α, β, α, β, γ, δ	type variables
n, m, i, j	index variables
x, y, z	term variables

n, m	$::=$			cohort index
	$\mid$	0		
	$\mid$	$n + 1$		
$\alpha^+, \beta^+, \gamma^+, \delta^+$	$::=$			positive variable
	$\mid$	α^+		
	$\mid$	α^{+n}		
	$\mid$	$\beta^\pm$		
$\alpha^-, \beta^-, \gamma^-, \delta^-$	$::=$			negative variable
	$\mid$	α^-		
	$\mid$	α^{-n}		
	$\mid$	$\alpha^\pm$		
$\alpha^\pm, \beta^\pm$	$::=$			positive or negative variable
	$\mid$	$\alpha^\pm$		
	$\mid$	$\alpha^{\pm n}$		
σ	$::=$			substitution
	$\mid$	$\cdot$		
	$\mid$	id		
	$\mid$	P/α^+		
	$\mid$	N/α^-		
	$\mid$	$\vec{P}/\overrightarrow{\alpha^+}$		
	$\mid$	$\vec{N}/\overrightarrow{\alpha^-}$		
	$\mid$	$\mathbf{pmas}\,/\overrightarrow{\alpha^+}$		
	$\mid$	$\mathbf{nmas}\,/\overrightarrow{\alpha^-}$		
	$\mid$	$\overrightarrow{\widetilde{\alpha^+}}/\overrightarrow{\alpha^+}$		
	$\mid$	$\overrightarrow{\widehat{\alpha^-}}/\overrightarrow{\alpha^-}$		
	$\mid$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\mid$	μ		
	$\mid$	$\sigma_1 \circ \sigma_2$		
	$\mid$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\mid$	(σ)	S	
	$\mid$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\mid$	$\mathbf{nf}\,(\sigma')$	M	
	$\mid$	$\sigma'\vert_{vars}$	M	
e	$::=$			entry of a subtyping constraint
	$\mid$	$\widehat{\alpha}^+ :\approx P$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		
	$\mid$	$\widehat{\alpha}^+ :\geqslant P$		
	$\mid$	(e)	S	
	$\mid$	$SC(\widehat{\alpha}^-)$	M	
	$\mid$	$e_1\ \&\ e_2$	M	
e	$::=$			entry of a unification constraint
	$\mid$	$\widehat{\alpha}^+ :\approx P$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		

	$\vert$	(e)	S	
	$\vert$	e_1 & e_2	M	
UC	$::=$			unification constraint
	$\vert$	$\cdot$		
	$\vert$	e		
	$\vert$	$UC \backslash vars$		
	$\vert$	$UC\vert vars$		
	$\vert$	$UC_1 \cup UC_2$		
	$\vert$	$\overline{UC_i}^{\,i}$		concatenate
	$\vert$	(UC)	S	
	$\vert$	$\mathbf{UC}\vert_{vars}$	M	
	$\vert$	UC_1 & UC_2	M	
	$\vert$	$UC_1 \cup UC_2$	M	
	$\vert$	$\vert SC\vert$	M	
SC	$::=$			subtyping constraint
	$\vert$	$\cdot$		
	$\vert$	e		
	$\vert$	$SC \backslash vars$		
	$\vert$	$SC\vert vars$		
	$\vert$	$SC_1 \cup SC_2$		
	$\vert$	UC		
	$\vert$	$\overline{SC_i}^{\,i}$		concatenate
	$\vert$	(SC)	S	
	$\vert$	$\mathbf{SC}\vert_{vars}$	M	
	$\vert$	SC_1 & SC_2	M	
$\widehat{\sigma}$	$::=$			unification substitution
	$\vert$	$\cdot$		
	$\vert$	$P/\widehat{\alpha}^+$		
	$\vert$	$N/\widehat{\alpha}^-$		
	$\vert$	$\overrightarrow{P}/\overrightarrow{\widetilde{\alpha^+}}$		
	$\vert$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	$(\widehat{\sigma})$	S	
	$\vert$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\vert$	$\mathbf{nf}\,(\widehat{\sigma}')$	M	
	$\vert$	$\widehat{\sigma}'\vert_{vars}$	M	
$\widehat{\tau},\ \widehat{\rho}$	$::=$			anti-unification substitution
	$\vert$	$\cdot$		
	$\vert$	$\widehat{\alpha}^- :\approx N$		
	$\vert$	$\widehat{\alpha}^- :\approx N$		
	$\vert$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
	$\vert$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	$\vert$	$(\widehat{\tau})$	S	
	$\vert$	$\widehat{\tau}'\vert_{vars}$	M	
	$\vert$	$\widehat{\tau}_1$ & $\widehat{\tau}_2$	M	

$P,\ Q,\ R$	$::=$		positive types
	$\mid\ \alpha^+$		
	$\mid\ \downarrow N$		
	$\mid\ \exists \alpha^-.P$		
	$\mid\ [\sigma]P$	M	

$N,\ M,\ K$	$::=$		negative types
	$\mid\ \alpha^-$		
	$\mid\ \uparrow P$		
	$\mid\ \forall \alpha^+.N$		
	$\mid\ P \to N$		
	$\mid\ [\sigma]N$	M	

$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+},\ \overrightarrow{\gamma^+},\ \overrightarrow{\delta^+}$	$::=$		positive variable list
	$\mid\ \cdot$		empty list
	$\mid\ \alpha^+$		a variable
	$\mid\ \overrightarrow{\alpha^+}$		a variable
	$\mid\ \overline{\overrightarrow{\alpha^+}_i}^{\,i}$		concatenate lists

$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$	$::=$		negative variables
	$\mid\ \cdot$		empty list
	$\mid\ \alpha^-$		a variable
	$\mid\ \overrightarrow{\alpha^-}$		variables
	$\mid\ \overline{\overrightarrow{\alpha^-}_i}^{\,i}$		concatenate lists

$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$	$::=$		positive or negative variable list
	$\mid\ \cdot$		empty list
	$\mid\ \alpha^\pm$		a variable
	$\mid\ \overrightarrow{\mathbf{pa}}$		variables
	$\mid\ \overline{\overrightarrow{\alpha^\pm}_i}^{\,i}$		concatenate lists

$P,\ Q,\ R$	$::=$		multi-quantified positive types
	$\mid\ \alpha^+$		
	$\mid\ \downarrow N$		
	$\mid\ \exists \overrightarrow{\alpha^-}.P$		$P \neq \exists \ldots$
	$\mid\ [\sigma]P$	M	
	$\mid\ [\hat{\tau}]\,P$	M	
	$\mid\ [\hat{\sigma}]\,P$	M	
	$\mid\ [\mu]P$	M	
	$\mid\ (P)$	S	
	$\mid\ P_1 \vee P_2$	M	
	$\mid\ \mathbf{nf}\,(P')$	M	

$N,\ M,\ K$	$::=$		multi-quantified negative types
	$\mid\ \alpha^-$		
	$\mid\ \uparrow P$		
	$\mid\ P \to N$		
	$\mid\ \forall \overrightarrow{\alpha^+}.N$		$N \neq \forall \ldots$
	$\mid\ [\sigma]N$	M	

	|	$[\widehat{\tau}]N$	M	
	|	$[\mu]N$	M	
	|	$[\widehat{\sigma}]N$	M	
	|	(N)	S	
	|	$\mathbf{nf}\,(N')$	M	
$\vec{P},\ \vec{Q}$	$::=$			list of positive types
	|	$\cdot$		empty list
	|	P		a singel type
	|	$[\sigma]\vec{P}$	M	
	|	$\overline{\vec{P}_i}^{\,i}$		concatenate lists
	|	$\mathbf{nf}\,(\vec{P}')$	M	
$\vec{N},\ \vec{M}$	$::=$			list of negative types
	|	$\cdot$		empty list
	|	N		a singel type
	|	$[\sigma]\vec{N}$	M	
	|	$\overline{\vec{N}_i}^{\,i}$		concatenate lists
	|	$\mathbf{nf}\,(\vec{N}')$	M	
$\Delta,\ \Gamma$	$::=$			declarative type context
	|	$\cdot$		empty context
	|	$\overrightarrow{\alpha^+}$		list of variables
	|	$\overrightarrow{\alpha^-}$		list of variables
	|	$\overrightarrow{\alpha^\pm}$		list of variables
	|	$vars$		
	|	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	|	(Γ)	S	
	|	$\Theta(\widehat{\alpha}^+)$	M	
	|	$\Theta(\widehat{\alpha}^-)$	M	
Θ	$::=$			unification type variable context
	|	$\cdot$		empty context
	|	$\vec{\widehat{\alpha}}\{\Delta\}$		from an ordered list of variables
	|	$\overrightarrow{\widehat{\alpha}^+\{\Delta\}}$		from a variable to a list
	|	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	|	(Θ)	S	
	|	$\Theta\rvert_{vars}$		leave only those variables that are in the set
	|	$\Theta_1 \cup \Theta_2$		
Ξ	$::=$			anti-unification type variable context
	|	$\cdot$		empty context
	|	$\overrightarrow{\widehat{\alpha}^-}$		list of variables
	|	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	|	(Ξ)	S	
	|	$\Xi_1 \cup \Xi_2$		
	|	$\Xi_1 \cap \Xi_2$		
	|	$\Xi'\rvert_{vars}$	M	

$\vec{\alpha},\ \vec{\beta}$	::=			ordered positive or negative variables
	|	$\cdot$		empty list
	|	$\overrightarrow{\alpha^{+}}$		list of variables
	|	$\overrightarrow{\alpha^{-}}$		list of variables
	|	$\overrightarrow{\alpha^{\pm}}$		list of variables
	|	$\overrightarrow{\widetilde{\alpha^{+}}}$		list of variables
	|	$\overrightarrow{\widehat{\alpha^{-}}}$		list of variables
	|	$\vec{\alpha}_1 \backslash vars$		setminus
	|	Γ		context
	|	$vars$		
	|	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	|	$(\vec{\alpha})$	S	parenthesis
	|	$[\mu]\vec{\alpha}$	M	apply moving to list
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ P$	M	
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ N$	M	
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ P$	M	
	|	$\mathbf{ord}\ vars\ \mathbf{in}\ N$	M	
$vars$	::=			set of variables
	|	$\varnothing$		empty set
	|	$\mathbf{fv}\ P$		free variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathbf{fv}\ \mathbf{imP}$		free variables
	|	$\mathbf{fv}\ \mathbf{imN}$		free variables
	|	$vars_1 \cap vars_2$		set intersection
	|	$vars_1 \cup vars_2$		set union
	|	$vars_1 \backslash vars_2$		set complement
	|	$\mathbf{mv}\ \mathbf{imP}$		movable variables
	|	$\mathbf{mv}\ \mathbf{imN}$		movable variables
	|	$\mathbf{uv}\ N$		unification variables
	|	$\mathbf{uv}\ P$		unification variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathbf{fv}\ P$		free variables
	|	$(vars)$	S	parenthesis
	|	$\vec{\alpha}$		ordered list of variables
	|	$[\mu]vars$	M	apply moving to varset
	|	$\mathbf{dom}\,(UC)$	M	
	|	$\mathbf{dom}\,(SC)$	M	
	|	$\mathbf{dom}\,(\widehat{\sigma})$	M	
	|	$\mathbf{dom}\,(\widehat{\tau})$	M	
	|	$\mathbf{dom}\,(\Theta)$	M	
μ	::=			
	|	$\cdot$		empty moving
	|	$pma1 \mapsto pma2$		Positive unit substitution
	|	$nma1 \mapsto nma2$		Positive unit substitution
	|	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	|	$\mu_1 \circ \mu_2$	M	Composition
	|	$\overline{\mu_i}^{\,i}$		concatenate movings
	|	$\mu\vert_{vars}$	M	restriction on a set

	$\|$	μ^{-1}	M	inversion
	$\|$	$\mathbf{nf}\,(\mu')$	M	
$\widehat{\alpha}^{\pm}$	$::=$			positive/negative unification variable
	$\|$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	$::=$			positive unification variable
	$\|$	$\widehat{\alpha}^{+}$		
	$\|$	$\widehat{\alpha}^{+}\{\Delta\}$		
	$\|$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	$::=$			negative unification variable
	$\|$	$\widehat{\alpha}^{-}$		
	$\|$	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	$\|$	$\widehat{\alpha}^{-}\{\Delta\}$		
	$\|$	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widetilde{\alpha}^{+}},\ \overrightarrow{\widetilde{\beta}^{+}}$	$::=$			positive unification variable list
	$\|$	$\cdot$		empty list
	$\|$	$\widehat{\alpha}^{+}$		a variable
	$\|$	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	$\|$	$\overline{\overrightarrow{\widetilde{\alpha}^{+}}_{i}}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}},\ \overrightarrow{\widehat{\beta}^{-}}$	$::=$			negative unification variable list
	$\|$	$\cdot$		empty list
	$\|$	$\widehat{\alpha}^{-}$		a variable
	$\|$	Ξ		from an antiunification context
	$\|$	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	$\|$	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	$\|$	$\overline{\overrightarrow{\widehat{\alpha}^{-}}_{i}}^{\,i}$		concatenate lists
$P,\ Q$	$::=$			a positive algorithmic type (potentially with metavariables)
	$\|$	α^{+}		
	$\|$	$\mathbf{pma}$		
	$\|$	$\widehat{\alpha}^{+}$		
	$\|$	$\downarrow N$		
	$\|$	$\exists\overrightarrow{\alpha^{-}}.\,P$		
	$\|$	$[\sigma]P$	M	
	$\|$	$[\widehat{\tau}]P$	M	
	$\|$	$[\mu]P$	M	
	$\|$	(P)	S	
	$\|$	$\mathbf{nf}\,(P')$	M	
$N,\ M$	$::=$			a negative algorithmic type (potentially with metavariables)
	$\|$	α^{-}		
	$\|$	$\widehat{\alpha}^{-}$		
	$\|$	$\uparrow P$		
	$\|$	$P \to N$		
	$\|$	$\forall\overrightarrow{\alpha^{+}}.\,N$		

	$\mid$	$[\sigma] N$	M
	$\mid$	$[\widehat{\tau}] N$	M
	$\mid$	$[\mu] N$	M
	$\mid$	(N)	S
	$\mid$	$\mathbf{nf}\,(N')$	M
$auSol$	$::=$		
	$\mid$	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	$\mid$	$(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$	
$terminals$	$::=$		
	$\mid$	$\exists$	
	$\mid$	$\forall$	
	$\mid$	$\uparrow$	
	$\mid$	$\downarrow$	
	$\mid$	$\rightarrow$	
	$\mid$	$\leftrightarrow$	
	$\mid$	$\in$	
	$\mid$	$\notin$	
	$\mid$	$\cdot$	
	$\mid$	$\vdash$	
	$\mid$	$\leqslant$	
	$\mid$	$\geqslant$	
	$\mid$	$\simeq$	
	$\mid$	$\cup$	
	$\mid$	$\cap$	
	$\mid$	$\setminus$	
	$\mid$	$\subseteq$	
	$\mid$	$\mapsto$	
	$\mid$	$\stackrel{u}{\simeq}$	
	$\mid$	$\stackrel{a}{\simeq}$	
	$\mid$	$\varnothing$	
	$\mid$	$\circ$	
	$\mid$	$\Rightarrow$	
	$\mid$	$\vDash$	
	$\mid$	$\Dashv$	
	$\mid$	$\neq$	
	$\mid$	$\equiv_n$	
	$\mid$	$\vee$	
	$\mid$	$\Downarrow$	
	$\mid$	$:\geqslant$	
	$\mid$	$:\simeq$	
	$\mid$	Λ	
	$\mid$	λ	
	$\mid$	$\mathbf{let}^{\exists}$	
	$\mid$	$\bullet$	
	$\mid$	$\Rrightarrow$	
$v,\ w$	$::=$		value terms
	$\mid$	x	

	$\mid$	$\{c\}$	
	$\mid$	$(v : P)$	
	$\mid$	(v)	M
$\vec{v}$	$::=$		list of arguments
	$\mid$	$\cdot$	
	$\mid$	v	
	$\mid$	$\overline{\vec{v}_i}^{\,i}$	concatenate
$c,\ d$	$::=$		computation terms
	$\mid$	$(c : N)$	
	$\mid$	$\lambda x : P.c$	
	$\mid$	$\Lambda\alpha^+.c$	
	$\mid$	$\textbf{return}\, v$	
	$\mid$	$\textbf{let}\, x = v; c$	
	$\mid$	$\textbf{let}\, x : P = v(\vec{v}); c$	
	$\mid$	$\textbf{let}\, x = v(\vec{v}); c$	
	$\mid$	$\textbf{let}^{\exists}(\alpha^-, x) = v; c$	
$vctx,\ \Phi$	$::=$		variable context
	$\mid$	$\cdot$	
	$\mid$	$x : P$	
	$\mid$	$\overline{\Phi_i}^{\,i}$	concatenate contexts
$formula$	$::=$		
	$\mid$	$judgement$	
	$\mid$	judgement $uniquely$	
	$\mid$	$formula_1 \quad .. \quad formula_n$	
	$\mid$	$\mu : vars_1 \leftrightarrow vars_2$	
	$\mid$	$\mu\, \textbf{is bijective}$	
	$\mid$	$v : P \in \Phi$	
	$\mid$	$UC_1 \subseteq UC_2$	
	$\mid$	$UC_1 = UC_2$	
	$\mid$	$SC_1 \subseteq SC_2$	
	$\mid$	$vars_1 \subseteq vars_2$	
	$\mid$	$vars_1 = vars_2$	
	$\mid$	$vars\, \textbf{is fresh}$	
	$\mid$	$\alpha^- \notin vars$	
	$\mid$	$\alpha^+ \notin vars$	
	$\mid$	$\alpha^- \in vars$	
	$\mid$	$\alpha^+ \in vars$	
	$\mid$	$\widehat{\alpha}^+ \in vars$	
	$\mid$	$\widehat{\alpha}^- \in vars$	
	$\mid$	$\widehat{\alpha}^- \in \Theta$	
	$\mid$	$\widehat{\alpha}^+ \in \Theta$	
	$\mid$	if any other rule is not applicable	
	$\mid$	$\vec{\alpha}_1 = \vec{\alpha}_2$	
	$\mid$	$e_1 = e_2$	
	$\mid$	$e_1 = e_2$	
	$\mid$	$\widehat{\sigma}_1 = \widehat{\sigma}_2$	

	|	$N = M$	
	|	$\Theta \subseteq \Theta'$	
	|	$\vec{v}_1 = \vec{v}_2$	
	|	$N \neq M$	
	|	$P \neq Q$	
	|	$N \neq M$	
	|	$P \neq Q$	
	|	$P \neq Q$	
	|	$N \neq M$	
	|	$\vec{v}_1 \neq \vec{v}_2$	
A	$::=$		
	|	$\Gamma; \Theta \models N \leqslant M \dashv SC$	Negative subtyping
	|	$\Gamma; \Theta \models P \geqslant Q \dashv SC$	Positive supertyping
AT	$::=$		
	|	$\Gamma; \Phi \models v : P$	Positive type inference
	|	$\Gamma; \Phi \models c : N$	Negative type inference
	|	$\Gamma; \Phi; \Theta_1 \models N \bullet \vec{v} \Longrightarrow\!\!> M \dashv \Theta_2; SC$	Application type inference
AU	$::=$		
	|	$\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	|	$\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$	
SCM	$::=$		
	|	$\Gamma \vdash e_1 \ \& \ e_2 = e_3$	Subtyping Constraint Entry Merge
	|	$\Theta \vdash SC_1 \& SC_2 = SC_3$	Merge of subtyping constraints
UCM	$::=$		
	|	$\Gamma \vdash e_1 \& e_2 = e_3$	
	|	$\Theta \vdash UC_1 \& UC_2 = UC_3$	
$SATSCE$	$::=$		
	|	$\Gamma \vdash P : e$	Positive type satisfies with the subtyping constr
	|	$\Gamma \vdash N : e$	Negative type satisfies with the subtyping const
$SING$	$::=$		
	|	e_1 **singular**	Subtyping Constraint Entry Is Singular
	|	SC **singular**	Subtyping Constraint Is Singular
$E1$	$::=$		
	|	$N \simeq_1^D M$	Negative multi-quantified type equivalence
	|	$P \simeq_1^D Q$	Positive multi-quantified type equivalence
	|	$P \simeq Q$	
$D1$	$::=$		
	|	$\Gamma \vdash N \simeq_1^{\leqslant} M$	Negative equivalence on MQ types
	|	$\Gamma \vdash P \simeq_1^{\leqslant} Q$	Positive equivalence on MQ types
	|	$\Gamma \vdash N \leqslant_1 M$	Negative subtyping

	$\mid$	$\Gamma \vdash P \geqslant_1 Q$	Positive supertyping
	$\mid$	$\Gamma_2 \vdash \sigma_1 \simeq^{\leqslant}_1 \sigma_2 : \Gamma_1$	Equivalence of substitutions
	$\mid$	$\Gamma \vdash \sigma_1 \simeq^{\leqslant}_1 \sigma_2 : \mathit{vars}$	Equivalence of substitutions
	$\mid$	$\Theta \vdash \widehat{\sigma}_1 \simeq^{\leqslant}_1 \widehat{\sigma}_2 : \mathit{vars}$	Equivalence of unification substitutions
$\mathit{D0}$	$::=$		
	$\mid$	$\Gamma \vdash N \simeq^{\leqslant}_0 M$	Negative equivalence
	$\mid$	$\Gamma \vdash P \simeq^{\leqslant}_0 Q$	Positive equivalence
	$\mid$	$\Gamma \vdash N \leqslant_0 M$	Negative subtyping
	$\mid$	$\Gamma \vdash P \geqslant_0 Q$	Positive supertyping
DT	$::=$		
	$\mid$	$\Gamma ; \Phi \vdash v : P$	Positive type inference
	$\mid$	$\Gamma ; \Phi \vdash c : N$	Negative type inference
	$\mid$	$\Gamma ; \Phi \vdash N \bullet \vec{v} \Rrightarrow M$	Application type inference
EQ	$::=$		
	$\mid$	$N = M$	Negative type equality (alpha-equivalence)
	$\mid$	$P = Q$	Positive type equuality (alphha-equivalence)
	$\mid$	$P = Q$	
LUBF	$::=$		
	$\mid$	$P_1 \vee P_2 === Q$	
	$\mid$	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P === \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N === \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P === \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N === \vec{\alpha}$	
	$\mid$	$\mathbf{nf}\,(N') === N$	
	$\mid$	$\mathbf{nf}\,(P') === P$	
	$\mid$	$\mathbf{nf}\,(N') === N$	
	$\mid$	$\mathbf{nf}\,(P') === P$	
	$\mid$	$\mathbf{nf}\,(\vec{N'}) === \vec{N}$	
	$\mid$	$\mathbf{nf}\,(\vec{P'}) === \vec{P}$	
	$\mid$	$\mathbf{nf}\,(\sigma') === \sigma$	
	$\mid$	$\mathbf{nf}\,(\widehat{\sigma}') === \widehat{\sigma}$	
	$\mid$	$\mathbf{nf}\,(\mu') === \mu$	
	$\mid$	$\sigma'\vert_{\mathit{vars}}$	
	$\mid$	$\widehat{\sigma}'\vert_{\mathit{vars}}$	
	$\mid$	$\widehat{\tau}'\vert_{\mathit{vars}}$	
	$\mid$	$\Xi'\vert_{\mathit{vars}}$	
	$\mid$	$\mathit{SC}\vert_{\mathit{vars}}$	
	$\mid$	$\mathit{UC}\vert_{\mathit{vars}}$	
	$\mid$	$e_1 \ \&\ e_2$	
	$\mid$	$e_1 \ \&\ e_2$	
	$\mid$	$\mathit{UC}_1 \ \&\ \mathit{UC}_2$	
	$\mid$	$\mathit{UC}_1 \cup \mathit{UC}_2$	
	$\mid$	$\mathit{SC}_1 \ \&\ \mathit{SC}_2$	
	$\mid$	$\widehat{\tau}_1 \ \&\ \widehat{\tau}_2$	
	$\mid$	$\mathbf{dom}\,(\mathit{UC}) === \mathit{vars}$	
	$\mid$	$\mathbf{dom}\,(\mathit{SC}) === \mathit{vars}$	

	$\mid$	$\mathbf{dom}\,(\widehat{\sigma}) === \mathit{vars}$	
	$\mid$	$\mathbf{dom}\,(\widehat{\tau}) === \mathit{vars}$	
	$\mid$	$\mathbf{dom}\,(\Theta) === \mathit{vars}$	
	$\mid$	$\lvert SC \rvert === UC$	
LUB	$::=$		
	$\mid$	$\Gamma \models P_1 \vee P_2 = Q$	Least Upper Bound (Least Common Supertype)
	$\mid$	$\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q$	
Nrm	$::=$		
	$\mid$	$\mathbf{nf}\,(N) = M$	
	$\mid$	$\mathbf{nf}\,(P) = Q$	
	$\mid$	$\mathbf{nf}\,(N) = M$	
	$\mid$	$\mathbf{nf}\,(P) = Q$	
$Order$	$::=$		
	$\mid$	$\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}$	
U	$::=$		
	$\mid$	$\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC$	Negative unification
	$\mid$	$\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC$	Positive unification
WF	$::=$		
	$\mid$	$\Gamma \vdash N$	Negative type well-formedness
	$\mid$	$\Gamma \vdash P$	Positive type well-formedness
	$\mid$	$\Gamma \vdash N$	Negative type well-formedness
	$\mid$	$\Gamma \vdash P$	Positive type well-formedness
	$\mid$	$\Gamma \vdash \vec{N}$	Negative type list well-formedness
	$\mid$	$\Gamma \vdash \vec{P}$	Positive type list well-formedness
	$\mid$	$\Gamma; \Theta \vdash N$	Negative unification type well-formedness
	$\mid$	$\Gamma; \Theta \vdash P$	Positive uniification type well-formedness
	$\mid$	$\Gamma; \Xi \vdash N$	Negative anti-unification type well-formedness
	$\mid$	$\Gamma; \Xi \vdash P$	Positive anti-unification type well-formedness
	$\mid$	$\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1$	Antiunification substitution well-formedness
	$\mid$	$\Gamma \vdash^{\supseteq} \Theta$	Unification context well-formedness
	$\mid$	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution well-formedness
	$\mid$	$\Theta \vdash \widehat{\sigma}$	Unification substitution well-formedness
	$\mid$	$\Theta \vdash \widehat{\sigma} : UC$	Unification substitution satisfies unification constraint
	$\mid$	$\Theta \vdash \widehat{\sigma} : SC$	Unification substitution satisfies subtyping constraint
	$\mid$	$\Gamma \vdash e$	Unification constraint entry well-formedness
	$\mid$	$\Gamma \vdash e$	Subtyping constraint entry well-formedness
	$\mid$	$\Gamma \vdash P : e$	Positive type satisfies unification constraint
	$\mid$	$\Gamma \vdash N : e$	Negative type satisfies unification constraint
	$\mid$	$\Gamma \vdash P : e$	Positive type satisfies subtyping constraint
	$\mid$	$\Gamma \vdash N : e$	Negative type satisfies subtyping constraint
	$\mid$	$\Theta \vdash UC$	Unification constraint well-formedness
	$\mid$	$\Theta \vdash SC$	Subtyping constraint well-formedness

$$
\begin{array}{lcl}
judgement & ::= & \\
& | & A \\
& | & AT \\
& | & AU \\
& | & SCM \\
& | & UCM \\
& | & SATSCE \\
& | & SING \\
& | & E1 \\
& | & D1 \\
& | & D0 \\
& | & DT \\
& | & EQ \\
& | & LUB \\
& | & Nrm \\
& | & Order \\
& | & U \\
& | & WF \\
\\
user\_syntax & ::= & \\
& | & \alpha \\
& | & n \\
& | & x \\
& | & n \\
& | & \alpha^{+} \\
& | & \alpha^{-} \\
& | & \alpha^{\pm} \\
& | & \sigma \\
& | & e \\
& | & e \\
& | & UC \\
& | & SC \\
& | & \widehat{\sigma} \\
& | & \widehat{\tau} \\
& | & P \\
& | & N \\
& | & \overrightarrow{\alpha^{+}} \\
& | & \overrightarrow{\alpha^{-}} \\
& | & \overrightarrow{\alpha^{\pm}} \\
& | & P \\
& | & N \\
& | & \vec{P} \\
& | & \vec{N} \\
& | & \Gamma \\
& | & \Theta \\
& | & \Xi \\
& | & \vec{\alpha} \\
& | & vars \\
& | & \mu \\
& | & \widehat{\alpha}^{\pm}
\end{array}
$$

$|\ \widehat{\alpha}^{+}$

$|\ \widehat{\alpha}^{-}$

$|\ \overrightarrow{\widetilde{\alpha}^{+}}$

$|\ \overrightarrow{\widehat{\alpha}^{-}}$

$|\ P$

$|\ N$

$|\ \mathit{auSol}$

$|\ \mathit{terminals}$

$|\ v$

$|\ \vec{v}$

$|\ c$

$|\ \mathit{vctx}$

$|\ \mathit{formula}$

$\boxed{\Gamma;\ \Theta \models N \leqslant M \dashv SC}$ Negative subtyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^{-} \leqslant \alpha^{-} \dashv \cdot}\ \textsc{ANVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(P) \stackrel{u}{\simeq} \mathbf{nf}\,(Q) \dashv UC}{\Gamma;\ \Theta \models \uparrow P \leqslant \uparrow Q \dashv UC}\ \textsc{AShiftU}$$

$$\frac{\Gamma;\ \Theta \models P \geqslant Q \dashv SC_1 \quad \Gamma;\ \Theta \models N \leqslant M \dashv SC_2 \quad \Theta \vdash SC_1 \& SC_2 = SC}{\Gamma;\ \Theta \models P \to N \leqslant Q \to M \dashv SC}\ \textsc{AArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma;\ \Theta \models \forall \overrightarrow{\alpha^{+}}.N \leqslant \forall \overrightarrow{\beta^{+}}.M \dashv SC \backslash \overrightarrow{\widehat{\alpha}^{+}}}\ \textsc{AForall}$$

$\boxed{\Gamma;\ \Theta \models P \geqslant Q \dashv SC}$ Positive supertyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^{+} \geqslant \alpha^{+} \dashv \cdot}\ \textsc{APVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(N) \stackrel{u}{\simeq} \mathbf{nf}\,(M) \dashv UC}{\Gamma;\ \Theta \models \downarrow N \geqslant \downarrow M \dashv UC}\ \textsc{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^{-}};\ \Theta, \overrightarrow{\widehat{\alpha}^{-}}\{\Gamma, \overrightarrow{\beta^{-}}\} \models [\overrightarrow{\widehat{\alpha}^{-}}/\overrightarrow{\alpha^{-}}]P \geqslant Q \dashv SC}{\Gamma;\ \Theta \models \exists \overrightarrow{\alpha^{-}}.P \geqslant \exists \overrightarrow{\beta^{-}}.Q \dashv SC \backslash \overrightarrow{\widehat{\alpha}^{-}}}\ \textsc{AExists}$$

$$\frac{\widehat{\alpha}^{+}\{\Delta\} \in \Theta \quad \mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}{\Gamma;\ \Theta \models \widehat{\alpha}^{+} \geqslant P \dashv (\widehat{\alpha}^{+} :\geqslant Q)}\ \textsc{APUVar}$$

$\boxed{\Gamma; \Phi \models v\colon P}$ Positive type inference

$$\frac{v : P \in \Phi}{\Gamma; \Phi \models v\colon P}\ \textsc{ATVar}$$

$$\frac{\Gamma; \Phi \models c\colon N}{\Gamma; \Phi \models \{c\}\colon \downarrow N}\ \textsc{ATThunk}$$

$$\frac{\Gamma; \Phi \models v\colon P \quad \Gamma;\ \cdot \models Q \geqslant P \dashv \cdot}{\Gamma; \Phi \models (v : Q)\colon Q}\ \textsc{ATPAnnot}$$

$\boxed{\Gamma; \Phi \models c\colon N}$ Negative type inference

$$\frac{\Gamma; \Phi \models c\colon N \quad \Gamma;\ \cdot \models N \leqslant M \dashv \cdot}{\Gamma; \Phi \models (c : M)\colon M}\ \textsc{ATNAnnot}$$

$$\frac{\Gamma;\Phi,x:P\models c\colon N}{\Gamma;\Phi\models\lambda x:P.c\colon P\to N}\ \textsc{ATtLam}$$

$$\frac{\Gamma,\alpha^+;\Phi\models c\colon N}{\Gamma;\Phi\models\Lambda\alpha^+.c\colon\forall\alpha^+.N}\ \textsc{ATTLam}$$

$$\frac{\Gamma;\Phi\models v\colon P}{\Gamma;\Phi\models\mathbf{return}\,v\colon\uparrow P}\ \textsc{ATReturn}$$

$$\frac{\Gamma;\Phi\models v\colon P\quad\Gamma;\Phi,x:P\models c\colon N}{\Gamma;\Phi\models\mathbf{let}\,x=v;c\colon N}\ \textsc{ATVarLet}$$

$$\frac{\begin{array}{l}\Gamma\vdash P\quad\Gamma;\Phi\models v\colon\downarrow M\quad\Gamma;\Phi;\cdot\models M\bullet\vec{v}\Rrightarrow\uparrow Q\dashv\Theta;SC_1\quad\Gamma;\Theta\models\uparrow Q\leqslant\uparrow P\dashv SC_2\\\Theta\vdash SC_1\&SC_2=SC\quad\Gamma;\Phi,x:P\models c\colon N\end{array}}{\Gamma;\Phi\models\mathbf{let}\,x:P=v(\vec{v});c\colon N}\ \textsc{ATAppLetAnn}$$

$\Gamma;\Phi\models v\colon\downarrow M\quad\Gamma;\Phi;\cdot\models M\bullet\vec{v}\Rrightarrow\uparrow Q\dashv\Theta;SC$

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<<no parses (char 57):  uv uQ  dom(SC) SC | uv uQ singular   u** :  SC >>
<<no parses (char 17):  ; , x:[u**]uQ  c :  iN >>
```

$$\overline{\Gamma;\Phi\models\mathbf{let}\,x=v(\vec{v});c\colon N}\ \textsc{ATAppLet}$$

$$\frac{\Gamma;\Phi\models v\colon\exists\alpha^-.P\quad\Gamma,\alpha^-;\Phi,x:P\models c\colon N\quad\Gamma\vdash N}{\Gamma;\Phi\models\mathbf{let}^{\exists}(\alpha^-,x)=v;c\colon N}\ \textsc{ATUnpack}$$

$\boxed{\Gamma;\Phi;\Theta_1\models N\bullet\vec{v}\Rrightarrow M\dashv\Theta_2;SC}$ Application type inference

$$\frac{}{\Gamma;\Phi;\Theta\models N\bullet\cdot\Rrightarrow N\dashv\Theta;\cdot}\ \textsc{ATEmptyApp}$$

$$\frac{\begin{array}{l}\Gamma;\Phi\models v\colon P\quad\Gamma;\Theta\models Q\geqslant P\dashv SC_1\quad\Gamma;\Phi;\Theta\models N\bullet\vec{v}\Rrightarrow M\dashv\Theta';SC_2\\\Theta\vdash SC_1\&SC_2=SC\end{array}}{\Gamma;\Phi;\Theta\models Q\to N\bullet v,\vec{v}\Rrightarrow M\dashv\Theta';SC}\ \textsc{ATArrowApp}$$

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<<multiple parses>>
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$$\frac{\vec{v}\neq\cdot}{\Gamma;\Phi;\Theta\models\forall\overrightarrow{\alpha^+}.N\bullet\vec{v}\Rrightarrow M\dashv\Theta';SC}\ \textsc{ATForallApp}$$

$\boxed{\Gamma\models P_1\overset{a}{\simeq}P_2\dashv(\Xi,Q,\widehat{\tau}_1,\widehat{\tau}_2)}$

$$\frac{}{\Gamma\models\alpha^+\overset{a}{\simeq}\alpha^+\dashv(\cdot,\alpha^+,\cdot,\cdot)}\ \textsc{AUPVar}$$

$$\frac{\Gamma\models N_1\overset{a}{\simeq}N_2\dashv(\Xi,M,\widehat{\tau}_1,\widehat{\tau}_2)}{\Gamma\models\downarrow N_1\overset{a}{\simeq}\downarrow N_2\dashv(\Xi,\downarrow M,\widehat{\tau}_1,\widehat{\tau}_2)}\ \textsc{AUShiftD}$$

$$\frac{\overrightarrow{\alpha^-}\cap\Gamma=\varnothing\quad\Gamma\models P_1\overset{a}{\simeq}P_2\dashv(\Xi,Q,\widehat{\tau}_1,\widehat{\tau}_2)}{\Gamma\models\exists\overrightarrow{\alpha^-}.P_1\overset{a}{\simeq}\exists\overrightarrow{\alpha^-}.P_2\dashv(\Xi,\exists\overrightarrow{\alpha^-}.Q,\widehat{\tau}_1,\widehat{\tau}_2)}\ \textsc{AUExists}$$

$\boxed{\Gamma\models N_1\overset{a}{\simeq}N_2\dashv(\Xi,M,\widehat{\tau}_1,\widehat{\tau}_2)}$

$$\frac{}{\Gamma\models\alpha^-\overset{a}{\simeq}\alpha^-\dashv(\cdot,\alpha^-,\cdot,\cdot)}\ \textsc{AUNVar}$$

$$\frac{\Gamma\models P_1\overset{a}{\simeq}P_2\dashv(\Xi,Q,\widehat{\tau}_1,\widehat{\tau}_2)}{\Gamma\models\uparrow P_1\overset{a}{\simeq}\uparrow P_2\dashv(\Xi,\uparrow Q,\widehat{\tau}_1,\widehat{\tau}_2)}\ \textsc{AUShiftU}$$

$$\frac{\overrightarrow{\alpha^+}\cap\Gamma=\varnothing\quad\Gamma\models N_1\overset{a}{\simeq}N_2\dashv(\Xi,M,\widehat{\tau}_1,\widehat{\tau}_2)}{\Gamma\models\forall\overrightarrow{\alpha^+}.N_1\overset{a}{\simeq}\forall\overrightarrow{\alpha^+}.N_2\dashv(\Xi,\forall\overrightarrow{\alpha^+}.M,\widehat{\tau}_1,\widehat{\tau}_2)}\ \textsc{AUForall}$$

$$\frac{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \widehat{\tau}_1, \widehat{\tau}_2) \quad \Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \widehat{\tau}'_1, \widehat{\tau}'_2)}{\Gamma \vDash P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \widehat{\tau}_1 \cup \widehat{\tau}'_1, \widehat{\tau}_2 \cup \widehat{\tau}'_2)} \quad \text{AUARROW}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \vDash N \overset{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, \widehat{\alpha}^-_{\{N,M\}}, (\widehat{\alpha}^-_{\{N,M\}} :\approx N), (\widehat{\alpha}^-_{\{N,M\}} :\approx M))} \quad \text{AUAU}$$

$\boxed{\Gamma \vdash e_1 \ \& \ e_2 = e_3}$ Subtyping Constraint Entry Merge

$$\frac{\Gamma \vDash P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1) \ \& \ (\widehat{\alpha}^+ :\geqslant P_2) = (\widehat{\alpha}^+ :\geqslant Q)} \quad \text{SCMESUPSUP}$$

$$\frac{\Gamma; \cdot \vDash P \geqslant Q \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \ \& \ (\widehat{\alpha}^+ :\geqslant Q) = (\widehat{\alpha}^+ :\approx P)} \quad \text{SCMEEQSUP}$$

$$\frac{\Gamma; \cdot \vDash Q \geqslant P \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P) \ \& \ (\widehat{\alpha}^+ :\approx Q) = (\widehat{\alpha}^+ :\approx Q)} \quad \text{SCMESUPEQ}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \ \& \ (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \text{SCMEPEQEQ}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \ \& \ (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \text{SCMENEQEQ}$$

$\boxed{\Theta \vdash SC_1 \& SC_2 = SC_3}$ Merge of subtyping constraints

$\boxed{\Gamma \vdash e_1 \& e_2 = e_3}$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \& (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \text{UCMEPEQEQ}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \& (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \text{UCMENEQEQ}$$

$\boxed{\Theta \vdash UC_1 \& UC_2 = UC_3}$

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies with the subtyping constraint entry

$$\frac{\Gamma \vdash P \geqslant_1 Q}{\Gamma \vdash P : (\widehat{\alpha}^+ :\geqslant Q)} \quad \text{SATSCESUP}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash P : (\widehat{\alpha}^+ :\approx Q)} \quad \text{SATSCEPEQ}$$

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies with the subtyping constraint entry

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash N : (\widehat{\alpha}^- :\approx M)} \quad \text{SATSCENEQ}$$

$\boxed{e_1 \ \mathbf{singular}}$ Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^+ :\approx P \ \mathbf{singular}} \quad \text{SINGPEQ}$$

$$\frac{}{\widehat{\alpha}^- :\approx N \ \mathbf{singular}} \quad \text{SINGNEQ}$$

$$\frac{}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.\alpha^+ \ \mathbf{singular}} \quad \text{SINGSUPVAR}$$

$$\frac{N \simeq_1^D \alpha_i^-}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.{\downarrow} N \ \mathbf{singular}} \quad \text{SINGSUPSHIFT}$$

$\boxed{SC\ \textbf{singular}}$ Subtyping Constraint Is Singular

$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-} \quad \text{E1NVar}$$

$$\frac{P \simeq_1^D Q}{\uparrow P \simeq_1^D \uparrow Q} \quad \text{E1ShiftU}$$

$$\frac{P \simeq_1^D Q \quad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M} \quad \text{E1Arrow}$$

$$\frac{\overrightarrow{\alpha^+} \cap \textbf{fv}\, M = \varnothing \quad \mu : (\overrightarrow{\beta^+} \cap \textbf{fv}\, M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \textbf{fv}\, N) \quad N \simeq_1^D [\mu]M}{\forall \overrightarrow{\alpha^+}.N \simeq_1^D \forall \overrightarrow{\beta^+}.M} \quad \text{E1Forall}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+} \quad \text{E1PVar}$$

$$\frac{N \simeq_1^D M}{\downarrow N \simeq_1^D \downarrow M} \quad \text{E1ShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \textbf{fv}\, Q = \varnothing \quad \mu : (\overrightarrow{\beta^-} \cap \textbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \textbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists \overrightarrow{\alpha^-}.P \simeq_1^D \exists \overrightarrow{\beta^-}.Q} \quad \text{E1Exists}$$

$\boxed{P \simeq Q}$

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M} \quad \text{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q} \quad \text{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-} \quad \text{D1NVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant_1 \uparrow Q} \quad \text{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M} \quad \text{D1Arrow}$$

$$\frac{\textbf{fv}\, N \cap \overrightarrow{\beta^+} = \varnothing \quad \Gamma, \overrightarrow{\beta^+} \vdash P_i \quad \Gamma, \overrightarrow{\beta^+} \vdash [\overrightarrow{P}/\overrightarrow{\alpha^+}]N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant_1 \forall \overrightarrow{\beta^+}.M} \quad \text{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+} \quad \text{D1PVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M} \quad \text{D1ShiftD}$$

$$\frac{\mathbf{fv}\,P \cap \overrightarrow{\beta^-} = \varnothing \quad \Gamma, \overrightarrow{\beta^-} \vdash N_i \quad \Gamma, \overrightarrow{\beta^-} \vdash [\overrightarrow{N}/\overrightarrow{\alpha^-}]P \geqslant_1 Q}{\Gamma \vdash \exists\overrightarrow{\alpha^-}.P \geqslant_1 \exists\overrightarrow{\beta^-}.Q} \quad \text{D1Exists}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1}$ Equivalence of substitutions

$\boxed{\Gamma \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \mathit{vars}}$ Equivalence of substitutions

$\boxed{\Theta \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : \mathit{vars}}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash N \simeq_0^{\leqslant} M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq_0^{\leqslant} M} \quad \text{D0NDef}$$

$\boxed{\Gamma \vdash P \simeq_0^{\leqslant} Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq_0^{\leqslant} Q} \quad \text{D0PDef}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_0 \alpha^-} \quad \text{D0NVar}$$

$$\frac{\Gamma \vdash P \simeq_0^{\leqslant} Q}{\Gamma \vdash \uparrow P \leqslant_0 \uparrow Q} \quad \text{D0ShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/\alpha^+]N \leqslant_0 M \quad M \neq \forall\beta^+.M'}{\Gamma \vdash \forall\alpha^+.N \leqslant_0 M} \quad \text{D0ForallL}$$

$$\frac{\Gamma, \alpha^+ \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall\alpha^+.M} \quad \text{D0ForallR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \to N \leqslant_0 Q \to M} \quad \text{D0Arrow}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_0 \alpha^+} \quad \text{D0PVar}$$

$$\frac{\Gamma \vdash N \simeq_0^{\leqslant} M}{\Gamma \vdash \downarrow N \geqslant_0 \downarrow M} \quad \text{D0ShiftD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/\alpha^-]P \geqslant_0 Q \quad Q \neq \exists\alpha^-.Q'}{\Gamma \vdash \exists\alpha^-.P \geqslant_0 Q} \quad \text{D0ExistsL}$$

$$\frac{\Gamma, \alpha^- \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists\alpha^-.Q} \quad \text{D0ExistsR}$$

$\boxed{\Gamma; \Phi \vdash v \colon P}$ Positive type inference

$$\frac{v : P \in \Phi}{\Gamma; \Phi \vdash v \colon P} \quad \text{DTVar}$$

$$\frac{\Gamma; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \{c\} \colon \downarrow N} \quad \text{DTThunk}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma; \Phi \vdash (v : Q) \colon Q} \quad \text{DTPAnnot}$$

$\boxed{\Gamma; \Phi \vdash c \colon N}$ Negative type inference

$$\frac{\Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \lambda x : P.c \colon P \to N} \quad \textsc{DTtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \Lambda\alpha^+.c \colon \forall\alpha^+.N} \quad \textsc{DTTLam}$$

$$\frac{\Gamma; \Phi \vdash v \colon P}{\Gamma; \Phi \vdash \mathbf{return}\, v \colon \uparrow P} \quad \textsc{DTReturn}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v; c \colon N} \quad \textsc{DTVarLet}$$

$$\frac{\Gamma; \Phi \vdash v \colon \downarrow M \quad \Gamma; \Phi \vdash M \bullet \vec{v} \Rightarrow\!\!> \uparrow Q \text{ uniquely} \quad \Gamma; \Phi, x : Q \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v(\vec{v}); c \colon N} \quad \textsc{DTAppLet}$$

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi \vdash v \colon \downarrow M \quad \Gamma; \Phi \vdash M \bullet \vec{v} \Rightarrow\!\!> \uparrow Q \quad \Gamma \vdash \uparrow Q \leqslant_1 \uparrow P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x : P = v(\vec{v}); c \colon N} \quad \textsc{DTAppLetAnn}$$

$$\frac{\Gamma; \Phi \vdash v \colon \exists\alpha^-.P \quad \Gamma, \alpha^-; \Phi, x : P \vdash c \colon N \quad \Gamma \vdash N}{\Gamma; \Phi \vdash \mathbf{let}^{\exists}(\alpha^-, x) = v; c \colon N} \quad \textsc{DTUnpack}$$

$$\frac{\Gamma; \Phi \vdash c \colon N \quad \Gamma \vdash N \leqslant_1 M}{\Gamma; \Phi \vdash (c : M) \colon M} \quad \textsc{DTNAnnot}$$

$\boxed{\Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!> M}$ Application type inference

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash N \bullet \cdot \Rightarrow\!\!> N'} \quad \textsc{DTEmptyApp}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P \quad \Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!> M}{\Gamma; \Phi \vdash Q \to N \bullet v, \vec{v} \Rightarrow\!\!> M} \quad \textsc{DTArrowApp}$$

$$\frac{\begin{array}{l}\Gamma \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma; \Phi \vdash [\sigma]N \bullet \vec{v} \Rightarrow\!\!> M \\ \vec{v} \neq \cdot\end{array}}{\Gamma; \Phi \vdash \forall\overrightarrow{\alpha^+}.N \bullet \vec{v} \Rightarrow\!\!> M} \quad \textsc{DTForallApp}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$

$\boxed{P_1 \vee P_2}$

$\boxed{\mathbf{ord}\ \mathit{vars}\, \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ \mathit{vars}\, \mathbf{in}\ N}$

$\boxed{\mathbf{ord}\ \mathit{vars}\, \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ \mathit{vars}\, \mathbf{in}\ N}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{N}')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{P}')}$

$\boxed{\mathbf{nf}\,(\sigma')}$

$\boxed{\mathbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\mathbf{nf}\,(\mu')}$

$\boxed{\sigma'|_{vars}}$

$\boxed{\widehat{\sigma}'|_{vars}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{\mathbf{SC}|_{vars}}$

$\boxed{\mathbf{UC}|_{vars}}$

$\boxed{e_1 \mathbin{\&} e_2}$

$\boxed{e_1 \mathbin{\&} e_2}$

$\boxed{UC_1 \mathbin{\&} UC_2}$

$\boxed{UC_1 \cup UC_2}$

$\boxed{SC_1 \mathbin{\&} SC_2}$

$\boxed{\widehat{\tau}_1 \mathbin{\&} \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(UC)}$

$\boxed{\mathbf{dom}\,(SC)}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{|SC|}$

$\boxed{\Gamma \vDash P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \vDash \alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBV}{\small \text{AR}}$$

$$\frac{\Gamma, \cdot \vDash \mathbf{nf}\,(\downarrow N) \overset{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}. [\overrightarrow{\alpha^-}/\Xi]\, P} \quad \text{LUBS}{\small \text{HIFT}}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \vDash P_1 \vee P_2 = Q}{\Gamma \vDash \exists \overrightarrow{\alpha^-}. P_1 \vee \exists \overrightarrow{\beta^-}. P_2 = Q} \quad \text{LUBE}{\small \text{XISTS}}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}$

$$\frac{\Gamma = \Delta, \overrightarrow{\alpha^{\pm}} \quad \overrightarrow{\beta^{\pm}}\ \mathbf{is\ fresh} \quad \overrightarrow{\gamma^{\pm}}\ \mathbf{is\ fresh} \\ \Delta, \overrightarrow{\beta^{\pm}}, \overrightarrow{\gamma^{\pm}} \models [\overrightarrow{\beta^{\pm}}/\overrightarrow{\alpha^{\pm}}]P \vee [\overrightarrow{\gamma^{\pm}}/\overrightarrow{\alpha^{\pm}}]P = Q}{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}\ \textsc{LUBUpgrade}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^{-}) = \alpha^{-}}\ \textsc{NrmNVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\uparrow P) = \uparrow Q}\ \textsc{NrmShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(P \to N) = Q \to M}\ \textsc{NrmArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\forall \overrightarrow{\alpha^{+}}.N) = \forall \overrightarrow{\alpha^{+}}'.N'}\ \textsc{NrmForall}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\alpha^{+}) = \alpha^{+}}\ \textsc{NrmPVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\downarrow N) = \downarrow M}\ \textsc{NrmShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\exists \overrightarrow{\alpha^{-}}.P) = \exists \overrightarrow{\alpha^{-}}'.P'}\ \textsc{NrmExists}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^{-}) = \widehat{\alpha}^{-}}\ \textsc{NrmNUVar}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^{+}) = \widehat{\alpha}^{+}}\ \textsc{NrmPUVar}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}$

$$\frac{\alpha^{-} \in \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^{-} = \alpha^{-}}\ \textsc{ONVarIn}$$

$$\frac{\alpha^{-} \notin \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^{-} = \cdot}\ \textsc{ONVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\uparrow P = \vec{\alpha}}\ \textsc{OShiftU}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}_1 \quad \mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}_2}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)}\ \textsc{OArrow}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^{+}} = \varnothing \quad \mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\forall \overrightarrow{\alpha^{+}}.N = \vec{\alpha}}\ \textsc{OForall}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}$

$$\frac{\alpha^+ \in vars}{\mathbf{ord}\ vars\ \mathbf{in}\ \alpha^+ = \alpha^+}\ \textsc{OPVarIn}$$

$$\frac{\alpha^+ \notin vars}{\mathbf{ord}\ vars\ \mathbf{in}\ \alpha^+ = \cdot}\ \textsc{OPVarNIn}$$

$$\frac{\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}}{\mathbf{ord}\ vars\ \mathbf{in}\ {\downarrow}N = \vec{\alpha}}\ \textsc{OShiftD}$$

$$\frac{vars \cap \overrightarrow{\alpha^-} = \varnothing \quad \mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}}{\mathbf{ord}\ vars\ \mathbf{in}\ \exists\overrightarrow{\alpha^-}.P = \vec{\alpha}}\ \textsc{OExists}$$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ vars\ \mathbf{in}\ \widehat{\alpha}^- = \cdot}\ \textsc{ONUVar}$$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\ vars\ \mathbf{in}\ \widehat{\alpha}^+ = \cdot}\ \textsc{OPUVar}$$

$\boxed{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC}$ Negative unification

$$\frac{}{\Gamma; \Theta \models \alpha^- \overset{u}{\simeq} \alpha^- \dashv \cdot}\ \textsc{UNVar}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models {\uparrow}P \overset{u}{\simeq} {\uparrow}Q \dashv UC}\ \textsc{UShiftU}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC_1 \quad \Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC_2}{\Gamma; \Theta \models P \to N \overset{u}{\simeq} Q \to M \dashv UC_1\ \&\ UC_2}\ \textsc{UArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \forall\overrightarrow{\alpha^+}.N \overset{u}{\simeq} \forall\overrightarrow{\alpha^+}.M \dashv UC}\ \textsc{UForall}$$

$$\frac{\widehat{\alpha}^-\{\Delta\} \in \Theta \quad \Delta \vdash N}{\Gamma; \Theta \models \widehat{\alpha}^- \overset{u}{\simeq} N \dashv (\widehat{\alpha}^- :\approx N)}\ \textsc{UNUVar}$$

$\boxed{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC}$ Positive unification

$$\frac{}{\Gamma; \Theta \models \alpha^+ \overset{u}{\simeq} \alpha^+ \dashv \cdot}\ \textsc{UPVar}$$

$$\frac{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models {\downarrow}N \overset{u}{\simeq} {\downarrow}M \dashv UC}\ \textsc{UShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \exists\overrightarrow{\alpha^-}.P \overset{u}{\simeq} \exists\overrightarrow{\alpha^-}.Q \dashv UC}\ \textsc{UExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \Delta \vdash P}{\Gamma; \Theta \models \widehat{\alpha}^+ \overset{u}{\simeq} P \dashv (\widehat{\alpha}^+ :\approx P)}\ \textsc{UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness
$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness
$\boxed{\Gamma ; \Theta \vdash N}$ Negative unification type well-formedness
$\boxed{\Gamma ; \Theta \vdash P}$ Positive unifiication type well-formedness
$\boxed{\Gamma ; \Xi \vdash N}$ Negative anti-unification type well-formedness
$\boxed{\Gamma ; \Xi \vdash P}$ Positive anti-unification type well-formedness
$\boxed{\Gamma ; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness
$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness
$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution well-formedness
$\boxed{\Theta \vdash \widehat{\sigma}}$ Unification substitution well-formedness
$\boxed{\Theta \vdash \widehat{\sigma} : UC}$ Unification substitution satisfies unification constraint
$\boxed{\Theta \vdash \widehat{\sigma} : SC}$ Unification substitution satisfies subtyping constraint
$\boxed{\Gamma \vdash e}$ Unification constraint entry well-formedness
$\boxed{\Gamma \vdash e}$ Subtyping constraint entry well-formedness
$\boxed{\Gamma \vdash P : e}$ Positive type satisfies unification constraint
$\boxed{\Gamma \vdash N : e}$ Negative type satisfies unification constraint
$\boxed{\Gamma \vdash P : e}$ Positive type satisfies subtyping constraint
$\boxed{\Gamma \vdash N : e}$ Negative type satisfies subtyping constraint
$\boxed{\Theta \vdash UC}$ Unification constraint well-formedness
$\boxed{\Theta \vdash SC}$ Subtyping constraint well-formedness

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Definition rules:        101 good    17 bad
Definition rule clauses: 202 good    18 bad
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