α, β, α, β, γ, δ	type variables
n, m, i, j	index variables
x, y, z	term variables

$n,\ m$	$::=$			cohort index
	$\|$	0		
	$\|$	$n+1$		
$\alpha^+,\ \beta^+,\ \gamma^+,\ \delta^+$	$::=$			positive variable
	$\|$	α^+		
	$\|$	α^{+n}		
	$\|$	$\beta^\pm$		
$\alpha^-,\ \beta^-,\ \gamma^-,\ \delta^-$	$::=$			negative variable
	$\|$	α^-		
	$\|$	α^{-n}		
	$\|$	$\alpha^\pm$		
$\alpha^\pm,\ \beta^\pm$	$::=$			positive or negative variable
	$\|$	$\alpha^\pm$		
	$\|$	$\alpha^{\pm n}$		
σ	$::=$			substitution
	$\|$	$\cdot$		
	$\|$	id		
	$\|$	P/α^+		
	$\|$	N/α^-		
	$\|$	$\overrightarrow{P}/\overrightarrow{\alpha^+}$		
	$\|$	$\overrightarrow{N}/\overrightarrow{\alpha^-}$		
	$\|$	$\mathbf{pmas}\,/\overrightarrow{\alpha^+}$		
	$\|$	$\mathbf{nmas}\,/\overrightarrow{\alpha^-}$		
	$\|$	$\overrightarrow{\widetilde{\alpha^+}}/\overrightarrow{\alpha^+}$		
	$\|$	$\overrightarrow{\widehat{\alpha^-}}/\overrightarrow{\alpha^-}$		
	$\|$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\|$	μ		
	$\|$	$\widehat{\sigma}$		
	$\|$	$\sigma_1 \circ \sigma_2$		
	$\|$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\|$	(σ)	S	
	$\|$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\|$	$\mathbf{nf}\,(\sigma')$	M	
	$\|$	$\sigma'\|_{vars}$	M	
e	$::=$			entry of a unification solution
	$\|$	$\widehat{\alpha}^+ :\approx P$		
	$\|$	$\widehat{\alpha}^- :\approx N$		
	$\|$	$\widehat{\alpha}^+ :\geqslant P$		
	$\|$	(e)	S	
	$\|$	$\widehat{\sigma}(\widehat{\alpha}^+)$	M	
	$\|$	$\widehat{\sigma}(\widehat{\alpha}^-)$	M	
	$\|$	$e_1\ \&\ e_2$	M	
$\widehat{\sigma}$	$::=$			unification solution (substitution)

	$\mid$	$\cdot$		
	$\mid$	e		
	$\mid$	$\widehat{\sigma} \backslash vars$		
	$\mid$	$\widehat{\sigma} \mid vars$		
	$\mid$	$\widehat{\sigma}_1 \cup \widehat{\sigma}_2$		
	$\mid$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\mid$	$(\widehat{\sigma})$	S	
	$\mid$	$\mathbf{nf}\,(\widehat{\sigma}')$	M	
	$\mid$	$\widehat{\sigma}'\vert_{vars}$	M	
	$\mid$	$\widehat{\sigma}_1 \;\&\; \widehat{\sigma}_2$	M	
$\widehat{\tau},\ \widehat{\rho}$	$::=$			anti-unification substitution
	$\mid$	$\cdot$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		
	$\mid$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha}^-}$		
	$\mid$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha}^-}$		
	$\mid$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
	$\mid$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	$\mid$	$(\widehat{\tau})$	S	
	$\mid$	$\widehat{\tau}'\vert_{vars}$	M	
	$\mid$	$\widehat{\tau}_1 \;\&\; \widehat{\tau}_2$	M	
$P,\ Q,\ R$	$::=$			positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists \alpha^-.P$		
	$\mid$	$[\sigma]P$	M	
$N,\ M,\ K$	$::=$			negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$\forall \alpha^+.N$		
	$\mid$	$P \to N$		
	$\mid$	$[\sigma]N$	M	
$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+},\ \overrightarrow{\gamma^+},\ \overrightarrow{\delta^+}$	$::=$			positive variable list
	$\mid$	$\cdot$		empty list
	$\mid$	α^+		a variable
	$\mid$	$\overrightarrow{\alpha^+}$		a variable
	$\mid$	$\overline{\overrightarrow{\alpha^+}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$	$::=$			negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	α^-		a variable
	$\mid$	$\overrightarrow{\alpha^-}$		variables
	$\mid$	$\overline{\overrightarrow{\alpha^-}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$	$::=$			positive or negative variable list

	$\mid$	$\cdot$		empty list
	$\mid$	$\alpha^{\pm}$		a variable
	$\mid$	$\overrightarrow{\mathbf{pa}}$		variables
	$\mid$	$\overline{\overrightarrow{\alpha^{\pm}}_i}^{\,i}$		concatenate lists
$P,\ Q,\ R$	$::=$			multi-quantified positive types
	$\mid$	α^{+}		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^{-}}.P$		$P \neq \exists \ldots$
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\hat{\tau}]\,P$	M	
	$\mid$	$[\hat{\sigma}]\,P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	(P)	S	
	$\mid$	$P_1 \vee P_2$	M	
	$\mid$	$\mathbf{nf}\,(P')$	M	
$N,\ M,\ K$	$::=$			multi-quantified negative types
	$\mid$	α^{-}		
	$\mid$	$\uparrow P$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$\forall\overrightarrow{\alpha^{+}}.N$		$N \neq \forall \ldots$
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\hat{\tau}]\,N$	M	
	$\mid$	$[\mu]N$	M	
	$\mid$	$[\hat{\sigma}]\,N$	M	
	$\mid$	(N)	S	
	$\mid$	$\mathbf{nf}\,(N')$	M	
$\vec{P},\ \vec{Q}$	$::=$			list of positive types
	$\mid$	$\cdot$		empty list
	$\mid$	P		a singel type
	$\mid$	$[\sigma]\vec{P}$	M	
	$\mid$	$\overline{\vec{P}_i}^{\,i}$		concatenate lists
	$\mid$	$\mathbf{nf}\,(\vec{P}')$	M	
$\vec{N},\ \vec{M}$	$::=$			list of negative types
	$\mid$	$\cdot$		empty list
	$\mid$	N		a singel type
	$\mid$	$[\sigma]\vec{N}$	M	
	$\mid$	$\overline{\vec{N}_i}^{\,i}$		concatenate lists
	$\mid$	$\mathbf{nf}\,(\vec{N}')$	M	
$\Delta,\ \Gamma$	$::=$			declarative type context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\alpha^{+}}$		list of variables
	$\mid$	$\overrightarrow{\alpha^{-}}$		list of variables
	$\mid$	$\overrightarrow{\alpha^{\pm}}$		list of variables
	$\mid$	$vars$		
	$\mid$	$\overline{\Gamma_i}^{\,i}$		concatenate contexts

	$\mid$	(Γ)	S	
	$\mid$	$\Theta(\widehat{\alpha}^+)$	M	
	$\mid$	$\Theta(\widehat{\alpha}^-)$	M	
Θ	$::=$			unification type variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\alpha}\{\Delta\}$		from an ordered list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^+\{\Delta\}}$		from a variable to a list
	$\mid$	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	$\mid$	(Θ)	S	
	$\mid$	$\Theta\rvert_{vars}$		leave only those variables that are in the set
	$\mid$	$\Theta_1 \cup \Theta_2$		
Ξ	$::=$			anti-unification type variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\widehat{\alpha}^-}$		list of variables
	$\mid$	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	$\mid$	(Ξ)	S	
	$\mid$	$\Xi_1 \cup \Xi_2$		
	$\mid$	$\Xi_1 \cap \Xi_2$		
	$\mid$	$\Xi'\rvert_{vars}$	M	
$\overrightarrow{\alpha},\ \overrightarrow{\beta}$	$::=$			ordered positive or negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	$\overrightarrow{\alpha^+}$		list of variables
	$\mid$	$\overrightarrow{\alpha^-}$		list of variables
	$\mid$	$\overrightarrow{\alpha^\pm}$		list of variables
	$\mid$	$\overrightarrow{\widetilde{\alpha}^+}$		list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^-}$		list of variables
	$\mid$	$\overrightarrow{\alpha}_1 \backslash vars$		setminus
	$\mid$	Γ		context
	$\mid$	$vars$		
	$\mid$	$\overline{\overrightarrow{\alpha}_i}^{\,i}$		concatenate contexts
	$\mid$	$(\overrightarrow{\alpha})$	S	parenthesis
	$\mid$	$[\mu]\overrightarrow{\alpha}$	M	apply moving to list
	$\mid$	$\mathbf{ord}\, vars\, \mathbf{in}\, P$	M	
	$\mid$	$\mathbf{ord}\, vars\, \mathbf{in}\, N$	M	
	$\mid$	$\mathbf{ord}\, vars\, \mathbf{in}\, P$	M	
	$\mid$	$\mathbf{ord}\, vars\, \mathbf{in}\, N$	M	
$vars$	$::=$			set of variables
	$\mid$	$\varnothing$		empty set
	$\mid$	$\mathbf{fv}\, P$		free variables
	$\mid$	$\mathbf{fv}\, N$		free variables
	$\mid$	$\mathbf{fv}\, \mathbf{imP}$		free variables
	$\mid$	$\mathbf{fv}\, \mathbf{imN}$		free variables
	$\mid$	$vars_1 \cap vars_2$		set intersection
	$\mid$	$vars_1 \cup vars_2$		set union
	$\mid$	$vars_1 \backslash vars_2$		set complement
	$\mid$	$\mathbf{mv}\, \mathbf{imP}$		movable variables

	$\mid$	$\mathbf{mv}\,\mathbf{imN}$		movable variables
	$\mid$	$\mathbf{uv}\ N$		unification variables
	$\mid$	$\mathbf{uv}\ P$		unification variables
	$\mid$	$\mathbf{fv}\ N$		free variables
	$\mid$	$\mathbf{fv}\ P$		free variables
	$\mid$	$(vars)$	S	parenthesis
	$\mid$	$\vec{\alpha}$		ordered list of variables
	$\mid$	$[\mu]vars$	M	apply moving to varset
	$\mid$	$\mathbf{dom}\,(\widehat{\sigma})$	M	
	$\mid$	$\mathbf{dom}\,(\widehat{\tau})$	M	
	$\mid$	$\mathbf{dom}\,(\Theta)$	M	
μ	$::=$			
	$\mid$	$\cdot$		empty moving
	$\mid$	$pma1 \mapsto pma2$		Positive unit substitution
	$\mid$	$nma1 \mapsto nma2$		Positive unit substitution
	$\mid$	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	$\mid$	$\mu_1 \circ \mu_2$	M	Composition
	$\mid$	$\overline{\mu_i}^{\,i}$		concatenate movings
	$\mid$	$\mu\vert_{vars}$	M	restriction on a set
	$\mid$	μ^{-1}	M	inversion
	$\mid$	$\mathbf{nf}\,(\mu')$	M	
$\widehat{\alpha}^{\pm}$	$::=$			positive/negative unification variable
	$\mid$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	$::=$			positive unification variable
	$\mid$	$\widehat{\alpha}^{+}$		
	$\mid$	$\widehat{\alpha}^{+}\{\Delta\}$		
	$\mid$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	$::=$			negative unification variable
	$\mid$	$\widehat{\alpha}^{-}$		
	$\mid$	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	$\mid$	$\widehat{\alpha}^{-}\{\Delta\}$		
	$\mid$	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widetilde{\alpha}^{+}},\ \overrightarrow{\widetilde{\beta}^{+}}$	$::=$			positive unification variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\widehat{\alpha}^{+}$		a variable
	$\mid$	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	$\mid$	$\overline{\overrightarrow{\widetilde{\alpha}^{+}}_i}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}},\ \overrightarrow{\widehat{\beta}^{-}}$	$::=$			negative unification variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\widehat{\alpha}^{-}$		a variable
	$\mid$	Ξ		from an antiunification context
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified

	$\vert$	$\overrightarrow{\widehat{\alpha}^-{}_i}^{\,i}$		concatenate lists
P, Q	$::=$			a positive algorithmic type (potentially with metavariables)
	$\vert$	α^+		
	$\vert$	$\mathbf{pma}$		
	$\vert$	$\widehat{\alpha}^+$		
	$\vert$	$\downarrow N$		
	$\vert$	$\exists \overrightarrow{\alpha^-}. P$		
	$\vert$	$[\sigma] P$	M	
	$\vert$	$[\widehat{\tau}] P$	M	
	$\vert$	$[\mu] P$	M	
	$\vert$	(P)	S	
	$\vert$	$\mathbf{nf}\,(P')$	M	
N, M	$::=$			a negative algorithmic type (potentially with metavariables)
	$\vert$	α^-		
	$\vert$	$\widehat{\alpha}^-$		
	$\vert$	$\uparrow P$		
	$\vert$	$P \to N$		
	$\vert$	$\forall \overrightarrow{\alpha^+}. N$		
	$\vert$	$[\sigma] N$	M	
	$\vert$	$[\widehat{\tau}] N$	M	
	$\vert$	$[\mu] N$	M	
	$\vert$	(N)	S	
	$\vert$	$\mathbf{nf}\,(N')$	M	
auSol	$::=$			
	$\vert$	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$		
	$\vert$	$(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$		
terminals	$::=$			
	$\vert$	$\exists$		
	$\vert$	$\forall$		
	$\vert$	$\uparrow$		
	$\vert$	$\downarrow$		
	$\vert$	$\to$		
	$\vert$	$\leftrightarrow$		
	$\vert$	$\in$		
	$\vert$	$\notin$		
	$\vert$	$\cdot$		
	$\vert$	$\vdash$		
	$\vert$	$\leqslant$		
	$\vert$	$\geqslant$		
	$\vert$	$\simeq$		
	$\vert$	$\cup$		
	$\vert$	$\cap$		
	$\vert$	$\setminus$		
	$\vert$	$\subseteq$		
	$\vert$	$\mapsto$		
	$\vert$	$\stackrel{u}{\simeq}$		

| $\overset{a}{\simeq}$
| $\varnothing$
| $\circ$
| $\Rightarrow$
| $\models$
| $\dashv$
| $\neq$
| $\equiv_n$
| $\vee$
| $\Downarrow$
| $:\geqslant$
| $:\simeq$
| Λ
| λ
| $\mathbf{let}^{\exists}$
| $\bullet$
| $\Rrightarrow$

$v,\ w$ ::= value terms
| x
| $\{c\}$
| $(v : P)$
| (v) M

$\vec{v}$ ::= list of arguments
| $\cdot$
| v
| $\overline{\vec{v}_i}^{\,i}$ concatenate

$c,\ d$ ::= computation terms
| $(c : N)$
| $\lambda x : P.c$
| $\Lambda\alpha^{+}.c$
| $\mathbf{return}\, v$
| $\mathbf{let}\, x : P = v(\vec{v}); c$
| $\mathbf{let}\, x = v(\vec{v}); c$
| $\mathbf{let}^{\exists}(\alpha^{-}, x) = v; c$

$vctx,\ \Phi$ ::= variable context
| $\cdot$
| $x : P$
| $\overline{\Phi_i}^{\,i}$ concatenate contexts

formula ::=
| *judgement*
| judgement *uniquely*
| $formula_1 \quad .. \quad formula_n$
| $\mu : vars_1 \leftrightarrow vars_2$
| μ **is bijective**
| $\widehat{\sigma}$ **is functional**

| $\widehat{\sigma}_1 \in \widehat{\sigma}_2$
| $v : P \in \Phi$
| $\widehat{\sigma}_1 \subseteq \widehat{\sigma}_2$
| $\mathit{vars}_1 \subseteq \mathit{vars}_2$
| $\mathit{vars}_1 = \mathit{vars}_2$
| vars **is fresh**
| $\alpha^- \notin \mathit{vars}$
| $\alpha^+ \notin \mathit{vars}$
| $\alpha^- \in \mathit{vars}$
| $\alpha^+ \in \mathit{vars}$
| $\widehat{\alpha}^+ \in \mathit{vars}$
| $\widehat{\alpha}^- \in \mathit{vars}$
| $\widehat{\alpha}^- \in \Theta$
| $\widehat{\alpha}^+ \in \Theta$
| if any other rule is not applicable
| $\vec{\alpha}_1 = \vec{\alpha}_2$
| $e_1 = e_2$
| $N = M$
| $N \neq M$
| $P \neq Q$
| $N \neq M$
| $P \neq Q$
| $P \neq Q$
| $N \neq M$

A ::=
| $\Gamma; \Theta \models N \leqslant M \dashv \widehat{\sigma}$ — Negative subtyping
| $\Gamma; \Theta \models P \geqslant Q \dashv \widehat{\sigma}$ — Positive supertyping

AT ::=
| $\Gamma; \Phi \models v : P$ — Positive type inference
| $\Gamma; \Phi \models c : N$ — Negative type inference
| $\Gamma; \Phi; \Theta \models N \bullet \vec{v} \Rightarrow\!\!\!> M \dashv \widehat{\sigma}$ — Application type inference

AU ::=
| $\Gamma \models P_1 \stackrel{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$
| $\Gamma \models N_1 \stackrel{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$

$E1$ ::=
| $N \simeq_1^D M$ — Negative multi-quantified type equivalence
| $P \simeq_1^D Q$ — Positive multi-quantified type equivalence
| $P \simeq Q$

$D1$ ::=
| $\Gamma \vdash N \simeq_1^{\leqslant} M$ — Negative equivalence on MQ types
| $\Gamma \vdash P \simeq_1^{\leqslant} Q$ — Positive equivalence on MQ types
| $\Gamma \vdash N \leqslant_1 M$ — Negative subtyping
| $\Gamma \vdash P \geqslant_1 Q$ — Positive supertyping
| $\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1$ — Equivalence of substitutions

D0	$::=$		
	$\vert$	$\Gamma \vdash N \simeq_0^{\leqslant} M$	Negative equivalence
	$\vert$	$\Gamma \vdash P \simeq_0^{\leqslant} Q$	Positive equivalence
	$\vert$	$\Gamma \vdash N \leqslant_0 M$	Negative subtyping
	$\vert$	$\Gamma \vdash P \geqslant_0 Q$	Positive supertyping
DT	$::=$		
	$\vert$	$\Gamma; \Phi \vdash v \colon P$	Positive type inference
	$\vert$	$\Gamma; \Phi \vdash c \colon N$	Negative type inference
	$\vert$	$\Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!\!> M$	Spin Application type inference
EQ	$::=$		
	$\vert$	$N = M$	Negative type equality (alpha-equivalence)
	$\vert$	$P = Q$	Positive type equuality (alphha-equivalence)
	$\vert$	$P = Q$	
LUBF	$::=$		
	$\vert$	$P_1 \vee P_2 === Q$	
	$\vert$	$\mathbf{ord}\ vars\ \mathbf{in}\ P === \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\ vars\ \mathbf{in}\ N === \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\ vars\ \mathbf{in}\ P === \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\ vars\ \mathbf{in}\ N === \vec{\alpha}$	
	$\vert$	$\mathbf{nf}\,(N') === N$	
	$\vert$	$\mathbf{nf}\,(P') === P$	
	$\vert$	$\mathbf{nf}\,(N') === N$	
	$\vert$	$\mathbf{nf}\,(P') === P$	
	$\vert$	$\mathbf{nf}\,(\vec{N}') === \vec{N}$	
	$\vert$	$\mathbf{nf}\,(\vec{P}') === \vec{P}$	
	$\vert$	$\mathbf{nf}\,(\sigma') === \sigma$	
	$\vert$	$\mathbf{nf}\,(\mu') === \mu$	
	$\vert$	$\mathbf{nf}\,(\widehat{\sigma}') === \widehat{\sigma}$	
	$\vert$	$\sigma'\vert_{vars}$	
	$\vert$	$\widehat{\sigma}'\vert_{vars}$	
	$\vert$	$\widehat{\tau}'\vert_{vars}$	
	$\vert$	$\Xi'\vert_{vars}$	
	$\vert$	$e_1\ \&\ e_2$	
	$\vert$	$\widehat{\sigma}_1\ \&\ \widehat{\sigma}_2$	
	$\vert$	$\widehat{\tau}_1\ \&\ \widehat{\tau}_2$	
	$\vert$	$\mathbf{dom}\,(\widehat{\sigma}) === vars$	
	$\vert$	$\mathbf{dom}\,(\widehat{\tau}) === vars$	
	$\vert$	$\mathbf{dom}\,(\Theta) === vars$	
LUB	$::=$		
	$\vert$	$\Gamma \models P_1 \vee P_2 = Q$	Least Upper Bound (Least Common Supertype)
	$\vert$	$\mathbf{upgrade}\,\Gamma \vdash P\ \mathbf{to}\,\Delta = Q$	
Nrm	$::=$		
	$\vert$	$\mathbf{nf}\,(N) = M$	
	$\vert$	$\mathbf{nf}\,(P) = Q$	
	$\vert$	$\mathbf{nf}\,(N) = M$	

| $\mathbf{nf}\,(P) = Q$

Order ::=

| $\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}$
| $\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}$
| $\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}$
| $\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}$

SM ::=

| $\Gamma \vdash e_1 \ \&\ e_2 = e_3$ — Unification Solution Entry Merge
| $\Theta \vdash \widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3$ — Merge unification solutions

SImp ::=

| $\Gamma \vdash e_1 \Rightarrow e_2$ — Weakening of unification solution entries
| $\Theta \vdash \widehat{\sigma}_1 \Rightarrow \widehat{\sigma}_2$ — Weakening of unification solutions
| $\Gamma \vdash e_1 \simeq e_2$
| $\Theta \vdash \widehat{\sigma}_1 \simeq \widehat{\sigma}_2$

U ::=

| $\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv \widehat{\sigma}$ — Negative unification
| $\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv \widehat{\sigma}$ — Positive unification

WF ::=

| $\Gamma \vdash N$ — Negative type well-formedness
| $\Gamma \vdash P$ — Positive type well-formedness
| $\Gamma \vdash N$ — Negative type well-formedness
| $\Gamma \vdash P$ — Positive type well-formedness
| $\Gamma \vdash \vec{N}$ — Negative type list well-formedness
| $\Gamma \vdash \vec{P}$ — Positive type list well-formedness
| $\Gamma; \Theta \vdash N$ — Negative unification type well-formedness
| $\Gamma; \Theta \vdash P$ — Positive unifiication type well-formedness
| $\Gamma; \Xi \vdash N$ — Negative anti-unification type well-formedness
| $\Gamma; \Xi \vdash P$ — Positive anti-unification type well-formedness
| $\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1$ — Antiunification substitution well-formedness
| $\widehat{\sigma} : \Theta$ — Unification substitution well-formedness
| $\Gamma \vdash^{\supseteq} \Theta$ — Unification context well-formedness
| $\Gamma_1 \vdash \sigma : \Gamma_2$ — Substitution well-formedness
| $\Gamma \vdash e$ — Unification solution entry well-formedness

judgement ::=

| *A*
| *AT*
| *AU*
| *E1*
| *D1*
| *D0*
| *DT*
| *EQ*
| *LUB*

	$\mid$	Nrm
	$\mid$	$Order$
	$\mid$	SM
	$\mid$	$SImp$
	$\mid$	U
	$\mid$	WF
$user\_syntax$	$::=$	
	$\mid$	α
	$\mid$	n
	$\mid$	x
	$\mid$	n
	$\mid$	α^+
	$\mid$	α^-
	$\mid$	$\alpha^\pm$
	$\mid$	σ
	$\mid$	e
	$\mid$	$\widehat{\sigma}$
	$\mid$	$\widehat{\tau}$
	$\mid$	P
	$\mid$	N
	$\mid$	$\overrightarrow{\alpha^+}$
	$\mid$	$\overrightarrow{\alpha^-}$
	$\mid$	$\overrightarrow{\alpha^\pm}$
	$\mid$	P
	$\mid$	N
	$\mid$	$\vec{P}$
	$\mid$	$\vec{N}$
	$\mid$	Γ
	$\mid$	Θ
	$\mid$	Ξ
	$\mid$	$\vec{\alpha}$
	$\mid$	$vars$
	$\mid$	μ
	$\mid$	$\widehat{\alpha}^\pm$
	$\mid$	$\widehat{\alpha}^+$
	$\mid$	$\widehat{\alpha}^-$
	$\mid$	$\overrightarrow{\widehat{\alpha}^+}$
	$\mid$	$\overrightarrow{\widehat{\alpha}^-}$
	$\mid$	P
	$\mid$	N
	$\mid$	$auSol$
	$\mid$	$terminals$
	$\mid$	v
	$\mid$	$\vec{v}$
	$\mid$	c
	$\mid$	$vctx$
	$\mid$	$formula$

$\boxed{\Gamma; \Theta \models N \leqslant M \dashv \widehat{\sigma}}$ Negative subtyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^- \leqslant \alpha^- \dashv \cdot}\ \textsc{ANVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(P) \overset{u}{\simeq} \mathbf{nf}\,(Q) \dashv \widehat{\sigma}}{\Gamma;\ \Theta \models \uparrow P \leqslant \uparrow Q \dashv \widehat{\sigma}}\ \textsc{AShiftU}$$

$$\frac{\Gamma;\ \Theta \models P \geqslant Q \dashv \widehat{\sigma}_1 \quad \Gamma;\ \Theta \models N \leqslant M \dashv \widehat{\sigma}_2 \quad \Theta \vdash \widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}}{\Gamma;\ \Theta \models P \to N \leqslant Q \to M \dashv \widehat{\sigma}}\ \textsc{AArrow}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma;\ \Theta \models \forall \overrightarrow{\alpha^+}.N \leqslant \forall \overrightarrow{\beta^+}.M \dashv \widehat{\sigma} \backslash \overrightarrow{\widehat{\alpha}^+}}\ \textsc{AForall}$$

$\boxed{\Gamma;\ \Theta \models P \geqslant Q \dashv \widehat{\sigma}}$ Positive supertyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^+ \geqslant \alpha^+ \dashv \cdot}\ \textsc{APVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(N) \overset{u}{\simeq} \mathbf{nf}\,(M) \dashv \widehat{\sigma}}{\Gamma;\ \Theta \models \downarrow N \geqslant \downarrow M \dashv \widehat{\sigma}}\ \textsc{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-};\ \Theta, \overrightarrow{\widehat{\alpha}^-}\{\Gamma, \overrightarrow{\beta^-}\} \models [\overrightarrow{\widehat{\alpha}^-}/\overrightarrow{\alpha^-}]P \geqslant Q \dashv \widehat{\sigma}}{\Gamma;\ \Theta \models \exists \overrightarrow{\alpha^-}.P \geqslant \exists \overrightarrow{\beta^-}.Q \dashv \widehat{\sigma} \backslash \overrightarrow{\widehat{\alpha}^-}}\ \textsc{AExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}{\Gamma;\ \Theta \models \widehat{\alpha}^+ \geqslant P \dashv (\widehat{\alpha}^+ :\geqslant Q)}\ \textsc{APUVar}$$

$\boxed{\Gamma; \Phi \models v\colon P}$ Positive type inference

$$\frac{v : P \in \Phi}{\Gamma; \Phi \models v\colon P}\ \textsc{ATVar}$$

$$\frac{\Gamma; \Phi \models c\colon N}{\Gamma; \Phi \models \{c\}\colon \downarrow N}\ \textsc{ATThunk}$$

$$\frac{\Gamma; \Phi \models v\colon P \quad \Gamma; \cdot \models Q \geqslant P \dashv \cdot}{\Gamma; \Phi \models (v : Q)\colon Q}\ \textsc{ATAnnot}$$

$\boxed{\Gamma; \Phi \models c\colon N}$ Negative type inference

$$\frac{\Gamma; \Phi \models c\colon N \quad \Gamma; \cdot \models N \leqslant M \dashv \cdot}{\Gamma; \Phi \models (c : M)\colon M}\ \textsc{ATAnnotN}$$

$$\frac{\Gamma; \Phi, x : P \models c\colon N}{\Gamma; \Phi \models \lambda x : P.c\colon P \to N}\ \textsc{ATtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \models c\colon N}{\Gamma; \Phi \models \Lambda \alpha^+.c\colon \forall \alpha^+.N}\ \textsc{ATTLam}$$

$$\frac{\Gamma; \Phi \models v\colon P}{\Gamma; \Phi \models \mathbf{return}\, v\colon \uparrow P}\ \textsc{ATReturn}$$

$$\frac{\begin{array}{c}\Gamma; \Phi \models v\colon \downarrow M \quad \Gamma; \Phi; \cdot \models M \bullet \overrightarrow{v} \Rrightarrow \uparrow Q \dashv \widehat{\sigma}_1 \quad \Gamma;\ \mathbf{uv}\ Q\{\Gamma\} \models \uparrow Q \leqslant \uparrow P \dashv \widehat{\sigma}_2 \\ \mathbf{uv}\ Q\{\Gamma\} \vdash \widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma} \quad \Gamma; \Phi, x : P \models c\colon N\end{array}}{\Gamma; \Phi \models \mathbf{let}\, x : P = v(\overrightarrow{v}); c\colon N}\ \textsc{ATLetAnn}$$

$$\frac{\Gamma; \Phi \models v\colon \downarrow M \quad \Gamma; \Phi; \cdot \models M \bullet \overrightarrow{v} \Rrightarrow \uparrow Q \dashv \widehat{\sigma} \quad \mathbf{uv}\,(Q) = \varnothing \quad \Gamma; \Phi, x : Q \models c\colon N}{\Gamma; \Phi \models \mathbf{let}\, x = v(\overrightarrow{v}); c\colon N}\ \textsc{ATLet}$$

$$\frac{\Gamma, \alpha^-; \Phi \models v \colon \exists \alpha^-.P \quad \Gamma, \alpha^-; \Phi, x : P \models c \colon N \quad \Gamma \vdash N}{\Gamma; \Phi \models \mathbf{let}^{\exists}(\alpha^-, x) = v; c \colon N} \text{ ATUNPACK}$$

$\boxed{\Gamma; \Phi; \Theta \models N \bullet \vec{v} \Longrightarrow\!\!\!> M \dashv \widehat{\sigma}}$ Application type inference

$$\frac{N \neq \forall \overrightarrow{\alpha^+}.M}{\Gamma; \Phi; \Theta \models N \bullet \cdot \Longrightarrow\!\!\!> N \dashv \cdot} \text{ ATEMTPTY}$$

$$\frac{\Gamma; \Phi \models v \colon P \quad \Gamma; \Theta \models Q \geqslant P \dashv \widehat{\sigma}_1 \quad \Gamma; \Phi; \Theta \models N \bullet \vec{v} \Longrightarrow\!\!\!> M \dashv \widehat{\sigma}_2}{\Gamma; \Phi; \Theta \models Q \to N \bullet v, \vec{v} \Longrightarrow\!\!\!> M \dashv \widehat{\sigma}_1 \,\&\, \widehat{\sigma}_2} \text{ ATARROW}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi; \Theta \models \forall \overrightarrow{\alpha^+}.N \bullet \vec{v} \Longrightarrow\!\!\!> M \dashv \widehat{\sigma}} \text{ ATFORALL}$$

$\boxed{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\cdot, \alpha^+, \cdot, \cdot)} \text{ AUPVAR}$$

$$\frac{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \downarrow N_1 \overset{a}{\simeq} \downarrow N_2 \dashv (\Xi, \downarrow M, \widehat{\tau}_1, \widehat{\tau}_2)} \text{ AUSHIFTD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \Gamma = \varnothing \quad \Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \exists \overrightarrow{\alpha^-}.P_1 \overset{a}{\simeq} \exists \overrightarrow{\alpha^-}.P_2 \dashv (\Xi, \exists \overrightarrow{\alpha^-}.Q, \widehat{\tau}_1, \widehat{\tau}_2)} \text{ AUEXISTS}$$

$\boxed{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^- \overset{a}{\simeq} \alpha^- \dashv (\cdot, \alpha^-, \cdot, \cdot)} \text{ AUNVAR}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \uparrow P_1 \overset{a}{\simeq} \uparrow P_2 \dashv (\Xi, \uparrow Q, \widehat{\tau}_1, \widehat{\tau}_2)} \text{ AUSHIFTU}$$

$$\frac{\overrightarrow{\alpha^+} \cap \Gamma = \varnothing \quad \Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \forall \overrightarrow{\alpha^+}.N_1 \overset{a}{\simeq} \forall \overrightarrow{\alpha^+}.N_2 \dashv (\Xi, \forall \overrightarrow{\alpha^+}.M, \widehat{\tau}_1, \widehat{\tau}_2)} \text{ AUFORALL}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \widehat{\tau}_1, \widehat{\tau}_2) \quad \Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \widehat{\tau}_1', \widehat{\tau}_2')}{\Gamma \models P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \widehat{\tau}_1 \cup \widehat{\tau}_1', \widehat{\tau}_2 \cup \widehat{\tau}_2')} \text{ AUARROW}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \models N \overset{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, \widehat{\alpha}^-_{\{N,M\}}, (\widehat{\alpha}^-_{\{N,M\}} :\approx N), (\widehat{\alpha}^-_{\{N,M\}} :\approx M))} \text{ AUAU}$$

$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-} \text{ E1NVAR}$$

$$\frac{P \simeq_1^D Q}{\uparrow P \simeq_1^D \uparrow Q} \text{ E1SHIFTU}$$

$$\frac{P \simeq_1^D Q \quad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M} \text{ E1ARROW}$$

$$\frac{\overrightarrow{\alpha^+} \cap \mathbf{fv}\, M = \varnothing \quad \mu : (\overrightarrow{\beta^+} \cap \mathbf{fv}\, M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \mathbf{fv}\, N) \quad N \simeq_1^D [\mu]M}{\forall \overrightarrow{\alpha^+}.N \simeq_1^D \forall \overrightarrow{\beta^+}.M} \text{ E1FORALL}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+} \quad \text{E1PVAR}$$

$$\frac{N \simeq_1^D M}{\downarrow N \simeq_1^D \downarrow M} \quad \text{E1SHIFTD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \mathbf{fv}\, Q = \varnothing \quad \mu : (\overrightarrow{\beta^-} \cap \mathbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \mathbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists \overrightarrow{\alpha^-}.P \simeq_1^D \exists \overrightarrow{\beta^-}.Q} \quad \text{E1EXISTS}$$

$\boxed{P \simeq Q}$

$\boxed{\Gamma \vdash N \simeq_1^\leqslant M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^\leqslant M} \quad \text{D1NDEF}$$

$\boxed{\Gamma \vdash P \simeq_1^\leqslant Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^\leqslant Q} \quad \text{D1PDEF}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-} \quad \text{D1NVAR}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant_1 \uparrow Q} \quad \text{D1SHIFTU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M} \quad \text{D1ARROW}$$

$$\frac{\mathbf{fv}\, N \cap \overrightarrow{\beta^+} = \varnothing \quad \Gamma, \overrightarrow{\beta^+} \vdash P_i \quad \Gamma, \overrightarrow{\beta^+} \vdash [\overrightarrow{P}/\overrightarrow{\alpha^+}]N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant_1 \forall \overrightarrow{\beta^+}.M} \quad \text{D1FORALL}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+} \quad \text{D1PVAR}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M} \quad \text{D1SHIFTD}$$

$$\frac{\mathbf{fv}\, P \cap \overrightarrow{\beta^-} = \varnothing \quad \Gamma, \overrightarrow{\beta^-} \vdash N_i \quad \Gamma, \overrightarrow{\beta^-} \vdash [\overrightarrow{N}/\overrightarrow{\alpha^-}]P \geqslant_1 Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P \geqslant_1 \exists \overrightarrow{\beta^-}.Q} \quad \text{D1EXISTS}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq_1^\leqslant \sigma_2 : \Gamma_1}$ Equivalence of substitutions

$\boxed{\Gamma \vdash N \simeq_0^\leqslant M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq_0^\leqslant M} \quad \text{D0NDEF}$$

$\boxed{\Gamma \vdash P \simeq_0^\leqslant Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq_0^\leqslant Q} \quad \text{D0PDEF}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_0 \alpha^-} \quad \text{D0NVar}$$

$$\frac{\Gamma \vdash P \simeq_0^{\leqslant} Q}{\Gamma \vdash \uparrow P \leqslant_0 \uparrow Q} \quad \text{D0ShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/\alpha^+]N \leqslant_0 M \quad M \neq \forall \beta^+.M'}{\Gamma \vdash \forall \alpha^+.N \leqslant_0 M} \quad \text{D0ForallL}$$

$$\frac{\Gamma, \alpha^+ \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall \alpha^+.M} \quad \text{D0ForallR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \to N \leqslant_0 Q \to M} \quad \text{D0Arrow}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_0 \alpha^+} \quad \text{D0PVar}$$

$$\frac{\Gamma \vdash N \simeq_0^{\leqslant} M}{\Gamma \vdash \downarrow N \geqslant_0 \downarrow M} \quad \text{D0ShiftD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/\alpha^-]P \geqslant_0 Q \quad Q \neq \exists \alpha^-.Q'}{\Gamma \vdash \exists \alpha^-.P \geqslant_0 Q} \quad \text{D0ExistsL}$$

$$\frac{\Gamma, \alpha^- \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists \alpha^-.Q} \quad \text{D0ExistsR}$$

$\boxed{\Gamma; \Phi \vdash v \colon P}$ Positive type inference

$$\frac{v : P \in \Phi}{\Gamma; \Phi \vdash v \colon P} \quad \text{DTVar}$$

$$\frac{\Gamma; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \{c\} \colon \downarrow N} \quad \text{DTThunk}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma; \Phi \vdash (v : Q) \colon Q} \quad \text{DTAnnotP}$$

$\boxed{\Gamma; \Phi \vdash c \colon N}$ Negative type inference

$$\frac{\Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \lambda x : P.c \colon P \to N} \quad \text{DTtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \Lambda \alpha^+.c \colon \forall \alpha^+.N} \quad \text{DTTLam}$$

$$\frac{\Gamma; \Phi \vdash v \colon P}{\Gamma; \Phi \vdash \mathbf{return}\, v \colon \uparrow P} \quad \text{DTReturn}$$

$$\frac{\Gamma; \Phi \vdash v \colon \downarrow M \quad \Gamma; \Phi \vdash M \bullet \vec{v} \Rightarrow\!\!\!> \uparrow Q \quad \Gamma \vdash \uparrow Q \leqslant_1 \uparrow P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x : P = v(\vec{v}); c \colon N} \quad \text{DTLetAnn}$$

$$\frac{\Gamma; \Phi \vdash v \colon \downarrow M \quad \Gamma; \Phi \vdash M \bullet \vec{v} \Rightarrow\!\!\!> \uparrow Q \text{ uniquely} \quad \Gamma; \Phi, x : Q \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v(\vec{v}); c \colon N} \quad \text{DTLet}$$

$$\frac{\Gamma, \alpha^-; \Phi \vdash v \colon \exists \alpha^-.P \quad \Gamma, \alpha^-; \Phi, x : P \vdash c \colon N \quad \Gamma \vdash N}{\Gamma; \Phi \vdash \mathbf{let}^{\exists}(\alpha^-, x) = v; c \colon N} \quad \text{DTUnpack}$$

$$\frac{\Gamma; \Phi \vdash c \colon N \quad \Gamma \vdash N \leqslant_1 M}{\Gamma; \Phi \vdash (c : M) \colon M} \quad \textsc{DTAnnotN}$$

$\boxed{\Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M}$ Spin Application type inference

$$\frac{N \neq \forall \overrightarrow{\alpha^+}.M}{\Gamma; \Phi \vdash N \bullet \cdot \Rrightarrow N} \quad \textsc{DTEmtpty}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P \quad \Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M}{\Gamma; \Phi \vdash Q \to N \bullet v, \vec{v} \Rrightarrow M} \quad \textsc{DTArrow}$$

$$\frac{\Gamma \vdash \vec{P} \quad \Gamma; \Phi \vdash [\vec{P}/\overrightarrow{\alpha^+}]N \bullet \vec{v} \Rrightarrow M}{\Gamma; \Phi \vdash \forall \overrightarrow{\alpha^+}.N \bullet \vec{v} \Rrightarrow M} \quad \textsc{DTForall}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$

$\boxed{P_1 \vee P_2}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ N}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ N}$

$\boxed{\mathbf{nf}\ (N')}$

$\boxed{\mathbf{nf}\ (P')}$

$\boxed{\mathbf{nf}\ (N')}$

$\boxed{\mathbf{nf}\ (P')}$

$\boxed{\mathbf{nf}\ (\vec{N'})}$

$\boxed{\mathbf{nf}\ (\vec{P'})}$

$\boxed{\mathbf{nf}\,(\sigma')}$

$\boxed{\mathbf{nf}\,(\mu')}$

$\boxed{\mathbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\sigma'|_{vars}}$

$\boxed{\widehat{\sigma}'|_{vars}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{\widehat{\sigma}_1 \ \& \ \widehat{\sigma}_2}$

$\boxed{\widehat{\tau}_1 \ \& \ \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{\Gamma \models P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \models \alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBVar}$$

$$\frac{\Gamma, \cdot \vDash \mathbf{nf}\,(\downarrow N) \overset{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \hat{\tau}_1, \hat{\tau}_2)}{\Gamma \vDash \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi]P} \quad \text{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \vDash P_1 \vee P_2 = Q}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P_1 \vee \exists \overrightarrow{\beta^-}.P_2 = Q} \quad \text{LUBExists}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}$

$$\frac{\Gamma = \Delta, \overrightarrow{\alpha^{\pm}} \quad \overrightarrow{\beta^{\pm}}\ \mathbf{is\ fresh} \quad \overrightarrow{\gamma^{\pm}}\ \mathbf{is\ fresh} \quad \Delta, \overrightarrow{\beta^{\pm}}, \overrightarrow{\gamma^{\pm}} \vDash [\overrightarrow{\beta^{\pm}}/\overrightarrow{\alpha^{\pm}}]P \vee [\overrightarrow{\gamma^{\pm}}/\overrightarrow{\alpha^{\pm}}]P = Q}{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q} \quad \text{LUBUpgrade}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^-) = \alpha^-} \quad \text{NrmNVar}$$

$$\frac{\text{<<multiple parses>>}}{\mathbf{nf}\,(\uparrow P) = \uparrow Q} \quad \text{NrmShiftU}$$

$$\frac{\text{<<multiple parses>>}}{\mathbf{nf}\,(P \to N) = Q \to M} \quad \text{NrmArrow}$$

$$\frac{\text{<<multiple parses>>}}{\mathbf{nf}\,(\forall \overrightarrow{\alpha^+}.N) = \forall \overrightarrow{\alpha^+}'.N'} \quad \text{NrmForall}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\alpha^+) = \alpha^+} \quad \text{NrmPVar}$$

$$\frac{\text{<<multiple parses>>}}{\mathbf{nf}\,(\downarrow N) = \downarrow M} \quad \text{NrmShiftD}$$

$$\frac{\text{<<multiple parses>>}}{\mathbf{nf}\,(\exists \overrightarrow{\alpha^-}.P) = \exists \overrightarrow{\alpha^-}'.P'} \quad \text{NrmExists}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\hat{\alpha}^-) = \hat{\alpha}^-} \quad \text{NrmNUVar}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\hat{\alpha}^+) = \hat{\alpha}^+} \quad \text{NrmPUVar}$$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}}$

$$\frac{\alpha^- \in vars}{\mathbf{ord}\ vars\ \mathbf{in}\ \alpha^- = \alpha^-} \quad \text{ONVarIn}$$

$$\frac{\alpha^- \notin vars}{\mathbf{ord}\ vars\ \mathbf{in}\ \alpha^- = \cdot} \quad \text{ONVarNIn}$$

$$\frac{\mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}}{\mathbf{ord}\ vars\ \mathbf{in}\ \uparrow P = \vec{\alpha}} \quad \text{OShiftU}$$

$$\frac{\mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}_1 \quad \mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}_2}{\mathbf{ord}\ vars\ \mathbf{in}\ P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)} \quad \text{OArrow}$$

$$\frac{vars \cap \overrightarrow{\alpha^+} = \varnothing \quad \mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}}{\mathbf{ord}\ vars\ \mathbf{in}\ \forall \overrightarrow{\alpha^+}. N = \vec{\alpha}} \quad \text{OForall}$$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}}$

$$\frac{\alpha^+ \in vars}{\mathbf{ord}\ vars\ \mathbf{in}\ \alpha^+ = \alpha^+} \quad \text{OPVarIn}$$

$$\frac{\alpha^+ \notin vars}{\mathbf{ord}\ vars\ \mathbf{in}\ \alpha^+ = \cdot} \quad \text{OPVarNIn}$$

$$\frac{\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}}{\mathbf{ord}\ vars\ \mathbf{in}\ {\downarrow} N = \vec{\alpha}} \quad \text{OShiftD}$$

$$\frac{vars \cap \overrightarrow{\alpha^-} = \varnothing \quad \mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}}{\mathbf{ord}\ vars\ \mathbf{in}\ \exists \overrightarrow{\alpha^-}. P = \vec{\alpha}} \quad \text{OExists}$$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ vars\ \mathbf{in}\ \widehat{\alpha}^- = \cdot} \quad \text{ONUVar}$$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ vars\ \mathbf{in}\ \widehat{\alpha}^+ = \cdot} \quad \text{OPUVar}$$

$\boxed{\Gamma \vdash e_1\ \&\ e_2 = e_3}$ Unification Solution Entry Merge

$$\frac{\Gamma \models P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1)\ \&\ (\widehat{\alpha}^+ :\geqslant P_2) = (\widehat{\alpha}^+ :\geqslant Q)} \quad \text{SMESupSup}$$

$$\frac{\Gamma; \cdot \models P \geqslant Q \dashv \widehat{\sigma}'}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P)\ \&\ (\widehat{\alpha}^+ :\geqslant Q) = (\widehat{\alpha}^+ :\approx P)} \quad \text{SMEEqSup}$$

$$\frac{\Gamma; \cdot \models Q \geqslant P \dashv \widehat{\sigma}'}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P)\ \&\ (\widehat{\alpha}^+ :\approx Q) = (\widehat{\alpha}^+ :\approx Q)} \quad \text{SMESupEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P)\ \&\ (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \text{SMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1)\ \&\ (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \text{SMENEqEq}$$

$\boxed{\Theta \vdash \widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3}$ Merge unification solutions

$\boxed{\Gamma \vdash e_1 \Rightarrow e_2}$ Weakening of unification solution entries

$$\frac{\Gamma \vdash P_1 \geqslant_1 P_2}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1) \Rightarrow (\widehat{\alpha}^+ :\geqslant P_2)} \quad \text{SImpESupSup}$$

$$\frac{\Gamma \vdash P_1 \geqslant_1 P_2}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P_1) \Rightarrow (\widehat{\alpha}^+ :\geqslant P_2)} \quad \text{SImpEEqSup}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P_1) \Rightarrow (\widehat{\alpha}^+ :\approx P_2)} \quad \text{SImpEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \Rightarrow (\widehat{\alpha}^- :\approx N_2)} \quad \text{SImpENEqEq}$$

$\boxed{\Theta \vdash \widehat{\sigma}_1 \Rightarrow \widehat{\sigma}_2}$ Weakening of unification solutions

$\boxed{\Gamma \vdash e_1 \simeq e_2}$

$$\frac{\text{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^{+} :\geqslant P_1) \simeq (\widehat{\alpha}^{+} :\geqslant P_2)} \quad \textsc{SImpEEqSupSup}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^{+} :\approx P_1) \simeq (\widehat{\alpha}^{+} :\approx P_2)} \quad \textsc{SImpEEqPEqEq}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^{-} :\approx N_1) \simeq (\widehat{\alpha}^{-} :\approx N_2)} \quad \textsc{SImpEEqNEqEq}$$

$\boxed{\Theta \vdash \widehat{\sigma}_1 \simeq \widehat{\sigma}_2}$

$\boxed{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv \widehat{\sigma}}$ Negative unification

$$\frac{}{\Gamma; \Theta \models \alpha^{-} \overset{u}{\simeq} \alpha^{-} \dashv \cdot} \quad \textsc{UNVar}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}{\Gamma; \Theta \models \uparrow P \overset{u}{\simeq} \uparrow Q \dashv \widehat{\sigma}} \quad \textsc{UShiftU}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv \widehat{\sigma}_1 \quad \Gamma; \Theta \models N \overset{u}{\simeq} M \dashv \widehat{\sigma}_2}{\Gamma; \Theta \models P \to N \overset{u}{\simeq} Q \to M \dashv \widehat{\sigma}_1 \;\&\; \widehat{\sigma}_2} \quad \textsc{UArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^{+}}; \Theta \models N \overset{u}{\simeq} M \dashv \widehat{\sigma}}{\Gamma; \Theta \models \forall \overrightarrow{\alpha^{+}}. N \overset{u}{\simeq} \forall \overrightarrow{\alpha^{+}}. M \dashv \widehat{\sigma}} \quad \textsc{UForall}$$

$$\frac{\widehat{\alpha}^{-}\{\Delta\} \in \Theta \quad \Delta \vdash N}{\Gamma; \Theta \models \widehat{\alpha}^{-} \overset{u}{\simeq} N \dashv (\widehat{\alpha}^{-} :\approx N)} \quad \textsc{UNUVar}$$

$\boxed{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}$ Positive unification

$$\frac{}{\Gamma; \Theta \models \alpha^{+} \overset{u}{\simeq} \alpha^{+} \dashv \cdot} \quad \textsc{UPVar}$$

$$\frac{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv \widehat{\sigma}}{\Gamma; \Theta \models \downarrow N \overset{u}{\simeq} \downarrow M \dashv \widehat{\sigma}} \quad \textsc{UShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^{-}}; \Theta \models P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}{\Gamma; \Theta \models \exists \overrightarrow{\alpha^{-}}. P \overset{u}{\simeq} \exists \overrightarrow{\alpha^{-}}. Q \dashv \widehat{\sigma}} \quad \textsc{UExists}$$

$$\frac{\widehat{\alpha}^{+}\{\Delta\} \in \Theta \quad \Delta \vdash P}{\Gamma; \Theta \models \widehat{\alpha}^{+} \overset{u}{\simeq} P \dashv (\widehat{\alpha}^{+} :\approx P)} \quad \textsc{UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness
$\boxed{\Gamma \vdash P}$ Positive type well-formedness
$\boxed{\Gamma \vdash N}$ Negative type well-formedness
$\boxed{\Gamma \vdash P}$ Positive type well-formedness
$\boxed{\Gamma \vdash \vec{N}}$ Negative type list well-formedness
$\boxed{\Gamma \vdash \vec{P}}$ Positive type list well-formedness
$\boxed{\Gamma; \Theta \vdash N}$ Negative unification type well-formedness
$\boxed{\Gamma; \Theta \vdash P}$ Positive unifiication type well-formedness
$\boxed{\Gamma; \Xi \vdash N}$ Negative anti-unification type well-formedness
$\boxed{\Gamma; \Xi \vdash P}$ Positive anti-unification type well-formedness
$\boxed{\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness
$\boxed{\widehat{\sigma} : \Theta}$ Unification substitution well-formedness
$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness

$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution well-formedness
$\boxed{\Gamma \vdash e}$ Unification solution entry well-formedness

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Definition rules:        98 good    16 bad
Definition rule clauses: 195 good    16 bad
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