi \vdash \forall \overrightarrow{\alpha^+}. N \bullet \vec{v} \Rightarrow\!\!> M} \quad \textsc{DTForallApp}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$

$\boxed{P_1 \vee P_2}$

$\boxed{\mathbf{ord}\; \mathit{vars}\, \mathbf{in}\; P}$

$\boxed{\mathbf{ord}\; \mathit{vars}\, \mathbf{in}\; N}$

$\boxed{\mathbf{ord}\; \mathit{vars}\, \mathbf{in}\; P}$

$\boxed{\mathbf{ord}\; \mathit{vars}\, \mathbf{in}\; N}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{N}')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{P}')}$

$\boxed{\mathbf{nf}\,(\sigma')}$

$\boxed{\mathbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\mathbf{nf}\,(\mu')}$

$\boxed{\sigma'|_{vars}}$

$\boxed{\widehat{\sigma}'|_{vars}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{\mathbf{SC}|_{vars}}$

$\boxed{\mathbf{UC}|_{vars}}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{UC_1 \ \& \ UC_2}$

$\boxed{UC_1 \cup UC_2}$

$\boxed{\Gamma_1 \cup \Gamma_2}$

$\boxed{SC_1 \ \& \ SC_2}$

$\boxed{\widehat{\tau}_1 \ \& \ \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(UC)}$

$\boxed{\mathbf{dom}\,(SC)}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{|SC|}$

$\boxed{\Gamma \models P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \models \alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBV}\textsc{ar}$$

$$\frac{\Gamma, \cdot \models \mathbf{nf}\,(\downarrow N) \overset{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi]\, P} \quad \text{LUBS}\textsc{hift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \models P_1 \vee P_2 = Q}{\Gamma \models \exists \overrightarrow{\alpha^-}.P_1 \vee \exists \overrightarrow{\beta^-}.P_2 = Q} \quad \text{LUBE}\textsc{xists}$$

$\boxed{\textbf{upgrade}\,\Gamma \vdash P\,\textbf{to}\,\Delta = Q}$

$$\frac{\Gamma = \Delta, \overrightarrow{\alpha^{\pm}} \quad \overrightarrow{\beta^{\pm}}\ \textbf{is fresh} \quad \overrightarrow{\gamma^{\pm}}\ \textbf{is fresh} \quad \Delta, \overrightarrow{\beta^{\pm}}, \overrightarrow{\gamma^{\pm}} \vDash [\overrightarrow{\beta^{\pm}}/\overrightarrow{\alpha^{\pm}}]P \vee [\overrightarrow{\gamma^{\pm}}/\overrightarrow{\alpha^{\pm}}]P = Q}{\textbf{upgrade}\,\Gamma \vdash P\,\textbf{to}\,\Delta = Q}\ \textsc{LUBUpgrade}$$

$\boxed{\textbf{nf}\,(N) = M}$

$$\frac{}{\textbf{nf}\,(\alpha^{-}) = \alpha^{-}}\ \textsc{NrmNVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\uparrow P) = \uparrow Q}\ \textsc{NrmShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(P \to N) = Q \to M}\ \textsc{NrmArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\forall \overrightarrow{\alpha^{+}}.N) = \forall \overrightarrow{\alpha^{+}}'.N'}\ \textsc{NrmForall}$$

$\boxed{\textbf{nf}\,(P) = Q}$

$$\frac{}{\textbf{nf}\,(\alpha^{+}) = \alpha^{+}}\ \textsc{NrmPVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\downarrow N) = \downarrow M}\ \textsc{NrmShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\exists \overrightarrow{\alpha^{-}}.P) = \exists \overrightarrow{\alpha^{-}}'.P'}\ \textsc{NrmExists}$$

$\boxed{\textbf{nf}\,(N) = M}$

$$\frac{}{\textbf{nf}\,(\widehat{\alpha}^{-}) = \widehat{\alpha}^{-}}\ \textsc{NrmNUVar}$$

$\boxed{\textbf{nf}\,(P) = Q}$

$$\frac{}{\textbf{nf}\,(\widehat{\alpha}^{+}) = \widehat{\alpha}^{+}}\ \textsc{NrmPUVar}$$

$\boxed{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,N = \vec{\alpha}}$

$$\frac{\alpha^{-} \in \mathit{vars}}{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,\alpha^{-} = \alpha^{-}}\ \textsc{ONVarIn}$$

$$\frac{\alpha^{-} \notin \mathit{vars}}{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,\alpha^{-} = \cdot}\ \textsc{ONVarNIn}$$

$$\frac{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,P = \vec{\alpha}}{\textbf{ord}\,\mathit{vars}\,\textbf{in}\uparrow P = \vec{\alpha}}\ \textsc{OShiftU}$$

$$\frac{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,P = \vec{\alpha}_1 \quad \textbf{ord}\,\mathit{vars}\,\textbf{in}\,N = \vec{\alpha}_2}{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)}\ \textsc{OArrow}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^{+}} = \varnothing \quad \textbf{ord}\,\mathit{vars}\,\textbf{in}\,N = \vec{\alpha}}{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,\forall \overrightarrow{\alpha^{+}}.N = \vec{\alpha}}\ \textsc{OForall}$$

$\boxed{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,P = \vec{\alpha}}$

$$\frac{\alpha^+ \in vars}{\mathbf{ord}\ vars\ \mathbf{in}\ \alpha^+ = \alpha^+}\ \text{OPVarIn}$$

$$\frac{\alpha^+ \notin vars}{\mathbf{ord}\ vars\ \mathbf{in}\ \alpha^+ = \cdot}\ \text{OPVarNIn}$$

$$\frac{\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}}{\mathbf{ord}\ vars\ \mathbf{in}\ \downarrow N = \vec{\alpha}}\ \text{OShiftD}$$

$$\frac{vars \cap \overrightarrow{\alpha^-} = \varnothing \quad \mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}}{\mathbf{ord}\ vars\ \mathbf{in}\ \exists\overrightarrow{\alpha^-}.P = \vec{\alpha}}\ \text{OExists}$$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ vars\ \mathbf{in}\ \widehat{\alpha}^- = \cdot}\ \text{ONUVar}$$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ vars\ \mathbf{in}\ \widehat{\alpha}^+ = \cdot}\ \text{OPUVar}$$

$\boxed{\Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv UC}$ Negative unification

$$\frac{}{\Gamma; \Theta \models \alpha^- \stackrel{u}{\simeq} \alpha^- \dashv \cdot}\ \text{UNVar}$$

$$\frac{\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \uparrow P \stackrel{u}{\simeq} \uparrow Q \dashv UC}\ \text{UShiftU}$$

$$\frac{\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC_1 \quad \Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv UC_2}{\Gamma; \Theta \models P \to N \stackrel{u}{\simeq} Q \to M \dashv UC_1 \ \&\ UC_2}\ \text{UArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Theta \models N \stackrel{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \forall\overrightarrow{\alpha^+}.N \stackrel{u}{\simeq} \forall\overrightarrow{\alpha^+}.M \dashv UC}\ \text{UForall}$$

$$\frac{\widehat{\alpha}^-\{\Delta\} \in \Theta \quad \Delta \vdash N}{\Gamma; \Theta \models \widehat{\alpha}^- \stackrel{u}{\simeq} N \dashv (\widehat{\alpha}^- :\approx N)}\ \text{UNUVar}$$

$\boxed{\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC}$ Positive unification

$$\frac{}{\Gamma; \Theta \models \alpha^+ \stackrel{u}{\simeq} \alpha^+ \dashv \cdot}\ \text{UPVar}$$

$$\frac{\Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \downarrow N \stackrel{u}{\simeq} \downarrow M \dashv UC}\ \text{UShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \exists\overrightarrow{\alpha^-}.P \stackrel{u}{\simeq} \exists\overrightarrow{\alpha^-}.Q \dashv UC}\ \text{UExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \Delta \vdash P}{\Gamma; \Theta \models \widehat{\alpha}^+ \stackrel{u}{\simeq} P \dashv (\widehat{\alpha}^+ :\approx P)}\ \text{UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma \vdash \alpha^-}\ \text{WFTNVar}$$

$$\frac{\Gamma \vdash P}{\Gamma \vdash \uparrow P}\ \textsc{WFTShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash N}{\Gamma \vdash P \to N}\ \textsc{WFTArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+} \vdash N}{\Gamma \vdash \forall \overrightarrow{\alpha^+}. N}\ \textsc{WFTForall}$$

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma \vdash \alpha^+}\ \textsc{WFTPVar}$$

$$\frac{\Gamma \vdash N}{\Gamma \vdash \downarrow N}\ \textsc{WFTShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-} \vdash P}{\Gamma \vdash \exists \overrightarrow{\alpha^-}. P}\ \textsc{WFTExists}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness
$\boxed{\Gamma \vdash P}$ Positive type well-formedness
$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness
$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness
$\boxed{\Gamma; \Xi \vdash N}$ Negative algorithmic type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma; \Xi \vdash \alpha^-}\ \textsc{WFATNVar}$$

$$\frac{\widehat{\alpha}^- \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^-}\ \textsc{WFATNUVar}$$

$$\frac{\Gamma; \Xi \vdash P}{\Gamma; \Xi \vdash \uparrow P}\ \textsc{WFATShiftU}$$

$$\frac{\Gamma; \Xi \vdash P \quad \Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash P \to N}\ \textsc{WFATArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Xi \vdash N}{\Gamma; \Xi \vdash \forall \overrightarrow{\alpha^+}. N}\ \textsc{WFATForall}$$

$\boxed{\Gamma; \Xi \vdash P}$ Positive algorithmic type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma; \Xi \vdash \alpha^+}\ \textsc{WFATPVar}$$

$$\frac{\widehat{\alpha}^+ \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^+}\ \textsc{WFATPUVar}$$

$$\frac{\Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash \downarrow N}\ \textsc{WFATShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Xi \vdash P}{\Gamma; \Xi \vdash \exists \overrightarrow{\alpha^-}. P}\ \textsc{WFATExists}$$

$\boxed{\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness
$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness
$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution signature

$\boxed{\Theta \vdash \widehat{\sigma}}$ Unification substitution signature

$\boxed{(\Theta \vdash UC) \vdash \widehat{\sigma}}$ Unification substitution satisfies unification constraint

$\boxed{(\Theta \vdash SC) \vdash \widehat{\sigma}}$ Unification substitution satisfies subtyping constraint

$\boxed{\Gamma \vdash \widehat{\sigma} : \Xi}$ Unification substitution general signature

$\boxed{\Gamma \vdash e}$ Unification constraint entry well-formedness

$\boxed{\Gamma \vdash e}$ Subtyping constraint entry well-formedness

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies unification constraint

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies unification constraint

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies subtyping constraint

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies subtyping constraint

$\boxed{\Theta \vdash UC}$ Unification constraint well-formedness

$\boxed{\Theta \vdash SC}$ Subtyping constraint well-formedness

$\boxed{\Gamma \vdash \overrightarrow{v}}$ Argument List well-formedness

$\boxed{\Gamma \vdash \Phi}$ Context well-formedness

$\boxed{\Gamma \vdash v}$ Value well-formedness

$$\frac{}{\Gamma \vdash x} \quad \textsc{WFATVar}$$

$\boxed{\Gamma \vdash c}$ Computation well-formedness

$$\frac{\Gamma \vdash v \quad \Gamma \vdash c \quad \Gamma \vdash \overrightarrow{v}}{\Gamma \vdash \mathbf{let}\, x = v(\overrightarrow{v}); c} \quad \textsc{WFATAppLet}$$

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Definition rules:        117 good    21 bad
Definition rule clauses: 241 good    21 bad
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