α, β, α, β, γ, δ	type variables
n, m, i, j	index variables
x, y, z	term variables

$n,\ m$	$::=$			cohort index
	$\mid$	0		
	$\mid$	$n+1$		
$\alpha^+,\ \beta^+,\ \gamma^+,\ \delta^+$	$::=$			positive variable
	$\mid$	α^+		
	$\mid$	α^{+n}		
	$\mid$	$\beta^\pm$		
$\alpha^-,\ \beta^-,\ \gamma^-,\ \delta^-$	$::=$			negative variable
	$\mid$	α^-		
	$\mid$	α^{-n}		
	$\mid$	$\alpha^\pm$		
$\alpha^\pm,\ \beta^\pm$	$::=$			positive or negative variable
	$\mid$	$\alpha^\pm$		
	$\mid$	$\alpha^{\pm n}$		
σ	$::=$			substitution
	$\mid$	$\cdot$		
	$\mid$	id		
	$\mid$	P/α^+		
	$\mid$	N/α^-		
	$\mid$	$\overrightarrow{P}/\overrightarrow{\alpha^+}$		
	$\mid$	$\overrightarrow{N}/\overrightarrow{\alpha^-}$		
	$\mid$	$\mathbf{pmas}\,/\overrightarrow{\alpha^+}$		
	$\mid$	$\mathbf{nmas}\,/\overrightarrow{\alpha^-}$		
	$\mid$	$\overrightarrow{\widetilde{\alpha^+}}/\overrightarrow{\alpha^+}$		
	$\mid$	$\overrightarrow{\widehat{\alpha^-}}/\overrightarrow{\alpha^-}$		
	$\mid$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\mid$	μ		
	$\mid$	$\sigma_1 \circ \sigma_2$		
	$\mid$	$\overrightarrow{\alpha}_1/\overrightarrow{\alpha}_2$		
	$\mid$	(σ)	S	
	$\mid$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\mid$	$\mathbf{nf}\,(\sigma')$	M	
	$\mid$	$\sigma'\rvert_{vars}$	M	
e	$::=$			entry of a subtyping constraint
	$\mid$	$\widehat{\alpha}^+ :\approx P$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		
	$\mid$	$\widehat{\alpha}^+ :\geqslant P$		
	$\mid$	(e)	S	
	$\mid$	$SC(\widehat{\alpha}^-)$	M	
	$\mid$	$e_1\ \&\ e_2$	M	
e	$::=$			entry of a unification constraint
	$\mid$	$\widehat{\alpha}^+ :\approx P$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		

	$\vert$	(e)	S	
	$\vert$	$e_1 \ \& \ e_2$	M	
UC	$::=$			unification constraint
	$\vert$	$\cdot$		
	$\vert$	e		
	$\vert$	$UC \backslash vars$		
	$\vert$	$UC \vert vars$		
	$\vert$	$UC_1 \cup UC_2$		
	$\vert$	$\overline{UC_i}^{\,i}$		concatenate
	$\vert$	(UC)	S	
	$\vert$	$\mathbf{UC}\vert_{vars}$	M	
	$\vert$	$UC_1 \ \& \ UC_2$	M	
	$\vert$	$UC_1 \cup UC_2$	M	
	$\vert$	$\vert SC \vert$	M	
SC	$::=$			subtyping constraint
	$\vert$	$\cdot$		
	$\vert$	e		
	$\vert$	$SC \backslash vars$		
	$\vert$	$SC \vert vars$		
	$\vert$	$SC_1 \cup SC_2$		
	$\vert$	UC		
	$\vert$	$\overline{SC_i}^{\,i}$		concatenate
	$\vert$	(SC)	S	
	$\vert$	$\mathbf{SC}\vert_{vars}$	M	
	$\vert$	$SC_1 \ \& \ SC_2$	M	
$\widehat{\sigma}$	$::=$			unification substitution
	$\vert$	$\cdot$		
	$\vert$	$P/\widehat{\alpha}^+$		
	$\vert$	$N/\widehat{\alpha}^-$		
	$\vert$	$\overrightarrow{P}/\overrightarrow{\widetilde{\alpha^+}}$		
	$\vert$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	$(\widehat{\sigma})$	S	
	$\vert$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\vert$	$\mathbf{nf}\,(\widehat{\sigma}')$	M	
	$\vert$	$\widehat{\sigma}'\vert_{vars}$	M	
$\widehat{\tau},\ \widehat{\rho}$	$::=$			anti-unification substitution
	$\vert$	$\cdot$		
	$\vert$	$\widehat{\alpha}^- :\approx N$		
	$\vert$	$\widehat{\alpha}^- :\approx N$		
	$\vert$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
	$\vert$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	$\vert$	$(\widehat{\tau})$	S	
	$\vert$	$\widehat{\tau}'\vert_{vars}$	M	
	$\vert$	$\widehat{\tau}_1 \ \& \ \widehat{\tau}_2$	M	

$P,\ Q,\ R$	$::=$			positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists \alpha^-.P$		
	$\mid$	$[\sigma]P$	M	
$N,\ M,\ K$	$::=$			negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$\forall \alpha^+.N$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$[\sigma]N$	M	
$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+},\ \overrightarrow{\gamma^+},\ \overrightarrow{\delta^+}$	$::=$			positive variable list
	$\mid$	$\cdot$		empty list
	$\mid$	α^+		a variable
	$\mid$	$\overrightarrow{\alpha^+}$		a variable
	$\mid$	$\overline{\overrightarrow{\alpha^+}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$	$::=$			negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	α^-		a variable
	$\mid$	$\overrightarrow{\alpha^-}$		variables
	$\mid$	$\overline{\overrightarrow{\alpha^-}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$	$::=$			positive or negative variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\alpha^\pm$		a variable
	$\mid$	$\overrightarrow{\mathbf{pa}}$		variables
	$\mid$	$\overline{\overrightarrow{\alpha^\pm}_i}^{\,i}$		concatenate lists
$P,\ Q,\ R$	$::=$			multi-quantified positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists \overrightarrow{\alpha^-}.P$		$P \neq \exists \ldots$
	$\mid$	$[\sigma]P$	M	
	$\mid$	$[\hat{\tau}]P$	M	
	$\mid$	$[\hat{\sigma}]P$	M	
	$\mid$	$[\mu]P$	M	
	$\mid$	(P)	S	
	$\mid$	$P_1 \vee P_2$	M	
	$\mid$	$\mathbf{nf}\,(P')$	M	
$N,\ M,\ K$	$::=$			multi-quantified negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$\forall \overrightarrow{\alpha^+}.N$		$N \neq \forall \ldots$
	$\mid$	$[\sigma]N$	M	

	$\vert$	$[\widehat{\tau}]N$	M	
	$\vert$	$[\mu]N$	M	
	$\vert$	$[\widehat{\sigma}]N$	M	
	$\vert$	(N)	S	
	$\vert$	$\mathbf{nf}\,(N')$	M	
$\overrightarrow{P},\ \overrightarrow{Q}$	$::=$			list of positive types
	$\vert$	$\cdot$		empty list
	$\vert$	P		a singel type
	$\vert$	$[\sigma]\overrightarrow{P}$	M	
	$\vert$	$\overline{\overrightarrow{P}_i}^{\,i}$		concatenate lists
	$\vert$	$\mathbf{nf}\,(\overrightarrow{P}')$	M	
$\overrightarrow{N},\ \overrightarrow{M}$	$::=$			list of negative types
	$\vert$	$\cdot$		empty list
	$\vert$	N		a singel type
	$\vert$	$[\sigma]\overrightarrow{N}$	M	
	$\vert$	$\overline{\overrightarrow{N}_i}^{\,i}$		concatenate lists
	$\vert$	$\mathbf{nf}\,(\overrightarrow{N}')$	M	
$\Delta,\ \Gamma$	$::=$			declarative type context
	$\vert$	$\cdot$		empty context
	$\vert$	$\overrightarrow{\alpha^+}$		list of variables
	$\vert$	$\overrightarrow{\alpha^-}$		list of variables
	$\vert$	$\overrightarrow{\alpha^\pm}$		list of variables
	$\vert$	$vars$		
	$\vert$	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	$\vert$	(Γ)	S	
	$\vert$	$\Theta(\widehat{\alpha}^+)$	M	
	$\vert$	$\Theta(\widehat{\alpha}^-)$	M	
Θ	$::=$			unification type variable context
	$\vert$	$\cdot$		empty context
	$\vert$	$\overrightarrow{\alpha}\{\Delta\}$		from an ordered list of variables
	$\vert$	$\overrightarrow{\widehat{\alpha}^+\{\Delta\}}$		from a variable to a list
	$\vert$	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	$\vert$	(Θ)	S	
	$\vert$	$\Theta\vert_{vars}$		leave only those variables that are in the set
	$\vert$	$\Theta_1 \cup \Theta_2$		
Ξ	$::=$			anti-unification type variable context
	$\vert$	$\cdot$		empty context
	$\vert$	$\overrightarrow{\widehat{\alpha}^-}$		list of variables
	$\vert$	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	$\vert$	(Ξ)	S	
	$\vert$	$\Xi_1 \cup \Xi_2$		
	$\vert$	$\Xi_1 \cap \Xi_2$		
	$\vert$	$\Xi'\vert_{vars}$	M	

$\vec{\alpha},\ \vec{\beta}$	::=			ordered positive or negative variables
	|	$\cdot$		empty list
	|	$\overrightarrow{\alpha^{+}}$		list of variables
	|	$\overrightarrow{\alpha^{-}}$		list of variables
	|	$\overrightarrow{\alpha^{\pm}}$		list of variables
	|	$\overrightarrow{\widetilde{\alpha^{+}}}$		list of variables
	|	$\overrightarrow{\widehat{\alpha^{-}}}$		list of variables
	|	$\vec{\alpha}_1 \backslash vars$		setminus
	|	Γ		context
	|	$vars$		
	|	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	|	$(\vec{\alpha})$	S	parenthesis
	|	$[\mu]\vec{\alpha}$	M	apply moving to list
	|	$\mathbf{ord}\, vars\, \mathbf{in}\, P$	M	
	|	$\mathbf{ord}\, vars\, \mathbf{in}\, N$	M	
	|	$\mathbf{ord}\, vars\, \mathbf{in}\, P$	M	
	|	$\mathbf{ord}\, vars\, \mathbf{in}\, N$	M	
$vars$	::=			set of variables
	|	$\varnothing$		empty set
	|	$\mathbf{fv}\, P$		free variables
	|	$\mathbf{fv}\, N$		free variables
	|	$\mathbf{fv}\, \mathbf{imP}$		free variables
	|	$\mathbf{fv}\, \mathbf{imN}$		free variables
	|	$vars_1 \cap vars_2$		set intersection
	|	$vars_1 \cup vars_2$		set union
	|	$vars_1 \backslash vars_2$		set complement
	|	$\mathbf{mv}\, \mathbf{imP}$		movable variables
	|	$\mathbf{mv}\, \mathbf{imN}$		movable variables
	|	$\mathbf{uv}\, N$		unification variables
	|	$\mathbf{uv}\, P$		unification variables
	|	$\mathbf{fv}\, N$		free variables
	|	$\mathbf{fv}\, P$		free variables
	|	$(vars)$	S	parenthesis
	|	$\vec{\alpha}$		ordered list of variables
	|	$[\mu]vars$	M	apply moving to varset
	|	$\mathbf{dom}\,(UC)$	M	
	|	$\mathbf{dom}\,(SC)$	M	
	|	$\mathbf{dom}\,(\widehat{\sigma})$	M	
	|	$\mathbf{dom}\,(\widehat{\tau})$	M	
	|	$\mathbf{dom}\,(\Theta)$	M	
μ	::=			
	|	$\cdot$		empty moving
	|	$pma1 \mapsto pma2$		Positive unit substitution
	|	$nma1 \mapsto nma2$		Positive unit substitution
	|	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	|	$\mu_1 \circ \mu_2$	M	Composition
	|	$\overline{\mu_i}^{\,i}$		concatenate movings
	|	$\mu\vert_{vars}$	M	restriction on a set

	$\vert$	μ^{-1}	M	inversion
	$\vert$	$\mathbf{nf}\,(\mu')$	M	
$\widehat{\alpha}^{\pm}$	$::=$			positive/negative unification variable
	$\vert$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	$::=$			positive unification variable
	$\vert$	$\widehat{\alpha}^{+}$		
	$\vert$	$\widehat{\alpha}^{+}\{\Delta\}$		
	$\vert$	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	$::=$			negative unification variable
	$\vert$	$\widehat{\alpha}^{-}$		
	$\vert$	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	$\vert$	$\widehat{\alpha}^{-}\{\Delta\}$		
	$\vert$	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widetilde{\alpha^{+}}},\ \overrightarrow{\widetilde{\beta^{+}}}$	$::=$			positive unification variable list
	$\vert$	$\cdot$		empty list
	$\vert$	$\widehat{\alpha}^{+}$		a variable
	$\vert$	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	$\vert$	$\overline{\overrightarrow{\widetilde{\alpha^{+}}}_{i}}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha^{-}}},\ \overrightarrow{\widehat{\beta^{-}}}$	$::=$			negative unification variable list
	$\vert$	$\cdot$		empty list
	$\vert$	$\widehat{\alpha}^{-}$		a variable
	$\vert$	Ξ		from an antiunification context
	$\vert$	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	$\vert$	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	$\vert$	$\overline{\overrightarrow{\widehat{\alpha^{-}}}_{i}}^{\,i}$		concatenate lists
$P,\ Q$	$::=$			a positive algorithmic type (potentially with metavariables)
	$\vert$	α^{+}		
	$\vert$	$\mathbf{pma}$		
	$\vert$	$\widehat{\alpha}^{+}$		
	$\vert$	$\downarrow N$		
	$\vert$	$\exists \overrightarrow{\alpha^{-}}.\, P$		
	$\vert$	$[\sigma]\, P$	M	
	$\vert$	$[\widehat{\tau}]\, P$	M	
	$\vert$	$[\mu]\, P$	M	
	$\vert$	(P)	S	
	$\vert$	$\mathbf{nf}\,(P')$	M	
$N,\ M$	$::=$			a negative algorithmic type (potentially with metavariables)
	$\vert$	α^{-}		
	$\vert$	$\widehat{\alpha}^{-}$		
	$\vert$	$\uparrow P$		
	$\vert$	$P \to N$		
	$\vert$	$\forall \overrightarrow{\alpha^{+}}.\, N$		

	$\mid$	$[\sigma]N$	M
	$\mid$	$[\widehat{\tau}]N$	M
	$\mid$	$[\mu]N$	M
	$\mid$	(N)	S
	$\mid$	$\mathbf{nf}\,(N')$	M
$auSol$	$::=$		
	$\mid$	$(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	$\mid$	$(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$	
$terminals$	$::=$		
	$\mid$	$\exists$	
	$\mid$	$\forall$	
	$\mid$	$\uparrow$	
	$\mid$	$\downarrow$	
	$\mid$	$\rightarrow$	
	$\mid$	$\leftrightarrow$	
	$\mid$	$\in$	
	$\mid$	$\notin$	
	$\mid$	$\cdot$	
	$\mid$	$\vdash$	
	$\mid$	$\leqslant$	
	$\mid$	$\geqslant$	
	$\mid$	$\simeq$	
	$\mid$	$\cup$	
	$\mid$	$\cap$	
	$\mid$	$\backslash$	
	$\mid$	$\subseteq$	
	$\mid$	$\mapsto$	
	$\mid$	$\stackrel{u}{\simeq}$	
	$\mid$	$\stackrel{a}{\simeq}$	
	$\mid$	$\varnothing$	
	$\mid$	$\circ$	
	$\mid$	$\Rightarrow$	
	$\mid$	$\vDash$	
	$\mid$	$\Dashv$	
	$\mid$	$\neq$	
	$\mid$	$\equiv_n$	
	$\mid$	$\vee$	
	$\mid$	$\Downarrow$	
	$\mid$	$:\geqslant$	
	$\mid$	$:\simeq$	
	$\mid$	Λ	
	$\mid$	λ	
	$\mid$	$\mathbf{let}^{\exists}$	
	$\mid$	$\bullet$	
	$\mid$	$\Rightarrow\!\!>$	
$v,\ w$	$::=$		value terms
	$\mid$	x	

	$\vert$	$\{c\}$	
	$\vert$	$(v : P)$	
	$\vert$	(v)	M
$\vec{v}$	::=		list of arguments
	$\vert$	$\cdot$	
	$\vert$	v	
	$\vert$	$\overline{\vec{v}_i}^{\,i}$	concatenate
$c,\ d$	::=		computation terms
	$\vert$	$(c : N)$	
	$\vert$	$\lambda x : P.c$	
	$\vert$	$\Lambda\alpha^{+}.c$	
	$\vert$	$\mathbf{return}\, v$	
	$\vert$	$\mathbf{let}\, x = v; c$	
	$\vert$	$\mathbf{let}\, x : P = v(\vec{v}); c$	
	$\vert$	$\mathbf{let}\, x = v(\vec{v}); c$	
	$\vert$	$\mathbf{let}^{\exists}(\overrightarrow{\alpha^{-}}, x) = v; c$	
$vctx,\ \Phi$	::=		variable context
	$\vert$	$\cdot$	
	$\vert$	$x : P$	
	$\vert$	$\overline{\Phi_i}^{\,i}$	concatenate contexts
$formula$	::=		
	$\vert$	$judgement$	
	$\vert$	$judgement$ unique	
	$\vert$	$formula_1 \quad .. \quad formula_n$	
	$\vert$	$\mu : vars_1 \leftrightarrow vars_2$	
	$\vert$	$\mu\,\mathbf{is\ bijective}$	
	$\vert$	$x : P \in \Phi$	
	$\vert$	$UC_1 \subseteq UC_2$	
	$\vert$	$UC_1 = UC_2$	
	$\vert$	$SC_1 \subseteq SC_2$	
	$\vert$	$vars_1 \subseteq vars_2$	
	$\vert$	$vars_1 = vars_2$	
	$\vert$	$vars\,\mathbf{is\ fresh}$	
	$\vert$	$\alpha^{-} \notin vars$	
	$\vert$	$\alpha^{+} \notin vars$	
	$\vert$	$\alpha^{-} \in vars$	
	$\vert$	$\alpha^{+} \in vars$	
	$\vert$	$\widehat{\alpha}^{+} \in vars$	
	$\vert$	$\widehat{\alpha}^{-} \in vars$	
	$\vert$	$\widehat{\alpha}^{-} \in \Theta$	
	$\vert$	$\widehat{\alpha}^{+} \in \Theta$	
	$\vert$	if any other rule is not applicable	
	$\vert$	$\vec{\alpha}_1 = \vec{\alpha}_2$	
	$\vert$	$e_1 = e_2$	
	$\vert$	$e_1 = e_2$	
	$\vert$	$\widehat{\sigma}_1 = \widehat{\sigma}_2$	

	|	$N = M$	
	|	$\Theta \subseteq \Theta'$	
	|	$\vec{v}_1 = \vec{v}_2$	
	|	$N \neq M$	
	|	$P \neq Q$	
	|	$N \neq M$	
	|	$P \neq Q$	
	|	$P \neq Q$	
	|	$N \neq M$	
	|	$\vec{v}_1 \neq \vec{v}_2$	
	|	$\overrightarrow{\alpha^+}_1 \neq \overrightarrow{\alpha^+}_2$	
A	::=		
	|	$\Gamma;\ \Theta \models N \leqslant M \dashv SC$	Negative subtyping
	|	$\Gamma;\ \Theta \models P \geqslant Q \dashv SC$	Positive supertyping
AT	::=		
	|	$\Gamma; \Phi \models v : P$	Positive type inference
	|	$\Gamma; \Phi \models c : N$	Negative type inference
	|	$\Gamma; \Phi; \Theta_1 \models N \bullet \vec{v} \Rightarrow\!\!\!> M \dashv \Theta_2; SC$	Application type inference
AU	::=		
	|	$\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	|	$\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$	
SCM	::=		
	|	$\Gamma \vdash e_1\ \&\ e_2 = e_3$	Subtyping Constraint Entry Merge
	|	$\Theta \vdash SC_1 \& SC_2 = SC_3$	Merge of subtyping constraints
UCM	::=		
	|	$\Gamma \vdash e_1 \& e_2 = e_3$	
	|	$\Theta \vdash UC_1 \& UC_2 = UC_3$	
$SATSCE$	::=		
	|	$\Gamma \vdash P : e$	Positive type satisfies with the subtyping constr
	|	$\Gamma \vdash N : e$	Negative type satisfies with the subtyping const
$SING$	::=		
	|	e_1 **singular with** P	Positive Subtyping Constraint Entry Is Singular
	|	e_1 **singular with** N	Negative Subtyping Constraint Entry Is Singula
	|	SC **singular with** $\widehat{\sigma}$	Subtyping Constraint Is Singular
$E1$	::=		
	|	$N \simeq_1^D M$	Negative multi-quantified type equivalence
	|	$P \simeq_1^D Q$	Positive multi-quantified type equivalence
	|	$P \simeq Q$	
$D1$	::=		
	|	$\Gamma \vdash N \simeq_1^{\leqslant} M$	Negative equivalence on MQ types

	$\mid$	$\Gamma \vdash P \simeq^{\leqslant}_1 Q$	Positive equivalence on MQ types
	$\mid$	$\Gamma \vdash N \leqslant_1 M$	Negative subtyping
	$\mid$	$\Gamma \vdash P \geqslant_1 Q$	Positive supertyping
	$\mid$	$\Gamma_2 \vdash \sigma_1 \simeq^{\leqslant}_1 \sigma_2 : \Gamma_1$	Equivalence of substitutions
	$\mid$	$\Gamma \vdash \sigma_1 \simeq^{\leqslant}_1 \sigma_2 : vars$	Equivalence of substitutions
	$\mid$	$\Theta \vdash \widehat{\sigma}_1 \simeq^{\leqslant}_1 \widehat{\sigma}_2 : vars$	Equivalence of unification substitutions
	$\mid$	$\Gamma \vdash \Phi_1 \simeq^{\leqslant}_1 \Phi_2$	Equivalence of contexts
$D0$	$::=$		
	$\mid$	$\Gamma \vdash N \simeq^{\leqslant}_0 M$	Negative equivalence
	$\mid$	$\Gamma \vdash P \simeq^{\leqslant}_0 Q$	Positive equivalence
	$\mid$	$\Gamma \vdash N \leqslant_0 M$	Negative subtyping
	$\mid$	$\Gamma \vdash P \geqslant_0 Q$	Positive supertyping
DT	$::=$		
	$\mid$	$\Gamma; \Phi \vdash v : P$	Positive type inference
	$\mid$	$\Gamma; \Phi \vdash c : N$	Negative type inference
	$\mid$	$\Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M$	Application type inference
EQ	$::=$		
	$\mid$	$N = M$	Negative type equality (alpha-equivalence)
	$\mid$	$P = Q$	Positive type equuality (alphha-equivalence)
	$\mid$	$P = Q$	
$LUBF$	$::=$		
	$\mid$	$P_1 \vee P_2 === Q$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P === \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N === \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ P === \vec{\alpha}$	
	$\mid$	$\mathbf{ord}\ vars\ \mathbf{in}\ N === \vec{\alpha}$	
	$\mid$	$\mathbf{nf}\,(N') === N$	
	$\mid$	$\mathbf{nf}\,(P') === P$	
	$\mid$	$\mathbf{nf}\,(N') === N$	
	$\mid$	$\mathbf{nf}\,(P') === P$	
	$\mid$	$\mathbf{nf}\,(\vec{N}') === \vec{N}$	
	$\mid$	$\mathbf{nf}\,(\vec{P}') === \vec{P}$	
	$\mid$	$\mathbf{nf}\,(\sigma') === \sigma$	
	$\mid$	$\mathbf{nf}\,(\widehat{\sigma}') === \widehat{\sigma}$	
	$\mid$	$\mathbf{nf}\,(\mu') === \mu$	
	$\mid$	$\sigma'\vert_{vars}$	
	$\mid$	$\widehat{\sigma}'\vert_{vars}$	
	$\mid$	$\widehat{\tau}'\vert_{vars}$	
	$\mid$	$\Xi'\vert_{vars}$	
	$\mid$	$SC\vert_{vars}$	
	$\mid$	$UC\vert_{vars}$	
	$\mid$	$e_1\ \&\ e_2$	
	$\mid$	$e_1\ \&\ e_2$	
	$\mid$	$UC_1\ \&\ UC_2$	
	$\mid$	$UC_1 \cup UC_2$	
	$\mid$	$SC_1\ \&\ SC_2$	

$$
\begin{array}{lll}
 & | \quad \widehat{\tau}_1 \ \& \ \widehat{\tau}_2 & \\
 & | \quad \mathbf{dom}\,(UC) === \mathit{vars} & \\
 & | \quad \mathbf{dom}\,(SC) === \mathit{vars} & \\
 & | \quad \mathbf{dom}\,(\widehat{\sigma}) === \mathit{vars} & \\
 & | \quad \mathbf{dom}\,(\widehat{\tau}) === \mathit{vars} & \\
 & | \quad \mathbf{dom}\,(\Theta) === \mathit{vars} & \\
 & | \quad |SC| === UC & \\
\\
\mathit{LUB} & ::= & \\
 & | \quad \Gamma \models P_1 \vee P_2 = Q & \text{Least Upper Bound (Least Common Supertype)} \\
 & | \quad \mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q & \\
\\
\mathit{Nrm} & ::= & \\
 & | \quad \mathbf{nf}\,(N) = M & \\
 & | \quad \mathbf{nf}\,(P) = Q & \\
 & | \quad \mathbf{nf}\,(N) = M & \\
 & | \quad \mathbf{nf}\,(P) = Q & \\
\\
\mathit{Order} & ::= & \\
 & | \quad \mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha} & \\
 & | \quad \mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P = \vec{\alpha} & \\
 & | \quad \mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,N = \vec{\alpha} & \\
 & | \quad \mathbf{ord}\,\mathit{vars}\,\mathbf{in}\,P = \vec{\alpha} & \\
\\
U & ::= & \\
 & | \quad \Gamma;\Theta \models N \stackrel{u}{\simeq} M \dashv UC & \text{Negative unification} \\
 & | \quad \Gamma;\Theta \models P \stackrel{u}{\simeq} Q \dashv UC & \text{Positive unification} \\
\\
\mathit{WF} & ::= & \\
 & | \quad \Gamma \vdash N & \text{Negative type well-formedness} \\
 & | \quad \Gamma \vdash P & \text{Positive type well-formedness} \\
 & | \quad \Gamma \vdash N & \text{Negative type well-formedness} \\
 & | \quad \Gamma \vdash P & \text{Positive type well-formedness} \\
 & | \quad \Gamma \vdash \vec{N} & \text{Negative type list well-formedness} \\
 & | \quad \Gamma \vdash \vec{P} & \text{Positive type list well-formedness} \\
 & | \quad \Gamma;\Theta \vdash N & \text{Negative unification type well-formedness} \\
 & | \quad \Gamma;\Theta \vdash P & \text{Positive unifiication type well-formedness} \\
 & | \quad \Gamma;\Xi \vdash N & \text{Negative anti-unification type well-formedness} \\
 & | \quad \Gamma;\Xi \vdash P & \text{Positive anti-unification type well-formedness} \\
 & | \quad \Gamma;\Xi_2 \vdash \widehat{\tau} : \Xi_1 & \text{Antiunification substitution well-formedness} \\
 & | \quad \Gamma \vdash^{\supseteq} \Theta & \text{Unification context well-formedness} \\
 & | \quad \Gamma_1 \vdash \sigma : \Gamma_2 & \text{Substitution well-formedness} \\
 & | \quad \Theta \vdash \widehat{\sigma} & \text{Unification substitution well-formedness} \\
 & | \quad \Theta \vdash \widehat{\sigma} : UC & \text{Unification substitution satisfies unification constraint} \\
 & | \quad \Theta \vdash \widehat{\sigma} : SC & \text{Unification substitution satisfies subtyping constraint} \\
 & | \quad \Gamma \vdash e & \text{Unification constraint entry well-formedness} \\
 & | \quad \Gamma \vdash e & \text{Subtyping constraint entry well-formedness} \\
 & | \quad \Gamma \vdash P : e & \text{Positive type satisfies unification constraint} \\
 & | \quad \Gamma \vdash N : e & \text{Negative type satisfies unification constraint} \\
 & | \quad \Gamma \vdash P : e & \text{Positive type satisfies subtyping constraint}
\end{array}
$$

|	$\Gamma \vdash N : e$	Negative type satisfies subtyping constraint
|	$\Theta \vdash UC$	Unification constraint well-formedness
|	$\Theta \vdash SC$	Subtyping constraint well-formedness
|	$\Gamma \vdash \Phi$	Context well-formedness

judgement ::=
| A
| AT
| AU
| SCM
| UCM
| $SATSCE$
| $SING$
| $E1$
| $D1$
| $D0$
| DT
| EQ
| LUB
| Nrm
| $Order$
| U
| WF

user_syntax ::=
| α
| n
| x
| n
| α^{+}
| α^{-}
| $\alpha^{\pm}$
| σ
| e
| e
| UC
| SC
| $\widehat{\sigma}$
| $\widehat{\tau}$
| P
| N
| $\overrightarrow{\alpha^{+}}$
| $\overrightarrow{\alpha^{-}}$
| $\overrightarrow{\alpha^{\pm}}$
| P
| N
| $\overrightarrow{P}$
| $\overrightarrow{N}$
| Γ
| Θ

| Ξ
| $\vec{\alpha}$
| $vars$
| μ
| $\widehat{\alpha}^{\pm}$
| $\widehat{\alpha}^{+}$
| $\widehat{\alpha}^{-}$
| $\overrightarrow{\widetilde{\alpha}^{+}}$
| $\overrightarrow{\widehat{\alpha}^{-}}$
| P
| N
| $auSol$
| $terminals$
| v
| $\vec{v}$
| c
| $vctx$
| $formula$

$\boxed{\Gamma;\, \Theta \vDash N \leqslant M \dashv SC}$ Negative subtyping

$$\frac{}{\Gamma;\, \Theta \vDash \alpha^- \leqslant \alpha^- \dashv \cdot}\ \textsc{ANVar}$$

$$\frac{\Gamma; \Theta \vDash \mathbf{nf}\,(P) \overset{u}{\simeq} \mathbf{nf}\,(Q) \dashv UC}{\Gamma;\, \Theta \vDash \uparrow P \leqslant \uparrow Q \dashv UC}\ \textsc{AShiftU}$$

$$\frac{\Gamma;\, \Theta \vDash P \geqslant Q \dashv SC_1 \quad \Gamma;\, \Theta \vDash N \leqslant M \dashv SC_2 \quad \Theta \vdash SC_1 \& SC_2 = SC}{\Gamma;\, \Theta \vDash P \to N \leqslant Q \to M \dashv SC}\ \textsc{AArrow}$$

$$\frac{\text{<<multiple parses>>}}{\Gamma;\, \Theta \vDash \forall \overrightarrow{\alpha^+}.N \leqslant \forall \overrightarrow{\beta^+}.M \dashv SC \backslash \overrightarrow{\widehat{\alpha}^+}}\ \textsc{AForall}$$

$\boxed{\Gamma;\, \Theta \vDash P \geqslant Q \dashv SC}$ Positive supertyping

$$\frac{}{\Gamma;\, \Theta \vDash \alpha^+ \geqslant \alpha^+ \dashv \cdot}\ \textsc{APVar}$$

$$\frac{\Gamma; \Theta \vDash \mathbf{nf}\,(N) \overset{u}{\simeq} \mathbf{nf}\,(M) \dashv UC}{\Gamma;\, \Theta \vDash \downarrow N \geqslant \downarrow M \dashv UC}\ \textsc{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-};\, \Theta, \overrightarrow{\widehat{\alpha}^-}\{\Gamma, \overrightarrow{\beta^-}\} \vDash [\overrightarrow{\widehat{\alpha}^-}/\overrightarrow{\alpha^-}]P \geqslant Q \dashv SC}{\Gamma;\, \Theta \vDash \exists \overrightarrow{\alpha^-}.P \geqslant \exists \overrightarrow{\beta^-}.Q \dashv SC \backslash \overrightarrow{\widehat{\alpha}^-}}\ \textsc{AExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}{\Gamma;\, \Theta \vDash \widehat{\alpha}^+ \geqslant P \dashv (\widehat{\alpha}^+ :\geqslant Q)}\ \textsc{APUVar}$$

$\boxed{\Gamma; \Phi \vDash v \colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \vDash x \colon \mathbf{nf}\,(P)}\ \textsc{ATVar}$$

$$\frac{\Gamma; \Phi \vDash c \colon N}{\Gamma; \Phi \vDash \{c\} \colon \downarrow N}\ \textsc{ATThunk}$$

$$\frac{\Gamma;\Phi \models v\colon P \quad \Gamma;\cdot \models Q \geqslant P \dashv \cdot}{\Gamma;\Phi \models (v : Q)\colon \mathbf{nf}\,(Q)} \quad \text{ATPAnnot}$$

$\boxed{\Gamma;\Phi \models c\colon N}$ Negative type inference

$$\frac{\Gamma;\Phi \models c\colon N \quad \Gamma;\cdot \models N \leqslant M \dashv \cdot}{\Gamma;\Phi \models (c : M)\colon \mathbf{nf}\,(M)} \quad \text{ATNAnnot}$$

$$\frac{\Gamma;\Phi, x : P \models c\colon N}{\Gamma;\Phi \models \lambda x : P.c\colon \mathbf{nf}\,(P \to N)} \quad \text{ATtLam}$$

$$\frac{\Gamma, \alpha^+;\Phi \models c\colon N}{\Gamma;\Phi \models \Lambda\alpha^+.c\colon \mathbf{nf}\,(\forall\alpha^+.N)} \quad \text{ATTLam}$$

$$\frac{\Gamma;\Phi \models v\colon P}{\Gamma;\Phi \models \mathbf{return}\, v\colon \uparrow P} \quad \text{ATReturn}$$

$$\frac{\Gamma;\Phi \models v\colon P \quad \Gamma;\Phi, x : P \models c\colon N}{\Gamma;\Phi \models \mathbf{let}\, x = v; c\colon N} \quad \text{ATVarLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma;\Phi \models v\colon \downarrow M \quad \Gamma;\Phi;\cdot \models M \bullet \vec{v} \Rightarrow\!\!\!> \uparrow Q \dashv \Theta; SC_1 \quad \Gamma;\Theta \models \uparrow Q \leqslant \uparrow P \dashv SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC \quad \Gamma;\Phi, x : P \models c\colon N\end{array}}{\Gamma;\Phi \models \mathbf{let}\, x : P = v(\vec{v}); c\colon N} \quad \text{ATAppLetAnn}$$

$$\frac{\begin{array}{l}\Gamma;\Phi \models v\colon \downarrow M \quad \Gamma;\Phi;\cdot \models M \bullet \vec{v} \Rightarrow\!\!\!> \uparrow Q \dashv \Theta; SC \\ \texttt{<<multiple parses>>} \\ \Gamma;\Phi, x : [\widehat{\sigma}]\, Q \models c\colon N\end{array}}{\Gamma;\Phi \models \mathbf{let}\, x = v(\vec{v}); c\colon N} \quad \text{ATAppLet}$$

$$\frac{\Gamma;\Phi \models v\colon \exists\overrightarrow{\alpha^-}.P \quad \Gamma, \overrightarrow{\alpha^-};\Phi, x : P \models c\colon N \quad \Gamma \vdash N}{\Gamma;\Phi \models \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c\colon N} \quad \text{ATUnpack}$$

$\boxed{\Gamma;\Phi;\Theta_1 \models N \bullet \vec{v} \Rightarrow\!\!\!> M \dashv \Theta_2; SC}$ Application type inference

$$\frac{}{\Gamma;\Phi;\Theta \models N \bullet \cdot \Rightarrow\!\!\!> \mathbf{nf}\,(N) \dashv \Theta; \cdot} \quad \text{ATEmptyApp}$$

$$\frac{\begin{array}{l}\Gamma;\Phi \models v\colon P \quad \Gamma;\Theta \models Q \geqslant P \dashv SC_1 \quad \Gamma;\Phi;\Theta \models N \bullet \vec{v} \Rightarrow\!\!\!> M \dashv \Theta'; SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC\end{array}}{\Gamma;\Phi;\Theta \models Q \to N \bullet v, \vec{v} \Rightarrow\!\!\!> M \dashv \Theta'; SC} \quad \text{ATArrowApp}$$

$$\frac{\begin{array}{c}\texttt{<<multiple parses>>} \\ \vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot\end{array}}{\Gamma;\Phi;\Theta \models \forall\overrightarrow{\alpha^+}.N \bullet \vec{v} \Rightarrow\!\!\!> M \dashv \Theta'; SC} \quad \text{ATForallApp}$$

$\boxed{\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \models \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\cdot, \alpha^+, \cdot, \cdot)} \quad \text{AUPVar}$$

$$\frac{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \downarrow N_1 \overset{a}{\simeq} \downarrow N_2 \dashv (\Xi, \downarrow M, \widehat{\tau}_1, \widehat{\tau}_2)} \quad \text{AUShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \Gamma = \varnothing \quad \Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \models \exists\overrightarrow{\alpha^-}.P_1 \overset{a}{\simeq} \exists\overrightarrow{\alpha^-}.P_2 \dashv (\Xi, \exists\overrightarrow{\alpha^-}.Q, \widehat{\tau}_1, \widehat{\tau}_2)} \quad \text{AUExists}$$

$\boxed{\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \vDash \alpha^- \stackrel{a}{\simeq} \alpha^- \dashv (\cdot, \alpha^-, \cdot, \cdot)} \quad \text{AUNVar}$$

$$\frac{\Gamma \vDash P_1 \stackrel{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \uparrow P_1 \stackrel{a}{\simeq} \uparrow P_2 \dashv (\Xi, \uparrow Q, \widehat{\tau}_1, \widehat{\tau}_2)} \quad \text{AUShiftU}$$

$$\frac{\overrightarrow{\alpha^+} \cap \Gamma = \varnothing \quad \Gamma \vDash N_1 \stackrel{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \forall \overrightarrow{\alpha^+}. N_1 \stackrel{a}{\simeq} \forall \overrightarrow{\alpha^+}. N_2 \dashv (\Xi, \forall \overrightarrow{\alpha^+}. M, \widehat{\tau}_1, \widehat{\tau}_2)} \quad \text{AUForall}$$

$$\frac{\Gamma \vDash P_1 \stackrel{a}{\simeq} P_2 \dashv (\Xi_1, Q, \widehat{\tau}_1, \widehat{\tau}_2) \quad \Gamma \vDash N_1 \stackrel{a}{\simeq} N_2 \dashv (\Xi_2, M, \widehat{\tau}'_1, \widehat{\tau}'_2)}{\Gamma \vDash P_1 \to N_1 \stackrel{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \widehat{\tau}_1 \cup \widehat{\tau}'_1, \widehat{\tau}_2 \cup \widehat{\tau}'_2)} \quad \text{AUArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \vDash N \stackrel{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, \widehat{\alpha}^-_{\{N,M\}}, (\widehat{\alpha}^-_{\{N,M\}} :\approx N), (\widehat{\alpha}^-_{\{N,M\}} :\approx M))} \quad \text{AUAU}$$

$\boxed{\Gamma \vdash e_1 \,\&\, e_2 = e_3}$ Subtyping Constraint Entry Merge

$$\frac{\Gamma \vDash P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1) \,\&\, (\widehat{\alpha}^+ :\geqslant P_2) = (\widehat{\alpha}^+ :\geqslant Q)} \quad \text{SCMESupSup}$$

$$\frac{\Gamma; \cdot \vDash P \geqslant Q \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \,\&\, (\widehat{\alpha}^+ :\geqslant Q) = (\widehat{\alpha}^+ :\approx P)} \quad \text{SCMEEqSup}$$

$$\frac{\Gamma; \cdot \vDash Q \geqslant P \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P) \,\&\, (\widehat{\alpha}^+ :\approx Q) = (\widehat{\alpha}^+ :\approx Q)} \quad \text{SCMESupEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \,\&\, (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \text{SCMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \,\&\, (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \text{SCMENEqEq}$$

$\boxed{\Theta \vdash SC_1 \& SC_2 = SC_3}$ Merge of subtyping constraints

$\boxed{\Gamma \vdash e_1 \& e_2 = e_3}$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \& (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)} \quad \text{UCMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \& (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)} \quad \text{UCMENEqEq}$$

$\boxed{\Theta \vdash UC_1 \& UC_2 = UC_3}$

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies with the subtyping constraint entry

$$\frac{\Gamma \vdash P \geqslant_1 Q}{\Gamma \vdash P : (\widehat{\alpha}^+ :\geqslant Q)} \quad \text{SATSCESup}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash P : (\widehat{\alpha}^+ :\approx Q)} \quad \text{SATSCEPEq}$$

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies with the subtyping constraint entry

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash N : (\widehat{\alpha}^- :\approx M)} \quad \text{SATSCENEq}$$

$\boxed{e_1 \textbf{ singular with } P}$ Positive Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^+ :\approx P \,\textbf{singular with}\, \mathbf{nf}\,(P)} \quad \text{SINGPEQ}$$

$$\frac{}{\widehat{\alpha}^+ :\geqslant \exists\overrightarrow{\alpha^-}.\alpha^+ \,\textbf{singular with}\, \alpha^+} \quad \text{SINGSUPVAR}$$

$$\frac{N \simeq_1^D \alpha_i^-}{\widehat{\alpha}^+ :\geqslant \exists\overrightarrow{\alpha^-}.{\downarrow}N \,\textbf{singular with}\, \exists\alpha^-.{\downarrow}\alpha^-} \quad \text{SINGSUPSHIFT}$$

$\boxed{e_1 \,\textbf{singular with}\, N}$ Negative Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^- :\approx N \,\textbf{singular with}\, \mathbf{nf}\,(N)} \quad \text{SINGNEQ}$$

$\boxed{SC \,\textbf{singular with}\, \widehat{\sigma}}$ Subtyping Constraint Is Singular

$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-} \quad \text{E1NVAR}$$

$$\frac{P \simeq_1^D Q}{{\uparrow}P \simeq_1^D {\uparrow}Q} \quad \text{E1SHIFTU}$$

$$\frac{P \simeq_1^D Q \quad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M} \quad \text{E1ARROW}$$

$$\frac{\overrightarrow{\alpha^+} \cap \mathbf{fv}\, M = \varnothing \quad \mu : (\overrightarrow{\beta^+} \cap \mathbf{fv}\, M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \mathbf{fv}\, N) \quad N \simeq_1^D [\mu]M}{\forall\overrightarrow{\alpha^+}.N \simeq_1^D \forall\overrightarrow{\beta^+}.M} \quad \text{E1FORALL}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+} \quad \text{E1PVAR}$$

$$\frac{N \simeq_1^D M}{{\downarrow}N \simeq_1^D {\downarrow}M} \quad \text{E1SHIFTD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \mathbf{fv}\, Q = \varnothing \quad \mu : (\overrightarrow{\beta^-} \cap \mathbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \mathbf{fv}\, P) \quad P \simeq_1^D [\mu]Q}{\exists\overrightarrow{\alpha^-}.P \simeq_1^D \exists\overrightarrow{\beta^-}.Q} \quad \text{E1EXISTS}$$

$\boxed{P \simeq Q}$

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M} \quad \text{D1NDEF}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q} \quad \text{D1PDEF}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-} \quad \text{D1NVAR}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash {\uparrow}P \leqslant_1 {\uparrow}Q} \quad \text{D1SHIFTU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M} \text{ D1Arrow}$$

$$\frac{\textbf{fv}\, N \cap \overrightarrow{\beta^+} = \varnothing \quad \Gamma, \overrightarrow{\beta^+} \vdash P_i \quad \Gamma, \overrightarrow{\beta^+} \vdash [\overrightarrow{P}/\overrightarrow{\alpha^+}]N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}.N \leqslant_1 \forall \overrightarrow{\beta^+}.M} \text{ D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+} \text{ D1PVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M} \text{ D1ShiftD}$$

$$\frac{\textbf{fv}\, P \cap \overrightarrow{\beta^-} = \varnothing \quad \Gamma, \overrightarrow{\beta^-} \vdash N_i \quad \Gamma, \overrightarrow{\beta^-} \vdash [\overrightarrow{N}/\overrightarrow{\alpha^-}]P \geqslant_1 Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P \geqslant_1 \exists \overrightarrow{\beta^-}.Q} \text{ D1Exists}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1}$ Equivalence of substitutions

$\boxed{\Gamma \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \mathit{vars}}$ Equivalence of substitutions

$\boxed{\Theta \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : \mathit{vars}}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash \Phi_1 \simeq_1^{\leqslant} \Phi_2}$ Equivalence of contexts

$\boxed{\Gamma \vdash N \simeq_0^{\leqslant} M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq_0^{\leqslant} M} \text{ D0NDef}$$

$\boxed{\Gamma \vdash P \simeq_0^{\leqslant} Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq_0^{\leqslant} Q} \text{ D0PDef}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_0 \alpha^-} \text{ D0NVar}$$

$$\frac{\Gamma \vdash P \simeq_0^{\leqslant} Q}{\Gamma \vdash \uparrow P \leqslant_0 \uparrow Q} \text{ D0ShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/\alpha^+]N \leqslant_0 M \quad M \neq \forall \beta^+.M'}{\Gamma \vdash \forall \alpha^+.N \leqslant_0 M} \text{ D0ForallL}$$

$$\frac{\Gamma, \alpha^+ \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall \alpha^+.M} \text{ D0ForallR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \to N \leqslant_0 Q \to M} \text{ D0Arrow}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_0 \alpha^+} \text{ D0PVar}$$

$$\frac{\Gamma \vdash N \simeq_0^{\leqslant} M}{\Gamma \vdash \downarrow N \geqslant_0 \downarrow M} \text{ D0ShiftD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/\alpha^-]P \geqslant_0 Q \quad Q \neq \exists \alpha^-.Q'}{\Gamma \vdash \exists \alpha^-.P \geqslant_0 Q} \text{ D0ExistsL}$$

$$\frac{\Gamma, \alpha^- \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists \alpha^-.Q} \text{ D0ExistsR}$$

$\boxed{\Gamma; \Phi \vdash v\colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \vdash x\colon P}\ \textsc{DTVar}$$

$$\frac{\Gamma; \Phi \vdash c\colon N}{\Gamma; \Phi \vdash \{c\}\colon {\downarrow}N}\ \textsc{DTThunk}$$

$$\frac{\Gamma; \Phi \vdash v\colon P \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma; \Phi \vdash (v : Q)\colon Q}\ \textsc{DTPAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash v\colon P'}\ \textsc{DTPEquiv}$$

$\boxed{\Gamma; \Phi \vdash c\colon N}$ Negative type inference

$$\frac{\Gamma; \Phi, x : P \vdash c\colon N}{\Gamma; \Phi \vdash \lambda x : P.c\colon P \to N}\ \textsc{DTtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \vdash c\colon N}{\Gamma; \Phi \vdash \Lambda\alpha^+.c\colon \forall\alpha^+.N}\ \textsc{DTTLam}$$

$$\frac{\Gamma; \Phi \vdash v\colon P}{\Gamma; \Phi \vdash \mathbf{return}\, v\colon {\uparrow}P}\ \textsc{DTReturn}$$

$$\frac{\Gamma; \Phi \vdash v\colon P \quad \Gamma; \Phi, x : P \vdash c\colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v; c\colon N}\ \textsc{DTVarLet}$$

$$\frac{\Gamma; \Phi \vdash v\colon {\downarrow}M \quad \Gamma; \Phi \vdash M \bullet \vec{v} \Rightarrow\!\!\!> {\uparrow}Q \text{ unique} \quad \Gamma; \Phi, x : Q \vdash c\colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v(\vec{v}); c\colon N}\ \textsc{DTAppLet}$$

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi \vdash v\colon {\downarrow}M \quad \Gamma; \Phi \vdash M \bullet \vec{v} \Rightarrow\!\!\!> {\uparrow}Q \quad \Gamma \vdash {\uparrow}Q \leqslant_1 {\uparrow}P \quad \Gamma; \Phi, x : P \vdash c\colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x : P = v(\vec{v}); c\colon N}\ \textsc{DTAppLetAnn}$$

$$\frac{\begin{array}{c}\texttt{<<multiple parses>>}\\ \Gamma, \overrightarrow{\alpha^-}; \Phi, x : P \vdash c\colon N \quad \Gamma \vdash N\end{array}}{\Gamma; \Phi \vdash \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c\colon N}\ \textsc{DTUnpack}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash (c : M)\colon M}\ \textsc{DTNAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash c\colon N'}\ \textsc{DTNEquiv}$$

$\boxed{\Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!\!> M}$ Application type inference

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash N \bullet \cdot \Rightarrow\!\!\!> N'}\ \textsc{DTEmptyApp}$$

$$\frac{\Gamma; \Phi \vdash v\colon P \quad \Gamma \vdash Q \geqslant_1 P \quad \Gamma; \Phi \vdash N \bullet \vec{v} \Rightarrow\!\!\!> M}{\Gamma; \Phi \vdash Q \to N \bullet v, \vec{v} \Rightarrow\!\!\!> M}\ \textsc{DTArrowApp}$$

$$\frac{\begin{array}{l}\Gamma \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma; \Phi \vdash [\sigma]N \bullet \vec{v} \Rightarrow\!\!\!> M\\ \vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot\end{array}}{\Gamma; \Phi \vdash \forall\overrightarrow{\alpha^+}.N \bullet \vec{v} \Rightarrow\!\!\!> M}\ \textsc{DTForallApp}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P = Q}$
$\boxed{P_1 \vee P_2}$

$\boxed{\mathbf{ord}\, \mathit{vars}\, \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\, \mathit{vars}\, \mathbf{in}\ N}$

$\boxed{\mathbf{ord}\, \mathit{vars}\, \mathbf{in}\, P}$

$\boxed{\mathbf{ord}\, \mathit{vars}\, \mathbf{in}\, N}$

$\boxed{\mathbf{nf}\,(N')}$

$\boxed{\mathbf{nf}\,(P')}$

$\boxed{\mathbf{nf}\,(\ N'\,)}$

$\boxed{\mathbf{nf}\,(\ P'\,)}$

$\boxed{\mathbf{nf}\,(\overrightarrow{N}')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{P}')}$

$\boxed{\mathbf{nf}\,(\sigma')}$

$\boxed{\mathbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\mathbf{nf}\,(\mu')}$

$\boxed{\sigma'|_{\mathit{vars}}}$

$\boxed{\widehat{\sigma}'|_{vars}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{\mathbf{SC}|_{vars}}$

$\boxed{\mathbf{UC}|_{vars}}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{e_1 \ \& \ e_2}$

$\boxed{UC_1 \ \& \ UC_2}$

$\boxed{UC_1 \cup UC_2}$

$\boxed{SC_1 \ \& \ SC_2}$

$\boxed{\widehat{\tau}_1 \ \& \ \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(UC)}$

$\boxed{\mathbf{dom}\,(SC)}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{|SC|}$

$\boxed{\Gamma \vDash P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \vDash \alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBVar}$$

$$\frac{\Gamma, \cdot \vDash \mathbf{nf}\,(\downarrow N) \overset{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \hat{\tau}_1, \hat{\tau}_2)}{\Gamma \vDash \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi] P} \quad \text{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \vDash P_1 \vee P_2 = Q}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P_1 \vee \exists \overrightarrow{\beta^-}.P_2 = Q} \quad \text{LUBExists}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}$

$$\frac{\Gamma = \Delta, \overrightarrow{\alpha^\pm} \quad \overrightarrow{\beta^\pm}\,\mathbf{is\,fresh} \quad \overrightarrow{\gamma^\pm}\,\mathbf{is\,fresh} \\ \Delta, \overrightarrow{\beta^\pm}, \overrightarrow{\gamma^\pm} \vDash [\overrightarrow{\beta^\pm}/\overrightarrow{\alpha^\pm}]P \vee [\overrightarrow{\gamma^\pm}/\overrightarrow{\alpha^\pm}]P = Q}{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q} \quad \text{LUBUpgrade}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^-) = \alpha^-} \quad \text{NrmNVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\uparrow P) = \uparrow Q} \quad \text{NrmShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(P \to N) = Q \to M} \quad \text{NrmArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\forall \overrightarrow{\alpha^+}.N) = \forall \overrightarrow{\alpha^+}'.N'} \quad \text{NrmForall}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\alpha^+) = \alpha^+} \quad \text{NrmPVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\downarrow N) = \downarrow M} \quad \text{NrmShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\exists \overrightarrow{\alpha^-}.P) = \exists \overrightarrow{\alpha^-}'.P'} \quad \text{NrmExists}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\hat{\alpha}^-) = \hat{\alpha}^-} \quad \text{NrmNUVar}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^{+}) = \widehat{\alpha}^{+}} \quad \text{NrmPUVar}$$

$\boxed{\mathbf{ord}\; vars\,\mathbf{in}\; N = \vec{\alpha}}$

$$\frac{\alpha^{-} \in vars}{\mathbf{ord}\; vars\,\mathbf{in}\; \alpha^{-} = \alpha^{-}} \quad \text{ONVarIn}$$

$$\frac{\alpha^{-} \notin vars}{\mathbf{ord}\; vars\,\mathbf{in}\; \alpha^{-} = \cdot} \quad \text{ONVarNIn}$$

$$\frac{\mathbf{ord}\; vars\,\mathbf{in}\; P = \vec{\alpha}}{\mathbf{ord}\; vars\,\mathbf{in}\; \uparrow P = \vec{\alpha}} \quad \text{OShiftU}$$

$$\frac{\mathbf{ord}\; vars\,\mathbf{in}\; P = \vec{\alpha}_1 \quad \mathbf{ord}\; vars\,\mathbf{in}\; N = \vec{\alpha}_2}{\mathbf{ord}\; vars\,\mathbf{in}\; P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)} \quad \text{OArrow}$$

$$\frac{vars \cap \overrightarrow{\alpha^{+}} = \varnothing \quad \mathbf{ord}\; vars\,\mathbf{in}\; N = \vec{\alpha}}{\mathbf{ord}\; vars\,\mathbf{in}\; \forall \overrightarrow{\alpha^{+}}.N = \vec{\alpha}} \quad \text{OForall}$$

$\boxed{\mathbf{ord}\; vars\,\mathbf{in}\; P = \vec{\alpha}}$

$$\frac{\alpha^{+} \in vars}{\mathbf{ord}\; vars\,\mathbf{in}\; \alpha^{+} = \alpha^{+}} \quad \text{OPVarIn}$$

$$\frac{\alpha^{+} \notin vars}{\mathbf{ord}\; vars\,\mathbf{in}\; \alpha^{+} = \cdot} \quad \text{OPVarNIn}$$

$$\frac{\mathbf{ord}\; vars\,\mathbf{in}\; N = \vec{\alpha}}{\mathbf{ord}\; vars\,\mathbf{in}\; \downarrow N = \vec{\alpha}} \quad \text{OShiftD}$$

$$\frac{vars \cap \overrightarrow{\alpha^{-}} = \varnothing \quad \mathbf{ord}\; vars\,\mathbf{in}\; P = \vec{\alpha}}{\mathbf{ord}\; vars\,\mathbf{in}\; \exists \overrightarrow{\alpha^{-}}.P = \vec{\alpha}} \quad \text{OExists}$$

$\boxed{\mathbf{ord}\; vars\,\mathbf{in}\; N = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\; vars\,\mathbf{in}\; \widehat{\alpha}^{-} = \cdot} \quad \text{ONUVar}$$

$\boxed{\mathbf{ord}\; vars\,\mathbf{in}\; P = \vec{\alpha}}$

$$\frac{}{\mathbf{ord}\; vars\,\mathbf{in}\; \widehat{\alpha}^{+} = \cdot} \quad \text{OPUVar}$$

$\boxed{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC}$ Negative unification

$$\frac{}{\Gamma; \Theta \models \alpha^{-} \overset{u}{\simeq} \alpha^{-} \dashv \cdot} \quad \text{UNVar}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \uparrow P \overset{u}{\simeq} \uparrow Q \dashv UC} \quad \text{UShiftU}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC_1 \quad \Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC_2}{\Gamma; \Theta \models P \to N \overset{u}{\simeq} Q \to M \dashv UC_1 \;\&\; UC_2} \quad \text{UArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^{+}}; \Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \forall \overrightarrow{\alpha^{+}}.N \overset{u}{\simeq} \forall \overrightarrow{\alpha^{+}}.M \dashv UC} \quad \text{UForall}$$

$$\frac{\widehat{\alpha}^{-}\{\Delta\} \in \Theta \quad \Delta \vdash N}{\Gamma; \Theta \models \widehat{\alpha}^{-} \overset{u}{\simeq} N \dashv (\widehat{\alpha}^{-} :\approx N)} \quad \text{UNUVar}$$

$\boxed{\Gamma;\Theta \models P \stackrel{u}{\simeq} Q \dashv UC}$ Positive unification

$$\frac{}{\Gamma;\Theta \models \alpha^+ \stackrel{u}{\simeq} \alpha^+ \dashv \cdot}\ \textsc{UPVar}$$

$$\frac{\Gamma;\Theta \models N \stackrel{u}{\simeq} M \dashv UC}{\Gamma;\Theta \models \downarrow N \stackrel{u}{\simeq} \downarrow M \dashv UC}\ \textsc{UShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-};\Theta \models P \stackrel{u}{\simeq} Q \dashv UC}{\Gamma;\Theta \models \exists\overrightarrow{\alpha^-}.P \stackrel{u}{\simeq} \exists\overrightarrow{\alpha^-}.Q \dashv UC}\ \textsc{UExists}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \Delta \vdash P}{\Gamma;\Theta \models \widehat{\alpha}^+ \stackrel{u}{\simeq} P \dashv (\widehat{\alpha}^+ :\approx P)}\ \textsc{UPUVar}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness
$\boxed{\Gamma \vdash P}$ Positive type well-formedness
$\boxed{\Gamma \vdash N}$ Negative type well-formedness
$\boxed{\Gamma \vdash P}$ Positive type well-formedness
$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness
$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness
$\boxed{\Gamma;\Theta \vdash N}$ Negative unification type well-formedness
$\boxed{\Gamma;\Theta \vdash P}$ Positive unifiication type well-formedness
$\boxed{\Gamma;\Xi \vdash N}$ Negative anti-unification type well-formedness
$\boxed{\Gamma;\Xi \vdash P}$ Positive anti-unification type well-formedness
$\boxed{\Gamma;\Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness
$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness
$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution well-formedness
$\boxed{\Theta \vdash \widehat{\sigma}}$ Unification substitution well-formedness
$\boxed{\Theta \vdash \widehat{\sigma} : UC}$ Unification substitution satisfies unification constraint
$\boxed{\Theta \vdash \widehat{\sigma} : SC}$ Unification substitution satisfies subtyping constraint
$\boxed{\Gamma \vdash e}$ Unification constraint entry well-formedness
$\boxed{\Gamma \vdash e}$ Subtyping constraint entry well-formedness
$\boxed{\Gamma \vdash P : e}$ Positive type satisfies unification constraint
$\boxed{\Gamma \vdash N : e}$ Negative type satisfies unification constraint
$\boxed{\Gamma \vdash P : e}$ Positive type satisfies subtyping constraint
$\boxed{\Gamma \vdash N : e}$ Negative type satisfies subtyping constraint
$\boxed{\Theta \vdash UC}$ Unification constraint well-formedness
$\boxed{\Theta \vdash SC}$ Subtyping constraint well-formedness
$\boxed{\Gamma \vdash \Phi}$ Context well-formedness

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Definition rules:        99 good    21 bad
Definition rule clauses: 204 good    21 bad
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