α, β, α, β, γ, δ	type variables
n, m, i, j	index variables

$n,\ m$	$::=$			cohort index
	$\mid$	0		
	$\mid$	$n+1$		
$\alpha^+,\ \beta^+,\ \gamma^+,\ \delta^+$	$::=$			positive variable
	$\mid$	α^+		
	$\mid$	α^{+n}		
	$\mid$	$\beta^{\pm}$		
$\alpha^-,\ \beta^-,\ \gamma^-,\ \delta^-$	$::=$			negative variable
	$\mid$	α^-		
	$\mid$	α^{-n}		
	$\mid$	$\alpha^{\pm}$		
$\alpha^{\pm},\ \beta^{\pm}$	$::=$			positive or negative variable
	$\mid$	$\alpha^{\pm}$		
	$\mid$	$\alpha^{\pm n}$		
σ	$::=$			substitution
	$\mid$	$\cdot$		
	$\mid$	id		
	$\mid$	P/α^+		
	$\mid$	N/α^-		
	$\mid$	$\overrightarrow{P}/\overrightarrow{\alpha^+}$		
	$\mid$	$\overrightarrow{N}/\overrightarrow{\alpha^-}$		
	$\mid$	$\mathbf{pmas}\,/\overrightarrow{\alpha^+}$		
	$\mid$	$\mathbf{nmas}\,/\overrightarrow{\alpha^-}$		
	$\mid$	$\overrightarrow{\widetilde{\alpha^+}}/\overrightarrow{\alpha^+}$		
	$\mid$	$\overrightarrow{\widehat{\alpha^-}}/\overrightarrow{\alpha^-}$		
	$\mid$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\mid$	μ		
	$\mid$	$\widehat{\sigma}$		
	$\mid$	$\sigma_1 \circ \sigma_2$		
	$\mid$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\mid$	(σ)	S	
	$\mid$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\mid$	$\mathbf{nf}\,(\sigma')$	M	
	$\mid$	$\sigma'\vert_{vars}$	M	
e	$::=$			entry of a unification solution
	$\mid$	$\Gamma \vdash \widehat{\alpha}^+ :\approx P$		
	$\mid$	$\Gamma \vdash \widehat{\alpha}^- :\approx N$		
	$\mid$	$\Gamma \vdash \widehat{\alpha}^+ :\geqslant P$		
	$\mid$	(e)	S	
	$\mid$	$\widehat{\sigma}(\widehat{\alpha}^+)$	M	
	$\mid$	$\widehat{\sigma}(\widehat{\alpha}^-)$	M	
	$\mid$	$e_1\ \&\ e_2$	M	
$\widehat{\sigma}$	$::=$			unification solution (substitution)

	$\mid$	$\cdot$		
	$\mid$	e		
	$\mid$	$\widehat{\sigma} \backslash vars$		
	$\mid$	$\widehat{\sigma}_1 \cup \widehat{\sigma}_2$		
	$\mid$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\mid$	$(\widehat{\sigma})$	S	
	$\mid$	$\widehat{\sigma}_1 \ \& \ \widehat{\sigma}_2$	M	
$\widehat{\tau}$	$::=$			anti-unification substitution
	$\mid$	$\cdot$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		
	$\mid$	$\overrightarrow{\alpha^-} / \overrightarrow{\widehat{\alpha}^-}$		
	$\mid$	$\overrightarrow{N} / \overrightarrow{\widehat{\alpha}^-}$		
	$\mid$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
	$\mid$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	$\mid$	$(\widehat{\tau})$	S	
$P, \ Q$	$::=$			positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists \alpha^- . P$		
	$\mid$	$[\sigma] P$	M	
$N, \ M$	$::=$			negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$\forall \alpha^+ . N$		
	$\mid$	$P \to N$		
	$\mid$	$[\sigma] N$	M	
$\overrightarrow{\alpha^+}, \ \overrightarrow{\beta^+}, \ \overrightarrow{\gamma^+}, \ \overrightarrow{\delta^+}$	$::=$			positive variable list
	$\mid$	$\cdot$		empty list
	$\mid$	α^+		a variable
	$\mid$	$\overrightarrow{\alpha^+}$		a variable
	$\mid$	$\overline{\overrightarrow{\alpha^+}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^-}, \ \overrightarrow{\beta^-}, \ \overrightarrow{\gamma^-}, \ \overrightarrow{\delta^-}$	$::=$			negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	α^-		a variable
	$\mid$	$\overrightarrow{\alpha^-}$		variables
	$\mid$	$\overline{\overrightarrow{\alpha^-}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^\pm}, \ \overrightarrow{\beta^\pm}, \ \overrightarrow{\gamma^\pm}, \ \overrightarrow{\delta^\pm}$	$::=$			positive or negative variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\alpha^\pm$		a variable
	$\mid$	$\overrightarrow{\mathbf{pa}}$		variables
	$\mid$	$\overline{\overrightarrow{\alpha^\pm}_i}^{\,i}$		concatenate lists

$P,\ Q$	$::=$			multi-quantified positive types
	$\vert$	α^+		
	$\vert$	$\downarrow N$		
	$\vert$	$\exists\overrightarrow{\alpha^-}.P$		$P \neq \exists\ldots$
	$\vert$	$[\sigma]P$	M	
	$\vert$	$[\widehat{\tau}]P$	M	
	$\vert$	$[\widehat{\sigma}]P$	M	
	$\vert$	$[\mu]P$	M	
	$\vert$	(P)	S	
	$\vert$	$\mathbf{nf}\,(P')$	M	
$N,\ M$	$::=$			multi-quantified negative types
	$\vert$	α^-		
	$\vert$	$\uparrow P$		
	$\vert$	$P \to N$		
	$\vert$	$\forall\overrightarrow{\alpha^+}.N$		$N \neq \forall\ldots$
	$\vert$	$[\sigma]N$	M	
	$\vert$	$[\mu]N$	M	
	$\vert$	$[\widehat{\sigma}]N$	M	
	$\vert$	(N)	S	
	$\vert$	$\mathbf{nf}\,(N')$	M	
$\vec{P},\ \vec{Q}$	$::=$			list of positive types
	$\vert$	$\cdot$		empty list
	$\vert$	P		a singel type
	$\vert$	$\overline{\vec{P}_i}^{\,i}$		concatenate lists
	$\vert$	$\mathbf{nf}\,(\vec{P}')$	M	
$\vec{N},\ \vec{M}$	$::=$			list of negative types
	$\vert$	$\cdot$		empty list
	$\vert$	N		a singel type
	$\vert$	$\overline{\vec{N}_i}^{\,i}$		concatenate lists
	$\vert$	$\mathbf{nf}\,(\vec{N}')$	M	
$\Delta,\ \Gamma$	$::=$			declarative type context
	$\vert$	$\cdot$		empty context
	$\vert$	$\overrightarrow{\alpha^+}$		list of variables
	$\vert$	$\overrightarrow{\alpha^-}$		list of variables
	$\vert$	$\overrightarrow{\alpha^\pm}$		list of variables
	$\vert$	$vars$		
	$\vert$	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	$\vert$	(Γ)	S	
Θ	$::=$			unification type variable context
	$\vert$	$\cdot$		empty context
	$\vert$	$\overrightarrow{\widetilde{\alpha}^+}$		list of variables
	$\vert$	$\overrightarrow{\widehat{\alpha}^-}$		list of variables
	$\vert$	$vars$		
	$\vert$	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	$\vert$	(Θ)	S	

	$\mid$	$\Theta \cap \mathit{vars}$		leave only those variables that are in the set
Ξ	$::=$			anti-unification type variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\widetilde{\alpha^+}}$		list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha^-}}$		list of variables
	$\mid$	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	$\mid$	(Ξ)	S	
	$\mid$	$\Xi_1 \cup \Xi_2$		
$\vec{\alpha},\ \vec{\beta}$	$::=$			ordered positive or negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	$\overrightarrow{\alpha^+}$		list of variables
	$\mid$	$\overrightarrow{\alpha^-}$		list of variables
	$\mid$	$\overrightarrow{\alpha^\pm}$		list of variables
	$\mid$	$\overrightarrow{\widetilde{\alpha^+}}$		list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha^-}}$		list of variables
	$\mid$	$\vec{\alpha}_1 \backslash \mathit{vars}$		setminus
	$\mid$	Γ		context
	$\mid$	vars		
	$\mid$	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	$\mid$	$(\vec{\alpha})$	S	parenthesis
	$\mid$	$[\mu]\vec{\alpha}$	M	apply moving to list
	$\mid$	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P$	M	
	$\mid$	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N$	M	
	$\mid$	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P$	M	
	$\mid$	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N$	M	
vars	$::=$			set of variables
	$\mid$	$\varnothing$		empty set
	$\mid$	$\mathbf{fv}\ P$		free variables
	$\mid$	$\mathbf{fv}\ N$		free variables
	$\mid$	$\mathbf{fv}\ \mathbf{imP}$		free variables
	$\mid$	$\mathbf{fv}\ \mathbf{imN}$		free variables
	$\mid$	$\mathit{vars}_1 \cap \mathit{vars}_2$		set intersection
	$\mid$	$\mathit{vars}_1 \cup \mathit{vars}_2$		set union
	$\mid$	$\mathit{vars}_1 \backslash \mathit{vars}_2$		set complement
	$\mid$	$\mathbf{mv}\ \mathbf{imP}$		movable variables
	$\mid$	$\mathbf{mv}\ \mathbf{imN}$		movable variables
	$\mid$	$\mathbf{uv}\ N$		unification variables
	$\mid$	$\mathbf{uv}\ P$		unification variables
	$\mid$	$\mathbf{fv}\ N$		free variables
	$\mid$	$\mathbf{fv}\ P$		free variables
	$\mid$	(vars)	S	parenthesis
	$\mid$	$\vec{\alpha}$		ordered list of variables
	$\mid$	$[\mu]\mathit{vars}$	M	apply moving to varset
μ	$::=$			
	$\mid$	$\cdot$		empty moving
	$\mid$	$\mathit{pma}1 \mapsto \mathit{pma}2$		Positive unit substitution

	|	$nma1 \mapsto nma2$		Positive unit substitution
	|	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	|	$\mu_1 \circ \mu_2$	M	Composition
	|	$\overline{\mu_i}^{\,i}$		concatenate movings
	|	$\mu\vert_{vars}$	M	restriction on a set
	|	μ^{-1}	M	inversion
	|	$\mathbf{nf}\,(\mu')$	M	
$\widehat{\alpha}^{\pm}$	::=			positive/negative unification variable
	|	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	::=			positive unification variable
	|	$\widehat{\alpha}^{+}$		
	|	$\widehat{\alpha}^{+}\{\Delta\}$		
	|	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	::=			negative unification variable
	|	$\widehat{\alpha}^{-}$		
	|	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	|	$\widehat{\alpha}^{-}\{\Delta\}$		
	|	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widetilde{\alpha}^{+}},\ \overrightarrow{\widetilde{\beta}^{+}}$	::=			positive unification variable list
	|	$\cdot$		empty list
	|	$\widehat{\alpha}^{+}$		a variable
	|	$\overrightarrow{\widehat{\alpha}^{+}\{\Delta\}}$		from a normal variable
	|	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	|	$\overline{\overrightarrow{\widetilde{\alpha}^{+}}_i}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}},\ \overrightarrow{\widehat{\beta}^{-}}$	::=			negative unification variable list
	|	$\cdot$		empty list
	|	$\widehat{\alpha}^{-}$		a variable
	|	Ξ		from an antiunification context
	|	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	|	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	|	$\overline{\overrightarrow{\widehat{\alpha}^{-}}_i}^{\,i}$		concatenate lists
$P,\ Q$	::=			a positive algorithmic type (potentially with metavariables)
	|	α^{+}		
	|	$\mathbf{pma}$		
	|	$\widehat{\alpha}^{+}$		
	|	$\downarrow N$		
	|	$\exists\overrightarrow{\alpha^{-}}.P$		
	|	$[\sigma]P$	M	
	|	$[\widehat{\tau}]P$	M	
	|	$[\mu]P$	M	
	|	(P)	S	
	|	$\mathbf{nf}\,(P')$	M	

N, M	::=			a negative algorithmic type (potentially with metavariables)
	|	α^-		
	|	$\widehat{\alpha}^-$		
	|	$\uparrow P$		
	|	$P \to N$		
	|	$\forall \overrightarrow{\alpha^+}. N$		
	|	$[\sigma] N$	M	
	|	$[\mu] N$	M	
	|	(N)	S	
	|	$\mathbf{nf}\,(N')$	M	
auSol	::=			
	|	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$		
terminals	::=			
	|	$\exists$		
	|	$\forall$		
	|	$\uparrow$		
	|	$\downarrow$		
	|	$\rightarrow$		
	|	$\leftrightarrow$		
	|	$\in$		
	|	$\notin$		
	|	$\cdot$		
	|	$\vdash$		
	|	$\leqslant$		
	|	$\geqslant$		
	|	$\simeq$		
	|	$\cup$		
	|	$\cap$		
	|	$\setminus$		
	|	$\subseteq$		
	|	$\mapsto$		
	|	$\stackrel{u}{\simeq}$		
	|	$\stackrel{a}{\simeq}$		
	|	$\varnothing$		
	|	$\circ$		
	|	$\Rightarrow$		
	|	$\vDash$		
	|	$\dashv$		
	|	$\neq$		
	|	$\equiv_n$		
	|	$\vee$		
	|	$\Downarrow$		
	|	$:\geqslant$		
	|	$:\simeq$		
formula	::=			
	|	*judgement*		
	|	$formula_1 \quad .. \quad formula_n$		

	$\mid$ $\mu : vars_1 \leftrightarrow vars_2$	
	$\mid$ μ **is bijective**	
	$\mid$ $\widehat{\sigma}$ **is functional**	
	$\mid$ $\widehat{\sigma}_1 \in \widehat{\sigma}_2$	
	$\mid$ $vars_1 \subseteq vars_2$	
	$\mid$ $vars_1 = vars_2$	
	$\mid$ $vars$ **is fresh**	
	$\mid$ $\alpha^- \notin vars$	
	$\mid$ $\alpha^+ \notin vars$	
	$\mid$ $\alpha^- \in vars$	
	$\mid$ $\alpha^+ \in vars$	
	$\mid$ $\widehat{\alpha}^- \in \Theta$	
	$\mid$ $\widehat{\alpha}^+ \in \Theta$	
	$\mid$ if any other rule is not applicable	
	$\mid$ $\vec{\alpha}_1 = \vec{\alpha}_2$	
	$\mid$ $N \neq M$	
	$\mid$ $P \neq Q$	
A	$::=$	
	$\mid$ $\Gamma; \Theta \models N \leqslant M \dashv \widehat{\sigma}$	Negative subtyping
	$\mid$ $\Gamma; \Theta \models P \geqslant Q \dashv \widehat{\sigma}$	Positive supertyping
AU	$::=$	
	$\mid$ $\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	$\mid$ $\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$	
$E1$	$::=$	
	$\mid$ $N \simeq_1^D M$	Negative multi-quantified type equivalence
	$\mid$ $P \simeq_1^D Q$	Positive multi-quantified type equivalence
	$\mid$ $P \simeq Q$	
$D1$	$::=$	
	$\mid$ $\Gamma \vdash N \simeq_1^{\leqslant} M$	Negative equivalence on MQ types
	$\mid$ $\Gamma \vdash P \simeq_1^{\leqslant} Q$	Positive equivalence on MQ types
	$\mid$ $\Gamma \vdash N \leqslant_1 M$	Negative subtyping
	$\mid$ $\Gamma \vdash P \geqslant_1 Q$	Positive supertyping
	$\mid$ $\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1$	Equivalence of substitutions
$D0$	$::=$	
	$\mid$ $\Gamma \vdash N \simeq_0^{\leqslant} M$	Negative equivalence
	$\mid$ $\Gamma \vdash P \simeq_0^{\leqslant} Q$	Positive equivalence
	$\mid$ $\Gamma \vdash N \leqslant_0 M$	Negative subtyping
	$\mid$ $\Gamma \vdash P \geqslant_0 Q$	Positive supertyping
EQ	$::=$	
	$\mid$ $N = M$	Negative type equality (alpha-equivalence)
	$\mid$ $P = Q$	Positive type equuality (alphha-equivalence)
	$\mid$ $P = Q$	

$LUBF$	$::=$		
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P\ ===\ \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N\ ===\ \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P\ ===\ \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N\ ===\ \vec{\alpha}$	
	$\mid$	$\mathbf{nf}\,(N')\ ===\ N$	
	$\mid$	$\mathbf{nf}\,(P')\ ===\ P$	
	$\mid$	$\mathbf{nf}\,(N')\ ===\ N$	
	$\mid$	$\mathbf{nf}\,(P')\ ===\ P$	
	$\mid$	$\mathbf{nf}\,(\vec{N}')\ ===\ \vec{N}$	
	$\mid$	$\mathbf{nf}\,(\vec{P}')\ ===\ \vec{P}$	
	$\mid$	$\mathbf{nf}\,(\sigma')\ ===\ \sigma$	
	$\mid$	$\mathbf{nf}\,(\mu')\ ===\ \mu$	
	$\mid$	$\sigma'\vert_{vars}$	
	$\mid$	$e_1\ \&\ e_2$	
	$\mid$	$\widehat{\sigma}_1\ \&\ \widehat{\sigma}_2$	
LUB	$::=$		
	$\mid$	$\Gamma \models P_1 \vee P_2 = Q$	Least Upper Bound (Least Common Supertype)
	$\mid$	$\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q$	
Nrm	$::=$		
	$\mid$	$\mathbf{nf}\,(N) = M$	
	$\mid$	$\mathbf{nf}\,(P) = Q$	
	$\mid$	$\mathbf{nf}\,(N) = M$	
	$\mid$	$\mathbf{nf}\,(P) = Q$	
$Order$	$::=$		
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}$	
SM	$::=$		
	$\mid$	$e_1\ \&\ e_2 = e_3$	Unification Solution Entry Merge
	$\mid$	$\widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3$	Merge unification solutions
$SWEAK$	$::=$		
	$\mid$	$e_1 \Rightarrow e_2$	Weakening of unification solution entries
	$\mid$	$\widehat{\sigma}_1 \Rightarrow \widehat{\sigma}_2$	Weakening of unification solutions
U	$::=$		
	$\mid$	$\Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv \widehat{\sigma}$	Negative unification
	$\mid$	$\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv \widehat{\sigma}$	Positive unification
WF	$::=$		
	$\mid$	$\Gamma \vdash N$	Negative type well-formedness
	$\mid$	$\Gamma \vdash P$	Positive type well-formedness
	$\mid$	$\Gamma \vdash N$	Negative type well-formedness

	$\vert$	$\Gamma \vdash P$	Positive type well-formedness
	$\vert$	$\Gamma \vdash \overrightarrow{N}$	Negative type list well-formedness
	$\vert$	$\Gamma \vdash \overrightarrow{P}$	Positive type list well-formedness
	$\vert$	$\Gamma ; \Theta \vdash N$	Negative unification type well-formedness
	$\vert$	$\Gamma ; \Theta \vdash P$	Positive unifiication type well-formedness
	$\vert$	$\Gamma ; \Xi \vdash P$	Positive anti-unification type well-formedness
	$\vert$	$\Gamma ; \Xi_2 \vdash \widehat{\tau} : \Xi_1$	Antiunification substitution well-formedness
	$\vert$	$\widehat{\sigma} : \Theta$	Unification substitution well-formedness
	$\vert$	$\Gamma \vdash^{\supseteq} \Theta$	Unification context well-formedness
	$\vert$	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution well-formedness
judgement	$::=$		
	$\vert$	A	
	$\vert$	AU	
	$\vert$	$E1$	
	$\vert$	$D1$	
	$\vert$	$D0$	
	$\vert$	EQ	
	$\vert$	LUB	
	$\vert$	Nrm	
	$\vert$	$Order$	
	$\vert$	SM	
	$\vert$	$SWEAK$	
	$\vert$	U	
	$\vert$	WF	
user_syntax	$::=$		
	$\vert$	α	
	$\vert$	n	
	$\vert$	n	
	$\vert$	α^{+}	
	$\vert$	α^{-}	
	$\vert$	$\alpha^{\pm}$	
	$\vert$	σ	
	$\vert$	e	
	$\vert$	$\widehat{\sigma}$	
	$\vert$	$\widehat{\tau}$	
	$\vert$	P	
	$\vert$	N	
	$\vert$	$\overrightarrow{\alpha^{+}}$	
	$\vert$	$\overrightarrow{\alpha^{-}}$	
	$\vert$	$\overrightarrow{\alpha^{\pm}}$	
	$\vert$	P	
	$\vert$	N	
	$\vert$	$\overrightarrow{P}$	
	$\vert$	$\overrightarrow{N}$	
	$\vert$	Γ	
	$\vert$	Θ	
	$\vert$	Ξ	
	$\vert$	$\overrightarrow{\alpha}$	

| *vars*
| μ
| $\widehat{\alpha}^{\pm}$
| $\widehat{\alpha}^{+}$
| $\widehat{\alpha}^{-}$
| $\overrightarrow{\widetilde{\alpha^{+}}}$
| $\overrightarrow{\widehat{\alpha^{-}}}$
| P
| N
| *auSol*
| *terminals*
| *formula*

$\boxed{\Gamma;\ \Theta \models N \leqslant M \dashv \widehat{\sigma}}$ Negative subtyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^{-} \leqslant \alpha^{-} \dashv \cdot}\ \text{ANVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(P) \overset{u}{\simeq} \mathbf{nf}\,(Q) \dashv \widehat{\sigma}}{\Gamma;\ \Theta \models \uparrow P \leqslant \uparrow Q \dashv \widehat{\sigma}}\ \text{AShiftU}$$

$$\frac{\Gamma;\ \Theta \models P \geqslant Q \dashv \widehat{\sigma}_1 \quad \Gamma;\ \Theta \models N \leqslant M \dashv \widehat{\sigma}_2}{\Gamma;\ \Theta \models P \to N \leqslant Q \to M \dashv \widehat{\sigma}_1\ \&\ \widehat{\sigma}_2}\ \text{AArrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^{+}};\ \Theta, \overrightarrow{\widehat{\alpha}^{+}\{\Gamma, \overrightarrow{\beta^{+}}\}} \models [\overrightarrow{\widehat{\alpha}^{+}}/\overrightarrow{\alpha^{+}}]N \leqslant M \dashv \widehat{\sigma}}{\Gamma;\ \Theta \models \forall\overrightarrow{\alpha^{+}}.N \leqslant \forall\overrightarrow{\beta^{+}}.M \dashv \widehat{\sigma}\backslash\overrightarrow{\widehat{\alpha}^{+}}}\ \text{AForall}$$

$\boxed{\Gamma;\ \Theta \models P \geqslant Q \dashv \widehat{\sigma}}$ Positive supertyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^{+} \geqslant \alpha^{+} \dashv \cdot}\ \text{APVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(N) \overset{u}{\simeq} \mathbf{nf}\,(M) \dashv \widehat{\sigma}}{\Gamma;\ \Theta \models \downarrow N \geqslant \downarrow M \dashv \widehat{\sigma}}\ \text{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^{-}};\ \Theta, \overrightarrow{\widehat{\alpha}^{-}\{\Gamma, \overrightarrow{\beta^{-}}\}} \models [\overrightarrow{\widehat{\alpha}^{-}}/\overrightarrow{\alpha^{-}}]P \geqslant Q \dashv \widehat{\sigma}}{\Gamma;\ \Theta \models \exists\overrightarrow{\alpha^{-}}.P \geqslant \exists\overrightarrow{\beta^{-}}.Q \dashv \widehat{\sigma}}\ \text{AExists}$$

$$\frac{\widehat{\alpha}^{+}\{\Delta\} \in \Theta \quad \mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}{\Gamma;\ \Theta \models \widehat{\alpha}^{+} \geqslant P \dashv (\Delta \vdash \widehat{\alpha}^{+} :\geqslant Q)}\ \text{APUVar}$$

$\boxed{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^{+} \overset{a}{\simeq} \alpha^{+} \dashv (\cdot, \alpha^{+}, \cdot, \cdot)}\ \text{AUPVar}$$

$$\frac{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \downarrow N_1 \overset{a}{\simeq} \downarrow N_2 \dashv (\Xi, \downarrow M, \widehat{\tau}_1, \widehat{\tau}_2)}\ \text{AUPShift}$$

$$\frac{\overrightarrow{\alpha^{-}} \cap \Gamma = \varnothing \quad \Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \exists\overrightarrow{\alpha^{-}}.P_1 \overset{a}{\simeq} \exists\overrightarrow{\alpha^{-}}.P_2 \dashv (\Xi, \exists\overrightarrow{\alpha^{-}}.Q, \widehat{\tau}_1, \widehat{\tau}_2)}\ \text{AUPExists}$$

$\boxed{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \vDash \alpha^- \stackrel{a}{\simeq} \alpha^- \dashv (\Xi, \alpha^-, \cdot, \cdot)}\ \textsc{AUNVar}$$

$$\frac{\Gamma \vDash P_1 \stackrel{a}{\simeq} P_2 \dashv (\Xi, Q, \hat{\tau}_1, \hat{\tau}_2)}{\Gamma \vDash {\uparrow}P_1 \stackrel{a}{\simeq} {\uparrow}P_2 \dashv (\Xi, {\uparrow}Q, \hat{\tau}_1, \hat{\tau}_2)}\ \textsc{AUNShift}$$

$$\frac{\Gamma \vDash P_1 \stackrel{a}{\simeq} P_2 \dashv (\Xi_1, Q, \hat{\tau}_1, \hat{\tau}_2) \quad \Gamma \vDash N_1 \stackrel{a}{\simeq} N_2 \dashv (\Xi_2, M, \hat{\tau}'_1, \hat{\tau}'_2)}{\Gamma \vDash P_1 \to N_1 \stackrel{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \hat{\tau}_1 \cup \hat{\tau}'_1, \hat{\tau}_2 \cup \hat{\tau}'_2)}\ \textsc{AUNArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \vDash N \stackrel{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, \widehat{\alpha}^-_{\{N,M\}}, (\widehat{\alpha}^-_{\{N,M\}} :\approx N), (\widehat{\alpha}^-_{\{N,M\}} :\approx M))}\ \textsc{AUNAU}$$

$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-}\ \textsc{E1NVar}$$

$$\frac{P \simeq_1^D Q}{{\uparrow}P \simeq_1^D {\uparrow}Q}\ \textsc{E1ShiftU}$$

$$\frac{P \simeq_1^D Q \quad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M}\ \textsc{E1Arrow}$$

$$\frac{\overrightarrow{\alpha^+} \cap \mathbf{fv}\, M = \varnothing \quad \mu : (\overrightarrow{\beta^+} \cap \mathbf{fv}\, M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \mathbf{fv}\, N) \quad N \simeq_1^D [\mu]M}{\forall \overrightarrow{\alpha^+}.N \simeq_1^D \forall \overrightarrow{\beta^+}.M}\ \textsc{E1Forall}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+}\ \textsc{E1PVar}$$

$$\frac{N \simeq_1^D M}{{\downarrow}N \simeq_1^D {\downarrow}M}\ \textsc{E1ShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \mathbf{fv}\, Q = \varnothing \quad \mu : (\overrightarrow{\beta^-} \cap \mathbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \mathbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists \overrightarrow{\alpha^-}.P \simeq_1^D \exists \overrightarrow{\beta^-}.Q}\ \textsc{E1Exists}$$

$\boxed{P \simeq Q}$

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M}\ \textsc{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q}\ \textsc{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-}\ \textsc{D1NVar}$$

<<multiple parses>>

$$\frac{}{\Gamma \vdash {\uparrow}P \leqslant_1 {\uparrow}Q}\ \textsc{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M}\ \textsc{D1Arrow}$$

$$\frac{\mathbf{fv}\ N \cap \overrightarrow{\beta^+} = \varnothing \quad \Gamma, \overrightarrow{\beta^+} \vdash P_i \quad \Gamma, \overrightarrow{\beta^+} \vdash [\overrightarrow{P}/\overrightarrow{\alpha^+}]N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant_1 \forall \overrightarrow{\beta^+}.M}\ \textsc{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+}\ \textsc{D1PVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M}\ \textsc{D1ShiftD}$$

$$\frac{\mathbf{fv}\ P \cap \overrightarrow{\beta^-} = \varnothing \quad \Gamma, \overrightarrow{\beta^-} \vdash N_i \quad \Gamma, \overrightarrow{\beta^-} \vdash [\overrightarrow{N}/\overrightarrow{\alpha^-}]P \geqslant_1 Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P \geqslant_1 \exists \overrightarrow{\beta^-}.Q}\ \textsc{D1Exists}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1}$ Equivalence of substitutions

$\boxed{\Gamma \vdash N \simeq_0^{\leqslant} M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq_0^{\leqslant} M}\ \textsc{D0NDef}$$

$\boxed{\Gamma \vdash P \simeq_0^{\leqslant} Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq_0^{\leqslant} Q}\ \textsc{D0PDef}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_0 \alpha^-}\ \textsc{D0NVar}$$

$$\frac{\Gamma \vdash P \simeq_0^{\leqslant} Q}{\Gamma \vdash \uparrow P \leqslant_0 \uparrow Q}\ \textsc{D0ShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/\alpha^+]N \leqslant_0 M \quad M \neq \forall \beta^+.M'}{\Gamma \vdash \forall \alpha^+.N \leqslant_0 M}\ \textsc{D0ForallL}$$

$$\frac{\Gamma, \alpha^+ \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall \alpha^+.M}\ \textsc{D0ForallR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \to N \leqslant_0 Q \to M}\ \textsc{D0Arrow}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_0 \alpha^+}\ \textsc{D0PVar}$$

$$\frac{\Gamma \vdash N \simeq_0^{\leqslant} M}{\Gamma \vdash \downarrow N \geqslant_0 \downarrow M}\ \textsc{D0ShiftD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/\alpha^-]P \geqslant_0 Q \quad Q \neq \exists \alpha^-.Q'}{\Gamma \vdash \exists \alpha^-.P \geqslant_0 Q}\ \textsc{D0ExistsL}$$

$$\frac{\Gamma, \alpha^- \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists \alpha^-.Q}\ \textsc{D0ExistsR}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$
$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ P}$

$\boxed{\textbf{ord}\ \mathit{vars}\ \textbf{in}\ N}$

$\boxed{\textbf{nf}\ (N')}$

$\boxed{\textbf{nf}\ (P')}$

$\boxed{\textbf{nf}\ (N')}$

$\boxed{\textbf{nf}\ (P')}$

$\boxed{\textbf{nf}\ (\overrightarrow{N'})}$

$\boxed{\textbf{nf}\ (\overrightarrow{P'})}$

$\boxed{\textbf{nf}\ (\sigma')}$

$\boxed{\textbf{nf}\ (\mu')}$

$\boxed{\sigma'|_{\mathit{vars}}}$

$\boxed{e_1\ \&\ e_2}$

$\boxed{\widehat{\sigma}_1\ \&\ \widehat{\sigma}_2}$

$\boxed{\Gamma \models P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \models \alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBVar}$$

$$\frac{\Gamma, \cdot \models \mathbf{nf}\,(\downarrow N) \overset{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi] P} \quad \text{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \models P_1 \vee P_2 = Q}{\Gamma \models \exists \overrightarrow{\alpha^-}.P_1 \vee \exists \overrightarrow{\beta^-}.P_2 = Q} \quad \text{LUBExists}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}$

$$\frac{\begin{array}{c}\Gamma = \Delta, \overrightarrow{\alpha^\pm} \quad \overrightarrow{\beta^\pm}\,\mathbf{is\ fresh} \quad \overrightarrow{\gamma^\pm}\,\mathbf{is\ fresh} \\ \Delta, \overrightarrow{\beta^\pm}, \overrightarrow{\gamma^\pm} \models [\overrightarrow{\beta^\pm}/\overrightarrow{\alpha^\pm}]P \vee [\overrightarrow{\gamma^\pm}/\overrightarrow{\alpha^\pm}]P = Q\end{array}}{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q} \quad \text{LUBUpgrade}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^-) = \alpha^-} \quad \text{NrmNVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\uparrow P) = \uparrow Q} \quad \text{NrmShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(P \to N) = Q \to M} \quad \text{NrmArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\forall \overrightarrow{\alpha^+}.N) = \forall \overrightarrow{\alpha^+}'.N'} \quad \text{NrmForall}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\alpha^+) = \alpha^+} \quad \text{NrmPVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\downarrow N) = \downarrow M} \quad \text{NrmShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\exists \overrightarrow{\alpha^-}.P) = \exists \overrightarrow{\alpha^-}'.P'} \quad \text{NrmExists}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^-) = \widehat{\alpha}^-} \quad \text{NrmNUVar}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^+) = \widehat{\alpha}^+} \quad \text{NrmPUVar}$$

$\boxed{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}$

$$\frac{\alpha^- \in \mathit{vars}}{\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,\alpha^- = \alpha^-} \quad \text{ONVarIn}$$

$$\frac{\alpha^- \notin \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \alpha^- = \cdot}\ \text{ONVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \uparrow P = \vec{\alpha}}\ \text{OShiftU}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}_1 \quad \mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}_2}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)}\ \text{OArrow}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^+} = \varnothing \quad \mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \forall \overrightarrow{\alpha^+}.N = \vec{\alpha}}\ \text{OForall}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}}$

$$\frac{\alpha^+ \in \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \alpha^+ = \alpha^+}\ \text{OPVarIn}$$

$$\frac{\alpha^+ \notin \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \alpha^+ = \cdot}\ \text{OPVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \downarrow N = \vec{\alpha}}\ \text{OShiftD}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^-} = \varnothing \quad \mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \exists \overrightarrow{\alpha^-}.P = \vec{\alpha}}\ \text{OExists}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \widehat{\alpha}^- = \cdot}\ \text{ONUVar}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ \widehat{\alpha}^+ = \cdot}\ \text{OPUVar}$$

$\boxed{e_1\ \&\ e_2 = e_3}$ Unification Solution Entry Merge

$$\frac{\Gamma \vDash P_1 \vee P_2 = Q}{(\Gamma \vdash \widehat{\alpha}^+ :\geqslant P_1)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\geqslant P_2) = (\Gamma \vdash \widehat{\alpha}^+ :\geqslant Q)}\ \text{SMESupSup}$$

$$\frac{\Gamma; \cdot \vDash P \geqslant Q \dashv \widehat{\sigma}'}{(\Gamma \vdash \widehat{\alpha}^+ :\approx P)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\geqslant Q) = (\Gamma \vdash \widehat{\alpha}^+ :\approx P)}\ \text{SMEEqSup}$$

$$\frac{\Gamma; \cdot \vDash Q \geqslant P \dashv \widehat{\sigma}'}{(\Gamma \vdash \widehat{\alpha}^+ :\geqslant P)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\approx Q) = (\Gamma \vdash \widehat{\alpha}^+ :\approx Q)}\ \text{SMESupEq}$$

$$\frac{}{(\Gamma \vdash \widehat{\alpha}^+ :\approx P)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\approx P) = (\Gamma \vdash \widehat{\alpha}^+ :\approx P)}\ \text{SMEPEqEq}$$

$$\frac{}{(\Gamma \vdash \widehat{\alpha}^- :\approx N)\ \&\ (\Gamma \vdash \widehat{\alpha}^- :\approx N) = (\Gamma \vdash \widehat{\alpha}^- :\approx N)}\ \text{SMENEqEq}$$

$\boxed{\widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3}$ Merge unification solutions

$\boxed{e_1 \Rightarrow e_2}$ Weakening of unification solution entries

$$\frac{\Gamma \vdash P_1 \geqslant_1 P_2}{(\Gamma \vdash \widehat{\alpha}^+ :\geqslant P_1) \Rightarrow (\Gamma \vdash \widehat{\alpha}^+ :\geqslant P_2)}\ \text{SWeakESupSup}$$

$$\frac{\Gamma \vdash P_1 \geqslant_1 P_2}{(\Gamma \vdash \widehat{\alpha}^+ :\approx P_1) \Rightarrow (\Gamma \vdash \widehat{\alpha}^+ :\geqslant P_2)}\ \text{SWeakEEqSup}$$

$$\frac{\text{<<multiple parses>>}}{(\Gamma \vdash \widehat{\alpha}^+ :\approx P_1) \Rightarrow (\Gamma \vdash \widehat{\alpha}^+ :\approx P_2)} \quad \text{SWEAKEEQEQ}$$

$\boxed{\widehat{\sigma}_1 \Rightarrow \widehat{\sigma}_2}$ Weakening of unification solutions

$\boxed{\Gamma; \Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}}$ Negative unification

$$\frac{}{\Gamma; \Theta \vDash \alpha^- \overset{u}{\simeq} \alpha^- \dashv \cdot} \quad \text{UNVAR}$$

$$\frac{\Gamma; \Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}{\Gamma; \Theta \vDash \uparrow P \overset{u}{\simeq} \uparrow Q \dashv \widehat{\sigma}} \quad \text{USHIFTU}$$

$$\frac{\Gamma; \Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}_1 \quad \Gamma; \Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}_2}{\Gamma; \Theta \vDash P \to N \overset{u}{\simeq} Q \to M \dashv \widehat{\sigma}_1 \ \&\ \widehat{\sigma}_2} \quad \text{UARROW}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}}{\Gamma; \Theta \vDash \forall \overrightarrow{\alpha^+}. N \overset{u}{\simeq} \forall \overrightarrow{\alpha^+}. M \dashv \widehat{\sigma}} \quad \text{UFORALL}$$

$$\frac{\widehat{\alpha}^-\{\Delta\} \in \Theta \quad \Delta \vdash N}{\Gamma; \Theta \vDash \widehat{\alpha}^- \overset{u}{\simeq} N \dashv (\Delta \vdash \widehat{\alpha}^- :\approx N)} \quad \text{UNUVAR}$$

$\boxed{\Gamma; \Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}$ Positive unification

$$\frac{}{\Gamma; \Theta \vDash \alpha^+ \overset{u}{\simeq} \alpha^+ \dashv \cdot} \quad \text{UPVAR}$$

$$\frac{\Gamma; \Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}}{\Gamma; \Theta \vDash \downarrow N \overset{u}{\simeq} \downarrow M \dashv \widehat{\sigma}} \quad \text{USHIFTD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}{\Gamma; \Theta \vDash \exists \overrightarrow{\alpha^-}. P \overset{u}{\simeq} \exists \overrightarrow{\alpha^-}. Q \dashv \widehat{\sigma}} \quad \text{UEXISTS}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \Delta \vdash P}{\Gamma; \Theta \vDash \widehat{\alpha}^+ \overset{u}{\simeq} P \dashv (\Delta \vdash \widehat{\alpha}^+ :\approx P)} \quad \text{UPUVAR}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness
$\boxed{\Gamma \vdash P}$ Positive type well-formedness
$\boxed{\Gamma \vdash N}$ Negative type well-formedness
$\boxed{\Gamma \vdash P}$ Positive type well-formedness
$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness
$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness
$\boxed{\Gamma; \Theta \vdash N}$ Negative unification type well-formedness
$\boxed{\Gamma; \Theta \vdash P}$ Positive unifiication type well-formedness
$\boxed{\Gamma; \Xi \vdash P}$ Positive anti-unification type well-formedness
$\boxed{\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness
$\boxed{\widehat{\sigma} : \Theta}$ Unification substitution well-formedness
$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness
$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution well-formedness

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Definition rules:        75 good    8 bad
Definition rule clauses: 138 good    8 bad
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