α, β, α, β, γ, δ	type variables
n, m, i, j	index variables
x, y, z	term variables

$n,\ m$	$::=$			cohort index
	$\|$	0		
	$\|$	$n+1$		
$\alpha^+,\ \beta^+,\ \gamma^+,\ \delta^+$	$::=$			positive variable
	$\|$	α^+		
	$\|$	α^{+n}		
	$\|$	$\beta^\pm$		
$\alpha^-,\ \beta^-,\ \gamma^-,\ \delta^-$	$::=$			negative variable
	$\|$	α^-		
	$\|$	α^{-n}		
	$\|$	$\alpha^\pm$		
$\alpha^\pm,\ \beta^\pm$	$::=$			positive or negative variable
	$\|$	$\alpha^\pm$		
	$\|$	$\alpha^{\pm n}$		
σ	$::=$			substitution
	$\|$	$\cdot$		
	$\|$	id		
	$\|$	P/α^+		
	$\|$	N/α^-		
	$\|$	$\overrightarrow{P}/\overrightarrow{\alpha^+}$		
	$\|$	$\overrightarrow{N}/\overrightarrow{\alpha^-}$		
	$\|$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha}^-}$		
	$\|$	μ		
	$\|$	$\sigma_1 \circ \sigma_2$		
	$\|$	$\sigma_1 \lll \sigma_2$		
	$\|$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\|$	(σ)	S	
	$\|$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\|$	$\mathbf{nf}\,(\sigma')$	M	
	$\|$	$\sigma'\|_{vars}$	M	
e	$::=$			entry of a subtyping constraint
	$\|$	$\widehat{\alpha}^+ :\approx P$		
	$\|$	$\widehat{\alpha}^- :\approx N$		
	$\|$	$\widehat{\alpha}^+ :\geqslant P$		
	$\|$	(e)	S	
	$\|$	$SC(\widehat{\alpha}^-)$	M	
	$\|$	$e_1\ \&\ e_2$	M	
e	$::=$			entry of a unification constraint
	$\|$	$\widehat{\alpha}^+ :\approx P$		
	$\|$	$\widehat{\alpha}^- :\approx N$		
	$\|$	(e)	S	
	$\|$	$e_1\ \&\ e_2$	M	

UC	$::=$			unification constraint
	$\|$	$\cdot$		
	$\|$	e		
	$\|$	$UC\backslash vars$		
	$\|$	$UC\|vars$		
	$\|$	$UC_1 \cup UC_2$		
	$\|$	$\overline{UC_i}^{\,i}$		concatenate
	$\|$	(UC)	S	
	$\|$	$UC'\|_{vars}$	M	
	$\|$	$UC_1 \mathbin{\&} UC_2$	M	
	$\|$	$UC_1 \cup UC_2$	M	
	$\|$	$\|SC\|$	M	
SC	$::=$			subtyping constraint
	$\|$	$\cdot$		
	$\|$	e		
	$\|$	$SC\backslash vars$		
	$\|$	$SC\|vars$		
	$\|$	$SC_1 \cup SC_2$		
	$\|$	UC		
	$\|$	$\overline{SC_i}^{\,i}$		concatenate
	$\|$	(SC)	S	
	$\|$	$SC'\|_{vars}$	M	
	$\|$	$SC_1 \mathbin{\&} SC_2$	M	
$\widehat{\sigma}$	$::=$			unification substitution
	$\|$	$\cdot$		
	$\|$	$P/\widehat{\alpha}^{+}$		
	$\|$	$N/\widehat{\alpha}^{-}$		
	$\|$	$\overrightarrow{P}/\overrightarrow{\widehat{\alpha}^{+}}$		
	$\|$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha}^{-}}$		
	$\|$	$(\widehat{\sigma})$	S	
	$\|$	$\widehat{\sigma}_1 \circ \widehat{\sigma}_2$		
	$\|$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\|$	$\mathbf{nf}\,(\widehat{\sigma}')$	M	
	$\|$	$\widehat{\sigma}'\|_{vars}$	M	
$\widehat{\tau},\ \widehat{\rho}$	$::=$			anti-unification substitution
	$\|$	$\cdot$		
	$\|$	$\widehat{\alpha}^{-} :\approx N$		
	$\|$	$\widehat{\alpha}^{-} :\approx N$		
	$\|$	$\overrightarrow{\alpha^{-}}/\overrightarrow{\widehat{\alpha}^{-}}$		
	$\|$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha}^{-}}$		
	$\|$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
	$\|$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	$\|$	$(\widehat{\tau})$	S	
	$\|$	$\widehat{\tau}'\|_{vars}$	M	
	$\|$	$\widehat{\tau}_1 \mathbin{\&} \widehat{\tau}_2$	M	
$P,\ Q,\ R$	$::=$			positive types

	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists \alpha^-.P$		
	$\mid$	$[\sigma]P$	M	
$N,\ M,\ K$	$::=$			negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$\forall \alpha^+.N$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$[\sigma]N$	M	
$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+},\ \overrightarrow{\gamma^+},\ \overrightarrow{\delta^+}$	$::=$			positive variable list
	$\mid$	$\cdot$		empty list
	$\mid$	α^+		a variable
	$\mid$	$\overrightarrow{\alpha^+}$		a variable
	$\mid$	$\overrightarrow{\overrightarrow{\alpha^+}_i}^{i}$		concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$	$::=$			negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	α^-		a variable
	$\mid$	$\overrightarrow{\alpha^-}$		variables
	$\mid$	$\overrightarrow{\overrightarrow{\alpha^-}_i}^{i}$		concatenate lists
$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$	$::=$			positive or negative variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\alpha^\pm$		a variable
	$\mid$	$\overrightarrow{\mathbf{pa}}$		variables
	$\mid$	$\overrightarrow{\overrightarrow{\alpha^\pm}_i}^{i}$		concatenate lists
$P,\ Q,\ R$	$::=$			multi-quantified positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists \overrightarrow{\alpha^-}.P$		
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\widehat{\tau}]\,P$	M	
	$\mid$	$[\widehat{\sigma}]\,P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	(P)	S	
	$\mid$	$P_1 \vee P_2$	M	
	$\mid$	$\mathbf{nf}\,(P')$	M	
$N,\ M,\ K$	$::=$			multi-quantified negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$\forall \overrightarrow{\alpha^+}.N$		
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\widehat{\tau}]\,N$	M	

	|	$[\mu]N$	M	
	|	$[\widehat{\sigma}]N$	M	
	|	(N)	S	
	|	$\mathbf{nf}\,(N')$	M	
$\vec{P},\ \vec{Q}$	::=			list of positive types
	|	$\cdot$		empty list
	|	P		a singel type
	|	$[\sigma]\vec{P}$	M	
	|	$\overline{\vec{P}_i}^{\,i}$		concatenate lists
	|	$(\vec{P})$	S	
	|	$\mathbf{nf}\,(\vec{P}')$	M	
$\vec{N},\ \vec{M}$	::=			list of negative types
	|	$\cdot$		empty list
	|	N		a singel type
	|	$[\sigma]\vec{N}$	M	
	|	$\overline{\vec{N}_i}^{\,i}$		concatenate lists
	|	$(\vec{N})$	S	
	|	$\mathbf{nf}\,(\vec{N}')$	M	
$\Delta,\ \Gamma$	::=			declarative type context
	|	$\cdot$		empty context
	|	$\overrightarrow{\alpha^{+}}$		list of variables
	|	$\overrightarrow{\alpha^{-}}$		list of variables
	|	$\overrightarrow{\alpha^{\pm}}$		list of variables
	|	$vars$		
	|	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	|	(Γ)	S	
	|	$\Theta(\widehat{\alpha}^{+})$	M	
	|	$\Theta(\widehat{\alpha}^{-})$	M	
	|	$\Gamma_1 \cup \Gamma_2$	M	
Θ	::=			algorithmic variable context
	|	$\cdot$		empty context
	|	$\vec{\alpha}\{\Delta\}$		from an ordered list of variables
	|	$\overrightarrow{\widehat{\alpha}^{+}\{\Delta\}}$		from a variable to a list
	|	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	|	(Θ)	S	
	|	$\Theta\vert_{vars}$		leave only those variables that are in the set
	|	$\Theta_1 \cup \Theta_2$		
Ξ	::=			anti-unification type variable context
	|	$\cdot$		empty context
	|	$\overrightarrow{\widehat{\alpha}^{+}}$		list of positive variables
	|	$\overrightarrow{\widehat{\alpha}^{-}}$		list of negative variables
	|	$\mathbf{uv}\ N$		unification variables
	|	$\mathbf{uv}\ P$		unification variables
	|	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	|	(Ξ)	S	

	|	$\Xi_1 \cup \Xi_2$		
	|	$\Xi_1 \cap \Xi_2$		
	|	$\Xi' \vert_{vars}$	M	
	|	$\mathbf{dom}\,(UC)$	M	
	|	$\mathbf{dom}\,(SC)$	M	
	|	$\mathbf{dom}\,(\widehat{\sigma})$	M	
	|	$\mathbf{dom}\,(\widehat{\tau})$	M	
	|	$\mathbf{dom}\,(\Theta)$	M	
$\vec{\alpha},\ \vec{\beta}$	::=			ordered positive or negative variables
	|	$\cdot$		empty list
	|	$\overrightarrow{\alpha^{+}}$		list of variables
	|	$\overrightarrow{\alpha^{-}}$		list of variables
	|	$\overrightarrow{\alpha^{\pm}}$		list of variables
	|	$\overrightarrow{\widehat{\alpha}^{+}}$		list of variables
	|	$\overrightarrow{\widehat{\alpha}^{-}}$		list of variables
	|	$\vec{\alpha}_1 \backslash vars$		setminus
	|	Γ		context
	|	$vars$		
	|	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	|	$(\vec{\alpha})$	S	parenthesis
	|	$[\mu]\vec{\alpha}$	M	apply moving to list
	|	$[\vec{\mu}]\vec{\alpha}$	M	apply umoving to list
	|	$\mathbf{ord}\ vars\,\mathbf{in}\ P$	M	
	|	$\mathbf{ord}\ vars\,\mathbf{in}\ N$	M	
	|	$\mathbf{ord}\ vars\,\mathbf{in}\ P$	M	
	|	$\mathbf{ord}\ vars\,\mathbf{in}\ N$	M	
$vars$	::=			set of variables
	|	$\varnothing$		empty set
	|	$\mathbf{fv}\ P$		free variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathbf{fv}\ \mathbf{imP}$		free variables
	|	$\mathbf{fv}\ \mathbf{imN}$		free variables
	|	$vars_1 \cap vars_2$		set intersection
	|	$vars_1 \cup vars_2$		set union
	|	$vars_1 \backslash vars_2$		set complement
	|	$\mathbf{mv}\ \mathbf{imP}$		movable variables
	|	$\mathbf{mv}\ \mathbf{imN}$		movable variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathbf{fv}\ P$		free variables
	|	$(vars)$	S	parenthesis
	|	$\vec{\alpha}$		ordered list of variables
	|	$[\mu]vars$	M	apply moving to varset
	|	Ξ		anti-unification context
μ	::=			
	|	$\cdot$		empty moving
	|	$pma1 \mapsto pma2$		Positive unit substitution
	|	$nma1 \mapsto nma2$		Positive unit substitution

	|	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	|	$\mu_1 \circ \mu_2$	M	Composition
	|	$\overline{\mu_i}^{\,i}$		concatenate movings
	|	$\mu\vert_{vars}$	M	restriction on a set
	|	μ^{-1}	M	inversion
	|	$\mathbf{nf}\,(\mu')$	M	
$\vec{\mu}$	::=			
	|	$\cdot$		empty moving
	|	$\overrightarrow{\widehat{\alpha}^{+}} / \overrightarrow{\alpha^{+}}$		
	|	$\overrightarrow{\widehat{\alpha}^{-}} / \overrightarrow{\alpha^{-}}$		
$\widehat{\alpha}^{\pm}$	::=			positive/negative unification variable
	|	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	::=			positive unification variable
	|	$\widehat{\alpha}^{+}$		
	|	$\widehat{\alpha}^{+}\{\Delta\}$		
	|	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	::=			negative unification variable
	|	$\widehat{\alpha}^{-}$		
	|	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	|	$\widehat{\alpha}^{-}\{\Delta\}$		
	|	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widehat{\alpha}^{+}},\ \overrightarrow{\widehat{\beta}^{+}}$	::=			positive unification variable list
	|	$\cdot$		empty list
	|	$\widehat{\alpha}^{+}$		a variable
	|	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	|	$\overline{\overrightarrow{\widehat{\alpha}^{+}}_i}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}},\ \overrightarrow{\widehat{\beta}^{-}}$	::=			negative unification variable list
	|	$\cdot$		empty list
	|	$\widehat{\alpha}^{-}$		a variable
	|	Ξ		from an antiunification context
	|	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	|	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	|	$\overline{\overrightarrow{\widehat{\alpha}^{-}}_i}^{\,i}$		concatenate lists
$P,\ Q$	::=			a positive algorithmic type (potentially with metavariables)
	|	$\widehat{\alpha}^{+}$		
	|	α^{+}		
	|	$\downarrow N$		
	|	$\exists\overrightarrow{\alpha^{-}}.\,P$		
	|	$[\sigma]P$	M	
	|	$[\widehat{\tau}]P$	M	
	|	$[\mu]P$	M	
	|	$[\widehat{\sigma}]P$	M	

	$\vert$	$[\vec{\mu}]P$	M
	$\vert$	(P)	S
	$\vert$	$\mathbf{nf}\,(P')$	M
$N,\ M$	$::=$		a negative algorithmic type (potentially with metavariables)
	$\vert$	$\widehat{\alpha}^-$	
	$\vert$	α^-	
	$\vert$	$\uparrow P$	
	$\vert$	$P \to N$	
	$\vert$	$\forall\overrightarrow{\alpha^+}.N$	
	$\vert$	$[\sigma]N$	M
	$\vert$	$[\widehat{\tau}]N$	M
	$\vert$	$[\mu]N$	M
	$\vert$	$[\widehat{\sigma}]N$	M
	$\vert$	$[\vec{\mu}]N$	M
	$\vert$	(N)	S
	$\vert$	$\mathbf{nf}\,(N')$	M
$auSol$	$::=$		
	$\vert$	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	$\vert$	$(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$	
$terminals$	$::=$		
	$\vert$	$\exists$	
	$\vert$	$\forall$	
	$\vert$	$\uparrow$	
	$\vert$	$\downarrow$	
	$\vert$	$\to$	
	$\vert$	$\leftrightarrow$	
	$\vert$	$\in$	
	$\vert$	$\notin$	
	$\vert$	$\cdot$	
	$\vert$	$\vdash$	
	$\vert$	$\leqslant$	
	$\vert$	$\geqslant$	
	$\vert$	$\simeq$	
	$\vert$	$\cup$	
	$\vert$	$\cap$	
	$\vert$	$\setminus$	
	$\vert$	$\subseteq$	
	$\vert$	$\mapsto$	
	$\vert$	$\stackrel{u}{\simeq}$	
	$\vert$	$\stackrel{a}{\simeq}$	
	$\vert$	$\varnothing$	
	$\vert$	$\circ$	
	$\vert$	$\Rightarrow$	
	$\vert$	$\models$	
	$\vert$	$\dashv$	
	$\vert$	$\neq$	
	$\vert$	$\equiv_n$	

	$\mid$	$\vee$	
	$\mid$	$\Downarrow$	
	$\mid$	$:\geqslant$	
	$\mid$	$:\simeq$	
	$\mid$	Λ	
	$\mid$	λ	
	$\mid$	$\mathbf{let}^{\exists}$	
	$\mid$	$\bullet$	
	$\mid$	$\Rrightarrow$	
	$\mid$	$\lll$	
$v,\ w$	$::=$		value terms
	$\mid$	x	
	$\mid$	$\{c\}$	
	$\mid$	$(v : P)$	
	$\mid$	(v)	M
$\overrightarrow{v}$	$::=$		list of arguments
	$\mid$	$\cdot$	
	$\mid$	v	
	$\mid$	$\overline{\overrightarrow{v}_i}^{\,i}$	concatenate
$c,\ d$	$::=$		computation terms
	$\mid$	$(c : N)$	
	$\mid$	$\lambda x : P.c$	
	$\mid$	$\Lambda\alpha^{+}.c$	
	$\mid$	$\mathbf{return}\, v$	
	$\mid$	$\mathbf{let}\, x = v; c$	
	$\mid$	$\mathbf{let}\, x : P = v(\overrightarrow{v}); c$	
	$\mid$	$\mathbf{let}\, x = v(\overrightarrow{v}); c$	
	$\mid$	$\mathbf{let}^{\exists}(\overrightarrow{\alpha^{-}}, x) = v; c$	
$vctx,\ \Phi$	$::=$		variable context
	$\mid$	$\cdot$	
	$\mid$	$x : P$	
	$\mid$	$\overline{\Phi_i}^{\,i}$	concatenate contexts
$formula$	$::=$		
	$\mid$	$judgement$	
	$\mid$	$judgement$ unique	
	$\mid$	$formula_1 \quad .. \quad formula_n$	
	$\mid$	$\mu : vars_1 \leftrightarrow vars_2$	
	$\mid$	$\mu\,\mathbf{is\ bijective}$	
	$\mid$	$x : P \in \Phi$	
	$\mid$	$UC_1 \subseteq UC_2$	
	$\mid$	$UC_1 = UC_2$	
	$\mid$	$SC_1 \subseteq SC_2$	
	$\mid$	$e \in SC$	
	$\mid$	$e \in UC$	
	$\mid$	$vars_1 \subseteq vars_2$	

| $vars_1 \subseteq vars_2 \subseteq vars_3$
| $vars_1 = vars_2$
| $vars$ **is fresh**
| $\alpha^- \notin vars$
| $\alpha^+ \notin vars$
| $\alpha^- \in vars$
| $\alpha^+ \in vars$
| $\widehat{\alpha}^+ \in vars$
| $\widehat{\alpha}^- \in vars$
| $\widehat{\alpha}^- \in \Theta$
| $\widehat{\alpha}^+ \in \Theta$
| $\widehat{\alpha}^- \notin vars$
| $\widehat{\alpha}^+ \notin vars$
| $\widehat{\alpha}^- \notin \Theta$
| $\widehat{\alpha}^+ \notin \Theta$
| $\widehat{\alpha}^- \in \Xi$
| $\widehat{\alpha}^- \notin \Xi$
| $\widehat{\alpha}^+ \in \Xi$
| $\widehat{\alpha}^+ \notin \Xi$
| if any other rule is not applicable
| $\vec{\alpha}_1 = \vec{\alpha}_2$
| $e_1 = e_2$
| $e_1 = e_2$
| $\widehat{\sigma}_1 = \widehat{\sigma}_2$
| $N = M$
| $\Theta \subseteq \Theta'$
| $\vec{v}_1 = \vec{v}_2$
| $N \neq M$
| $P \neq Q$
| $N \neq M$
| $P \neq Q$
| $P \neq Q$
| $N \neq M$
| $\vec{v}_1 \neq \vec{v}_2$
| $\overrightarrow{\alpha^+}_1 \neq \overrightarrow{\alpha^+}_2$
| $|\overrightarrow{\alpha^-}| + |\overrightarrow{\beta^-}| > 0$
| $|\overrightarrow{\alpha^+}| + |\overrightarrow{\beta^+}| > 0$

A ::=
| $\Gamma; \Theta \models N \leqslant M \dashv SC$ — Negative subtyping
| $\Gamma; \Theta \models P \geqslant Q \dashv SC$ — Positive supertyping

AT ::=
| $\Gamma; \Phi \models v : P$ — Positive type inference
| $\Gamma; \Phi \models c : N$ — Negative type inference
| $\Gamma; \Phi; \Theta_1 \models N \bullet \vec{v} \Rrightarrow M \dashv \Theta_2; SC$ — Application type inference

AU ::=
| $\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$
| $\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$

SCM	::=		
	|	$\Gamma \vdash e_1 \ \& \ e_2 = e_3$	Subtyping Constraint Entry Merge
	|	$\Theta \vdash SC_1 \& SC_2 = SC_3$	Merge of subtyping constraints
UCM	::=		
	|	$\Gamma \vdash e_1 \& e_2 = e_3$	
	|	$\Theta \vdash UC_1 \& UC_2 = UC_3$	Merge of unification constraints
SATSCE	::=		
	|	$\Gamma \vdash P : e$	Positive type satisfies with the subtyping constraint entry
	|	$\Gamma \vdash N : e$	Negative type satisfies with the subtyping constraint entry
SING	::=		
	|	e_1 **singular with** P	Positive Subtyping Constraint Entry Is Singular
	|	e_1 **singular with** N	Negative Subtyping Constraint Entry Is Singular
	|	SC **singular with** $\widehat{\sigma}$	Subtyping Constraint Is Singular
E1	::=		
	|	$N \simeq_1^D M$	Negative multi-quantified type equivalence
	|	$P \simeq_1^D Q$	Positive multi-quantified type equivalence
	|	$P \simeq_1^D Q$	Positive unification type equivalence
	|	$N \simeq_1^D M$	Positive unification type equivalence
D1	::=		
	|	$\Gamma \vdash N \simeq_1^{\leqslant} M$	Negative equivalence on MQ types
	|	$\Gamma \vdash P \simeq_1^{\leqslant} Q$	Positive equivalence on MQ types
	|	$\Gamma \vdash N \leqslant_1 M$	Negative subtyping
	|	$\Gamma \vdash P \geqslant_1 Q$	Positive supertyping
	|	$\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1$	Equivalence of substitutions
	|	$\Gamma \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : vars$	Equivalence of substitutions
	|	$\Theta \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions
	|	$\Gamma \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions
	|	$\Gamma \vdash \Phi_1 \simeq_1^{\leqslant} \Phi_2$	Equivalence of contexts
D0	::=		
	|	$\Gamma \vdash N \simeq_0^{\leqslant} M$	Negative equivalence
	|	$\Gamma \vdash P \simeq_0^{\leqslant} Q$	Positive equivalence
	|	$\Gamma \vdash N \leqslant_0 M$	Negative subtyping
	|	$\Gamma \vdash P \geqslant_0 Q$	Positive supertyping
DT	::=		
	|	$\Gamma; \Phi \vdash v : P$	Positive type inference
	|	$\Gamma; \Phi \vdash c : N$	Negative type inference
	|	$\Gamma; \Phi \vdash N \bullet \overrightarrow{v} \Longrightarrow M$	Application type inference
EQ	::=		
	|	$N = M$	Negative type equality (alpha-equivalence)
	|	$P = Q$	Positive type equuality (alphha-equivalence)
	|	$P = Q$	

$LUBF$	$::=$		
	$\vert$	$P_1 \vee P_2 === Q$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, P === \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, N === \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, P === \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, N === \vec{\alpha}$	
	$\vert$	$\mathbf{nf}\,(N') === N$	
	$\vert$	$\mathbf{nf}\,(P') === P$	
	$\vert$	$\mathbf{nf}\,(N') === N$	
	$\vert$	$\mathbf{nf}\,(P') === P$	
	$\vert$	$\mathbf{nf}\,(\vec{N}') === \vec{N}$	
	$\vert$	$\mathbf{nf}\,(\vec{P}') === \vec{P}$	
	$\vert$	$\mathbf{nf}\,(\sigma') === \sigma$	
	$\vert$	$\mathbf{nf}\,(\widehat{\sigma}') === \widehat{\sigma}$	
	$\vert$	$\mathbf{nf}\,(\mu') === \mu$	
	$\vert$	$\sigma'\vert_{vars}$	
	$\vert$	$\widehat{\sigma}'\vert_{vars}$	
	$\vert$	$\widehat{\tau}'\vert_{vars}$	
	$\vert$	$\Xi'\vert_{vars}$	
	$\vert$	$SC'\vert_{vars}$	
	$\vert$	$UC'\vert_{vars}$	
	$\vert$	$e_1 \;\&\; e_2$	
	$\vert$	$e_1 \;\&\; e_2$	
	$\vert$	$UC_1 \;\&\; UC_2$	
	$\vert$	$UC_1 \cup UC_2$	
	$\vert$	$\Gamma_1 \cup \Gamma_2$	
	$\vert$	$SC_1 \;\&\; SC_2$	
	$\vert$	$\widehat{\tau}_1 \;\&\; \widehat{\tau}_2$	
	$\vert$	$\mathbf{dom}\,(UC) === \Xi$	
	$\vert$	$\mathbf{dom}\,(SC) === \Xi$	
	$\vert$	$\mathbf{dom}\,(\widehat{\sigma}) === \Xi$	
	$\vert$	$\mathbf{dom}\,(\widehat{\tau}) === \Xi$	
	$\vert$	$\mathbf{dom}\,(\Theta) === \Xi$	
	$\vert$	$\vert SC\vert === UC$	
LUB	$::=$		
	$\vert$	$\Gamma \models P_1 \vee P_2 = Q$	Least Upper Bound (Least Common Supertype)
	$\vert$	$\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q$	
Nrm	$::=$		
	$\vert$	$\mathbf{nf}\,(N) = M$	
	$\vert$	$\mathbf{nf}\,(P) = Q$	
	$\vert$	$\mathbf{nf}\,(N) = M$	
	$\vert$	$\mathbf{nf}\,(P) = Q$	
$Order$	$::=$		
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, N = \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, P = \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, N = \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\, vars\, \mathbf{in}\, P = \vec{\alpha}$	

U	::=		
	|	$\Gamma; \Theta \vDash N \overset{u}{\simeq} M \dashv UC$	Negative unification
	|	$\Gamma; \Theta \vDash P \overset{u}{\simeq} Q \dashv UC$	Positive unification
WFT	::=		
	|	$\Gamma \vdash N$	Negative type well-formedness
	|	$\Gamma \vdash P$	Positive type well-formedness
	|	$\Gamma \vdash N$	Negative type well-formedness
	|	$\Gamma \vdash P$	Positive type well-formedness
	|	$\Gamma \vdash \overrightarrow{N}$	Negative type list well-formedness
	|	$\Gamma \vdash \overrightarrow{P}$	Positive type list well-formedness
$WFAT$	::=		
	|	$\Gamma; \Xi \vdash N$	Negative algorithmic type well-formedness
	|	$\Gamma; \Xi \vdash P$	Positive algorithmic type well-formedness
	|	$\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1$	Antiunification substitution well-formedness
	|	$\Gamma \vdash^{\sqsupseteq} \Theta$	Unification context well-formedness
	|	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution signature
	|	$\Theta \vdash \widehat{\sigma} : \Xi$	Unification substitution signature
	|	$\Gamma \vdash \widehat{\sigma} : \Xi$	Unification substitution general signature
	|	$\Theta \vdash \widehat{\sigma} : UC$	Unification substitution satisfies unification constraint
	|	$\Theta \vdash \widehat{\sigma} : SC$	Unification substitution satisfies subtyping constraint
	|	$\Gamma \vdash e$	Unification constraint entry well-formedness
	|	$\Gamma \vdash e$	Subtyping constraint entry well-formedness
	|	$\Gamma \vdash P : e$	Positive type satisfies unification constraint
	|	$\Gamma \vdash N : e$	Negative type satisfies unification constraint
	|	$\Gamma \vdash P : e$	Positive type satisfies subtyping constraint
	|	$\Gamma \vdash N : e$	Negative type satisfies subtyping constraint
	|	$\Theta \vdash UC : \Xi$	Unification constraint well-formedness with specified domai
	|	$\Theta \vdash SC : \Xi$	Subtyping constraint well-formedness with specified domain
	|	$\Theta \vdash UC$	Unification constraint well-formedness
	|	$\Theta \vdash SC$	Subtyping constraint well-formedness
	|	$\Gamma \vdash \overrightarrow{v}$	Argument List well-formedness
	|	$\Gamma \vdash \Phi$	Context well-formedness
	|	$\Gamma \vdash v$	Value well-formedness
	|	$\Gamma \vdash c$	Computation well-formedness
$judgement$	::=		
	|	A	
	|	AT	
	|	AU	
	|	SCM	
	|	UCM	
	|	$SATSCE$	
	|	$SING$	
	|	$E1$	
	|	$D1$	
	|	$D0$	
	|	DT	
	|	EQ	

$$
\begin{array}{lll}
 & | & LUB \\
 & | & Nrm \\
 & | & Order \\
 & | & U \\
 & | & WFT \\
 & | & WFAT \\
 & & \\
user\_syntax & ::= & \\
 & | & \alpha \\
 & | & n \\
 & | & x \\
 & | & n \\
 & | & \alpha^{+} \\
 & | & \alpha^{-} \\
 & | & \alpha^{\pm} \\
 & | & \sigma \\
 & | & e \\
 & | & e \\
 & | & UC \\
 & | & SC \\
 & | & \widehat{\sigma} \\
 & | & \widehat{\tau} \\
 & | & P \\
 & | & N \\
 & | & \overrightarrow{\alpha^{+}} \\
 & | & \overrightarrow{\alpha^{-}} \\
 & | & \overrightarrow{\alpha^{\pm}} \\
 & | & P \\
 & | & N \\
 & | & \overrightarrow{P} \\
 & | & \overrightarrow{N} \\
 & | & \Gamma \\
 & | & \Theta \\
 & | & \Xi \\
 & | & \overrightarrow{\alpha} \\
 & | & vars \\
 & | & \mu \\
 & | & \overrightarrow{\mu} \\
 & | & \widehat{\alpha}^{\pm} \\
 & | & \widehat{\alpha}^{+} \\
 & | & \widehat{\alpha}^{-} \\
 & | & \overrightarrow{\widehat{\alpha}^{+}} \\
 & | & \overrightarrow{\widehat{\alpha}^{-}} \\
 & | & P \\
 & | & N \\
 & | & auSol \\
 & | & terminals \\
 & | & v \\
 & | & \overrightarrow{v} \\
 & | & c
\end{array}
$$

| *vctx*
| *formula*

$\boxed{\Gamma;\ \Theta \vDash N \leqslant M \dashv SC}$ Negative subtyping

$$\frac{}{\Gamma;\ \Theta \vDash \alpha^- \leqslant \alpha^- \dashv \cdot}\quad \text{ANVar}$$

$$\frac{\Gamma;\Theta \vDash \mathbf{nf}\,(P) \overset{u}{\simeq} \mathbf{nf}\,(Q) \dashv UC}{\Gamma;\ \Theta \vDash \uparrow P \leqslant \uparrow Q \dashv UC}\quad \text{AShiftU}$$

$$\frac{\Gamma;\ \Theta \vDash P \geqslant Q \dashv SC_1 \quad \Gamma;\ \Theta \vDash N \leqslant M \dashv SC_2 \quad \Theta \vdash SC_1 \& SC_2 = SC}{\Gamma;\ \Theta \vDash P \to N \leqslant Q \to M \dashv SC}\quad \text{AArrow}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma;\ \Theta \vDash \forall \overrightarrow{\alpha^+}.N \leqslant \forall \overrightarrow{\beta^+}.M \dashv SC \backslash \overrightarrow{\widehat{\alpha}^+}}\quad \text{AForall}$$

$\boxed{\Gamma;\ \Theta \vDash P \geqslant Q \dashv SC}$ Positive supertyping

$$\frac{}{\Gamma;\ \Theta \vDash \alpha^+ \geqslant \alpha^+ \dashv \cdot}\quad \text{APVar}$$

$$\frac{\Gamma;\Theta \vDash \mathbf{nf}\,(N) \overset{u}{\simeq} \mathbf{nf}\,(M) \dashv UC}{\Gamma;\ \Theta \vDash \downarrow N \geqslant \downarrow M \dashv UC}\quad \text{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-};\ \Theta, \overrightarrow{\widehat{\alpha}^-}\{\Gamma, \overrightarrow{\beta^-}\} \vDash [\overrightarrow{\widehat{\alpha}^-}/\overrightarrow{\alpha^-}]P \geqslant Q \dashv SC}{\Gamma;\ \Theta \vDash \exists \overrightarrow{\alpha^-}.P \geqslant \exists \overrightarrow{\beta^-}.Q \dashv SC \backslash \overrightarrow{\widehat{\alpha}^-}}\quad \text{AExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}{\Gamma;\ \Theta \vDash \widehat{\alpha}^+ \geqslant P \dashv (\widehat{\alpha}^+ :\geqslant Q)}\quad \text{APUVar}$$

$\boxed{\Gamma;\Phi \vDash v\colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma;\Phi \vDash x\colon \mathbf{nf}\,(P)}\quad \text{ATVar}$$

$$\frac{\Gamma;\Phi \vDash c\colon N}{\Gamma;\Phi \vDash \{c\}\colon \downarrow N}\quad \text{ATThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma;\Phi \vDash v\colon P \quad \Gamma;\cdot \vDash Q \geqslant P \dashv \cdot}{\Gamma;\Phi \vDash (v : Q)\colon \mathbf{nf}\,(Q)}\quad \text{ATPAnnot}$$

$\boxed{\Gamma;\Phi \vDash c\colon N}$ Negative type inference

$$\frac{\Gamma \vdash M \quad \Gamma;\Phi \vDash c\colon N \quad \Gamma;\cdot \vDash N \leqslant M \dashv \cdot}{\Gamma;\Phi \vDash (c : M)\colon \mathbf{nf}\,(M)}\quad \text{ATNAnnot}$$

$$\frac{\Gamma \vdash P \quad \Gamma;\Phi, x : P \vDash c\colon N}{\Gamma;\Phi \vDash \lambda x : P.c\colon \mathbf{nf}\,(P \to N)}\quad \text{ATtLam}$$

$$\frac{\Gamma, \alpha^+;\Phi \vDash c\colon N}{\Gamma;\Phi \vDash \Lambda\alpha^+.c\colon \mathbf{nf}\,(\forall\alpha^+.N)}\quad \text{ATTLam}$$

$$\frac{\Gamma;\Phi \vDash v\colon P}{\Gamma;\Phi \vDash \mathbf{return}\,v\colon \uparrow P}\quad \text{ATReturn}$$

$$\frac{\Gamma;\Phi \vDash v\colon P \quad \Gamma;\Phi, x : P \vDash c\colon N}{\Gamma;\Phi \vDash \mathbf{let}\,x = v; c\colon N}\quad \text{ATVarLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma; \Phi \vDash v \colon {\downarrow} M \\ \Gamma; \Phi; \cdot \vDash M \bullet \vec{v} \Longrightarrow\!\!\!> {\uparrow} Q \dashv \Theta; SC_1 \quad \Gamma; \Theta \vDash {\uparrow} Q \leqslant {\uparrow} P \dashv SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC \quad \Gamma; \Phi, x : P \vDash c \colon N\end{array}}{\Gamma; \Phi \vDash \mathbf{let}\, x : P = v(\vec{v}); c \colon N} \ \textsc{ATAppLetAnn}$$

$$\frac{\begin{array}{l}\Gamma; \Phi \vDash v \colon {\downarrow} M \quad \Gamma; \Phi; \cdot \vDash M \bullet \vec{v} \Longrightarrow\!\!\!> {\uparrow} Q \dashv \Theta; SC \\ \mathbf{uv}\, Q = \mathbf{dom}\,(SC) \quad SC\, \mathbf{singular\ with}\, \widehat{\sigma} \\ \Gamma; \Phi, x : [\widehat{\sigma}] Q \vDash c \colon N\end{array}}{\Gamma; \Phi \vDash \mathbf{let}\, x = v(\vec{v}); c \colon N} \ \textsc{ATAppLet}$$

$$\frac{\Gamma; \Phi \vDash v \colon \exists \overrightarrow{\alpha^-}.P \quad \Gamma, \overrightarrow{\alpha^-}; \Phi, x : P \vDash c \colon N \quad \Gamma \vdash N}{\Gamma; \Phi \vDash \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c \colon N} \ \textsc{ATUnpack}$$

$\boxed{\Gamma; \Phi; \Theta_1 \vDash N \bullet \vec{v} \Longrightarrow\!\!\!> M \dashv \Theta_2; SC}$ Application type inference

$$\frac{}{\Gamma; \Phi; \Theta \vDash N \bullet \cdot \Longrightarrow\!\!\!> \mathbf{nf}\,(N) \dashv \Theta; \cdot} \ \textsc{ATEmptyApp}$$

$$\frac{\begin{array}{l}\Gamma; \Phi \vDash v \colon P \quad \Gamma; \Theta \vDash Q \geqslant P \dashv SC_1 \quad \Gamma; \Phi; \Theta \vDash N \bullet \vec{v} \Longrightarrow\!\!\!> M \dashv \Theta'; SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC\end{array}}{\Gamma; \Phi; \Theta \vDash Q \to N \bullet v, \vec{v} \Longrightarrow\!\!\!> M \dashv \Theta'; SC} \ \textsc{ATArrowApp}$$

<<multiple parses>>

$$\frac{\vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot}{} \ \textsc{ATForallApp}$$

<<multiple parses>>

$\boxed{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \vDash \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\cdot, \alpha^+, \cdot, \cdot)} \ \textsc{AUPVar}$$

$$\frac{\Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash {\downarrow} N_1 \overset{a}{\simeq} {\downarrow} N_2 \dashv (\Xi, {\downarrow} M, \widehat{\tau}_1, \widehat{\tau}_2)} \ \textsc{AUShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \Gamma = \varnothing \quad \Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P_1 \overset{a}{\simeq} \exists \overrightarrow{\alpha^-}.P_2 \dashv (\Xi, \exists \overrightarrow{\alpha^-}.Q, \widehat{\tau}_1, \widehat{\tau}_2)} \ \textsc{AUExists}$$

$\boxed{\Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \vDash \alpha^- \overset{a}{\simeq} \alpha^- \dashv (\cdot, \alpha^-, \cdot, \cdot)} \ \textsc{AUNVar}$$

$$\frac{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash {\uparrow} P_1 \overset{a}{\simeq} {\uparrow} P_2 \dashv (\Xi, {\uparrow} Q, \widehat{\tau}_1, \widehat{\tau}_2)} \ \textsc{AUShiftU}$$

$$\frac{\overrightarrow{\alpha^+} \cap \Gamma = \varnothing \quad \Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \forall \overrightarrow{\alpha^+}.N_1 \overset{a}{\simeq} \forall \overrightarrow{\alpha^+}.N_2 \dashv (\Xi, \forall \overrightarrow{\alpha^+}.M, \widehat{\tau}_1, \widehat{\tau}_2)} \ \textsc{AUForall}$$

$$\frac{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \widehat{\tau}_1, \widehat{\tau}_2) \quad \Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \widehat{\tau}'_1, \widehat{\tau}'_2)}{\Gamma \vDash P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \widehat{\tau}_1 \cup \widehat{\tau}'_1, \widehat{\tau}_2 \cup \widehat{\tau}'_2)} \ \textsc{AUArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \vDash N \overset{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, \widehat{\alpha}^-_{\{N,M\}}, (\widehat{\alpha}^-_{\{N,M\}} :\approx N), (\widehat{\alpha}^-_{\{N,M\}} :\approx M))} \ \textsc{AUAU}$$

$\boxed{\Gamma \vdash e_1 \ \&\ e_2 = e_3}$ Subtyping Constraint Entry Merge

$$\frac{\Gamma \vDash P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1) \;\&\; (\widehat{\alpha}^+ :\geqslant P_2) = (\widehat{\alpha}^+ :\geqslant Q)} \quad \textsc{SCMESupSup}$$

$$\frac{\Gamma; \cdot \vDash P \geqslant Q \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \;\&\; (\widehat{\alpha}^+ :\geqslant Q) = (\widehat{\alpha}^+ :\approx P)} \quad \textsc{SCMEEqSup}$$

$$\frac{\Gamma; \cdot \vDash Q \geqslant P \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P) \;\&\; (\widehat{\alpha}^+ :\approx Q) = (\widehat{\alpha}^+ :\approx Q)} \quad \textsc{SCMESupEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \;\&\; (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \textsc{SCMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \;\&\; (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \textsc{SCMENEqEq}$$

$\boxed{\Theta \vdash SC_1 \& SC_2 = SC_3}$ Merge of subtyping constraints

$\boxed{\Gamma \vdash e_1 \& e_2 = e_3}$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \& (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \textsc{UCMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \& (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \textsc{UCMENEqEq}$$

$\boxed{\Theta \vdash UC_1 \& UC_2 = UC_3}$ Merge of unification constraints

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies with the subtyping constraint entry

$$\frac{\Gamma \vdash P \geqslant_1 Q}{\Gamma \vdash P : (\widehat{\alpha}^+ :\geqslant Q)} \quad \textsc{SATSCESup}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash P : (\widehat{\alpha}^+ :\approx Q)} \quad \textsc{SATSCEPEq}$$

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies with the subtyping constraint entry

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash N : (\widehat{\alpha}^- :\approx M)} \quad \textsc{SATSCENEq}$$

$\boxed{e_1 \,\mathbf{singular\ with}\, P}$ Positive Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^+ :\approx P \,\mathbf{singular\ with\ nf}\, (P)} \quad \textsc{SingPEq}$$

$$\frac{}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.\alpha^+ \,\mathbf{singular\ with}\, \alpha^+} \quad \textsc{SingSupVar}$$

$$\frac{N \simeq_1^D \alpha_i^-}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.\downarrow N \,\mathbf{singular\ with}\, \exists \alpha^-.\downarrow \alpha^-} \quad \textsc{SingSupShift}$$

$\boxed{e_1 \,\mathbf{singular\ with}\, N}$ Negative Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^- :\approx N \,\mathbf{singular\ with\ nf}\, (N)} \quad \textsc{SingNEq}$$

$\boxed{SC \,\mathbf{singular\ with}\, \widehat{\sigma}}$ Subtyping Constraint Is Singular

$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-} \quad \textsc{E1NVar}$$

$$\frac{P \simeq_1^D Q}{\uparrow P \simeq_1^D \uparrow Q} \quad \text{E1ShiftU}$$

$$\frac{P \simeq_1^D Q \quad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M} \quad \text{E1Arrow}$$

$$\frac{\overrightarrow{\alpha^+} \cap \mathbf{fv}\, M = \varnothing \quad \mu : (\overrightarrow{\beta^+} \cap \mathbf{fv}\, M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \mathbf{fv}\, N) \quad N \simeq_1^D [\mu]M}{\forall \overrightarrow{\alpha^+}.N \simeq_1^D \forall \overrightarrow{\beta^+}.M} \quad \text{E1Forall}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+} \quad \text{E1PVar}$$

$$\frac{N \simeq_1^D M}{\downarrow N \simeq_1^D \downarrow M} \quad \text{E1ShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \mathbf{fv}\, Q = \varnothing \quad \mu : (\overrightarrow{\beta^-} \cap \mathbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \mathbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists \overrightarrow{\alpha^-}.P \simeq_1^D \exists \overrightarrow{\beta^-}.Q} \quad \text{E1Exists}$$

$\boxed{P \simeq_1^D Q}$ Positive unification type equivalence

$\boxed{N \simeq_1^D M}$ Positive unification type equivalence

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M} \quad \text{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q} \quad \text{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-} \quad \text{D1NVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant_1 \uparrow Q} \quad \text{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M} \quad \text{D1Arrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma, \overrightarrow{\beta^+} \vdash [\sigma]N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant_1 \forall \overrightarrow{\beta^+}.M} \quad \text{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+} \quad \text{D1PVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M} \quad \text{D1ShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vdash \sigma : \overrightarrow{\alpha^-} \quad \Gamma, \overrightarrow{\beta^-} \vdash [\sigma]P \geqslant_1 Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P \geqslant_1 \exists \overrightarrow{\beta^-}.Q} \quad \text{D1Exists}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1}$ Equivalence of substitutions

$\boxed{\Gamma \vdash \sigma_1 \simeq^{\leqslant}_1 \sigma_2 : \mathit{vars}}$ Equivalence of substitutions

$\boxed{\Theta \vdash \widehat{\sigma}_1 \simeq^{\leqslant}_1 \widehat{\sigma}_2 : \mathit{vars}}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash \widehat{\sigma}_1 \simeq^{\leqslant}_1 \widehat{\sigma}_2 : \mathit{vars}}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash \Phi_1 \simeq^{\leqslant}_1 \Phi_2}$ Equivalence of contexts

$\boxed{\Gamma \vdash N \simeq^{\leqslant}_0 M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq^{\leqslant}_0 M} \quad \text{D0NDef}$$

$\boxed{\Gamma \vdash P \simeq^{\leqslant}_0 Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq^{\leqslant}_0 Q} \quad \text{D0PDef}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_0 \alpha^-} \quad \text{D0NVar}$$

$$\frac{\Gamma \vdash P \simeq^{\leqslant}_0 Q}{\Gamma \vdash \uparrow P \leqslant_0 \uparrow Q} \quad \text{D0ShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/\alpha^+]N \leqslant_0 M \quad M \neq \forall \beta^+.M'}{\Gamma \vdash \forall \alpha^+.N \leqslant_0 M} \quad \text{D0ForallL}$$

$$\frac{\Gamma, \alpha^+ \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall \alpha^+.M} \quad \text{D0ForallR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \to N \leqslant_0 Q \to M} \quad \text{D0Arrow}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_0 \alpha^+} \quad \text{D0PVar}$$

$$\frac{\Gamma \vdash N \simeq^{\leqslant}_0 M}{\Gamma \vdash \downarrow N \geqslant_0 \downarrow M} \quad \text{D0ShiftD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/\alpha^-]P \geqslant_0 Q \quad Q \neq \exists \alpha^-.Q'}{\Gamma \vdash \exists \alpha^-.P \geqslant_0 Q} \quad \text{D0ExistsL}$$

$$\frac{\Gamma, \alpha^- \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists \alpha^-.Q} \quad \text{D0ExistsR}$$

$\boxed{\Gamma; \Phi \vdash v \colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \vdash x \colon P} \quad \text{DTVar}$$

$$\frac{\Gamma; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \{c\} \colon \downarrow N} \quad \text{DTThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma; \Phi \vdash (v : Q) \colon Q} \quad \text{DTPAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash v \colon P'} \quad \text{DTPEquiv}$$

$\boxed{\Gamma; \Phi \vdash c \colon N}$ Negative type inference

$$\frac{\Gamma \vdash P \quad \Gamma ; \Phi , x : P \vdash c \colon N}{\Gamma ; \Phi \vdash \lambda x : P . c \colon P \rightarrow N} \quad \textsc{DTtLam}$$

$$\frac{\Gamma , \alpha^{+} ; \Phi \vdash c \colon N}{\Gamma ; \Phi \vdash \Lambda \alpha^{+} . c \colon \forall \alpha^{+} . N} \quad \textsc{DTTLam}$$

$$\frac{\Gamma ; \Phi \vdash v \colon P}{\Gamma ; \Phi \vdash \mathbf{return}\, v \colon \uparrow P} \quad \textsc{DTReturn}$$

$$\frac{\Gamma ; \Phi \vdash v \colon P \quad \Gamma ; \Phi , x : P \vdash c \colon N}{\Gamma ; \Phi \vdash \mathbf{let}\, x = v ; c \colon N} \quad \textsc{DTVarLet}$$

$$\frac{\Gamma ; \Phi \vdash v \colon \downarrow M \quad \Gamma ; \Phi \vdash M \bullet \vec{v} \Rrightarrow \uparrow Q \text{ unique} \quad \Gamma ; \Phi , x : Q \vdash c \colon N}{\Gamma ; \Phi \vdash \mathbf{let}\, x = v (\vec{v}) ; c \colon N} \quad \textsc{DTAppLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma ; \Phi \vdash v \colon \downarrow M \\ \Gamma ; \Phi \vdash M \bullet \vec{v} \Rrightarrow \uparrow Q \quad \Gamma \vdash \uparrow Q \leqslant_1 \uparrow P \quad \Gamma ; \Phi , x : P \vdash c \colon N\end{array}}{\Gamma ; \Phi \vdash \mathbf{let}\, x : P = v (\vec{v}) ; c \colon N} \quad \textsc{DTAppLetAnn}$$

<<multiple parses>>

$$\frac{\Gamma , \overrightarrow{\alpha^{-}} ; \Phi , x : P \vdash c \colon N \quad \Gamma \vdash N}{\Gamma ; \Phi \vdash \mathbf{let}^{\exists} (\overrightarrow{\alpha^{-}} , x) = v ; c \colon N} \quad \textsc{DTUnpack}$$

<<multiple parses>>

$$\frac{}{\Gamma ; \Phi \vdash (c : M) \colon M} \quad \textsc{DTNAnnot}$$

<<multiple parses>>

$$\frac{}{\Gamma ; \Phi \vdash c \colon N'} \quad \textsc{DTNEquiv}$$

$\boxed{\Gamma ; \Phi \vdash N \bullet \vec{v} \Rrightarrow M}$ Application type inference

<<multiple parses>>

$$\frac{}{\Gamma ; \Phi \vdash N \bullet \cdot \Rrightarrow N'} \quad \textsc{DTEmptyApp}$$

$$\frac{\Gamma ; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P \quad \Gamma ; \Phi \vdash N \bullet \vec{v} \Rrightarrow M}{\Gamma ; \Phi \vdash Q \rightarrow N \bullet v , \vec{v} \Rrightarrow M} \quad \textsc{DTArrowApp}$$

$$\frac{\begin{array}{l}\Gamma \vdash \sigma : \overrightarrow{\alpha^{+}} \quad \Gamma ; \Phi \vdash [\sigma] N \bullet \vec{v} \Rrightarrow M \\ \vec{v} \neq \cdot \quad \overrightarrow{\alpha^{+}} \neq \cdot\end{array}}{\Gamma ; \Phi \vdash \forall \overrightarrow{\alpha^{+}} . N \bullet \vec{v} \Rrightarrow M} \quad \textsc{DTForallApp}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$

$\boxed{P_1 \vee P_2}$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N}$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\, vars\, \mathbf{in}\, N}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{N}')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{P}')}$

$\boxed{\mathbf{nf}\,(\sigma')}$

$\boxed{\mathbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\mathbf{nf}\,(\mu')}$

$\boxed{\sigma'|_{vars}}$

$\boxed{\widehat{\sigma}'|_{vars}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{SC'|_{vars}}$

$\boxed{UC'|_{vars}}$

$\boxed{e_1 \ \&\ e_2}$

$\boxed{e_1 \ \&\ e_2}$

$\boxed{UC_1 \ \&\ UC_2}$

$\boxed{UC_1 \cup UC_2}$

$\boxed{\Gamma_1 \cup \Gamma_2}$

$\boxed{SC_1 \ \&\ SC_2}$

$\boxed{\widehat{\tau}_1 \ \&\ \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(UC)}$

$\boxed{\mathbf{dom}\,(SC)}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{|SC|}$

$\boxed{\Gamma \models P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \models \alpha^+ \vee \alpha^+ = \alpha^+}\ \textsc{LUBVar}$$

$$\frac{\Gamma, \cdot \models \mathbf{nf}\,(\downarrow N) \overset{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi]\, P}\ \textsc{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \models P_1 \vee P_2 = Q}{\Gamma \models \exists \overrightarrow{\alpha^-}.P_1 \vee \exists \overrightarrow{\beta^-}.P_2 = Q}\ \textsc{LUBExists}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}$

$$\frac{\Gamma = \Delta, \overrightarrow{\alpha^\pm} \quad \overrightarrow{\beta^\pm}\,\mathbf{is\ fresh} \quad \overrightarrow{\gamma^\pm}\,\mathbf{is\ fresh} \qquad \Delta, \overrightarrow{\beta^\pm}, \overrightarrow{\gamma^\pm} \models [\overrightarrow{\beta^\pm}/\overrightarrow{\alpha^\pm}]P \vee [\overrightarrow{\gamma^\pm}/\overrightarrow{\alpha^\pm}]P = Q}{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}\ \textsc{LUBUpgrade}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^-) = \alpha^-}\ \textsc{NrmNVar}$$

$$\frac{\text{<<multiple parses>>}}{\mathbf{nf}\,(\uparrow P) = \uparrow Q}\ \textsc{NrmShiftU}$$

$$\frac{\text{<<multiple parses>>}}{\mathbf{nf}\,(P \to N) = Q \to M}\ \textsc{NrmArrow}$$

$$\frac{\text{<<multiple parses>>}}{\mathbf{nf}\,(\forall \overrightarrow{\alpha^+}.N) = \forall \overrightarrow{\alpha^+}'.N'}\ \textsc{NrmForall}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\alpha^+) = \alpha^+}\ \textsc{NrmPVar}$$

$$\frac{\text{<<multiple parses>>}}{\mathbf{nf}\,(\downarrow N) = \downarrow M}\ \textsc{NrmShiftD}$$

$$\frac{\text{<<multiple parses>>}}{\mathbf{nf}\,(\exists \overrightarrow{\alpha^-}.P) = \exists \overrightarrow{\alpha^-}'.P'}\ \textsc{NrmExists}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^-) = \widehat{\alpha}^-}\ \textsc{NrmNUVar}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^+) = \widehat{\alpha}^+}\ \textsc{NrmPUVar}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}$

$$\frac{\alpha^- \in \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^- = \alpha^-}\ \textsc{ONVarIn}$$

$$\frac{\alpha^- \notin \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^- = \cdot}\ \textsc{ONVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\uparrow P = \vec{\alpha}}\ \textsc{OShiftU}$$

$$\frac{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P = \vec{\alpha}_1 \quad \textbf{ord}\ \mathit{vars}\ \textbf{in}\ N = \vec{\alpha}_2}{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)}\ \textsc{OArrow}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^+} = \varnothing \quad \textbf{ord}\ \mathit{vars}\ \textbf{in}\ N = \vec{\alpha}}{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ \forall\overrightarrow{\alpha^+}.N = \vec{\alpha}}\ \textsc{OForall}$$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P = \vec{\alpha}}$

$$\frac{\alpha^+ \in \mathit{vars}}{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ \alpha^+ = \alpha^+}\ \textsc{OPVarIn}$$

$$\frac{\alpha^+ \notin \mathit{vars}}{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ \alpha^+ = \cdot}\ \textsc{OPVarNIn}$$

$$\frac{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N = \vec{\alpha}}{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ \downarrow N = \vec{\alpha}}\ \textsc{OShiftD}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^-} = \varnothing \quad \textbf{ord}\ \mathit{vars}\ \textbf{in}\ P = \vec{\alpha}}{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ \exists\overrightarrow{\alpha^-}.P = \vec{\alpha}}\ \textsc{OExists}$$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N = \vec{\alpha}}$

$$\frac{}{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ \widehat{\alpha}^- = \cdot}\ \textsc{ONUVar}$$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P = \vec{\alpha}}$

$$\frac{}{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ \widehat{\alpha}^+ = \cdot}\ \textsc{OPUVar}$$

$\boxed{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC}$ Negative unification

$$\frac{}{\Gamma; \Theta \models \alpha^- \overset{u}{\simeq} \alpha^- \dashv \cdot}\ \textsc{UNVar}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \uparrow P \overset{u}{\simeq} \uparrow Q \dashv UC}\ \textsc{UShiftU}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC_1 \quad \Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC_2}{\Gamma; \Theta \models P \to N \overset{u}{\simeq} Q \to M \dashv UC_1\ \&\ UC_2}\ \textsc{UArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \forall\overrightarrow{\alpha^+}.N \overset{u}{\simeq} \forall\overrightarrow{\alpha^+}.M \dashv UC}\ \textsc{UForall}$$

$$\frac{\widehat{\alpha}^-\{\Delta\} \in \Theta \quad \Delta \vdash N}{\Gamma; \Theta \models \widehat{\alpha}^- \overset{u}{\simeq} N \dashv (\widehat{\alpha}^- :\approx N)}\ \textsc{UNUVar}$$

$\boxed{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC}$ Positive unification

$$\frac{}{\Gamma; \Theta \models \alpha^+ \overset{u}{\simeq} \alpha^+ \dashv \cdot}\ \textsc{UPVar}$$

$$\frac{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \downarrow N \overset{u}{\simeq} \downarrow M \dashv UC}\ \textsc{UShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \exists\overrightarrow{\alpha^-}.P \overset{u}{\simeq} \exists\overrightarrow{\alpha^-}.Q \dashv UC}\ \textsc{UExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \Delta \vdash P}{\Gamma; \Theta \models \widehat{\alpha}^+ \overset{u}{\simeq} P \dashv (\widehat{\alpha}^+ :\approx P)} \quad \text{UPUVAR}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma \vdash \alpha^-} \quad \text{WFTNVAR}$$

$$\frac{\Gamma \vdash P}{\Gamma \vdash \uparrow P} \quad \text{WFTSHIFTU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash N}{\Gamma \vdash P \to N} \quad \text{WFTARROW}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+} \vdash N}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N} \quad \text{WFTFORALL}$$

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma \vdash \alpha^+} \quad \text{WFTPVAR}$$

$$\frac{\Gamma \vdash N}{\Gamma \vdash \downarrow N} \quad \text{WFTSHIFTD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-} \vdash P}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P} \quad \text{WFTEXISTS}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness
$\boxed{\Gamma \vdash P}$ Positive type well-formedness
$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness
$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness
$\boxed{\Gamma; \Xi \vdash N}$ Negative algorithmic type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma; \Xi \vdash \alpha^-} \quad \text{WFATNVAR}$$

$$\frac{\widehat{\alpha}^- \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^-} \quad \text{WFATNUVAR}$$

$$\frac{\Gamma; \Xi \vdash P}{\Gamma; \Xi \vdash \uparrow P} \quad \text{WFATSHIFTU}$$

$$\frac{\Gamma; \Xi \vdash P \quad \Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash P \to N} \quad \text{WFATARROW}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Xi \vdash N}{\Gamma; \Xi \vdash \forall \overrightarrow{\alpha^+}.N} \quad \text{WFATFORALL}$$

$\boxed{\Gamma; \Xi \vdash P}$ Positive algorithmic type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma; \Xi \vdash \alpha^+} \quad \text{WFATPVAR}$$

$$\frac{\widehat{\alpha}^+ \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^+} \quad \text{WFATPUVAR}$$

$$\frac{\Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash \downarrow N} \quad \text{WFATSHIFTD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Xi \vdash P}{\Gamma; \Xi \vdash \exists \overrightarrow{\alpha^-}. P} \quad \text{WFATEXISTS}$$

$\boxed{\Gamma; \Xi_2 \vdash \hat{\tau} : \Xi_1}$ Antiunification substitution well-formedness
$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness
$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution signature
$\boxed{\Theta \vdash \hat{\sigma} : \Xi}$ Unification substitution signature
$\boxed{\Gamma \vdash \hat{\sigma} : \Xi}$ Unification substitution general signature
$\boxed{\Theta \vdash \hat{\sigma} : UC}$ Unification substitution satisfies unification constraint
$\boxed{\Theta \vdash \hat{\sigma} : SC}$ Unification substitution satisfies subtyping constraint
$\boxed{\Gamma \vdash e}$ Unification constraint entry well-formedness
$\boxed{\Gamma \vdash e}$ Subtyping constraint entry well-formedness
$\boxed{\Gamma \vdash P : e}$ Positive type satisfies unification constraint
$\boxed{\Gamma \vdash N : e}$ Negative type satisfies unification constraint
$\boxed{\Gamma \vdash P : e}$ Positive type satisfies subtyping constraint
$\boxed{\Gamma \vdash N : e}$ Negative type satisfies subtyping constraint
$\boxed{\Theta \vdash UC : \Xi}$ Unification constraint well-formedness with specified domain
$\boxed{\Theta \vdash SC : \Xi}$ Subtyping constraint well-formedness with specified domain
$\boxed{\Theta \vdash UC}$ Unification constraint well-formedness
$\boxed{\Theta \vdash SC}$ Subtyping constraint well-formedness
$\boxed{\Gamma \vdash \vec{v}}$ Argument List well-formedness
$\boxed{\Gamma \vdash \Phi}$ Context well-formedness
$\boxed{\Gamma \vdash v}$ Value well-formedness

$$\frac{}{\Gamma \vdash x} \quad \text{WFATVAR}$$

$\boxed{\Gamma \vdash c}$ Computation well-formedness

$$\frac{\Gamma \vdash v \quad \Gamma \vdash c \quad \Gamma \vdash \vec{v}}{\Gamma \vdash \mathbf{let}\, x = v(\vec{v}); c} \quad \text{WFATAPPLET}$$

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Definition rules:        118 good    20 bad
Definition rule clauses: 241 good    21 bad
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