α, β, α, β, γ, δ	type variables
n, m, i, j	index variables
x, y, z	term variables

$n,\ m$	$::=$			cohort index
	$\|$	0		
	$\|$	$n+1$		
$\alpha^+,\ \beta^+,\ \gamma^+,\ \delta^+$	$::=$			positive variable
	$\|$	α^+		
	$\|$	α^{+n}		
	$\|$	$\beta^\pm$		
$\alpha^-,\ \beta^-,\ \gamma^-,\ \delta^-$	$::=$			negative variable
	$\|$	α^-		
	$\|$	α^{-n}		
	$\|$	$\alpha^\pm$		
$\alpha^\pm,\ \beta^\pm$	$::=$			positive or negative variable
	$\|$	$\alpha^\pm$		
	$\|$	$\alpha^{\pm n}$		
σ	$::=$			substitution
	$\|$	$\cdot$		
	$\|$	id		
	$\|$	$\mathbf{P}\,/\alpha^+$		
	$\|$	$\mathbf{N}\,/\alpha^-$		
	$\|$	$\vec{P}/\overrightarrow{\alpha^+}$		
	$\|$	$\vec{N}/\overrightarrow{\alpha^-}$		
	$\|$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha}^-}$		
	$\|$	μ		
	$\|$	$\sigma_1 \circ \sigma_2$		
	$\|$	$\sigma_1 \lll \sigma_2$		
	$\|$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\|$	(σ)	S	
	$\|$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\|$	$\mathbf{nf}\,(\sigma')$	M	
	$\|$	$\sigma'\|_{vars}$	M	
e	$::=$			entry of a subtyping constraint
	$\|$	$\widehat{\alpha}^+ :\approx P$		
	$\|$	$\widehat{\alpha}^- :\approx N$		
	$\|$	$\widehat{\alpha}^+ :\geqslant P$		
	$\|$	(e)	S	
	$\|$	$SC(\widehat{\alpha}^-)$	M	
	$\|$	$e_1\ \&\ e_2$	M	
e	$::=$			entry of a unification constraint
	$\|$	$\widehat{\alpha}^+ :\approx P$		
	$\|$	$\widehat{\alpha}^- :\approx N$		
	$\|$	(e)	S	
	$\|$	$e_1\ \&\ e_2$	M	

UC	::=			unification constraint
	|	$\cdot$		
	|	e		
	|	$UC \backslash vars$		
	|	$UC \vert vars$		
	|	$UC_1 \cup UC_2$		
	|	$\overline{UC_i}^{\,i}$		concatenate
	|	(UC)	S	
	|	$UC' \vert_{vars}$	M	
	|	$UC_1 \mathbin{\&} UC_2$	M	
	|	$UC_1 \cup UC_2$	M	
	|	$\vert SC \vert$	M	
SC	::=			subtyping constraint
	|	$\cdot$		
	|	e		
	|	$SC \backslash vars$		
	|	$SC \vert vars$		
	|	$SC_1 \cup SC_2$		
	|	UC		
	|	$\overline{SC_i}^{\,i}$		concatenate
	|	(SC)	S	
	|	$SC' \vert_{vars}$	M	
	|	$SC_1 \mathbin{\&} SC_2$	M	
$\widehat{\sigma}$	::=			unification substitution
	|	$\cdot$		
	|	$P / \widehat{\alpha}^{+}$		
	|	$N / \widehat{\alpha}^{-}$		
	|	$\overrightarrow{P} / \overrightarrow{\widehat{\alpha}^{+}}$		
	|	$\overrightarrow{N} / \overrightarrow{\widehat{\alpha}^{-}}$		
	|	$(\widehat{\sigma})$	S	
	|	$\widehat{\sigma}_1 \circ \widehat{\sigma}_2$		
	|	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	|	$\mathbf{nf}\,(\widehat{\sigma}')$	M	
	|	$\widehat{\sigma}' \vert_{vars}$	M	
$\widehat{\tau},\ \widehat{\rho}$	::=			anti-unification substitution
	|	$\cdot$		
	|	$\widehat{\alpha}^{-} :\approx N$		
	|	$\widehat{\alpha}^{-} :\approx N$		
	|	$\overrightarrow{\alpha^{-}} / \overrightarrow{\widehat{\alpha}^{-}}$		
	|	$\overrightarrow{N} / \overrightarrow{\widehat{\alpha}^{-}}$		
	|	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
	|	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	|	$(\widehat{\tau})$	S	
	|	$\widehat{\tau}' \vert_{vars}$	M	
	|	$\widehat{\tau}_1 \mathbin{\&} \widehat{\tau}_2$	M	
$\overrightarrow{\alpha^{+}},\ \overrightarrow{\beta^{+}},\ \overrightarrow{\gamma^{+}},\ \overrightarrow{\delta^{+}}$	::=			positive variable list

	$\mid$	$\cdot$		empty list
	$\mid$	α^+		a variable
	$\mid$	$\overrightarrow{\alpha^+}$		a variable
	$\mid$	$\overline{\overrightarrow{\alpha^+}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$	$::=$			negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	α^-		a variable
	$\mid$	$\overrightarrow{\alpha^-}$		variables
	$\mid$	$\overline{\overrightarrow{\alpha^-}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$	$::=$			positive or negative variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\alpha^\pm$		a variable
	$\mid$	$\overrightarrow{\mathbf{pa}}$		variables
	$\mid$	$\overline{\overrightarrow{\alpha^\pm}_i}^{\,i}$		concatenate lists
$P,\ Q,\ R$	$::=$			multi-quantified positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^-}.P$		
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\widehat{\tau}]\,P$	M	
	$\mid$	$[\widehat{\sigma}]\,P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	(P)	S	
	$\mid$	$P_1 \vee P_2$	M	
	$\mid$	$\mathbf{nf}\,(P')$	M	
$N,\ M,\ K$	$::=$			multi-quantified negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$\forall\overrightarrow{\alpha^+}.N$		
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\widehat{\tau}]\,N$	M	
	$\mid$	$[\mu]N$	M	
	$\mid$	$[\widehat{\sigma}]\,N$	M	
	$\mid$	(N)	S	
	$\mid$	$\mathbf{nf}\,(N')$	M	
$\overrightarrow{P},\ \overrightarrow{Q}$	$::=$			list of positive types
	$\mid$	$\cdot$		empty list
	$\mid$	P		a singel type
	$\mid$	$[\sigma]\overrightarrow{P}$	M	
	$\mid$	$\overline{\overrightarrow{P}_i}^{\,i}$		concatenate lists
	$\mid$	$(\overrightarrow{P})$	S	
	$\mid$	$\mathbf{nf}\,(\overrightarrow{P}')$	M	

$\overrightarrow{N},\ \overrightarrow{M}$	$::=$			list of negative types
	$\mid$	$\cdot$		empty list
	$\mid$	N		a singel type
	$\mid$	$[\sigma]\overrightarrow{N}$	M	
	$\mid$	$\overline{\overrightarrow{N}_i}^{\,i}$		concatenate lists
	$\mid$	$(\overrightarrow{N})$	S	
	$\mid$	$\mathbf{nf}\,(\overrightarrow{N}')$	M	
$\Delta,\ \Gamma$	$::=$			declarative type context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\alpha^+}$		list of variables
	$\mid$	$\overrightarrow{\alpha^-}$		list of variables
	$\mid$	$\overrightarrow{\alpha^\pm}$		list of variables
	$\mid$	$vars$		
	$\mid$	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	$\mid$	(Γ)	S	
	$\mid$	$\Theta(\widehat{\alpha}^+)$	M	
	$\mid$	$\Theta(\widehat{\alpha}^-)$	M	
	$\mid$	$\Gamma_1 \cup \Gamma_2$	M	
Θ	$::=$			algorithmic variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\alpha}\{\Delta\}$		from an ordered list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^+\{\Delta\}}$		from a variable to a list
	$\mid$	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	$\mid$	(Θ)	S	
	$\mid$	$\Theta\vert_{vars}$		leave only those variables that are in the set
	$\mid$	$\Theta_1 \cup \Theta_2$		
Ξ	$::=$			anti-unification type variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\widehat{\alpha}^+}$		list of positive variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^-}$		list of negative variables
	$\mid$	$\mathbf{uv}\ N$		unification variables
	$\mid$	$\mathbf{uv}\ P$		unification variables
	$\mid$	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	$\mid$	(Ξ)	S	
	$\mid$	$\Xi_1 \cup \Xi_2$		
	$\mid$	$\Xi_1 \cap \Xi_2$		
	$\mid$	$\Xi'\vert_{vars}$	M	
	$\mid$	$\mathbf{dom}\,(UC)$	M	
	$\mid$	$\mathbf{dom}\,(SC)$	M	
	$\mid$	$\mathbf{dom}\,(\widehat{\sigma})$	M	
	$\mid$	$\mathbf{dom}\,(\widehat{\tau})$	M	
	$\mid$	$\mathbf{dom}\,(\Theta)$	M	
$\overrightarrow{\alpha},\ \overrightarrow{\beta}$	$::=$			ordered positive or negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	$\overrightarrow{\alpha^+}$		list of variables
	$\mid$	$\overrightarrow{\alpha^-}$		list of variables

	$\vert$	$\overrightarrow{\alpha^{\pm}}$		list of variables
	$\vert$	$\overrightarrow{\widehat{\alpha}^{+}}$		list of variables
	$\vert$	$\overrightarrow{\widehat{\alpha}^{-}}$		list of variables
	$\vert$	$\vec{\alpha}_1 \backslash vars$		setminus
	$\vert$	Γ		context
	$\vert$	$vars$		
	$\vert$	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	$\vert$	$(\vec{\alpha})$	S	parenthesis
	$\vert$	$[\mu]\vec{\alpha}$	M	apply moving to list
	$\vert$	$[\vec{\mu}]\vec{\alpha}$	M	apply umoving to list
	$\vert$	$\mathbf{ord}\ vars\ \mathbf{in}\ P$	M	
	$\vert$	$\mathbf{ord}\ vars\ \mathbf{in}\ N$	M	
	$\vert$	$\mathbf{ord}\ vars\ \mathbf{in}\ P$	M	
	$\vert$	$\mathbf{ord}\ vars\ \mathbf{in}\ N$	M	
$vars$	$::=$			set of variables
	$\vert$	$\varnothing$		empty set
	$\vert$	$\mathbf{fv}\ P$		free variables
	$\vert$	$\mathbf{fv}\ N$		free variables
	$\vert$	$\mathbf{fv}\ \mathbf{imP}$		free variables
	$\vert$	$\mathbf{fv}\ \mathbf{imN}$		free variables
	$\vert$	$vars_1 \cap vars_2$		set intersection
	$\vert$	$vars_1 \cup vars_2$		set union
	$\vert$	$vars_1 \backslash vars_2$		set complement
	$\vert$	$\mathbf{mv}\ \mathbf{imP}$		movable variables
	$\vert$	$\mathbf{mv}\ \mathbf{imN}$		movable variables
	$\vert$	$\mathbf{fv}\ N$		free variables
	$\vert$	$\mathbf{fv}\ P$		free variables
	$\vert$	$(vars)$	S	parenthesis
	$\vert$	$\vec{\alpha}$		ordered list of variables
	$\vert$	$[\mu]vars$	M	apply moving to varset
	$\vert$	Ξ		anti-unification context
μ	$::=$			
	$\vert$	$\cdot$		empty moving
	$\vert$	$pma1 \mapsto pma2$		Positive unit substitution
	$\vert$	$nma1 \mapsto nma2$		Positive unit substitution
	$\vert$	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	$\vert$	$\mu_1 \circ \mu_2$	M	Composition
	$\vert$	$\overline{\mu_i}^{\,i}$		concatenate movings
	$\vert$	$\mu\vert_{vars}$	M	restriction on a set
	$\vert$	μ^{-1}	M	inversion
	$\vert$	$\mathbf{nf}\ (\mu')$	M	
$\vec{\mu}$	$::=$			
	$\vert$	$\cdot$		empty moving
	$\vert$	$\overrightarrow{\widehat{\alpha}^{+}}/\overrightarrow{\alpha^{+}}$		
	$\vert$	$\overrightarrow{\widehat{\alpha}^{-}}/\overrightarrow{\alpha^{-}}$		
$\widehat{\alpha}^{\pm}$	$::=$			positive/negative unification variable

	$\|$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	$::=$			positive unification variable
	$\|$	$\widehat{\alpha}^{+}$		
	$\|$	$\widehat{\alpha}^{+}\{\Delta\}$		
	$\|$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	$::=$			negative unification variable
	$\|$	$\widehat{\alpha}^{-}$		
	$\|$	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	$\|$	$\widehat{\alpha}^{-}\{\Delta\}$		
	$\|$	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widehat{\alpha}^{+}},\ \overrightarrow{\widehat{\beta}^{+}}$	$::=$			positive unification variable list
	$\|$	$\cdot$		empty list
	$\|$	$\widehat{\alpha}^{+}$		a variable
	$\|$	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	$\|$	$\overline{\overrightarrow{\widehat{\alpha}^{+}}_{i}}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}},\ \overrightarrow{\widehat{\beta}^{-}}$	$::=$			negative unification variable list
	$\|$	$\cdot$		empty list
	$\|$	$\widehat{\alpha}^{-}$		a variable
	$\|$	Ξ		from an antiunification context
	$\|$	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	$\|$	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	$\|$	$\overline{\overrightarrow{\widehat{\alpha}^{-}}_{i}}^{\,i}$		concatenate lists
$P,\ Q$	$::=$			a positive algorithmic type (potentially with metavariables)
	$\|$	$\widehat{\alpha}^{+}$		
	$\|$	α^{+}		
	$\|$	$\downarrow N$		
	$\|$	$\exists\overrightarrow{\alpha^{-}}.P$		
	$\|$	$[\sigma]P$	M	
	$\|$	$[\widehat{\tau}]P$	M	
	$\|$	$[\mu]P$	M	
	$\|$	$[\widehat{\sigma}]P$	M	
	$\|$	$[\vec{\mu}]P$	M	
	$\|$	(P)	S	
	$\|$	$\mathbf{nf}\,(P')$	M	
$N,\ M$	$::=$			a negative algorithmic type (potentially with metavariables)
	$\|$	$\widehat{\alpha}^{-}$		
	$\|$	α^{-}		
	$\|$	$\uparrow P$		
	$\|$	$P \rightarrow N$		
	$\|$	$\forall\overrightarrow{\alpha^{+}}.N$		
	$\|$	$[\sigma]N$	M	
	$\|$	$[\widehat{\tau}]N$	M	
	$\|$	$[\mu]N$	M	

	$\mid$	$[\widehat{\sigma}]N$	M
	$\mid$	$[\vec{\mu}]N$	M
	$\mid$	(N)	S
	$\mid$	$\mathbf{nf}\,(N')$	M
$auSol$	$::=$		
	$\mid$	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	$\mid$	$(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$	
$terminals$	$::=$		
	$\mid$	$\exists$	
	$\mid$	$\forall$	
	$\mid$	$\uparrow$	
	$\mid$	$\downarrow$	
	$\mid$	$\rightarrow$	
	$\mid$	$\leftrightarrow$	
	$\mid$	$\in$	
	$\mid$	$\notin$	
	$\mid$	$\cdot$	
	$\mid$	$\vdash$	
	$\mid$	$\leqslant$	
	$\mid$	$\geqslant$	
	$\mid$	$\simeq$	
	$\mid$	$\cup$	
	$\mid$	$\cap$	
	$\mid$	$\setminus$	
	$\mid$	$\subseteq$	
	$\mid$	$\mapsto$	
	$\mid$	$\stackrel{u}{\simeq}$	
	$\mid$	$\stackrel{a}{\simeq}$	
	$\mid$	$\varnothing$	
	$\mid$	$\circ$	
	$\mid$	$\Rightarrow$	
	$\mid$	$\vDash$	
	$\mid$	$\Dashv$	
	$\mid$	$\neq$	
	$\mid$	$\equiv_n$	
	$\mid$	$\vee$	
	$\mid$	$\Downarrow$	
	$\mid$	$:\geqslant$	
	$\mid$	$:\simeq$	
	$\mid$	Λ	
	$\mid$	λ	
	$\mid$	$\mathbf{let}^{\exists}$	
	$\mid$	$\bullet$	
	$\mid$	$\Rightarrow\!\!\!>$	
	$\mid$	$<\!\!\!\Leftarrow$	
$v,\ w$	$::=$		value terms
	$\mid$	x	

	|	$\{c\}$	
	|	$(v : P)$	
	|	(v)	M
$\overrightarrow{v}$	::=		list of arguments
	|	$\cdot$	
	|	v	
	|	$\overline{\overrightarrow{v}_i}^{\,i}$	concatenate
$c,\ d$	::=		computation terms
	|	$(c : N)$	
	|	$\lambda x : P.c$	
	|	$\Lambda \alpha^+.c$	
	|	$\mathbf{return}\, v$	
	|	$\mathbf{let}\, x = v; c$	
	|	$\mathbf{let}\, x : P = v(\overrightarrow{v}); c$	
	|	$\mathbf{let}\, x = v(\overrightarrow{v}); c$	
	|	$\mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c$	
$vctx,\ \Phi$	::=		variable context
	|	$\cdot$	
	|	$x : P$	
	|	$\overline{\Phi_i}^{\,i}$	concatenate contexts
$formula$	::=		
	|	$judgement$	
	|	$judgement$ unique	
	|	$formula_1 \quad .. \quad formula_n$	
	|	$\mu : vars_1 \leftrightarrow vars_2$	
	|	$\mu\, \mathbf{is\, bijective}$	
	|	$x : P \in \Phi$	
	|	$UC_1 \subseteq UC_2$	
	|	$UC_1 = UC_2$	
	|	$SC_1 \subseteq SC_2$	
	|	$e \in SC$	
	|	$e \in UC$	
	|	$vars_1 \subseteq vars_2$	
	|	$vars_1 \subseteq vars_2 \subseteq vars_3$	
	|	$vars_1 = vars_2$	
	|	$vars\, \mathbf{is\, fresh}$	
	|	$\alpha^- \notin vars$	
	|	$\alpha^+ \notin vars$	
	|	$\alpha^- \in vars$	
	|	$\alpha^+ \in vars$	
	|	$\widehat{\alpha}^+ \in vars$	
	|	$\widehat{\alpha}^- \in vars$	
	|	$\widehat{\alpha}^- \in \Theta$	
	|	$\widehat{\alpha}^+ \in \Theta$	
	|	$\widehat{\alpha}^- \notin vars$	
	|	$\widehat{\alpha}^+ \notin vars$	

	|	$\widehat{\alpha}^- \notin \Theta$	
	|	$\widehat{\alpha}^+ \notin \Theta$	
	|	$\widehat{\alpha}^- \in \Xi$	
	|	$\widehat{\alpha}^- \notin \Xi$	
	|	$\widehat{\alpha}^+ \in \Xi$	
	|	$\widehat{\alpha}^+ \notin \Xi$	
	|	if any other rule is not applicable	
	|	$\vec{\alpha}_1 = \vec{\alpha}_2$	
	|	$e_1 = e_2$	
	|	$e_1 = e_2$	
	|	$\widehat{\sigma}_1 = \widehat{\sigma}_2$	
	|	$N = M$	
	|	$\Theta \subseteq \Theta'$	
	|	$\vec{v}_1 = \vec{v}_2$	
	|	$\mathbf{N} \neq \mathbf{M}$	
	|	$\mathbf{P} \neq \mathbf{Q}$	
	|	$N \neq M$	
	|	$P \neq Q$	
	|	$P \neq Q$	
	|	$N \neq M$	
	|	$\vec{v}_1 \neq \vec{v}_2$	
	|	$\overrightarrow{\alpha^+}_1 \neq \overrightarrow{\alpha^+}_2$	
	|	$\lvert\overrightarrow{\alpha^-}\rvert + \lvert\overrightarrow{\beta^-}\rvert > 0$	
	|	$\lvert\overrightarrow{\alpha^+}\rvert + \lvert\overrightarrow{\beta^+}\rvert > 0$	
A	::=		
	|	$\Gamma; \Theta \models N \leqslant M \dashv SC$	Negative subtyping
	|	$\Gamma; \Theta \models P \geqslant Q \dashv SC$	Positive supertyping
AT	::=		
	|	$\Gamma; \Phi \models v : P$	Positive type inference
	|	$\Gamma; \Phi \models c : N$	Negative type inference
	|	$\Gamma; \Phi; \Theta_1 \models N \bullet \vec{v} \Longrightarrow\!\!\!> M \dashv \Theta_2; SC$	Application type inference
AU	::=		
	|	$\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	|	$\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$	
SCM	::=		
	|	$\Gamma \vdash e_1 \mathbin{\&} e_2 = e_3$	Subtyping Constraint Entry Merge
	|	$\Theta \vdash SC_1 \& SC_2 = SC_3$	Merge of subtyping constraints
UCM	::=		
	|	$\Gamma \vdash e_1 \& e_2 = e_3$	
	|	$\Theta \vdash UC_1 \& UC_2 = UC_3$	Merge of unification constraints
$SATSCE$	::=		
	|	$\Gamma \vdash P : e$	Positive type satisfies with the subtyping constr
	|	$\Gamma \vdash N : e$	Negative type satisfies with the subtyping const

$SING$	$::=$		
	$\mid$	e_1 **singular with** P	Positive Subtyping Constraint Entry Is Singular
	$\mid$	e_1 **singular with** N	Negative Subtyping Constraint Entry Is Singular
	$\mid$	SC **singular with** $\widehat{\sigma}$	Subtyping Constraint Is Singular
$E1$	$::=$		
	$\mid$	$N \simeq_1^{D} M$	Negative multi-quantified type equivalence
	$\mid$	$P \simeq_1^{D} Q$	Positive multi-quantified type equivalence
	$\mid$	$\boxed{P} \simeq_1^{D} \boxed{Q}$	Positive unification type equivalence
	$\mid$	$\boxed{N} \simeq_1^{D} \boxed{M}$	Positive unification type equivalence
$D1$	$::=$		
	$\mid$	$\Gamma \vdash N \simeq_1^{\leqslant} M$	Negative equivalence on MQ types
	$\mid$	$\Gamma \vdash P \simeq_1^{\leqslant} Q$	Positive equivalence on MQ types
	$\mid$	$\Gamma \vdash N \leqslant_1 M$	Negative subtyping
	$\mid$	$\Gamma \vdash P \geqslant_1 Q$	Positive supertyping
	$\mid$	$\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1$	Equivalence of substitutions
	$\mid$	$\Gamma \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : vars$	Equivalence of substitutions
	$\mid$	$\Theta \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions
	$\mid$	$\Gamma \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions
	$\mid$	$\Gamma \vdash \Phi_1 \simeq_1^{\leqslant} \Phi_2$	Equivalence of contexts
DT	$::=$		
	$\mid$	$\Gamma; \Phi \vdash v \colon P$	Positive type inference
	$\mid$	$\Gamma; \Phi \vdash c \colon N$	Negative type inference
	$\mid$	$\Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M$	Application type inference
EQ	$::=$		
	$\mid$	$N = M$	Negative type equality (alpha-equivalence)
	$\mid$	$P = Q$	Positive type equuality (alphha-equivalence)
	$\mid$	$\boxed{P} = \boxed{Q}$	
$LUBF$	$::=$		
	$\mid$	$P_1 \vee P_2 === Q$	
	$\mid$	**ord** $vars$ **in** $\boxed{P} === \vec{\alpha}$	
	$\mid$	**ord** $vars$ **in** $\boxed{N} === \vec{\alpha}$	
	$\mid$	**ord** $vars$ **in** $P === \vec{\alpha}$	
	$\mid$	**ord** $vars$ **in** $N === \vec{\alpha}$	
	$\mid$	**nf** $(N') === N$	
	$\mid$	**nf** $(P') === P$	
	$\mid$	**nf** $(\boxed{N'}) === \boxed{N}$	
	$\mid$	**nf** $(\boxed{P'}) === \boxed{P}$	
	$\mid$	**nf** $(\vec{N'}) === \vec{N}$	
	$\mid$	**nf** $(\vec{P'}) === \vec{P}$	
	$\mid$	**nf** $(\sigma') === \sigma$	
	$\mid$	**nf** $(\widehat{\sigma}') === \widehat{\sigma}$	
	$\mid$	**nf** $(\mu') === \mu$	
	$\mid$	$\sigma'\vert_{vars}$	
	$\mid$	$\widehat{\sigma}'\vert_{vars}$	
	$\mid$	$\widehat{\tau}'\vert_{vars}$	

	$\mid$	$\Xi'\|_{vars}$	
	$\mid$	$SC'\|_{vars}$	
	$\mid$	$UC'\|_{vars}$	
	$\mid$	$e_1 \mathbin{\&} e_2$	
	$\mid$	$e_1 \mathbin{\&} e_2$	
	$\mid$	$UC_1 \mathbin{\&} UC_2$	
	$\mid$	$UC_1 \cup UC_2$	
	$\mid$	$\Gamma_1 \cup \Gamma_2$	
	$\mid$	$SC_1 \mathbin{\&} SC_2$	
	$\mid$	$\widehat{\tau}_1 \mathbin{\&} \widehat{\tau}_2$	
	$\mid$	$\mathbf{dom}\,(UC) === \Xi$	
	$\mid$	$\mathbf{dom}\,(SC) === \Xi$	
	$\mid$	$\mathbf{dom}\,(\widehat{\sigma}) === \Xi$	
	$\mid$	$\mathbf{dom}\,(\widehat{\tau}) === \Xi$	
	$\mid$	$\mathbf{dom}\,(\Theta) === \Xi$	
	$\mid$	$\|SC\| === UC$	
LUB	$::=$		
	$\mid$	$\Gamma \models P_1 \vee P_2 = Q$	Least Upper Bound (Least Common Supertype)
	$\mid$	$\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q$	
Nrm	$::=$		
	$\mid$	$\mathbf{nf}\,(N) = M$	
	$\mid$	$\mathbf{nf}\,(P) = Q$	
	$\mid$	$\mathbf{nf}\,(N) = M$	
	$\mid$	$\mathbf{nf}\,(P) = Q$	
Order	$::=$		
	$\mid$	$\mathbf{ord}\,vars\,\mathbf{in}\,N = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\,vars\,\mathbf{in}\,P = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\,vars\,\mathbf{in}\,N = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\,vars\,\mathbf{in}\,P = \vec{\alpha}$	
U	$::=$		
	$\mid$	$\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC$	Negative unification
	$\mid$	$\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC$	Positive unification
WFT	$::=$		
	$\mid$	$\Gamma \vdash N$	Negative type well-formedness
	$\mid$	$\Gamma \vdash P$	Positive type well-formedness
	$\mid$	$\Gamma \vdash \mathbf{N}$	Negative type well-formedness
	$\mid$	$\Gamma \vdash \mathbf{P}$	Positive type well-formedness
	$\mid$	$\Gamma \vdash \vec{N}$	Negative type list well-formedness
	$\mid$	$\Gamma \vdash \vec{P}$	Positive type list well-formedness
WFAT	$::=$		
	$\mid$	$\Gamma; \Xi \vdash N$	Negative algorithmic type well-formedness
	$\mid$	$\Gamma; \Xi \vdash P$	Positive algorithmic type well-formedness
	$\mid$	$\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1$	Antiunification substitution well-formedness

|	$\Gamma \vdash^{\supseteq} \Theta$	Unification context well-formedness
|	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution signature
|	$\Theta \vdash \widehat{\sigma} : \Xi$	Unification substitution signature
|	$\Gamma \vdash \widehat{\sigma} : \Xi$	Unification substitution general signature
|	$\Theta \vdash \widehat{\sigma} : UC$	Unification substitution satisfies unification constraint
|	$\Theta \vdash \widehat{\sigma} : SC$	Unification substitution satisfies subtyping constraint
|	$\Gamma \vdash e$	Unification constraint entry well-formedness
|	$\Gamma \vdash e$	Subtyping constraint entry well-formedness
|	$\Gamma \vdash P : e$	Positive type satisfies unification constraint
|	$\Gamma \vdash N : e$	Negative type satisfies unification constraint
|	$\Gamma \vdash P : e$	Positive type satisfies subtyping constraint
|	$\Gamma \vdash N : e$	Negative type satisfies subtyping constraint
|	$\Theta \vdash UC : \Xi$	Unification constraint well-formedness with specified domain
|	$\Theta \vdash SC : \Xi$	Subtyping constraint well-formedness with specified domain
|	$\Theta \vdash UC$	Unification constraint well-formedness
|	$\Theta \vdash SC$	Subtyping constraint well-formedness
|	$\Gamma \vdash \overrightarrow{v}$	Argument List well-formedness
|	$\Gamma \vdash \Phi$	Context well-formedness
|	$\Gamma \vdash v$	Value well-formedness
|	$\Gamma \vdash c$	Computation well-formedness

judgement	::=	
	|	A
	|	AT
	|	AU
	|	SCM
	|	UCM
	|	$SATSCE$
	|	$SING$
	|	$E1$
	|	$D1$
	|	DT
	|	EQ
	|	LUB
	|	Nrm
	|	$Order$
	|	U
	|	WFT
	|	$WFAT$

user_syntax	::=	
	|	α
	|	n
	|	x
	|	n
	|	α^{+}
	|	α^{-}
	|	$\alpha^{\pm}$
	|	σ
	|	e

| e
| UC
| SC
| $\widehat{\sigma}$
| $\widehat{\tau}$
| $\overrightarrow{\alpha^+}$
| $\overrightarrow{\alpha^-}$
| $\overrightarrow{\alpha^\pm}$
| P
| N
| $\vec{P}$
| $\vec{N}$
| Γ
| Θ
| Ξ
| $\vec{\alpha}$
| $vars$
| μ
| $\vec{\mu}$
| $\widehat{\alpha}^\pm$
| $\widehat{\alpha}^+$
| $\widehat{\alpha}^-$
| $\overrightarrow{\widehat{\alpha}^+}$
| $\overrightarrow{\widehat{\alpha}^-}$
| P
| N
| $auSol$
| $terminals$
| v
| $\vec{v}$
| c
| $vctx$
| $formula$

$\boxed{\Gamma;\, \Theta \models N \leqslant M \dashv SC}$ Negative subtyping

$$\frac{}{\Gamma;\, \Theta \models \alpha^- \leqslant \alpha^- \dashv \cdot}\ \text{ANVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(P) \overset{u}{\simeq} \mathbf{nf}\,(Q) \dashv UC}{\Gamma;\, \Theta \models \uparrow P \leqslant \uparrow Q \dashv UC}\ \text{AShiftU}$$

$$\frac{\Gamma;\, \Theta \models P \geqslant Q \dashv SC_1 \quad \Gamma;\, \Theta \models N \leqslant M \dashv SC_2 \quad \Theta \vdash SC_1 \& SC_2 = SC}{\Gamma;\, \Theta \models P \to N \leqslant Q \to M \dashv SC}\ \text{AArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma;\, \Theta \models \forall \overrightarrow{\alpha^+}.N \leqslant \forall \overrightarrow{\beta^+}.M \dashv SC \backslash \overrightarrow{\widehat{\alpha}^+}}\ \text{AForall}$$

$\boxed{\Gamma;\, \Theta \models P \geqslant Q \dashv SC}$ Positive supertyping

$$\frac{}{\Gamma;\, \Theta \models \alpha^+ \geqslant \alpha^+ \dashv \cdot}\ \text{APVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(N) \overset{u}{\simeq} \mathbf{nf}\,(M) \dashv UC}{\Gamma; \Theta \models {\downarrow}N \geqslant {\downarrow}M \dashv UC}\ \textsc{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-}; \Theta, \overrightarrow{\widehat{\alpha}^-}\{\Gamma, \overrightarrow{\beta^-}\} \models [\overrightarrow{\widehat{\alpha}^-}/\overrightarrow{\alpha^-}]P \geqslant Q \dashv SC}{\Gamma; \Theta \models \exists\overrightarrow{\alpha^-}.P \geqslant \exists\overrightarrow{\beta^-}.Q \dashv SC\backslash\overrightarrow{\widehat{\alpha}^-}}\ \textsc{AExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}{\Gamma; \Theta \models \widehat{\alpha}^+ \geqslant P \dashv (\widehat{\alpha}^+ :\geqslant Q)}\ \textsc{APUVar}$$

$\boxed{\Gamma; \Phi \models v\colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \models x\colon \mathbf{nf}\,(P)}\ \textsc{ATVar}$$

$$\frac{\Gamma; \Phi \models c\colon N}{\Gamma; \Phi \models \{c\}\colon {\downarrow}N}\ \textsc{ATThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma; \Phi \models v\colon P \quad \Gamma; \cdot \models Q \geqslant P \dashv \cdot}{\Gamma; \Phi \models (v : Q)\colon \mathbf{nf}\,(Q)}\ \textsc{ATPAnnot}$$

$\boxed{\Gamma; \Phi \models c\colon N}$ Negative type inference

$$\frac{\Gamma \vdash M \quad \Gamma; \Phi \models c\colon N \quad \Gamma; \cdot \models N \leqslant M \dashv \cdot}{\Gamma; \Phi \models (c : M)\colon \mathbf{nf}\,(M)}\ \textsc{ATNAnnot}$$

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi, x : P \models c\colon N}{\Gamma; \Phi \models \lambda x : P.c\colon \mathbf{nf}\,(P \to N)}\ \textsc{ATtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \models c\colon N}{\Gamma; \Phi \models \Lambda\alpha^+.c\colon \mathbf{nf}\,(\forall\alpha^+.N)}\ \textsc{ATTLam}$$

$$\frac{\Gamma; \Phi \models v\colon P}{\Gamma; \Phi \models \mathbf{return}\,v\colon {\uparrow}P}\ \textsc{ATReturn}$$

$$\frac{\Gamma; \Phi \models v\colon P \quad \Gamma; \Phi, x : P \models c\colon N}{\Gamma; \Phi \models \mathbf{let}\,x = v; c\colon N}\ \textsc{ATVarLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma; \Phi \models v\colon {\downarrow}M \\ \Gamma; \Phi; \cdot \models M \bullet \overrightarrow{v} \Longrightarrow {\uparrow}Q \dashv \Theta; SC_1 \quad \Gamma; \Theta \models {\uparrow}Q \leqslant {\uparrow}P \dashv SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC \quad \Gamma; \Phi, x : P \models c\colon N\end{array}}{\Gamma; \Phi \models \mathbf{let}\,x : P = v(\overrightarrow{v}); c\colon N}\ \textsc{ATAppLetAnn}$$

$$\frac{\begin{array}{l}\Gamma; \Phi \models v\colon {\downarrow}M \quad \Gamma; \Phi; \cdot \models M \bullet \overrightarrow{v} \Longrightarrow {\uparrow}Q \dashv \Theta; SC \\ \mathbf{uv}\,Q = \mathbf{dom}\,(SC) \quad SC\,\mathbf{singular\,with}\,\widehat{\sigma} \\ \Gamma; \Phi, x : [\widehat{\sigma}]Q \models c\colon N\end{array}}{\Gamma; \Phi \models \mathbf{let}\,x = v(\overrightarrow{v}); c\colon N}\ \textsc{ATAppLet}$$

$$\frac{\Gamma; \Phi \models v\colon \exists\overrightarrow{\alpha^-}.P \quad \Gamma, \overrightarrow{\alpha^-}; \Phi, x : P \models c\colon N \quad \Gamma \vdash N}{\Gamma; \Phi \models \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c\colon N}\ \textsc{ATUnpack}$$

$\boxed{\Gamma; \Phi; \Theta_1 \models N \bullet \overrightarrow{v} \Longrightarrow M \dashv \Theta_2; SC}$ Application type inference

$$\frac{}{\Gamma; \Phi; \Theta \models N \bullet \cdot \Longrightarrow \mathbf{nf}\,(N) \dashv \Theta; \cdot}\ \textsc{ATEmptyApp}$$

$$\frac{\begin{array}{l}\Gamma; \Phi \models v\colon P \quad \Gamma; \Theta \models Q \geqslant P \dashv SC_1 \quad \Gamma; \Phi; \Theta \models N \bullet \overrightarrow{v} \Longrightarrow M \dashv \Theta'; SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC\end{array}}{\Gamma; \Phi; \Theta \models Q \to N \bullet v, \overrightarrow{v} \Longrightarrow M \dashv \Theta'; SC}\ \textsc{ATArrowApp}$$

<<multiple parses>>

$$\frac{\overrightarrow{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot}{\text{<<multiple parses>>}} \quad \textsc{ATForallApp}$$

$\boxed{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\cdot, \alpha^+, \cdot, \cdot)} \quad \textsc{AUPVar}$$

$$\frac{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models {\downarrow} N_1 \overset{a}{\simeq} {\downarrow} N_2 \dashv (\Xi, {\downarrow} M, \widehat{\tau}_1, \widehat{\tau}_2)} \quad \textsc{AUShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \Gamma = \varnothing \quad \Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \exists \overrightarrow{\alpha^-}. P_1 \overset{a}{\simeq} \exists \overrightarrow{\alpha^-}. P_2 \dashv (\Xi, \exists \overrightarrow{\alpha^-}. Q, \widehat{\tau}_1, \widehat{\tau}_2)} \quad \textsc{AUExists}$$

$\boxed{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^- \overset{a}{\simeq} \alpha^- \dashv (\cdot, \alpha^-, \cdot, \cdot)} \quad \textsc{AUNVar}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models {\uparrow} P_1 \overset{a}{\simeq} {\uparrow} P_2 \dashv (\Xi, {\uparrow} Q, \widehat{\tau}_1, \widehat{\tau}_2)} \quad \textsc{AUShiftU}$$

$$\frac{\overrightarrow{\alpha^+} \cap \Gamma = \varnothing \quad \Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \forall \overrightarrow{\alpha^+}. N_1 \overset{a}{\simeq} \forall \overrightarrow{\alpha^+}. N_2 \dashv (\Xi, \forall \overrightarrow{\alpha^+}. M, \widehat{\tau}_1, \widehat{\tau}_2)} \quad \textsc{AUForall}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \widehat{\tau}_1, \widehat{\tau}_2) \quad \Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \widehat{\tau}'_1, \widehat{\tau}'_2)}{\Gamma \models P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \widehat{\tau}_1 \cup \widehat{\tau}'_1, \widehat{\tau}_2 \cup \widehat{\tau}'_2)} \quad \textsc{AUArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \models N \overset{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, \widehat{\alpha}^-_{\{N,M\}}, (\widehat{\alpha}^-_{\{N,M\}} :\approx N), (\widehat{\alpha}^-_{\{N,M\}} :\approx M))} \quad \textsc{AUAU}$$

$\boxed{\Gamma \vdash e_1 \;\&\; e_2 = e_3}$ Subtyping Constraint Entry Merge

$$\frac{\Gamma \models P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1) \;\&\; (\widehat{\alpha}^+ :\geqslant P_2) = (\widehat{\alpha}^+ :\geqslant Q)} \quad \textsc{SCMESupSup}$$

$$\frac{\Gamma; \cdot \models P \geqslant Q \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \;\&\; (\widehat{\alpha}^+ :\geqslant Q) = (\widehat{\alpha}^+ :\approx P)} \quad \textsc{SCMEEqSup}$$

$$\frac{\Gamma; \cdot \models Q \geqslant P \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P) \;\&\; (\widehat{\alpha}^+ :\approx Q) = (\widehat{\alpha}^+ :\approx Q)} \quad \textsc{SCMESupEq}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \;\&\; (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \textsc{SCMEPEqEq}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \;\&\; (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \textsc{SCMENEqEq}$$

$\boxed{\Theta \vdash SC_1 \& SC_2 = SC_3}$ Merge of subtyping constraints

$\boxed{\Gamma \vdash e_1 \& e_2 = e_3}$

$$\frac{\text{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \& (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \textsc{UCMEPEqEq}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \& (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \textsc{UCMENEqEq}$$

$\boxed{\Theta \vdash UC_1 \& UC_2 = UC_3}$ Merge of unification constraints

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies with the subtyping constraint entry

$$\frac{\Gamma \vdash P \geqslant_1 Q}{\Gamma \vdash P : (\widehat{\alpha}^+ :\geqslant Q)} \quad \text{SATSCESUP}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash P : (\widehat{\alpha}^+ :\approx Q)} \quad \text{SATSCEPEQ}$$

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies with the subtyping constraint entry

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash N : (\widehat{\alpha}^- :\approx M)} \quad \text{SATSCENEQ}$$

$\boxed{e_1 \textbf{ singular with } P}$ Positive Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^+ :\approx P \textbf{ singular with nf } (P)} \quad \text{SINGPEQ}$$

$$\frac{}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.\alpha^+ \textbf{ singular with } \alpha^+} \quad \text{SINGSUPVAR}$$

$$\frac{N \simeq_1^D \alpha_i^-}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.\downarrow N \textbf{ singular with } \exists \alpha^-.\downarrow \alpha^-} \quad \text{SINGSUPSHIFT}$$

$\boxed{e_1 \textbf{ singular with } N}$ Negative Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^- :\approx N \textbf{ singular with nf } (N)} \quad \text{SINGNEQ}$$

$\boxed{SC \textbf{ singular with } \widehat{\sigma}}$ Subtyping Constraint Is Singular

$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-} \quad \text{E1NVAR}$$

$$\frac{P \simeq_1^D Q}{\uparrow P \simeq_1^D \uparrow Q} \quad \text{E1SHIFTU}$$

$$\frac{P \simeq_1^D Q \quad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M} \quad \text{E1ARROW}$$

$$\frac{\overrightarrow{\alpha^+} \cap \textbf{fv}\, M = \varnothing \quad \mu : (\overrightarrow{\beta^+} \cap \textbf{fv}\, M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \textbf{fv}\, N) \quad N \simeq_1^D [\mu]M}{\forall \overrightarrow{\alpha^+}.N \simeq_1^D \forall \overrightarrow{\beta^+}.M} \quad \text{E1FORALL}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+} \quad \text{E1PVAR}$$

$$\frac{N \simeq_1^D M}{\downarrow N \simeq_1^D \downarrow M} \quad \text{E1SHIFTD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \textbf{fv}\, Q = \varnothing \quad \mu : (\overrightarrow{\beta^-} \cap \textbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \textbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists \overrightarrow{\alpha^-}.P \simeq_1^D \exists \overrightarrow{\beta^-}.Q} \quad \text{E1EXISTS}$$

$\boxed{P \simeq_1^D Q}$ Positive unification type equivalence

$\boxed{N \simeq_1^D M}$ Positive unification type equivalence

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M} \text{ D1NDEF}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q} \text{ D1PDEF}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-} \text{ D1NVAR}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant_1 \uparrow Q} \text{ D1SHIFTU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M} \text{ D1ARROW}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma, \overrightarrow{\beta^+} \vdash [\sigma]N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant_1 \forall \overrightarrow{\beta^+}.M} \text{ D1FORALL}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+} \text{ D1PVAR}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M} \text{ D1SHIFTD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vdash \sigma : \overrightarrow{\alpha^-} \quad \Gamma, \overrightarrow{\beta^-} \vdash [\sigma]P \geqslant_1 Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P \geqslant_1 \exists \overrightarrow{\beta^-}.Q} \text{ D1EXISTS}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1}$ Equivalence of substitutions
$\boxed{\Gamma \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : vars}$ Equivalence of substitutions
$\boxed{\Theta \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : vars}$ Equivalence of unification substitutions
$\boxed{\Gamma \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : vars}$ Equivalence of unification substitutions
$\boxed{\Gamma \vdash \Phi_1 \simeq_1^{\leqslant} \Phi_2}$ Equivalence of contexts
$\boxed{\Gamma; \Phi \vdash v \colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \vdash x \colon P} \text{ DTVAR}$$

$$\frac{\Gamma; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \{c\} \colon \downarrow N} \text{ DTTHUNK}$$

$$\frac{\Gamma \vdash Q \quad \Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma; \Phi \vdash (v : Q) \colon Q} \text{ DTPANNOT}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash v \colon P'} \text{ DTPEQUIV}$$

$\boxed{\Gamma; \Phi \vdash c \colon N}$ Negative type inference

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \lambda x : P.c \colon P \to N} \text{ DTTLAM}$$

$$\frac{\Gamma, \alpha^+; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \Lambda \alpha^+.c \colon \forall \alpha^+.N} \text{ DTTLAM}$$

$$\frac{\Gamma;\Phi \vdash v\colon P}{\Gamma;\Phi \vdash \mathbf{return}\, v\colon \uparrow P}\ \textsc{DTReturn}$$

$$\frac{\Gamma;\Phi \vdash v\colon P \quad \Gamma;\Phi, x : P \vdash c\colon N}{\Gamma;\Phi \vdash \mathbf{let}\, x = v; c\colon N}\ \textsc{DTVarLet}$$

$$\frac{\Gamma;\Phi \vdash v\colon \downarrow M \quad \Gamma;\Phi \vdash M \bullet \vec{v} \Rightarrow\!\!\!> \uparrow Q \text{ unique} \quad \Gamma;\Phi, x : Q \vdash c\colon N}{\Gamma;\Phi \vdash \mathbf{let}\, x = v(\vec{v}); c\colon N}\ \textsc{DTAppLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma;\Phi \vdash v\colon \downarrow M \\ \Gamma;\Phi \vdash M \bullet \vec{v} \Rightarrow\!\!\!> \uparrow Q \quad \Gamma \vdash \uparrow Q \leqslant_1 \uparrow P \quad \Gamma;\Phi, x : P \vdash c\colon N\end{array}}{\Gamma;\Phi \vdash \mathbf{let}\, x : P = v(\vec{v}); c\colon N}\ \textsc{DTAppLetAnn}$$

$$\frac{\begin{array}{l}\texttt{<<multiple parses>>} \\ \Gamma, \overrightarrow{\alpha^-};\Phi, x : P \vdash c\colon N \quad \Gamma \vdash N\end{array}}{\Gamma;\Phi \vdash \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c\colon N}\ \textsc{DTUnpack}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma;\Phi \vdash (c : M)\colon M}\ \textsc{DTNAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma;\Phi \vdash c\colon N'}\ \textsc{DTNEquiv}$$

$\boxed{\Gamma;\Phi \vdash N \bullet \vec{v} \Rightarrow\!\!\!> M}$ Application type inference

$$\frac{\texttt{<<multiple parses>>}}{\Gamma;\Phi \vdash N \bullet \cdot \Rightarrow\!\!\!> N'}\ \textsc{DTEmptyApp}$$

$$\frac{\Gamma;\Phi \vdash v\colon P \quad \Gamma \vdash Q \geqslant_1 P \quad \Gamma;\Phi \vdash N \bullet \vec{v} \Rightarrow\!\!\!> M}{\Gamma;\Phi \vdash Q \to N \bullet v, \vec{v} \Rightarrow\!\!\!> M}\ \textsc{DTArrowApp}$$

$$\frac{\begin{array}{l}\Gamma \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma;\Phi \vdash [\sigma]N \bullet \vec{v} \Rightarrow\!\!\!> M \\ \vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot\end{array}}{\Gamma;\Phi \vdash \forall\overrightarrow{\alpha^+}.N \bullet \vec{v} \Rightarrow\!\!\!> M}\ \textsc{DTForallApp}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$

$\boxed{P_1 \vee P_2}$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N}$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{N}')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{P}')}$

$\boxed{\mathbf{nf}\,(\sigma')}$

$\boxed{\mathbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\mathbf{nf}\,(\mu')}$

$\boxed{\sigma'|_{vars}}$

$\boxed{\widehat{\sigma}'|_{vars}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{SC'|_{vars}}$

$\boxed{UC'|_{vars}}$

$\boxed{e_1 \mathrel{\&} e_2}$

$\boxed{e_1 \mathrel{\&} e_2}$

$\boxed{UC_1 \mathrel{\&} UC_2}$

$\boxed{UC_1 \cup UC_2}$

$\boxed{\Gamma_1 \cup \Gamma_2}$

$\boxed{SC_1 \mathrel{\&} SC_2}$

$\boxed{\widehat{\tau}_1 \mathrel{\&} \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(UC)}$

$\boxed{\mathbf{dom}\,(SC)}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{|SC|}$

$\boxed{\Gamma \models P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \models \alpha^+ \vee \alpha^+ = \alpha^+}\ \textsc{LUBVar}$$

$$\frac{\Gamma, \cdot \models \mathbf{nf}\,(\downarrow N) \stackrel{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi]\,P}\ \textsc{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \vDash P_1 \vee P_2 = Q}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P_1 \vee \exists \overrightarrow{\beta^-}.P_2 = Q} \quad \text{LUBEXISTS}$$

$\boxed{\textbf{upgrade}\,\Gamma \vdash P\,\textbf{to}\,\Delta = Q}$

$$\frac{\Gamma = \Delta, \overrightarrow{\alpha^\pm} \quad \overrightarrow{\beta^\pm}\,\textbf{is fresh} \quad \overrightarrow{\gamma^\pm}\,\textbf{is fresh} \quad \Delta, \overrightarrow{\beta^\pm}, \overrightarrow{\gamma^\pm} \vDash [\overrightarrow{\beta^\pm}/\overrightarrow{\alpha^\pm}]P \vee [\overrightarrow{\gamma^\pm}/\overrightarrow{\alpha^\pm}]P = Q}{\textbf{upgrade}\,\Gamma \vdash P\,\textbf{to}\,\Delta = Q} \quad \text{LUBUPGRADE}$$

$\boxed{\textbf{nf}\,(N) = M}$

$$\frac{}{\textbf{nf}\,(\alpha^-) = \alpha^-} \quad \text{NRMNVAR}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\uparrow P) = \uparrow Q} \quad \text{NRMSHIFTU}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(P \to N) = Q \to M} \quad \text{NRMARROW}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\forall \overrightarrow{\alpha^+}.N) = \forall \overrightarrow{\alpha^{+\prime}}.N'} \quad \text{NRMFORALL}$$

$\boxed{\textbf{nf}\,(P) = Q}$

$$\frac{}{\textbf{nf}\,(\alpha^+) = \alpha^+} \quad \text{NRMPVAR}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\downarrow N) = \downarrow M} \quad \text{NRMSHIFTD}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\exists \overrightarrow{\alpha^-}.P) = \exists \overrightarrow{\alpha^{-\prime}}.P'} \quad \text{NRMEXISTS}$$

$\boxed{\textbf{nf}\,(N) = M}$

$$\frac{}{\textbf{nf}\,(\widehat{\alpha}^-) = \widehat{\alpha}^-} \quad \text{NRMNUVAR}$$

$\boxed{\textbf{nf}\,(P) = Q}$

$$\frac{}{\textbf{nf}\,(\widehat{\alpha}^+) = \widehat{\alpha}^+} \quad \text{NRMPUVAR}$$

$\boxed{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,N = \vec{\alpha}}$

$$\frac{\alpha^- \in \mathit{vars}}{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,\alpha^- = \alpha^-} \quad \text{ONVARIN}$$

$$\frac{\alpha^- \notin \mathit{vars}}{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,\alpha^- = \cdot} \quad \text{ONVARNIN}$$

$$\frac{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,P = \vec{\alpha}}{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,\uparrow P = \vec{\alpha}} \quad \text{OSHIFTU}$$

$$\frac{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,P = \vec{\alpha}_1 \quad \textbf{ord}\,\mathit{vars}\,\textbf{in}\,N = \vec{\alpha}_2}{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)} \quad \text{OARROW}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^+} = \varnothing \quad \textbf{ord}\,\mathit{vars}\,\textbf{in}\,N = \vec{\alpha}}{\textbf{ord}\,\mathit{vars}\,\textbf{in}\,\forall \overrightarrow{\alpha^+}.N = \vec{\alpha}} \quad \text{OFORALL}$$

$\boxed{\textbf{ord}\, vars\, \textbf{in}\, P = \vec{\alpha}}$

$$\frac{\alpha^+ \in vars}{\textbf{ord}\, vars\, \textbf{in}\, \alpha^+ = \alpha^+} \quad \text{OPVarIn}$$

$$\frac{\alpha^+ \notin vars}{\textbf{ord}\, vars\, \textbf{in}\, \alpha^+ = \cdot} \quad \text{OPVarNIn}$$

$$\frac{\textbf{ord}\, vars\, \textbf{in}\, N = \vec{\alpha}}{\textbf{ord}\, vars\, \textbf{in}\, {\downarrow}N = \vec{\alpha}} \quad \text{OShiftD}$$

$$\frac{vars \cap \overrightarrow{\alpha^-} = \varnothing \quad \textbf{ord}\, vars\, \textbf{in}\, P = \vec{\alpha}}{\textbf{ord}\, vars\, \textbf{in}\, \exists\overrightarrow{\alpha^-}.P = \vec{\alpha}} \quad \text{OExists}$$

$\boxed{\textbf{ord}\, vars\, \textbf{in}\, N = \vec{\alpha}}$

$$\frac{}{\textbf{ord}\, vars\, \textbf{in}\, \widehat{\alpha}^- = \cdot} \quad \text{ONUVar}$$

$\boxed{\textbf{ord}\, vars\, \textbf{in}\, P = \vec{\alpha}}$

$$\frac{}{\textbf{ord}\, vars\, \textbf{in}\, \widehat{\alpha}^+ = \cdot} \quad \text{OPUVar}$$

$\boxed{\Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv UC}$ Negative unification

$$\frac{}{\Gamma; \Theta \models \alpha^- \stackrel{u}{\simeq} \alpha^- \dashv \cdot} \quad \text{UNVar}$$

$$\frac{\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models {\uparrow}P \stackrel{u}{\simeq} {\uparrow}Q \dashv UC} \quad \text{UShiftU}$$

$$\frac{\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC_1 \quad \Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv UC_2}{\Gamma; \Theta \models P \to N \stackrel{u}{\simeq} Q \to M \dashv UC_1 \,\&\, UC_2} \quad \text{UArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Theta \models N \stackrel{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \forall\overrightarrow{\alpha^+}.N \stackrel{u}{\simeq} \forall\overrightarrow{\alpha^+}.M \dashv UC} \quad \text{UForall}$$

$$\frac{\widehat{\alpha}^-\{\Delta\} \in \Theta \quad \Delta \vdash N}{\Gamma; \Theta \models \widehat{\alpha}^- \stackrel{u}{\simeq} N \dashv (\widehat{\alpha}^- :\approx N)} \quad \text{UNUVar}$$

$\boxed{\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC}$ Positive unification

$$\frac{}{\Gamma; \Theta \models \alpha^+ \stackrel{u}{\simeq} \alpha^+ \dashv \cdot} \quad \text{UPVar}$$

$$\frac{\Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models {\downarrow}N \stackrel{u}{\simeq} {\downarrow}M \dashv UC} \quad \text{UShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \exists\overrightarrow{\alpha^-}.P \stackrel{u}{\simeq} \exists\overrightarrow{\alpha^-}.Q \dashv UC} \quad \text{UExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \Delta \vdash P}{\Gamma; \Theta \models \widehat{\alpha}^+ \stackrel{u}{\simeq} P \dashv (\widehat{\alpha}^+ :\approx P)} \quad \text{UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma \vdash \alpha^-} \quad \text{WFTNVar}$$

$$\frac{\Gamma \vdash P}{\Gamma \vdash \uparrow P} \quad \text{WFTShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash N}{\Gamma \vdash P \to N} \quad \text{WFTArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+} \vdash N}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N} \quad \text{WFTForall}$$

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma \vdash \alpha^+} \quad \text{WFTPVar}$$

$$\frac{\Gamma \vdash N}{\Gamma \vdash \downarrow N} \quad \text{WFTShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-} \vdash P}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P} \quad \text{WFTExists}$$

$\boxed{\Gamma \vdash \mathbf{N}}$ Negative type well-formedness

$\boxed{\Gamma \vdash \mathbf{P}}$ Positive type well-formedness

$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness

$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness

$\boxed{\Gamma; \Xi \vdash N}$ Negative algorithmic type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma; \Xi \vdash \alpha^-} \quad \text{WFATNVar}$$

$$\frac{\widehat{\alpha}^- \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^-} \quad \text{WFATNUVar}$$

$$\frac{\Gamma; \Xi \vdash P}{\Gamma; \Xi \vdash \uparrow P} \quad \text{WFATShiftU}$$

$$\frac{\Gamma; \Xi \vdash P \quad \Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash P \to N} \quad \text{WFATArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Xi \vdash N}{\Gamma; \Xi \vdash \forall \overrightarrow{\alpha^+}.N} \quad \text{WFATForall}$$

$\boxed{\Gamma; \Xi \vdash P}$ Positive algorithmic type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma; \Xi \vdash \alpha^+} \quad \text{WFATPVar}$$

$$\frac{\widehat{\alpha}^+ \in \Xi}{\Gamma; \Xi \vdash \widehat{\alpha}^+} \quad \text{WFATPUVar}$$

$$\frac{\Gamma; \Xi \vdash N}{\Gamma; \Xi \vdash \downarrow N} \quad \text{WFATShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Xi \vdash P}{\Gamma; \Xi \vdash \exists \overrightarrow{\alpha^-}.P} \quad \text{WFATExists}$$

$\boxed{\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness

$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness

$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution signature

$\boxed{\Theta \vdash \widehat{\sigma} : \Xi}$ Unification substitution signature
$\boxed{\Gamma \vdash \widehat{\sigma} : \Xi}$ Unification substitution general signature
$\boxed{\Theta \vdash \widehat{\sigma} : UC}$ Unification substitution satisfies unification constraint
$\boxed{\Theta \vdash \widehat{\sigma} : SC}$ Unification substitution satisfies subtyping constraint
$\boxed{\Gamma \vdash e}$ Unification constraint entry well-formedness
$\boxed{\Gamma \vdash e}$ Subtyping constraint entry well-formedness
$\boxed{\Gamma \vdash P : e}$ Positive type satisfies unification constraint
$\boxed{\Gamma \vdash N : e}$ Negative type satisfies unification constraint
$\boxed{\Gamma \vdash P : e}$ Positive type satisfies subtyping constraint
$\boxed{\Gamma \vdash N : e}$ Negative type satisfies subtyping constraint
$\boxed{\Theta \vdash UC : \Xi}$ Unification constraint well-formedness with specified domain
$\boxed{\Theta \vdash SC : \Xi}$ Subtyping constraint well-formedness with specified domain
$\boxed{\Theta \vdash UC}$ Unification constraint well-formedness
$\boxed{\Theta \vdash SC}$ Subtyping constraint well-formedness
$\boxed{\Gamma \vdash \overrightarrow{v}}$ Argument List well-formedness
$\boxed{\Gamma \vdash \Phi}$ Context well-formedness
$\boxed{\Gamma \vdash v}$ Value well-formedness

$$\frac{}{\Gamma \vdash x} \quad \text{WFATV{\small AR}}$$

$\boxed{\Gamma \vdash c}$ Computation well-formedness

$$\frac{\Gamma \vdash v \quad \Gamma \vdash c \quad \Gamma \vdash \overrightarrow{v}}{\Gamma \vdash \textbf{let}\, x = v(\overrightarrow{v});\, c} \quad \text{WFATA{\small PP}L{\small ET}}$$

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Definition rules:        107 good    20 bad
Definition rule clauses: 221 good    21 bad
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