1 Vanilla System

1.1 Grammar

$P,\ Q$	$::=$		positive types
	$\mid$	$a+$	
	$\mid$	$\downarrow N$	
	$\mid$	$\exists \alpha^{-}.P$	
$N,\ M$	$::=$		negative types
	$\mid$	$a-$	
	$\mid$	$\uparrow P$	
	$\mid$	$\forall \alpha^{+}.N$	
	$\mid$	$P \rightarrow N$	

1.2 Declarative Subtyping

$\boxed{\Gamma \vdash N \simeq_0^{\leqslant} M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq_0^{\leqslant} M} \ \textsc{D0NDef}$$

$\boxed{\Gamma \vdash P \simeq_0^{\leqslant} Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq_0^{\leqslant} Q} \ \textsc{D0PDef}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash a- \leqslant_0 a-} \ \textsc{D0NVar}$$

$$\frac{\Gamma \vdash P \simeq_0^{\leqslant} Q}{\Gamma \vdash \uparrow P \leqslant_0 \uparrow Q} \ \textsc{D0ShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/a+]N \leqslant_0 M \quad M \neq \forall \beta^{+}.M'}{\Gamma \vdash \forall \alpha^{+}.N \leqslant_0 M} \ \textsc{D0ForallL}$$

$$\frac{\Gamma, \alpha^{+} \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall \alpha^{+}.M} \ \textsc{D0ForallR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \rightarrow N \leqslant_0 Q \rightarrow M} \ \textsc{D0Arrow}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash a+ \geqslant_0 a+} \ \textsc{D0PVar}$$

$$\frac{\Gamma \vdash N \simeq_0^{\leqslant} M}{\Gamma \vdash \downarrow N \geqslant_0 \downarrow M} \ \textsc{D0ShiftD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/a-]P \geqslant_0 Q \quad Q \neq \exists \alpha^{-}.Q'}{\Gamma \vdash \exists \alpha^{-}.P \geqslant_0 Q} \ \textsc{D0ExistsL}$$

$$\frac{\Gamma, \alpha^{-} \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists \alpha^{-}.Q} \ \textsc{D0ExistsR}$$

2 Multi-Quantified System

2.1 Grammar

$P,\ Q$	$::=$		multi-quantified positive types
	$\mid$	α^{+}	

$$
\begin{array}{llll}
 & | & \downarrow N & \\
 & | & \exists\overrightarrow{\alpha^-}.P & P \neq \exists\ldots \\
N,\ M & ::= & & \text{multi-quantified negative types} \\
 & | & \alpha^- & \\
 & | & \uparrow P & \\
 & | & P \to N & \\
 & | & \forall\overrightarrow{\alpha^+}.N & N \neq \forall\ldots
\end{array}
$$

2.2 Declarative Subtyping

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M}\ \text{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q}\ \text{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-}\ \text{D1NVar}$$

$$\frac{\Gamma \vdash P \simeq_1^{\leqslant} Q}{\Gamma \vdash \uparrow P \leqslant_1 \uparrow Q}\ \text{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M}\ \text{D1Arrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vdash P_i \quad \Gamma, \overrightarrow{\beta^+} \vdash [\overrightarrow{P}/\overrightarrow{\alpha^+}]N \leqslant_1 M}{\Gamma \vdash \forall\overrightarrow{\alpha^+}.N \leqslant_1 \forall\overrightarrow{\beta^+}.M}\ \text{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+}\ \text{D1PVar}$$

$$\frac{\Gamma \vdash N \simeq_1^{\leqslant} M}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M}\ \text{D1ShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vdash N_i \quad \Gamma, \overrightarrow{\beta^-} \vdash [\overrightarrow{N}/\overrightarrow{\alpha^-}]P \geqslant_1 Q'}{\Gamma \vdash \exists\overrightarrow{\alpha^-}.P \geqslant_1 \exists\overrightarrow{\beta^-}.Q}\ \text{D1ExistsL}$$

2.3 Declarative Equivalence

$\boxed{N \simeq_1^{D} M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^{D} \alpha^-}\ \text{E1NVar}$$

$$\frac{P \simeq_1^{D} Q}{\uparrow P \simeq_1^{D} \uparrow Q}\ \text{E1ShiftU}$$

$$\frac{P \simeq_1^{D} Q \quad N \simeq_1^{D} M}{P \to N \simeq_1^{D} Q \to M}\ \text{E1Arrow}$$

$$\frac{\mu : (\overrightarrow{\beta^+} \cap \mathbf{fv}\, M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \mathbf{fv}\, N) \quad N \simeq_1^{D} [\mu]M}{\forall\overrightarrow{\alpha^+}.N \simeq_1^{D} \forall\overrightarrow{\beta^+}.M}\ \text{E1Forall}$$

$\boxed{P \simeq_1^{D} Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+}\ \textsc{E1PVar}$$

$$\frac{N \simeq_1^D M}{\downarrow N \simeq_1^D \downarrow M}\ \textsc{E1ShiftD}$$

$$\frac{\mu : (\overrightarrow{\beta^-} \cap \mathbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \mathbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists \overrightarrow{\alpha^-}. P \simeq_1^D \exists \overrightarrow{\beta^-}. Q}\ \textsc{E1Exists}$$

3 Algorithm

3.1 Algorithmic Equivalence

$\boxed{n \vDash N \simeq_1^A M \dashv \mu}$ Negative multi-quantified type equivalence (algorithmic)

$$\frac{}{n \vDash \alpha^- \simeq_1^A \alpha^- \dashv \cdot}\ \textsc{E1ANVar}$$

$$\frac{n \vDash P \simeq_1^A Q \dashv \mu}{n \vDash \uparrow P \simeq_1^A \uparrow Q \dashv \mu}\ \textsc{E1AShiftU}$$

$$\frac{n \vDash P \simeq_1^A Q \dashv \mu_1 \quad n \vDash N \simeq_1^A M \dashv \mu_2 \quad \mu_1 \cup \mu_2\ \textbf{is bijective}}{n \vDash P \to N \simeq_1^A Q \to M \dashv \mu_1 \cup \mu_2}\ \textsc{E1AArrow}$$

$$\frac{n+1 \vDash [\overrightarrow{\widetilde{\alpha}^{+n}} / \overrightarrow{\alpha^+}]N \simeq_1^A [\overrightarrow{\widetilde{\beta}^{+n}} / \overrightarrow{\beta^+}]M \dashv \mu}{n \vDash \forall \overrightarrow{\alpha^+}. N \simeq_1^A \forall \overrightarrow{\beta^+}. M \dashv \mu|_{\mathbf{mv}\, M}}\ \textsc{E1AForall}$$

$$\frac{}{n \vDash \widetilde{\alpha}^{-n} \simeq_1^A \widetilde{\beta}^{-n} \dashv \widetilde{\beta}^{-n} \mapsto \widetilde{\alpha}^{-n}}\ \textsc{E1ANMVar}$$

$\boxed{n \vDash P \simeq_1^A Q \dashv \mu}$ Positive multi-quantified type equivalence (algorithmic)

$$\frac{}{n \vDash \alpha^+ \simeq_1^A \alpha^+ \dashv \cdot}\ \textsc{E1APVar}$$

$$\frac{n \vDash N \simeq_1^A M \dashv \mu}{n \vDash \downarrow N \simeq_1^A \downarrow M \dashv \mu}\ \textsc{E1AShiftD}$$

$$\frac{n+1 \vDash [\overrightarrow{\widetilde{\alpha}^{-n}} / \overrightarrow{\alpha^-}]P \simeq_1^A [\overrightarrow{\widetilde{\beta}^{-n}} / \overrightarrow{\beta^-}]Q \dashv \mu}{n \vDash \exists \overrightarrow{\alpha^-}. P \simeq_1^A \exists \overrightarrow{\beta^-}. Q \dashv \mu|_{\mathbf{mv}\, Q}}\ \textsc{E1AExists}$$

$$\frac{}{n \vDash \widetilde{\alpha}^{+n} \simeq_1^A \widetilde{\beta}^{+n} \dashv \widetilde{\beta}^{+n} \mapsto \widetilde{\alpha}^{+n}}\ \textsc{E1APMVar}$$

3.2 Unification

$\boxed{n \vDash N \overset{u}{\simeq} M \dashv \mu;\ \widehat{\sigma}}$ Negative unification

$$\frac{}{n \vDash \alpha^- \overset{u}{\simeq} \alpha^- \dashv \cdot;\ \cdot}\ \textsc{UNVar}$$

$$\frac{n \vDash P \overset{u}{\simeq} Q \dashv \mu;\ \widehat{\sigma}}{n \vDash \uparrow P \overset{u}{\simeq} \uparrow Q \dashv \mu;\ \widehat{\sigma}}\ \textsc{UShiftU}$$

$$\frac{\begin{array}{l} n \vDash P \overset{u}{\simeq} Q \dashv \mu_1;\ \widehat{\sigma}_1 \quad n \vDash N \overset{u}{\simeq} M \dashv \mu_2;\ \widehat{\sigma}_2 \\ \mu_1 \cup \mu_2\ \textbf{is bijective} \end{array}}{n \vDash P \to N \overset{u}{\simeq} Q \to M \dashv \mu_1 \cup \mu_2;\ \widehat{\sigma}_1 \& \widehat{\sigma}_2}\ \textsc{UArrow}$$

$$\frac{n+1 \vDash [\overrightarrow{\widetilde{\alpha}^{+n}} / \overrightarrow{\alpha^+}]N \overset{u}{\simeq} [\overrightarrow{\widetilde{\beta}^{+n}} / \overrightarrow{\beta^+}]M \dashv \mu;\ \widehat{\sigma}}{n \vDash \forall \overrightarrow{\alpha^+}. N \overset{u}{\simeq} \forall \overrightarrow{\beta^+}. M \dashv \mu|_{\mathbf{mv}\, M};\ \widehat{\sigma}}\ \textsc{UForall}$$

$$\frac{}{n \vDash \widetilde{\alpha}^{-n} \overset{u}{\simeq} \widetilde{\beta}^{-n} \dashv \widetilde{\beta}^{-n} \mapsto \widetilde{\alpha}^{-n};\ \cdot}\ \textsc{UNMVar}$$

$$\frac{\mathbf{fv}\, N \subseteq \mathit{vars} \quad \mathbf{mv}\, N = \varnothing}{n \vDash \widehat{\alpha}^-\{\mathit{vars}\} \stackrel{u}{\simeq} N \Dashv \cdot;\, \widehat{\alpha}^- :\approx N}\ \textsc{UNUVar}$$

$\boxed{n \vDash P \stackrel{u}{\simeq} Q \Dashv \mu;\, \widehat{\sigma}}$ Positive unification

$$\frac{}{n \vDash \alpha^+ \stackrel{u}{\simeq} \alpha^+ \Dashv \cdot;\, \cdot}\ \textsc{UPVar}$$

$$\frac{n \vDash N \stackrel{u}{\simeq} M \Dashv \mu;\, \widehat{\sigma}}{n \vDash {\downarrow} N \stackrel{u}{\simeq} {\downarrow} M \Dashv \mu;\, \widehat{\sigma}}\ \textsc{UShiftD}$$

$$\frac{n+1 \vDash [\overrightarrow{\widetilde{\alpha}^{-n}/\alpha^-}]P \stackrel{u}{\simeq} [\overrightarrow{\widetilde{\beta}^{-n}/\beta^-}]Q \Dashv \mu;\, \widehat{\sigma}}{n \vDash \exists \overrightarrow{\alpha^-}.P \stackrel{u}{\simeq} \exists \overrightarrow{\beta^-}.Q \Dashv \mu|_{\mathbf{mv}\, Q};\, \widehat{\sigma}}\ \textsc{UExists}$$

$$\frac{}{n \vDash \widetilde{\alpha}^{+n} \stackrel{u}{\simeq} \widetilde{\beta}^{+n} \Dashv \widetilde{\beta}^{+n} \mapsto \widetilde{\alpha}^{+n};\, \cdot}\ \textsc{UPMVar}$$

$$\frac{\mathbf{fv}\, P \subseteq \mathit{vars} \quad \mathbf{mv}\, P = \varnothing}{n \vDash \widehat{\alpha}^+\{\mathit{vars}\} \stackrel{u}{\simeq} P \Dashv \cdot;\, \widehat{\alpha}^+ :\approx P}\ \textsc{UPUVar}$$

3.3 Algorithmic Subtyping

$\boxed{\Gamma \vDash N \leqslant M \Dashv \widehat{\sigma}}$ Negative subtyping

$$\frac{}{\Gamma \vDash \alpha^- \leqslant \alpha^- \Dashv \cdot}\ \textsc{ANVar}$$

$$\frac{0 \vDash P \stackrel{u}{\simeq} Q \Dashv \mu;\, \widehat{\sigma}}{\Gamma \vDash {\uparrow} P \leqslant {\uparrow} Q \Dashv \widehat{\sigma}}\ \textsc{AShiftU}$$

$$\frac{\Gamma \vDash P \geqslant Q \Dashv \widehat{\sigma}_1 \quad \Gamma \vDash N \leqslant M \Dashv \widehat{\sigma}_2}{\Gamma \vDash P \to N \leqslant Q \to M \Dashv \widehat{\sigma}_1 \& \widehat{\sigma}_2}\ \textsc{AArrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vDash [\overrightarrow{\widehat{\alpha}^+\{\Gamma, \overrightarrow{\beta^+}\}/\alpha^+}]N \leqslant M \Dashv \widehat{\sigma}}{\Gamma \vDash \forall \overrightarrow{\alpha^+}.N \leqslant \forall \overrightarrow{\beta^+}.M \Dashv \widehat{\sigma} \backslash \overrightarrow{\widehat{\alpha}^+}}\ \textsc{AForall}$$

$\boxed{\Gamma \vDash P \geqslant Q \Dashv \widehat{\sigma}}$ Positive supertyping

$$\frac{}{\Gamma \vDash \alpha^+ \geqslant \alpha^+ \Dashv \cdot}\ \textsc{APVar}$$

$$\frac{0 \vDash N \stackrel{u}{\simeq} M \Dashv \mu;\, \widehat{\sigma}}{\Gamma \vDash {\downarrow} N \geqslant {\downarrow} M \Dashv \widehat{\sigma}}\ \textsc{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vDash [\overrightarrow{\widehat{\alpha}^-\{\Gamma, \overrightarrow{\beta^-}\}/\alpha^-}]P \geqslant Q \Dashv \widehat{\sigma}}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P \geqslant \exists \overrightarrow{\beta^-}.Q \Dashv \widehat{\sigma}}\ \textsc{AExists}$$

$$\frac{\mathit{vars}_1 = \mathbf{fv}\, P \backslash \mathit{vars} \quad \mathit{vars}_2\ \textbf{is fresh}}{\Gamma \vDash \widehat{\alpha}^+\{\mathit{vars}\} \geqslant P \Dashv (\widehat{\alpha}^+ :\geqslant P \vee [\mathit{vars}_2/\mathit{vars}_1]P)}\ \textsc{APUVar}$$

3.4 Unification Solution Merge

$\boxed{e_1 \& e_2 = e_3}$ Unification Solution Entry Merge

$$\frac{}{\widehat{\alpha}^+ :\geqslant P \& \widehat{\alpha}^+ :\geqslant Q = \widehat{\alpha}^+ :\geqslant P \vee Q}\ \textsc{SMEPSupSup}$$

$$\frac{\mathbf{fv}\, P \cup \mathbf{fv}\, Q \vDash P \geqslant Q \Dashv \widehat{\sigma}'}{\widehat{\alpha}^+ :\approx P \& \widehat{\alpha}^+ :\geqslant Q = \widehat{\alpha}^+ :\approx P}\ \textsc{SMEPEqSup}$$

$$\frac{\mathbf{fv}\, P \cup \mathbf{fv}\, Q \vDash Q \geqslant P \Dashv \widehat{\sigma}'}{\widehat{\alpha}^+ :\geqslant P \& \widehat{\alpha}^+ :\approx Q = \widehat{\alpha}^+ :\approx Q}\ \textsc{SMEPSupEq}$$

$$\frac{0 \vDash P \simeq_1^A Q \Dashv \mu}{\widehat{\alpha}^+ :\approx P \& \widehat{\alpha}^+ :\approx Q = \widehat{\alpha}^+ :\approx Q}\ \textsc{SMEPEqEq}$$

$$\frac{0 \vDash N \simeq_1^A M \dashv \mu}{\widehat{\alpha}^- :\approx N \& \widehat{\alpha}^- :\approx M = \widehat{\alpha}^+ :\approx Q} \quad \text{SMENEqEq}$$

$\boxed{\widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3}$ Merge unification solutions

$$\frac{}{\cdot \& \widehat{\sigma} = \widehat{\sigma}} \quad \text{SMEmpty}$$

$$\frac{(\widehat{\alpha}^+ :\approx Q) \in \widehat{\sigma}_2 \quad 0 \vDash P \simeq_1^A Q \dashv \mu \quad \widehat{\sigma}_1 \& (\widehat{\sigma}_2 \backslash \widehat{\alpha}^+) = \widehat{\sigma}_3}{(\widehat{\alpha}^+ :\approx P, \widehat{\sigma}_1) \& \widehat{\sigma}_2 = (\widehat{\alpha}^+ :\approx P, \widehat{\sigma}_3)} \quad \text{SMPEqEq}$$

$$\frac{(\widehat{\alpha}^+ :\geqslant Q) \in \widehat{\sigma}_2 \quad \widehat{\sigma}_1 \& (\widehat{\sigma}_2 \backslash \widehat{\alpha}^+) = \widehat{\sigma}_3}{(\widehat{\alpha}^+ :\geqslant P, \widehat{\sigma}_1) \& \widehat{\sigma}_2 = (\widehat{\alpha}^+ :\geqslant P \vee Q, \widehat{\sigma}_3)} \quad \text{SMPSupSup}$$

$$\frac{(\widehat{\alpha}^+ :\approx Q) \in \widehat{\sigma}_2 \quad \mathbf{fv}\, Q \cup \mathbf{fv}\, P \vDash Q \geqslant P \dashv \widehat{\sigma}' \quad \widehat{\sigma}_1 \& (\widehat{\sigma}_2 \backslash \widehat{\alpha}^+) = \widehat{\sigma}_3}{(\widehat{\alpha}^+ :\geqslant P, \widehat{\sigma}_1) \& \widehat{\sigma}_2 = (\widehat{\alpha}^+ :\approx Q, \widehat{\sigma}_3)} \quad \text{SMPSupEq}$$

$$\frac{(\widehat{\alpha}^+ :\geqslant Q) \in \widehat{\sigma}_2 \quad \mathbf{fv}\, Q \cup \mathbf{fv}\, P \vDash P \geqslant Q \dashv \widehat{\sigma}' \quad \widehat{\sigma}_1 \& (\widehat{\sigma}_2 \backslash \widehat{\alpha}^+) = \widehat{\sigma}_3}{(\widehat{\alpha}^+ :\approx P, \widehat{\sigma}_1) \& \widehat{\sigma}_2 = (\widehat{\alpha}^+ :\approx P, \widehat{\sigma}_3)} \quad \text{SMPEqSup}$$

$$\frac{(\widehat{\alpha}^- :\approx M) \in \widehat{\sigma}_2 \quad 0 \vDash N \simeq_1^A M \dashv \mu \quad \widehat{\sigma}_1 \& (\widehat{\sigma}_2 \backslash \widehat{\alpha}^-) = \widehat{\sigma}_3}{(\widehat{\alpha}^- :\approx N, \widehat{\sigma}_1) \& \widehat{\sigma}_2 = (\widehat{\alpha}^- :\approx N, \widehat{\sigma}_3)} \quad \text{SMNEqEq}$$

3.5 Least Upper Bound

$\boxed{P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBVar}$$

$$\frac{0 \vDash {\downarrow} N \overset{a}{\simeq} {\downarrow} M \dashv (P, \widehat{\sigma}_1, \widehat{\sigma}_2); \mu}{{\downarrow} N \vee {\downarrow} M = \exists \overrightarrow{\alpha^-}. [\overrightarrow{\alpha^-} / \mathbf{uv}\, P] P} \quad \text{LUBShift}$$

$$\frac{\overrightarrow{\alpha^-} \cap \overrightarrow{\beta^-} = \varnothing}{\exists \overrightarrow{\alpha^-}. P_1 \vee \exists \overrightarrow{\beta^-}. P_2 = P_1 \vee P_2} \quad \text{LUBExists}$$

3.6 Antiunification

$\boxed{n \vDash P_1 \overset{a}{\simeq} P_2 \dashv (Q, \widehat{\sigma}_1, \widehat{\sigma}_2); \mu}$

$$\frac{}{n \vDash \widetilde{\alpha}^{+n} \overset{a}{\simeq} \widetilde{\beta}^{+n} \dashv (\alpha^+, \cdot, \cdot); \widetilde{\beta}^{+n} \mapsto \widetilde{\alpha}^{+n}} \quad \text{AUPPVar}$$

$\boxed{n \vDash N_1 \overset{a}{\simeq} N_2 \dashv (M, \widehat{\sigma}_1, \widehat{\sigma}_2); \mu}$

$$\frac{}{n \vDash \widetilde{\alpha}^{-n} \overset{a}{\simeq} \widetilde{\beta}^{-n} \dashv (\alpha^-, \cdot, \cdot); \widetilde{\beta}^{-n} \mapsto \widetilde{\alpha}^{-n}} \quad \text{AUNNVar}$$

$$\frac{n \vDash P_1 \overset{a}{\simeq} P_2 \dashv (Q, \widehat{\sigma}_1, \widehat{\sigma}_2); \mu \quad n \vDash N_1 \overset{a}{\simeq} N_2 \dashv (M, \widehat{\sigma}'_1, \widehat{\sigma}'_2); \mu'}{n \vDash P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (Q \to M, , \cdot); \mu \cup \mu'} \quad \text{AUNArrow}$$