α, β, α, β	type variables
n, m, i, j	index variables

$\alpha^+,\ \beta^+$	$::=$			positive variable
	$\vert$	α^+		
$\alpha^-,\ \beta^-$	$::=$			negative variable
	$\vert$	α^-		
σ	$::=$			substitution
	$\vert$	$\cdot$		
	$\vert$	P/α^+		
	$\vert$	N/α^-		
	$\vert$	$\overrightarrow{P}/\overrightarrow{\alpha^+}$		
	$\vert$	$\overrightarrow{N}/\overrightarrow{\alpha^-}$		
	$\vert$	$\overrightarrow{\widetilde{\alpha^+}}/\overrightarrow{\alpha^+}$		
	$\vert$	$\overrightarrow{\widetilde{\alpha^-}}/\overrightarrow{\alpha^-}$		
	$\vert$	$\overrightarrow{\widetilde{\alpha^+}}/\overrightarrow{\alpha^+}$		
	$\vert$	$\overrightarrow{\widehat{\alpha^-}}/\overrightarrow{\alpha^-}$		
	$\vert$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	$\sigma_1 \circ \sigma_2$		
	$\vert$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\vert$	(σ)	S	
	$\vert$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\vert$	$\mathbf{nf}\,(\sigma')$	M	
	$\vert$	$\sigma'\vert_{vars}$	M	
e	$::=$			entry of a unification solution
	$\vert$	$\Gamma \vdash \widehat{\alpha}^+ :\approx P$		
	$\vert$	$\Gamma \vdash \widehat{\alpha}^- :\approx N$		
	$\vert$	$\Gamma \vdash \widehat{\alpha}^+ :\geqslant P$		
	$\vert$	(e)	S	
	$\vert$	$e_1\ \&\ e_2$	M	
$\widehat{\sigma}$	$::=$			unification solution (substitution)
	$\vert$	$\cdot$		
	$\vert$	e		
	$\vert$	$\widehat{\sigma}\backslash\overrightarrow{\widetilde{\alpha^+}}$		
	$\vert$	$\widehat{\sigma}\backslash\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	$\widehat{\sigma}\backslash\widehat{\alpha}^+$		
	$\vert$	$\widehat{\sigma}\backslash\widehat{\alpha}^-$		
	$\vert$	$\widehat{\sigma}_1 \cup \widehat{\sigma}_2$		
	$\vert$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\vert$	$(\widehat{\sigma})$	S	
	$\vert$	$\widehat{\sigma}_1\ \&\ \widehat{\sigma}_2$	M	
$\widehat{\tau}$	$::=$			anti-unification substitution
	$\vert$	$\cdot$		
	$\vert$	$\widehat{\alpha}^- :\approx N$		
	$\vert$	$\widehat{\alpha}^- :\approx N$		
	$\vert$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		

	$\mid$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	$\mid$	$(\widehat{\tau})$	S	
$P,\ Q$	$::=$			positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\alpha^-.P$		
	$\mid$	$[\sigma]P$	M	
$N,\ M$	$::=$			negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$\forall\alpha^+.N$		
	$\mid$	$P \to N$		
	$\mid$	$[\sigma]N$	M	
$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+}$	$::=$			positive variable list
	$\mid$	$\cdot$		empty list
	$\mid$	α^+		a variable
	$\mid$	$\overline{\overrightarrow{\alpha^+}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-}$	$::=$			negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	α^-		a variable
	$\mid$	$\overline{\overrightarrow{\alpha^-}_i}^{\,i}$		concatenate lists
$P,\ Q$	$::=$			multi-quantified positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^-}.P$		$P \neq \exists\ldots$
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\widehat{\tau}]P$	M	
	$\mid$	$[\widehat{\sigma}]P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	(P)	S	
	$\mid$	$\mathbf{nf}\,(P')$	M	
$N,\ M$	$::=$			multi-quantified negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$P \to N$		
	$\mid$	$\forall\overrightarrow{\alpha^+}.N$		$N \neq \forall\ldots$
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\mu]N$	M	
	$\mid$	$[\widehat{\sigma}]N$	M	
	$\mid$	(N)	S	
	$\mid$	$\mathbf{nf}\,(N')$	M	
$\overrightarrow{P}$	$::=$			list of positive types

	$\mid$	$\cdot$		empty list
	$\mid$	P		a singel type
	$\mid$	$\overline{\overrightarrow{P}_i}^{\,i}$		concatenate lists
	$\mid$	$\mathbf{nf}\,(\overrightarrow{P'})$	M	
$\overrightarrow{N},\ \overrightarrow{M}$	$::=$			list of negative types
	$\mid$	$\cdot$		empty list
	$\mid$	N		a singel type
	$\mid$	$\overline{\overrightarrow{N}_i}^{\,i}$		concatenate lists
	$\mid$	$\mathbf{nf}\,(\overrightarrow{N'})$	M	
$\Delta,\ \Gamma$	$::=$			declarative type context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\alpha^+}$		list of variables
	$\mid$	$\overrightarrow{\alpha^-}$		list of variables
	$\mid$	$vars$		
	$\mid$	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	$\mid$	(Γ)	S	
Θ	$::=$			unification type variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\widetilde{\alpha}^+}$		list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^-}$		list of variables
	$\mid$	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	$\mid$	(Θ)	S	
Ξ	$::=$			anti-unification type variable context
	$\mid$	$\cdot$		empty context
	$\mid$	$\overrightarrow{\widetilde{\alpha}^+}$		list of variables
	$\mid$	$\overrightarrow{\widehat{\alpha}^-}$		list of variables
	$\mid$	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	$\mid$	(Ξ)	S	
	$\mid$	$\Xi_1 \cup \Xi_2$		
$\vec{\alpha},\ \vec{\beta}$	$::=$			ordered positive or negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	$\overrightarrow{\alpha^+}$		list of variables
	$\mid$	$\overrightarrow{\alpha^-}$		list of variables
	$\mid$	$\vec{\alpha}_1 \backslash vars$		setminus
	$\mid$	Γ		context
	$\mid$	$vars$		
	$\mid$	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	$\mid$	$(\vec{\alpha})$	S	parenthesis
	$\mid$	$[\mu]\vec{\alpha}$	M	apply moving to list
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P$	M	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N$	M	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P$	M	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N$	M	

$vars$	$::=$			set of variables
	$\mid$	$\varnothing$		empty set
	$\mid$	$\mathbf{fv}\, P$		free variables
	$\mid$	$\mathbf{fv}\, N$		free variables
	$\mid$	$\mathbf{fv}\, P$		free variables
	$\mid$	$\mathbf{fv}\, N$		free variables
	$\mid$	$vars_1 \cap vars_2$		set intersection
	$\mid$	$vars_1 \cup vars_2$		set union
	$\mid$	$vars_1 \backslash vars_2$		set complement
	$\mid$	$\mathbf{mv}\, P$		movable variables
	$\mid$	$\mathbf{mv}\, N$		movable variables
	$\mid$	$\mathbf{uv}\, N$		unification variables
	$\mid$	$\mathbf{uv}\, P$		unification variables
	$\mid$	$\mathbf{fv}\, N$		free variables
	$\mid$	$\mathbf{fv}\, P$		free variables
	$\mid$	$(vars)$	S	parenthesis
	$\mid$	$\{\vec{\alpha}\}$		ordered list of variables
	$\mid$	$[\mu]vars$	M	apply moving to varset
μ	$::=$			
	$\mid$	$\cdot$		empty moving
	$\mid$	$\widetilde{\alpha}^{+}_1 \mapsto \widetilde{\alpha}^{+}_2$		Positive unit substitution
	$\mid$	$\widetilde{\alpha}^{-}_1 \mapsto \widetilde{\alpha}^{-}_2$		Positive unit substitution
	$\mid$	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	$\mid$	$\overline{\mu_i}^{\,i}$		concatenate movings
	$\mid$	$\mu\vert_{vars}$	M	restriction on a set
	$\mid$	μ^{-1}	M	inversion
	$\mid$	$\mathbf{nf}\,(\mu')$	M	
n	$::=$			cohort index
	$\mid$	0		
	$\mid$	$n+1$		
$\widetilde{\alpha}^{+}$	$::=$			positive movable variable
	$\mid$	$\widetilde{\alpha}^{+n}$		
$\widetilde{\alpha}^{-}$	$::=$			negative movable variable
	$\mid$	$\widetilde{\alpha}^{-n}$		
$\overrightarrow{\widetilde{\alpha}^{+}},\ \overrightarrow{\widetilde{\beta}^{+}}$	$::=$			positive movable variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\widetilde{\alpha}^{+}$		a variable
	$\mid$	$\overrightarrow{\widetilde{\alpha}^{+n}}$		from a non-movable variable
	$\mid$	$\overline{\overrightarrow{\widetilde{\alpha}^{+}}_i}^{\,i}$		concatenate lists
$\overrightarrow{\widetilde{\alpha}^{-}},\ \overrightarrow{\widetilde{\beta}^{-}}$	$::=$			negatiive movable variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\widetilde{\alpha}^{-}$		a variable
	$\mid$	$\overrightarrow{\widetilde{\alpha}^{-n}}$		from a non-movable variable

	$\mid$	$\overline{\overrightarrow{\widetilde{\alpha}^{-}}_{i}}^{\,i}$		concatenate lists
$P,\ Q$	$::=$			multi-quantified positive types with movable variables
	$\mid$	α^{+}		
	$\mid$	$\widetilde{\alpha}^{+}$		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^{-}}.P$		
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\mu]P$	M	
$N,\ M$	$::=$			multi-quantified negative types with movable variables
	$\mid$	α^{-}		
	$\mid$	$\widetilde{\alpha}^{-}$		
	$\mid$	$\uparrow P$		
	$\mid$	$P \to N$		
	$\mid$	$\forall\overrightarrow{\alpha^{+}}.N$		
	$\mid$	$[\sigma]N$	M	
	$\mid$	$[\mu]N$	M	
$\widehat{\alpha}^{+}$	$::=$			positive unification variable
	$\mid$	$\widehat{\alpha}^{+}$		
	$\mid$	$\widehat{\alpha}^{+}\{\Delta\}$		
$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	$::=$			negative unification variable
	$\mid$	$\widehat{\alpha}^{-}$		
	$\mid$	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	$\mid$	$\widehat{\alpha}^{-}\{\Delta\}$		
$\overrightarrow{\widetilde{\alpha}^{+}},\ \overrightarrow{\widetilde{\beta}^{+}}$	$::=$			positive unification variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\widehat{\alpha}^{+}$		a variable
	$\mid$	$\overrightarrow{\widehat{\alpha}^{+}\{\Delta\}}$		from a normal variable
	$\mid$	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	$\mid$	$\overline{\overrightarrow{\widetilde{\alpha}^{+}}_{i}}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}},\ \overrightarrow{\widehat{\beta}^{-}}$	$::=$			negative unification variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\widehat{\alpha}^{-}$		a variable
	$\mid$	Ξ		from an antiunification context
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	$\mid$	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	$\mid$	$\overline{\overrightarrow{\widehat{\alpha}^{-}}_{i}}^{\,i}$		concatenate lists
$P,\ Q$	$::=$			a positive algorithmic type (potentially with metavariables)
	$\mid$	α^{+}		
	$\mid$	$\widetilde{\alpha}^{+}$		
	$\mid$	$\widehat{\alpha}^{+}$		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists\overrightarrow{\alpha^{-}}.P$		

	$\vert$	$[\sigma]P$	M	
	$\vert$	$[\widehat{\tau}]P$	M	
	$\vert$	$[\mu]P$	M	
	$\vert$	$\mathbf{nf}\,(P')$	M	
$N,\ M$	$::=$			a negative algorithmic type (potentially with metavariables)
	$\vert$	α^{-}		
	$\vert$	$\widehat{\alpha}^{-}$		
	$\vert$	$\uparrow P$		
	$\vert$	$P \to N$		
	$\vert$	$\forall \overrightarrow{\alpha^{+}}.\,N$		
	$\vert$	$[\sigma]N$	M	
	$\vert$	$[\mu]N$	M	
	$\vert$	$\mathbf{nf}\,(N')$	M	
auSol	$::=$			
	$\vert$	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$		
terminals	$::=$			
	$\vert$	$\exists$		
	$\vert$	$\forall$		
	$\vert$	$\uparrow$		
	$\vert$	$\downarrow$		
	$\vert$	$\to$		
	$\vert$	$\leftrightarrow$		
	$\vert$	$\in$		
	$\vert$	$\notin$		
	$\vert$	$\cdot$		
	$\vert$	$\vdash$		
	$\vert$	$\leqslant$		
	$\vert$	$\geqslant$		
	$\vert$	$\simeq$		
	$\vert$	$\cup$		
	$\vert$	$\cap$		
	$\vert$	$\setminus$		
	$\vert$	$\subseteq$		
	$\vert$	$\mapsto$		
	$\vert$	$\stackrel{u}{\simeq}$		
	$\vert$	$\stackrel{a}{\simeq}$		
	$\vert$	$\varnothing$		
	$\vert$	$\circ$		
	$\vert$	$\models$		
	$\vert$	$\dashv$		
	$\vert$	$\neq$		
	$\vert$	$\equiv_n$		
	$\vert$	$\vee$		
	$\vert$	$\Downarrow$		
	$\vert$	$:\geqslant$		
	$\vert$	$:\simeq$		

formula	::=		
	|	*judgement*	
	|	$formula_1 \quad .. \quad formula_n$	
	|	$\mu : vars_1 \leftrightarrow vars_2$	
	|	μ **is bijective**	
	|	$\widehat{\sigma}$ **is functional**	
	|	$\widehat{\sigma}_1 \in \widehat{\sigma}_2$	
	|	$vars_1 \subseteq vars_2$	
	|	$vars_1 = vars_2$	
	|	$vars$ **is fresh**	
	|	$\alpha^- \notin vars$	
	|	$\alpha^+ \notin vars$	
	|	$\alpha^- \in vars$	
	|	$\alpha^+ \in vars$	
	|	$\widehat{\alpha}^- \in \Theta$	
	|	$\widehat{\alpha}^+ \in \Theta$	
	|	if any other rule is not applicable	
	|	$N \neq M$	
	|	$P \neq Q$	
A	::=		
	|	$\Gamma; \Theta \models N \leqslant M \dashv \widehat{\sigma}$	Negative subtyping
	|	$\Gamma; \Theta \models P \geqslant Q \dashv \widehat{\sigma}$	Positive supertyping
AU	::=		
	|	$\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	|	$\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$	
E1	::=		
	|	$N \simeq_1^D M$	Negative multi-quantified type equivalence
	|	$P \simeq_1^D Q$	Positive multi-quantified type equivalence
	|	$P \simeq Q$	
D1	::=		
	|	$\Gamma \vdash N \simeq_1^{\leqslant} M$	Negative equivalence on MQ types
	|	$\Gamma \vdash P \simeq_1^{\leqslant} Q$	Positive equivalence on MQ types
	|	$\Gamma \vdash N \leqslant_1 M$	Negative subtyping
	|	$\Gamma \vdash P \geqslant_1 Q$	Positive supertyping
D0	::=		
	|	$\Gamma \vdash N \simeq_0^{\leqslant} M$	Negative equivalence
	|	$\Gamma \vdash P \simeq_0^{\leqslant} Q$	Positive equivalence
	|	$\Gamma \vdash N \leqslant_0 M$	Negative subtyping
	|	$\Gamma \vdash P \geqslant_0 Q$	Positive supertyping
EQ	::=		
	|	$N = M$	Negative type equality (alpha-equivalence)
	|	$P = Q$	Positive type equuality (alphha-equivalence)
	|	$P = Q$	

$LUBF$	$::=$		
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P === \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N === \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P === \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N === \vec{\alpha}$	
	$\mid$	$\mathbf{nf}\ (N') === N$	
	$\mid$	$\mathbf{nf}\ (P') === P$	
	$\mid$	$\mathbf{nf}\ (N') === N$	
	$\mid$	$\mathbf{nf}\ (P') === P$	
	$\mid$	$\mathbf{nf}\ (\vec{N}') === \vec{N}$	
	$\mid$	$\mathbf{nf}\ (\vec{P}') === \vec{P}$	
	$\mid$	$\mathbf{nf}\ (\sigma') === \sigma$	
	$\mid$	$\mathbf{nf}\ (\mu') === \mu$	
	$\mid$	$\sigma'\vert_{vars}$	
	$\mid$	$e_1\ \&\ e_2$	
	$\mid$	$\widehat{\sigma}_1\ \&\ \widehat{\sigma}_2$	
LUB	$::=$		
	$\mid$	$\Gamma \models P_1 \vee P_2 = Q$	Least Upper Bound (Least Common Supertype)
	$\mid$	$\mathbf{upgrade}\ \Gamma \vdash P\ \mathbf{to}\ \Delta = Q$	
Nrm	$::=$		
	$\mid$	$\mathbf{nf}\ (N) = M$	
	$\mid$	$\mathbf{nf}\ (P) = Q$	
	$\mid$	$\mathbf{nf}\ (N) = M$	
	$\mid$	$\mathbf{nf}\ (P) = Q$	
$Order$	$::=$		
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N = \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P = \vec{\alpha}$	
SM	$::=$		
	$\mid$	$e_1\ \&\ e_2 = e_3$	Unification Solution Entry Merge
	$\mid$	$\widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3$	Merge unification solutions
U	$::=$		
	$\mid$	$\Theta \models N \overset{u}{\simeq} M \dashv \widehat{\sigma}$	Negative unification
	$\mid$	$\Theta \models P \overset{u}{\simeq} Q \dashv \widehat{\sigma}$	Positive unification
WF	$::=$		
	$\mid$	$\Gamma \vdash N$	Negative type well-formedness
	$\mid$	$\Gamma \vdash P$	Positive type well-formedness
	$\mid$	$\Gamma \vdash N$	Negative type well-formedness
	$\mid$	$\Gamma \vdash P$	Positive type well-formedness
	$\mid$	$\Gamma \vdash \vec{N}$	Negative type list well-formedness
	$\mid$	$\Gamma \vdash \vec{P}$	Positive type list well-formedness
	$\mid$	$\Gamma; \Xi \vdash P$	Positive anti-unification type well-formedness

	$\mid$	$\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1$	Antiunification substitution well-formedness
	$\mid$	$\Theta \vdash \widehat{\sigma}$	Unification substitution well-formedness
	$\mid$	$\Gamma \vdash \Theta$	Unification context well-formedness
	$\mid$	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution well-formedness

judgement	::=	
	$\mid$	A
	$\mid$	AU
	$\mid$	$E1$
	$\mid$	$D1$
	$\mid$	$D0$
	$\mid$	EQ
	$\mid$	LUB
	$\mid$	Nrm
	$\mid$	$Order$
	$\mid$	SM
	$\mid$	U
	$\mid$	WF

user_syntax	::=	
	$\mid$	α
	$\mid$	n
	$\mid$	α^+
	$\mid$	α^-
	$\mid$	σ
	$\mid$	e
	$\mid$	$\widehat{\sigma}$
	$\mid$	$\widehat{\tau}$
	$\mid$	P
	$\mid$	N
	$\mid$	$\overrightarrow{\alpha^+}$
	$\mid$	$\overrightarrow{\alpha^-}$
	$\mid$	P
	$\mid$	N
	$\mid$	$\overrightarrow{P}$
	$\mid$	$\overrightarrow{N}$
	$\mid$	Γ
	$\mid$	Θ
	$\mid$	Ξ
	$\mid$	$\overrightarrow{\alpha}$
	$\mid$	$vars$
	$\mid$	μ
	$\mid$	n
	$\mid$	$\widetilde{\alpha}^+$
	$\mid$	$\widetilde{\alpha}^-$
	$\mid$	$\overrightarrow{\widetilde{\alpha}^+}$
	$\mid$	$\overrightarrow{\widetilde{\alpha}^-}$
	$\mid$	P
	$\mid$	N
	$\mid$	$\widehat{\alpha}^+$

| $\widehat{\alpha}^-$
| $\overrightarrow{\widetilde{\alpha}^+}$
| $\overrightarrow{\widehat{\alpha}^-}$
| P
| N
| $auSol$
| $terminals$
| $formula$

$\boxed{\Gamma;\, \Theta \models N \leqslant M \dashv \widehat{\sigma}}$ Negative subtyping

$$\frac{}{\Gamma;\, \Theta \models \alpha^- \leqslant \alpha^- \dashv \cdot}\ \textsc{ANVar}$$

$$\frac{\Theta \models \mathbf{nf}\,(P) \overset{u}{\simeq} \mathbf{nf}\,(Q) \dashv \widehat{\sigma}}{\Gamma;\, \Theta \models {\uparrow}P \leqslant {\uparrow}Q \dashv \widehat{\sigma}}\ \textsc{AShiftU}$$

$$\frac{\Gamma;\, \Theta \models P \geqslant Q \dashv \widehat{\sigma}_1 \quad \Gamma;\, \Theta \models N \leqslant M \dashv \widehat{\sigma}_2}{\Gamma;\, \Theta \models P \to N \leqslant Q \to M \dashv \widehat{\sigma}_1 \,\&\, \widehat{\sigma}_2}\ \textsc{AArrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^+};\, \Theta, \overrightarrow{\widehat{\alpha}^+\{\Gamma, \overrightarrow{\beta^+}\}} \models [\overrightarrow{\widehat{\alpha}^+}/\overrightarrow{\alpha^+}]N \leqslant M \dashv \widehat{\sigma}}{\Gamma;\, \Theta \models \forall\overrightarrow{\alpha^+}.N \leqslant \forall\overrightarrow{\beta^+}.M \dashv \widehat{\sigma}\backslash\overrightarrow{\widehat{\alpha}^+}}\ \textsc{AForall}$$

$\boxed{\Gamma;\, \Theta \models P \geqslant Q \dashv \widehat{\sigma}}$ Positive supertyping

$$\frac{}{\Gamma;\, \Theta \models \alpha^+ \geqslant \alpha^+ \dashv \cdot}\ \textsc{APVar}$$

$$\frac{\Theta \models \mathbf{nf}\,(N) \overset{u}{\simeq} \mathbf{nf}\,(M) \dashv \widehat{\sigma}}{\Gamma;\, \Theta \models {\downarrow}N \geqslant {\downarrow}M \dashv \widehat{\sigma}}\ \textsc{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-};\, \Theta, \overrightarrow{\widehat{\alpha}^-\{\Gamma, \overrightarrow{\beta^-}\}} \models [\overrightarrow{\widehat{\alpha}^-}/\overrightarrow{\alpha^-}]P \geqslant Q \dashv \widehat{\sigma}}{\Gamma;\, \Theta \models \exists\overrightarrow{\alpha^-}.P \geqslant \exists\overrightarrow{\beta^-}.Q \dashv \widehat{\sigma}}\ \textsc{AExists}$$

$$\frac{\mathbf{upgrade}\,\Gamma \vdash \mathbf{nf}\,(P)\,\mathbf{to}\,\Delta = Q}{\Gamma;\, \Theta \models \widehat{\alpha}^+\{\Delta\} \geqslant P \dashv (\Delta \vdash \widehat{\alpha}^+ :\geqslant Q)}\ \textsc{APUVar}$$

$\boxed{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\cdot, \alpha^+, \cdot, \cdot)}\ \textsc{AUPVar}$$

$$\frac{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models {\downarrow}N_1 \overset{a}{\simeq} {\downarrow}N_2 \dashv (\Xi, {\downarrow}M, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUPShift}$$

$$\frac{\{\overrightarrow{\alpha^-}\} \cap \{\Gamma\} = \varnothing \quad \Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \exists\overrightarrow{\alpha^-}.P_1 \overset{a}{\simeq} \exists\overrightarrow{\alpha^-}.P_2 \dashv (\Xi, \exists\overrightarrow{\alpha^-}.Q, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUPExists}$$

$\boxed{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^- \overset{a}{\simeq} \alpha^- \dashv (\Xi, \alpha^-, \cdot, \cdot)}\ \textsc{AUNVar}$$

$$\frac{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models {\uparrow}P_1 \overset{a}{\simeq} {\uparrow}P_2 \dashv (\Xi, {\uparrow}Q, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUNShift}$$

$$\frac{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \hat{\tau}_1, \hat{\tau}_2) \quad \Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \hat{\tau}_1', \hat{\tau}_2')}{\Gamma \vDash P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \hat{\tau}_1 \cup \hat{\tau}_1', \hat{\tau}_2 \cup \hat{\tau}_2')} \ \text{AUNArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \vDash N \overset{a}{\simeq} M \dashv (\hat{\alpha}^-_{\{N,M\}}, \hat{\alpha}^-_{\{N,M\}}, (\hat{\alpha}^-_{\{N,M\}} :\approx N), (\hat{\alpha}^-_{\{N,M\}} :\approx M))} \ \text{AUNAU}$$

$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-} \ \text{E1NVar}$$

$$\frac{P \simeq_1^D Q}{\uparrow P \simeq_1^D \uparrow Q} \ \text{E1ShiftU}$$

$$\frac{P \simeq_1^D Q \quad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M} \ \text{E1Arrow}$$

$$\frac{\{\overrightarrow{\alpha^+}\} \cap \mathbf{fv}\, M = \varnothing \quad \mu : (\{\overrightarrow{\beta^+}\} \cap \mathbf{fv}\, M) \leftrightarrow (\{\overrightarrow{\alpha^+}\} \cap \mathbf{fv}\, N) \quad N \simeq_1^D [\mu]M}{\forall \overrightarrow{\alpha^+}.N \simeq_1^D \forall \overrightarrow{\beta^+}.M} \ \text{E1Forall}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+} \ \text{E1PVar}$$

$$\frac{N \simeq_1^D M}{\downarrow N \simeq_1^D \downarrow M} \ \text{E1ShiftD}$$

$$\frac{\{\overrightarrow{\alpha^-}\} \cap \mathbf{fv}\, Q = \varnothing \quad \mu : (\{\overrightarrow{\beta^-}\} \cap \mathbf{fv}\, Q) \leftrightarrow (\{\overrightarrow{\alpha^-}\} \cap \mathbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists \overrightarrow{\alpha^-}.P \simeq_1^D \exists \overrightarrow{\beta^-}.Q} \ \text{E1Exists}$$

$\boxed{P \simeq Q}$

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M} \ \text{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q} \ \text{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-} \ \text{D1NVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant_1 \uparrow Q} \ \text{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M} \ \text{D1Arrow}$$

$$\frac{\mathbf{fv}\, N \cap \{\overrightarrow{\beta^+}\} = \varnothing \quad \Gamma, \overrightarrow{\beta^+} \vdash P_i \quad \Gamma, \overrightarrow{\beta^+} \vdash [\overrightarrow{P}/\overrightarrow{\alpha^+}]N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant_1 \forall \overrightarrow{\beta^+}.M} \ \text{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+} \quad \text{D1PVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M} \quad \text{D1ShiftD}$$

$$\frac{\mathbf{fv}\, P \cap \{\overrightarrow{\beta^-}\} = \varnothing \quad \Gamma, \overrightarrow{\beta^-} \vdash N_i \quad \Gamma, \overrightarrow{\beta^-} \vdash [\overrightarrow{N}/\overrightarrow{\alpha^-}]P \geqslant_1 Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P \geqslant_1 \exists \overrightarrow{\beta^-}.Q} \quad \text{D1Exists}$$

$\boxed{\Gamma \vdash N \simeq_0^{\leqslant} M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq_0^{\leqslant} M} \quad \text{D0NDef}$$

$\boxed{\Gamma \vdash P \simeq_0^{\leqslant} Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq_0^{\leqslant} Q} \quad \text{D0PDef}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_0 \alpha^-} \quad \text{D0NVar}$$

$$\frac{\Gamma \vdash P \simeq_0^{\leqslant} Q}{\Gamma \vdash \uparrow P \leqslant_0 \uparrow Q} \quad \text{D0ShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/\alpha^+]N \leqslant_0 M \quad M \neq \forall \beta^+.M'}{\Gamma \vdash \forall \alpha^+.N \leqslant_0 M} \quad \text{D0ForallL}$$

$$\frac{\Gamma, \alpha^+ \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall \alpha^+.M} \quad \text{D0ForallR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \to N \leqslant_0 Q \to M} \quad \text{D0Arrow}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_0 \alpha^+} \quad \text{D0PVar}$$

$$\frac{\Gamma \vdash N \simeq_0^{\leqslant} M}{\Gamma \vdash \downarrow N \geqslant_0 \downarrow M} \quad \text{D0ShiftD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/\alpha^-]P \geqslant_0 Q \quad Q \neq \exists \alpha^-.Q'}{\Gamma \vdash \exists \alpha^-.P \geqslant_0 Q} \quad \text{D0ExistsL}$$

$$\frac{\Gamma, \alpha^- \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists \alpha^-.Q} \quad \text{D0ExistsR}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P \;=\; Q}$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ \mathit{vars}\ \mathbf{in}\ N}$

$\boxed{\textbf{ord}\, vars\, \textbf{in}\, P}$

$\boxed{\textbf{ord}\, vars\, \textbf{in}\, N}$

$\boxed{\textbf{nf}\,(N')}$

$\boxed{\textbf{nf}\,(P')}$

$\boxed{\textbf{nf}\,(N')}$

$\boxed{\textbf{nf}\,(P')}$

$\boxed{\textbf{nf}\,(\overrightarrow{N'})}$

$\boxed{\textbf{nf}\,(\overrightarrow{P'})}$

$\boxed{\textbf{nf}\,(\sigma')}$

$\boxed{\textbf{nf}\,(\mu')}$

$\boxed{\sigma'|_{vars}}$

$\boxed{e_1 \,\&\, e_2}$

$\boxed{\widehat{\sigma}_1 \,\&\, \widehat{\sigma}_2}$

$\boxed{\Gamma \vDash P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \vDash \alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBVar}$$

$$\frac{\Gamma, \cdot \vDash {\downarrow}N \overset{a}{\simeq} {\downarrow}M \dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash {\downarrow}N \vee {\downarrow}M = \exists\overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi]P} \quad \text{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \models P_1 \vee P_2 = Q}{\Gamma \models \exists\overrightarrow{\alpha^-}.P_1 \vee \exists\overrightarrow{\beta^-}.P_2 = Q}\ \textsc{LUBExists}$$

$\boxed{\textbf{upgrade}\,\Gamma \vdash P\,\textbf{to}\,\Delta = Q}$

$\boxed{\textbf{nf}\,(N) = M}$

$$\frac{}{\textbf{nf}\,(\alpha^-) = \alpha^-}\ \textsc{NrmNVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\uparrow P) = \uparrow Q}\ \textsc{NrmShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(P \to N) = Q \to M}\ \textsc{NrmArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\forall\overrightarrow{\alpha^+}.N) = \forall\overrightarrow{\alpha^{+\prime}}.N'}\ \textsc{NrmForall}$$

$\boxed{\textbf{nf}\,(P) = Q}$

$$\frac{}{\textbf{nf}\,(\alpha^+) = \alpha^+}\ \textsc{NrmPVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\downarrow N) = \downarrow M}\ \textsc{NrmShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\textbf{nf}\,(\exists\overrightarrow{\alpha^-}.P) = \exists\overrightarrow{\alpha^{-\prime}}.P'}\ \textsc{NrmExists}$$

$\boxed{\textbf{nf}\,(N) = M}$

$$\frac{}{\textbf{nf}\,(\widehat{\alpha}^-) = \widehat{\alpha}^-}\ \textsc{NrmNUVar}$$

$\boxed{\textbf{nf}\,(P) = Q}$

$$\frac{}{\textbf{nf}\,(\widehat{\alpha}^+) = \widehat{\alpha}^+}\ \textsc{NrmPUVar}$$

$\boxed{\textbf{ord}\,vars\,\textbf{in}\,N = \vec{\alpha}}$

$$\frac{\alpha^- \in vars}{\textbf{ord}\,vars\,\textbf{in}\,\alpha^- = \alpha^-}\ \textsc{ONVarIn}$$

$$\frac{\alpha^- \notin vars}{\textbf{ord}\,vars\,\textbf{in}\,\alpha^- = \cdot}\ \textsc{ONVarNIn}$$

$$\frac{\textbf{ord}\,vars\,\textbf{in}\,P = \vec{\alpha}}{\textbf{ord}\,vars\,\textbf{in}\,\uparrow P = \vec{\alpha}}\ \textsc{OShiftU}$$

$$\frac{\textbf{ord}\,vars\,\textbf{in}\,P = \vec{\alpha}_1 \quad \textbf{ord}\,vars\,\textbf{in}\,N = \vec{\alpha}_2}{\textbf{ord}\,vars\,\textbf{in}\,P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \{\vec{\alpha}_1\})}\ \textsc{OArrow}$$

$$\frac{vars \cap \{\overrightarrow{\alpha^+}\} = \varnothing \quad \textbf{ord}\,vars\,\textbf{in}\,N = \vec{\alpha}}{\textbf{ord}\,vars\,\textbf{in}\,\forall\overrightarrow{\alpha^+}.N = \vec{\alpha}}\ \textsc{OForall}$$

$\boxed{\textbf{ord}\,vars\,\textbf{in}\,P = \vec{\alpha}}$

$$\frac{\alpha^+ \in vars}{\textbf{ord}\,vars\,\textbf{in}\,\alpha^+ = \alpha^+}\ \textsc{OPVarIn}$$

$$\frac{\alpha^+ \notin vars}{\mathbf{ord}\, vars\, \mathbf{in}\, \alpha^+ = \cdot}\ \textsc{OPVarNIn}$$

$$\frac{\mathbf{ord}\, vars\, \mathbf{in}\, N = \vec{\alpha}}{\mathbf{ord}\, vars\, \mathbf{in}\, {\downarrow} N = \vec{\alpha}}\ \textsc{OShiftD}$$

$$\frac{vars \cap \{\overrightarrow{\alpha^-}\} = \varnothing \quad \mathbf{ord}\, vars\, \mathbf{in}\, P = \vec{\alpha}}{\mathbf{ord}\, vars\, \mathbf{in}\, \exists \overrightarrow{\alpha^-}. P = \vec{\alpha}}\ \textsc{OExists}$$

$\boxed{\mathbf{ord}\, vars\, \mathbf{in}\, N = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\, vars\, \mathbf{in}\, \widehat{\alpha}^- = \cdot}\ \textsc{ONUVar}$$

$\boxed{\mathbf{ord}\, vars\, \mathbf{in}\, P = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\, vars\, \mathbf{in}\, \widehat{\alpha}^+ = \cdot}\ \textsc{OPUVar}$$

$\boxed{e_1 \ \&\ e_2 = e_3}$ Unification Solution Entry Merge

$$\frac{\Gamma \vDash P_1 \vee P_2 = Q}{(\Gamma \vdash \widehat{\alpha}^+ :\geqslant P_1)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\geqslant P_2) = (\Gamma \vdash \widehat{\alpha}^+ :\geqslant Q)}\ \textsc{SMESupSup}$$

$$\frac{\Gamma; \cdot \vDash P \geqslant Q \dashv \widehat{\sigma}'}{(\Gamma \vdash \widehat{\alpha}^+ :\approx P)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\geqslant Q) = (\Gamma \vdash \widehat{\alpha}^+ :\approx P)}\ \textsc{SMEEqSup}$$

$$\frac{\Gamma; \cdot \vDash Q \geqslant P \dashv \widehat{\sigma}'}{(\Gamma \vdash \widehat{\alpha}^+ :\geqslant P)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\approx Q) = (\Gamma \vdash \widehat{\alpha}^+ :\approx Q)}\ \textsc{SMESupEq}$$

$$\frac{}{(\Gamma \vdash \widehat{\alpha}^+ :\approx P)\ \&\ (\Gamma \vdash \widehat{\alpha}^+ :\approx P) = (\Gamma \vdash \widehat{\alpha}^+ :\approx P)}\ \textsc{SMEPEqEq}$$

$$\frac{}{(\Gamma \vdash \widehat{\alpha}^- :\approx N)\ \&\ (\Gamma \vdash \widehat{\alpha}^- :\approx N) = (\Gamma \vdash \widehat{\alpha}^- :\approx N)}\ \textsc{SMENEqEq}$$

$\boxed{\widehat{\sigma}_1 \& \widehat{\sigma}_2 = \widehat{\sigma}_3}$ Merge unification solutions

$\boxed{\Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}}$ Negative unification

$$\frac{}{\Theta \vDash \alpha^- \overset{u}{\simeq} \alpha^- \dashv \cdot}\ \textsc{UNVar}$$

$$\frac{\Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}{\Theta \vDash {\uparrow} P \overset{u}{\simeq} {\uparrow} Q \dashv \widehat{\sigma}}\ \textsc{UShiftU}$$

$$\frac{\Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}_1 \quad \Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}_2}{\Theta \vDash P \to N \overset{u}{\simeq} Q \to M \dashv \widehat{\sigma}_1 \ \&\ \widehat{\sigma}_2}\ \textsc{UArrow}$$

$$\frac{\Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}}{\Theta \vDash \forall \overrightarrow{\alpha^+}. N \overset{u}{\simeq} \forall \overrightarrow{\alpha^+}. M \dashv \widehat{\sigma}}\ \textsc{UForall}$$

$$\frac{\widehat{\alpha}^-\{\Delta\} \in \Theta \quad \Delta \vdash N}{\Theta \vDash \widehat{\alpha}^- \overset{u}{\simeq} N \dashv (\Delta \vdash \widehat{\alpha}^- :\approx N)}\ \textsc{UNUVar}$$

$\boxed{\Theta \vDash P \overset{u}{\simeq} Q \dashv \widehat{\sigma}}$ Positive unification

$$\frac{}{\Theta \vDash \alpha^+ \overset{u}{\simeq} \alpha^+ \dashv \cdot}\ \textsc{UPVar}$$

$$\frac{\Theta \vDash N \overset{u}{\simeq} M \dashv \widehat{\sigma}}{\Theta \vDash {\downarrow} N \overset{u}{\simeq} {\downarrow} M \dashv \widehat{\sigma}}\ \textsc{UShiftD}$$

$$\frac{\Theta \models \boxed{P} \overset{u}{\simeq} Q \dashv \widehat{\sigma}}{\Theta \models \exists\overrightarrow{\alpha^-}.\boxed{P} \overset{u}{\simeq} \exists\overrightarrow{\alpha^-}.Q \dashv \widehat{\sigma}} \quad \textsc{UExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \Delta \vdash P}{\Theta \models \widehat{\alpha}^+ \overset{u}{\simeq} P \dashv (\Delta \vdash \widehat{\alpha}^+ :\approx P)} \quad \textsc{UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness

$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness

$\boxed{\Gamma; \Xi \vdash P}$ Positive anti-unification type well-formedness

$\boxed{\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness

$\boxed{\Theta \vdash \widehat{\sigma}}$ Unification substitution well-formedness

$\boxed{\Gamma \vdash \Theta}$ Unification context well-formedness

$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution well-formedness

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Definition rules:        72 good    7 bad
Definition rule clauses: 130 good    7 bad
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