α, β, α, β, γ, δ	type variables
n, m, i, j	index variables
x, y, z	term variables

$n,\ m$	$::=$			cohort index
	$\vert$	0		
	$\vert$	$n+1$		
$\alpha^+,\ \beta^+,\ \gamma^+,\ \delta^+$	$::=$			positive variable
	$\vert$	α^+		
	$\vert$	α^{+n}		
	$\vert$	$\beta^{\pm}$		
$\alpha^-,\ \beta^-,\ \gamma^-,\ \delta^-$	$::=$			negative variable
	$\vert$	α^-		
	$\vert$	α^{-n}		
	$\vert$	$\alpha^{\pm}$		
$\alpha^{\pm},\ \beta^{\pm}$	$::=$			positive or negative variable
	$\vert$	$\alpha^{\pm}$		
	$\vert$	$\alpha^{\pm n}$		
σ	$::=$			substitution
	$\vert$	$\cdot$		
	$\vert$	id		
	$\vert$	P/α^+		
	$\vert$	N/α^-		
	$\vert$	$\vec{P}/\overrightarrow{\alpha^+}$		
	$\vert$	$\vec{N}/\overrightarrow{\alpha^-}$		
	$\vert$	$\mathbf{pmas}\,/\overrightarrow{\alpha^+}$		
	$\vert$	$\mathbf{nmas}\,/\overrightarrow{\alpha^-}$		
	$\vert$	$\overrightarrow{\widetilde{\alpha^+}}/\overrightarrow{\alpha^+}$		
	$\vert$	$\overrightarrow{\widehat{\alpha^-}}/\overrightarrow{\alpha^-}$		
	$\vert$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\vert$	μ		
	$\vert$	$\sigma_1 \circ \sigma_2$		
	$\vert$	$\sigma_1 \lll \sigma_2$		
	$\vert$	$\vec{\alpha}_1/\vec{\alpha}_2$		
	$\vert$	(σ)	S	
	$\vert$	$\overline{\sigma_i}^{\,i}$		concatenate
	$\vert$	$\mathbf{nf}\,(\sigma')$	M	
	$\vert$	$\sigma'\vert_{vars}$	M	
e	$::=$			entry of a subtyping constraint
	$\vert$	$\widehat{\alpha}^+ :\approx P$		
	$\vert$	$\widehat{\alpha}^- :\approx N$		
	$\vert$	$\widehat{\alpha}^+ :\geqslant P$		
	$\vert$	(e)	S	
	$\vert$	$SC(\widehat{\alpha}^-)$	M	
	$\vert$	$e_1\ \&\ e_2$	M	
e	$::=$			entry of a unification constraint
	$\vert$	$\widehat{\alpha}^+ :\approx P$		

	$\mid$	$\widehat{\alpha}^- :\approx N$		
	$\mid$	(e)	S	
	$\mid$	$e_1 \ \&\ e_2$	M	
UC	$::=$			unification constraint
	$\mid$	$\cdot$		
	$\mid$	e		
	$\mid$	$UC \backslash vars$		
	$\mid$	$UC\vert vars$		
	$\mid$	$UC_1 \cup UC_2$		
	$\mid$	$\overline{UC_i}^{\,i}$		concatenate
	$\mid$	(UC)	S	
	$\mid$	$\mathbf{UC}\vert_{vars}$	M	
	$\mid$	$UC_1 \ \&\ UC_2$	M	
	$\mid$	$UC_1 \cup UC_2$	M	
	$\mid$	$\vert SC\vert$	M	
SC	$::=$			subtyping constraint
	$\mid$	$\cdot$		
	$\mid$	e		
	$\mid$	$SC \backslash vars$		
	$\mid$	$SC\vert vars$		
	$\mid$	$SC_1 \cup SC_2$		
	$\mid$	UC		
	$\mid$	$\overline{SC_i}^{\,i}$		concatenate
	$\mid$	(SC)	S	
	$\mid$	$\mathbf{SC}\vert_{vars}$	M	
	$\mid$	$SC_1 \ \&\ SC_2$	M	
$\widehat{\sigma}$	$::=$			unification substitution
	$\mid$	$\cdot$		
	$\mid$	$P/\widehat{\alpha}^+$		
	$\mid$	$N/\widehat{\alpha}^-$		
	$\mid$	$\overrightarrow{P}/\overrightarrow{\widetilde{\alpha^+}}$		
	$\mid$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha^-}}$		
	$\mid$	$(\widehat{\sigma})$	S	
	$\mid$	$\overline{\widehat{\sigma}_i}^{\,i}$		concatenate
	$\mid$	$\mathbf{nf}\,(\widehat{\sigma}')$	M	
	$\mid$	$\widehat{\sigma}'\vert_{vars}$	M	
$\widehat{\tau},\ \widehat{\rho}$	$::=$			anti-unification substitution
	$\mid$	$\cdot$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		
	$\mid$	$\widehat{\alpha}^- :\approx N$		
	$\mid$	$\overrightarrow{\alpha^-}/\overrightarrow{\widehat{\alpha^-}}$		
	$\mid$	$\overrightarrow{N}/\overrightarrow{\widehat{\alpha^-}}$		
	$\mid$	$\widehat{\tau}_1 \cup \widehat{\tau}_2$		
	$\mid$	$\overline{\widehat{\tau}_i}^{\,i}$		concatenate
	$\mid$	$(\widehat{\tau})$	S	
	$\mid$	$\widehat{\tau}'\vert_{vars}$	M	

	$\mid$	$\widehat{\tau}_1 \mathrel{\&} \widehat{\tau}_2$	M	
$P,\ Q,\ R$	$::=$			positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists \alpha^- . P$		
	$\mid$	$[\sigma] P$	M	
$N,\ M,\ K$	$::=$			negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$\forall \alpha^+ . N$		
	$\mid$	$P \rightarrow N$		
	$\mid$	$[\sigma] N$	M	
$\overrightarrow{\alpha^+},\ \overrightarrow{\beta^+},\ \overrightarrow{\gamma^+},\ \overrightarrow{\delta^+}$	$::=$			positive variable list
	$\mid$	$\cdot$		empty list
	$\mid$	α^+		a variable
	$\mid$	$\overrightarrow{\alpha^+}$		a variable
	$\mid$	$\overline{\overrightarrow{\alpha^+}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^-},\ \overrightarrow{\beta^-},\ \overrightarrow{\gamma^-},\ \overrightarrow{\delta^-}$	$::=$			negative variables
	$\mid$	$\cdot$		empty list
	$\mid$	α^-		a variable
	$\mid$	$\overrightarrow{\alpha^-}$		variables
	$\mid$	$\overline{\overrightarrow{\alpha^-}_i}^{\,i}$		concatenate lists
$\overrightarrow{\alpha^\pm},\ \overrightarrow{\beta^\pm},\ \overrightarrow{\gamma^\pm},\ \overrightarrow{\delta^\pm}$	$::=$			positive or negative variable list
	$\mid$	$\cdot$		empty list
	$\mid$	$\alpha^\pm$		a variable
	$\mid$	$\overrightarrow{\mathbf{pa}}$		variables
	$\mid$	$\overline{\overrightarrow{\alpha^\pm}_i}^{\,i}$		concatenate lists
$P,\ Q,\ R$	$::=$			multi-quantified positive types
	$\mid$	α^+		
	$\mid$	$\downarrow N$		
	$\mid$	$\exists \overrightarrow{\alpha^-} . P$		$P \neq \exists \ldots$
	$\mid$	$[\sigma] P$	M	
	$\mid$	$[\widehat{\tau}] P$	M	
	$\mid$	$[\widehat{\sigma}] P$	M	
	$\mid$	$[\mu] P$	M	
	$\mid$	(P)	S	
	$\mid$	$P_1 \vee P_2$	M	
	$\mid$	$\mathbf{nf}\,(P')$	M	
$N,\ M,\ K$	$::=$			multi-quantified negative types
	$\mid$	α^-		
	$\mid$	$\uparrow P$		
	$\mid$	$P \rightarrow N$		

	|	$\forall\overrightarrow{\alpha^+}.N$		$N \neq \forall\ldots$
	|	$[\sigma]N$	M	
	|	$[\widehat{\tau}]N$	M	
	|	$[\mu]N$	M	
	|	$[\widehat{\sigma}]N$	M	
	|	(N)	S	
	|	$\mathbf{nf}\,(N')$	M	
$\overrightarrow{P},\ \overrightarrow{Q}$	$::=$			list of positive types
	|	$\cdot$		empty list
	|	P		a singel type
	|	$[\sigma]\overrightarrow{P}$	M	
	|	$\overline{\overrightarrow{P}_i}^{\,i}$		concatenate lists
	|	$(\overrightarrow{P})$	S	
	|	$\mathbf{nf}\,(\overrightarrow{P}')$	M	
$\overrightarrow{N},\ \overrightarrow{M}$	$::=$			list of negative types
	|	$\cdot$		empty list
	|	N		a singel type
	|	$[\sigma]\overrightarrow{N}$	M	
	|	$\overline{\overrightarrow{N}_i}^{\,i}$		concatenate lists
	|	$(\overrightarrow{N})$	S	
	|	$\mathbf{nf}\,(\overrightarrow{N}')$	M	
$\Delta,\ \Gamma$	$::=$			declarative type context
	|	$\cdot$		empty context
	|	$\overrightarrow{\alpha^+}$		list of variables
	|	$\overrightarrow{\alpha^-}$		list of variables
	|	$\overrightarrow{\alpha^\pm}$		list of variables
	|	vars		
	|	$\overline{\Gamma_i}^{\,i}$		concatenate contexts
	|	(Γ)	S	
	|	$\Theta(\widehat{\alpha}^+)$	M	
	|	$\Theta(\widehat{\alpha}^-)$	M	
	|	$\Gamma_1 \cup \Gamma_2$	M	
Θ	$::=$			unification type variable context
	|	$\cdot$		empty context
	|	$\vec{\alpha}\{\Delta\}$		from an ordered list of variables
	|	$\overrightarrow{\widehat{\alpha}^+\{\Delta\}}$		from a variable to a list
	|	$\overline{\Theta_i}^{\,i}$		concatenate contexts
	|	(Θ)	S	
	|	$\Theta\rvert_{\mathit{vars}}$		leave only those variables that are in the set
	|	$\Theta_1 \cup \Theta_2$		
Ξ	$::=$			anti-unification type variable context
	|	$\cdot$		empty context
	|	$\overrightarrow{\widehat{\alpha^-}}$		list of variables
	|	$\overline{\Xi_i}^{\,i}$		concatenate contexts
	|	(Ξ)	S	

	|	$\Xi_1 \cup \Xi_2$		
	|	$\Xi_1 \cap \Xi_2$		
	|	$\Xi' \|_{vars}$	M	
$\vec{\alpha},\ \vec{\beta}$	::=			ordered positive or negative variables
	|	$\cdot$		empty list
	|	$\overrightarrow{\alpha^{+}}$		list of variables
	|	$\overrightarrow{\alpha^{-}}$		list of variables
	|	$\overrightarrow{\alpha^{\pm}}$		list of variables
	|	$\overrightarrow{\widetilde{\alpha}^{+}}$		list of variables
	|	$\overrightarrow{\widehat{\alpha}^{-}}$		list of variables
	|	$\vec{\alpha}_1 \backslash vars$		setminus
	|	Γ		context
	|	$vars$		
	|	$\overline{\vec{\alpha}_i}^{\,i}$		concatenate contexts
	|	$(\vec{\alpha})$	S	parenthesis
	|	$[\mu]\vec{\alpha}$	M	apply moving to list
	|	$\mathbf{ord}\, vars\, \mathbf{in}\ P$	M	
	|	$\mathbf{ord}\, vars\, \mathbf{in}\ N$	M	
	|	$\mathbf{ord}\, vars\, \mathbf{in}\, P$	M	
	|	$\mathbf{ord}\, vars\, \mathbf{in}\, N$	M	
$vars$	::=			set of variables
	|	$\varnothing$		empty set
	|	$\mathbf{fv}\, P$		free variables
	|	$\mathbf{fv}\, N$		free variables
	|	$\mathbf{fv}\, \mathbf{imP}$		free variables
	|	$\mathbf{fv}\, \mathbf{imN}$		free variables
	|	$vars_1 \cap vars_2$		set intersection
	|	$vars_1 \cup vars_2$		set union
	|	$vars_1 \backslash vars_2$		set complement
	|	$\mathbf{mv}\, \mathbf{imP}$		movable variables
	|	$\mathbf{mv}\, \mathbf{imN}$		movable variables
	|	$\mathbf{uv}\ N$		unification variables
	|	$\mathbf{uv}\ P$		unification variables
	|	$\mathbf{fv}\ N$		free variables
	|	$\mathbf{fv}\ P$		free variables
	|	$(vars)$	S	parenthesis
	|	$\vec{\alpha}$		ordered list of variables
	|	$[\mu]vars$	M	apply moving to varset
	|	$\mathbf{dom}\,(UC)$	M	
	|	$\mathbf{dom}\,(SC)$	M	
	|	$\mathbf{dom}\,(\widehat{\sigma})$	M	
	|	$\mathbf{dom}\,(\widehat{\tau})$	M	
	|	$\mathbf{dom}\,(\Theta)$	M	
μ	::=			
	|	$\cdot$		empty moving
	|	$pma1 \mapsto pma2$		Positive unit substitution
	|	$nma1 \mapsto nma2$		Positive unit substitution

	|	$\mu_1 \cup \mu_2$	M	Set-like union of movings
	|	$\mu_1 \circ \mu_2$	M	Composition
	|	$\overline{\mu_i}^{\,i}$		concatenate movings
	|	$\mu\vert_{vars}$	M	restriction on a set
	|	μ^{-1}	M	inversion
	|	$\mathbf{nf}\,(\mu')$	M	
$\widehat{\alpha}^{\pm}$	::=			positive/negative unification variable
	|	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{+}$	::=			positive unification variable
	|	$\widehat{\alpha}^{+}$		
	|	$\widehat{\alpha}^{+}\{\Delta\}$		
	|	$\widehat{\alpha}^{\pm}$		
$\widehat{\alpha}^{-},\ \widehat{\beta}^{-}$	::=			negative unification variable
	|	$\widehat{\alpha}^{-}$		
	|	$\widehat{\alpha}^{-}_{\{N,M\}}$		
	|	$\widehat{\alpha}^{-}\{\Delta\}$		
	|	$\widehat{\alpha}^{\pm}$		
$\overrightarrow{\widetilde{\alpha}^{+}},\ \overrightarrow{\widetilde{\beta}^{+}}$	::=			positive unification variable list
	|	$\cdot$		empty list
	|	$\widehat{\alpha}^{+}$		a variable
	|	$\overrightarrow{\widehat{\alpha}^{+}}$		from a normal variable, context unspecified
	|	$\overline{\overrightarrow{\widetilde{\alpha}^{+}}_i}^{\,i}$		concatenate lists
$\overrightarrow{\widehat{\alpha}^{-}},\ \overrightarrow{\widehat{\beta}^{-}}$	::=			negative unification variable list
	|	$\cdot$		empty list
	|	$\widehat{\alpha}^{-}$		a variable
	|	Ξ		from an antiunification context
	|	$\overrightarrow{\widehat{\alpha}^{-}\{\Delta\}}$		from a normal variable
	|	$\overrightarrow{\widehat{\alpha}^{-}}$		from a normal variable, context unspecified
	|	$\overline{\overrightarrow{\widehat{\alpha}^{-}}_i}^{\,i}$		concatenate lists
$P,\ Q$	::=			a positive algorithmic type (potentially with metavariables)
	|	$\widehat{\alpha}^{+}$		
	|	α^{+}		
	|	$\downarrow N$		
	|	$\exists\overrightarrow{\alpha^{-}}.\,P$		
	|	$[\sigma]P$	M	
	|	$[\widehat{\tau}]P$	M	
	|	$[\mu]P$	M	
	|	(P)	S	
	|	$\mathbf{nf}\,(P')$	M	
$N,\ M$	::=			a negative algorithmic type (potentially with metavariables)
	|	$\widehat{\alpha}^{-}$		
	|	α^{-}		

	$\mid$ $\uparrow P$	
	$\mid$ $P \rightarrow N$	
	$\mid$ $\forall \overrightarrow{\alpha^{+}}. N$	
	$\mid$ $[\sigma] N$	M
	$\mid$ $[\widehat{\tau}] N$	M
	$\mid$ $[\mu] N$	M
	$\mid$ (N)	S
	$\mid$ $\mathbf{nf}\,(N')$	M
$auSol$	$::=$	
	$\mid$ $(\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	$\mid$ $(\Xi, N, \widehat{\tau}_1, \widehat{\tau}_2)$	
$terminals$	$::=$	
	$\mid$ $\exists$	
	$\mid$ $\forall$	
	$\mid$ $\uparrow$	
	$\mid$ $\downarrow$	
	$\mid$ $\rightarrow$	
	$\mid$ $\leftrightarrow$	
	$\mid$ $\in$	
	$\mid$ $\notin$	
	$\mid$ $\cdot$	
	$\mid$ $\vdash$	
	$\mid$ $\leqslant$	
	$\mid$ $\geqslant$	
	$\mid$ $\simeq$	
	$\mid$ $\cup$	
	$\mid$ $\cap$	
	$\mid$ $\setminus$	
	$\mid$ $\subseteq$	
	$\mid$ $\mapsto$	
	$\mid$ $\stackrel{u}{\simeq}$	
	$\mid$ $\stackrel{a}{\simeq}$	
	$\mid$ $\varnothing$	
	$\mid$ $\circ$	
	$\mid$ $\Rightarrow$	
	$\mid$ $\models$	
	$\mid$ $\dashv$	
	$\mid$ $\neq$	
	$\mid$ $\equiv_n$	
	$\mid$ $\vee$	
	$\mid$ $\Downarrow$	
	$\mid$ $:\geqslant$	
	$\mid$ $:\simeq$	
	$\mid$ Λ	
	$\mid$ λ	
	$\mid$ $\mathbf{let}^{\exists}$	
	$\mid$ $\bullet$	
	$\mid$ $\Rrightarrow$	

	$\mid$	$\lll$	
$v,\ w$	$::=$		value terms
	$\mid$	x	
	$\mid$	$\{c\}$	
	$\mid$	$(v : P)$	
	$\mid$	(v)	M
$\vec{v}$	$::=$		list of arguments
	$\mid$	$\cdot$	
	$\mid$	v	
	$\mid$	$\overline{\vec{v}_i}^{\,i}$	concatenate
$c,\ d$	$::=$		computation terms
	$\mid$	$(c : N)$	
	$\mid$	$\lambda x : P.c$	
	$\mid$	$\Lambda\alpha^{+}.c$	
	$\mid$	$\mathbf{return}\, v$	
	$\mid$	$\mathbf{let}\, x = v; c$	
	$\mid$	$\mathbf{let}\, x : P = v(\vec{v}); c$	
	$\mid$	$\mathbf{let}\, x = v(\vec{v}); c$	
	$\mid$	$\mathbf{let}^{\exists}(\overrightarrow{\alpha^{-}}, x) = v; c$	
$vctx,\ \Phi$	$::=$		variable context
	$\mid$	$\cdot$	
	$\mid$	$x : P$	
	$\mid$	$\overline{\Phi_i}^{\,i}$	concatenate contexts
$formula$	$::=$		
	$\mid$	$judgement$	
	$\mid$	$judgement$ unique	
	$\mid$	$formula_1 \quad .. \quad formula_n$	
	$\mid$	$\mu : vars_1 \leftrightarrow vars_2$	
	$\mid$	$\mu\, \mathbf{is\ bijective}$	
	$\mid$	$x : P \in \Phi$	
	$\mid$	$UC_1 \subseteq UC_2$	
	$\mid$	$UC_1 = UC_2$	
	$\mid$	$SC_1 \subseteq SC_2$	
	$\mid$	$vars_1 \subseteq vars_2$	
	$\mid$	$vars_1 = vars_2$	
	$\mid$	$vars\, \mathbf{is\ fresh}$	
	$\mid$	$\alpha^{-} \notin vars$	
	$\mid$	$\alpha^{+} \notin vars$	
	$\mid$	$\alpha^{-} \in vars$	
	$\mid$	$\alpha^{+} \in vars$	
	$\mid$	$\widehat{\alpha}^{+} \in vars$	
	$\mid$	$\widehat{\alpha}^{-} \in vars$	
	$\mid$	$\widehat{\alpha}^{-} \in \Theta$	
	$\mid$	$\widehat{\alpha}^{+} \in \Theta$	
	$\mid$	$\widehat{\alpha}^{-} \notin vars$	

	$\mid$	$\widehat{\alpha}^{+} \notin vars$
	$\mid$	$\widehat{\alpha}^{-} \notin \Theta$
	$\mid$	$\widehat{\alpha}^{+} \notin \Theta$
	$\mid$	$\widehat{\alpha}^{-} \in \Xi$
	$\mid$	$\widehat{\alpha}^{-} \notin \Xi$
	$\mid$	if any other rule is not applicable
	$\mid$	$\vec{\alpha}_1 = \vec{\alpha}_2$
	$\mid$	$e_1 = e_2$
	$\mid$	$e_1 = e_2$
	$\mid$	$\widehat{\sigma}_1 = \widehat{\sigma}_2$
	$\mid$	$N = M$
	$\mid$	$\Theta \subseteq \Theta'$
	$\mid$	$\vec{v}_1 = \vec{v}_2$
	$\mid$	$N \neq M$
	$\mid$	$P \neq Q$
	$\mid$	$N \neq M$
	$\mid$	$P \neq Q$
	$\mid$	$P \neq Q$
	$\mid$	$N \neq M$
	$\mid$	$\vec{v}_1 \neq \vec{v}_2$
	$\mid$	$\overrightarrow{\alpha^{+}}_1 \neq \overrightarrow{\alpha^{+}}_2$
	$\mid$	$\lvert\overrightarrow{\alpha^{-}}\rvert + \lvert\overrightarrow{\beta^{-}}\rvert > 0$
	$\mid$	$\lvert\overrightarrow{\alpha^{+}}\rvert + \lvert\overrightarrow{\beta^{+}}\rvert > 0$

A	$::=$		
	$\mid$	$\Gamma;\ \Theta \models N \leqslant M \dashv SC$	Negative subtyping
	$\mid$	$\Gamma;\ \Theta \models P \geqslant Q \dashv SC$	Positive supertyping
AT	$::=$		
	$\mid$	$\Gamma; \Phi \models v : P$	Positive type inference
	$\mid$	$\Gamma; \Phi \models c : N$	Negative type inference
	$\mid$	$\Gamma; \Phi; \Theta_1 \models N \bullet \vec{v} \Rrightarrow M \dashv \Theta_2; SC$	Application type inference
AU	$::=$		
	$\mid$	$\Gamma \models P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)$	
	$\mid$	$\Gamma \models N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)$	
SCM	$::=$		
	$\mid$	$\Gamma \vdash e_1\ \&\ e_2 = e_3$	Subtyping Constraint Entry Merge
	$\mid$	$\Theta \vdash SC_1 \& SC_2 = SC_3$	Merge of subtyping constraints
UCM	$::=$		
	$\mid$	$\Gamma \vdash e_1 \& e_2 = e_3$	
	$\mid$	$\Theta \vdash UC_1 \& UC_2 = UC_3$	
$SATSCE$	$::=$		
	$\mid$	$\Gamma \vdash P : e$	Positive type satisfies with the subtyping constr
	$\mid$	$\Gamma \vdash N : e$	Negative type satisfies with the subtyping const

SING	::=		
	|	e_1 **singular with** P	Positive Subtyping Constraint Entry Is Singular
	|	e_1 **singular with** N	Negative Subtyping Constraint Entry Is Singular
	|	SC **singular with** $\widehat{\sigma}$	Subtyping Constraint Is Singular
E1	::=		
	|	$N \simeq^{D}_{1} M$	Negative multi-quantified type equivalence
	|	$P \simeq^{D}_{1} Q$	Positive multi-quantified type equivalence
	|	$P \simeq^{D}_{1} Q$	Positive unification type equivalence
	|	$N \simeq^{D}_{1} M$	Positive unification type equivalence
D1	::=		
	|	$\Gamma \vdash N \simeq^{\leqslant}_{1} M$	Negative equivalence on MQ types
	|	$\Gamma \vdash P \simeq^{\leqslant}_{1} Q$	Positive equivalence on MQ types
	|	$\Gamma \vdash N \leqslant_{1} M$	Negative subtyping
	|	$\Gamma \vdash P \geqslant_{1} Q$	Positive supertyping
	|	$\Gamma_2 \vdash \sigma_1 \simeq^{\leqslant}_{1} \sigma_2 : \Gamma_1$	Equivalence of substitutions
	|	$\Gamma \vdash \sigma_1 \simeq^{\leqslant}_{1} \sigma_2 : \mathit{vars}$	Equivalence of substitutions
	|	$\Theta \vdash \widehat{\sigma}_1 \simeq^{\leqslant}_{1} \widehat{\sigma}_2 : \mathit{vars}$	Equivalence of unification substitutions
	|	$\Gamma \vdash \Phi_1 \simeq^{\leqslant}_{1} \Phi_2$	Equivalence of contexts
D0	::=		
	|	$\Gamma \vdash N \simeq^{\leqslant}_{0} M$	Negative equivalence
	|	$\Gamma \vdash P \simeq^{\leqslant}_{0} Q$	Positive equivalence
	|	$\Gamma \vdash N \leqslant_{0} M$	Negative subtyping
	|	$\Gamma \vdash P \geqslant_{0} Q$	Positive supertyping
DT	::=		
	|	$\Gamma; \Phi \vdash v : P$	Positive type inference
	|	$\Gamma; \Phi \vdash c : N$	Negative type inference
	|	$\Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M$	Application type inference
EQ	::=		
	|	$N = M$	Negative type equality (alpha-equivalence)
	|	$P = Q$	Positive type equuality (alphha-equivalence)
	|	$P = Q$	
LUBF	::=		
	|	$P_1 \vee P_2 === Q$	
	|	**ord** vars **in** $P === \vec{\alpha}$	
	|	**ord** vars **in** $N === \vec{\alpha}$	
	|	**ord** vars **in** $P === \vec{\alpha}$	
	|	**ord** vars **in** $N === \vec{\alpha}$	
	|	**nf** $(N') === N$	
	|	**nf** $(P') === P$	
	|	**nf** $(N') === N$	
	|	**nf** $(P') === P$	
	|	**nf** $(\vec{N}') === \vec{N}$	
	|	**nf** $(\vec{P}') === \vec{P}$	

	$\vert$	$\mathbf{nf}\,(\sigma') === \sigma$	
	$\vert$	$\mathbf{nf}\,(\widehat{\sigma}') === \widehat{\sigma}$	
	$\vert$	$\mathbf{nf}\,(\mu') === \mu$	
	$\vert$	$\sigma'\vert_{vars}$	
	$\vert$	$\widehat{\sigma}'\vert_{vars}$	
	$\vert$	$\widehat{\tau}'\vert_{vars}$	
	$\vert$	$\Xi'\vert_{vars}$	
	$\vert$	$SC\vert_{vars}$	
	$\vert$	$UC\vert_{vars}$	
	$\vert$	$e_1\ \&\ e_2$	
	$\vert$	$e_1\ \&\ e_2$	
	$\vert$	$UC_1\ \&\ UC_2$	
	$\vert$	$UC_1 \cup UC_2$	
	$\vert$	$\Gamma_1 \cup \Gamma_2$	
	$\vert$	$SC_1\ \&\ SC_2$	
	$\vert$	$\widehat{\tau}_1\ \&\ \widehat{\tau}_2$	
	$\vert$	$\mathbf{dom}\,(UC) === vars$	
	$\vert$	$\mathbf{dom}\,(SC) === vars$	
	$\vert$	$\mathbf{dom}\,(\widehat{\sigma}) === vars$	
	$\vert$	$\mathbf{dom}\,(\widehat{\tau}) === vars$	
	$\vert$	$\mathbf{dom}\,(\Theta) === vars$	
	$\vert$	$\vert SC\vert === UC$	
LUB	$::=$		
	$\vert$	$\Gamma \models P_1 \vee P_2 = Q$	Least Upper Bound (Least Common Supertype)
	$\vert$	$\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q$	
Nrm	$::=$		
	$\vert$	$\mathbf{nf}\,(N) = M$	
	$\vert$	$\mathbf{nf}\,(P) = Q$	
	$\vert$	$\mathbf{nf}\,(N) = M$	
	$\vert$	$\mathbf{nf}\,(P) = Q$	
$Order$	$::=$		
	$\vert$	$\mathbf{ord}\ vars\,\mathbf{in}\,N = \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\ vars\,\mathbf{in}\,P = \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\ vars\,\mathbf{in}\,N = \vec{\alpha}$	
	$\vert$	$\mathbf{ord}\ vars\,\mathbf{in}\,P = \vec{\alpha}$	
U	$::=$		
	$\vert$	$\Gamma; \Theta \models N \stackrel{u}{\simeq} M \dashv UC$	Negative unification
	$\vert$	$\Gamma; \Theta \models P \stackrel{u}{\simeq} Q \dashv UC$	Positive unification
WFT	$::=$		
	$\vert$	$\Gamma \vdash N$	Negative type well-formedness
	$\vert$	$\Gamma \vdash P$	Positive type well-formedness
	$\vert$	$\Gamma \vdash N$	Negative type well-formedness
	$\vert$	$\Gamma \vdash P$	Positive type well-formedness
	$\vert$	$\Gamma \vdash \vec{N}$	Negative type list well-formedness
	$\vert$	$\Gamma \vdash \vec{P}$	Positive type list well-formedness

WFAT	::=		
	|	$\Gamma ; \Theta \vdash \boxed{N}$	Negative unification type well-formedness
	|	$\Gamma ; \Theta \vdash \boxed{P}$	Positive unifiication type well-formedness
	|	$\Gamma ; \Xi \vdash \boxed{N}$	Negative anti-unification type well-formedness
	|	$\Gamma ; \Xi \vdash \boxed{P}$	Positive anti-unification type well-formedness
	|	$\Gamma ; \Xi_2 \vdash \widehat{\tau} : \Xi_1$	Antiunification substitution well-formedness
	|	$\Gamma \vdash^{\supseteq} \Theta$	Unification context well-formedness
	|	$\Gamma_1 \vdash \sigma : \Gamma_2$	Substitution well-formedness
	|	$\Theta \vdash \widehat{\sigma}$	Unification substitution well-formedness
	|	$\Theta \vdash \widehat{\sigma} : UC$	Unification substitution satisfies unification constraint
	|	$\Theta \vdash \widehat{\sigma} : SC$	Unification substitution satisfies subtyping constraint
	|	$\Gamma \vdash e$	Unification constraint entry well-formedness
	|	$\Gamma \vdash e$	Subtyping constraint entry well-formedness
	|	$\Gamma \vdash P : e$	Positive type satisfies unification constraint
	|	$\Gamma \vdash N : e$	Negative type satisfies unification constraint
	|	$\Gamma \vdash P : e$	Positive type satisfies subtyping constraint
	|	$\Gamma \vdash N : e$	Negative type satisfies subtyping constraint
	|	$\Theta \vdash UC$	Unification constraint well-formedness
	|	$\Theta \vdash SC$	Subtyping constraint well-formedness
	|	$\Gamma \vdash \overrightarrow{v}$	Argument List well-formedness
	|	$\Gamma \vdash \Phi$	Context well-formedness
	|	$\Gamma \vdash v$	Value well-formedness
	|	$\Gamma \vdash c$	Computation well-formedness
judgement	::=		
	|	A	
	|	AT	
	|	AU	
	|	SCM	
	|	UCM	
	|	$SATSCE$	
	|	$SING$	
	|	$E1$	
	|	$D1$	
	|	$D0$	
	|	DT	
	|	EQ	
	|	LUB	
	|	Nrm	
	|	$Order$	
	|	U	
	|	WFT	
	|	$WFAT$	
user_syntax	::=		
	|	α	
	|	n	
	|	x	
	|	n	
	|	α^{+}	

$|\quad \alpha^-$
$|\quad \alpha^\pm$
$|\quad \sigma$
$|\quad e$
$|\quad e$
$|\quad UC$
$|\quad SC$
$|\quad \widehat{\sigma}$
$|\quad \widehat{\tau}$
$|\quad P$
$|\quad N$
$|\quad \overrightarrow{\alpha^+}$
$|\quad \overrightarrow{\alpha^-}$
$|\quad \overrightarrow{\alpha^\pm}$
$|\quad P$
$|\quad N$
$|\quad \overrightarrow{P}$
$|\quad \overrightarrow{N}$
$|\quad \Gamma$
$|\quad \Theta$
$|\quad \Xi$
$|\quad \overrightarrow{\alpha}$
$|\quad vars$
$|\quad \mu$
$|\quad \widehat{\alpha}^\pm$
$|\quad \widehat{\alpha}^+$
$|\quad \widehat{\alpha}^-$
$|\quad \overrightarrow{\widetilde{\alpha^+}}$
$|\quad \overrightarrow{\widehat{\alpha^-}}$
$|\quad \boxed{P}$
$|\quad \boxed{N}$
$|\quad auSol$
$|\quad terminals$
$|\quad v$
$|\quad \overrightarrow{v}$
$|\quad c$
$|\quad vctx$
$|\quad formula$

$\boxed{\Gamma;\ \Theta \models N \leqslant M \dashv SC}$ Negative subtyping

$$\frac{}{\Gamma;\ \Theta \models \alpha^- \leqslant \alpha^- \dashv \cdot} \quad \text{ANVar}$$

$$\frac{\Gamma; \Theta \models \mathbf{nf}\,(P) \overset{u}{\simeq} \mathbf{nf}\,(Q) \dashv UC}{\Gamma;\ \Theta \models {\uparrow}P \leqslant {\uparrow}Q \dashv UC} \quad \text{AShiftU}$$

$$\frac{\Gamma;\ \Theta \models P \geqslant Q \dashv SC_1 \quad \Gamma;\ \Theta \models N \leqslant M \dashv SC_2 \quad \Theta \vdash SC_1 \& SC_2 = SC}{\Gamma;\ \Theta \models P \to N \leqslant Q \to M \dashv SC} \quad \text{AArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma;\ \Theta \models \forall\overrightarrow{\alpha^+}.N \leqslant \forall\overrightarrow{\beta^+}.M \dashv SC \backslash \overrightarrow{\widehat{\alpha}^+}} \quad \text{AForall}$$

$\boxed{\Gamma;\,\Theta \vDash P \geqslant Q \dashv SC}$ Positive supertyping

$$\frac{}{\Gamma;\ \Theta \vDash \alpha^{+} \geqslant \alpha^{+} \dashv \cdot}\ \textsc{APVar}$$

$$\frac{\Gamma;\Theta \vDash \mathbf{nf}\,(N) \overset{u}{\simeq} \mathbf{nf}\,(M) \dashv UC}{\Gamma;\ \Theta \vDash {\downarrow}N \geqslant {\downarrow}M \dashv UC}\ \textsc{AShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^{-}};\ \Theta, \overrightarrow{\widehat{\alpha}^{-}}\{\Gamma, \overrightarrow{\beta^{-}}\} \vDash [\overrightarrow{\widehat{\alpha}^{-}}/\overrightarrow{\alpha^{-}}]P \geqslant Q \dashv SC}{\Gamma;\ \Theta \vDash \exists\overrightarrow{\alpha^{-}}.P \geqslant \exists\overrightarrow{\beta^{-}}.Q \dashv SC\backslash\overrightarrow{\widehat{\alpha}^{-}}}\ \textsc{AExists}$$

$$\frac{\widehat{\alpha}^{+}\{\Delta\} \in \Theta \quad \mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}{\Gamma;\ \Theta \vDash \widehat{\alpha}^{+} \geqslant P \dashv (\widehat{\alpha}^{+} :\geqslant Q)}\ \textsc{APUVar}$$

$\boxed{\Gamma;\Phi \vDash v\colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma;\Phi \vDash x\colon \mathbf{nf}\,(P)}\ \textsc{ATVar}$$

$$\frac{\Gamma;\Phi \vDash c\colon N}{\Gamma;\Phi \vDash \{c\}\colon {\downarrow}N}\ \textsc{ATThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma;\Phi \vDash v\colon P \quad \Gamma;\ \cdot \vDash Q \geqslant P \dashv \cdot}{\Gamma;\Phi \vDash (v : Q)\colon \mathbf{nf}\,(Q)}\ \textsc{ATPAnnot}$$

$\boxed{\Gamma;\Phi \vDash c\colon N}$ Negative type inference

$$\frac{\Gamma \vdash M \quad \Gamma;\Phi \vDash c\colon N \quad \Gamma;\ \cdot \vDash N \leqslant M \dashv \cdot}{\Gamma;\Phi \vDash (c : M)\colon \mathbf{nf}\,(M)}\ \textsc{ATNAnnot}$$

$$\frac{\Gamma \vdash P \quad \Gamma;\Phi, x : P \vDash c\colon N}{\Gamma;\Phi \vDash \lambda x : P.c\colon \mathbf{nf}\,(P \to N)}\ \textsc{ATtLam}$$

$$\frac{\Gamma, \alpha^{+};\Phi \vDash c\colon N}{\Gamma;\Phi \vDash \Lambda\alpha^{+}.c\colon \mathbf{nf}\,(\forall\alpha^{+}.N)}\ \textsc{ATTLam}$$

$$\frac{\Gamma;\Phi \vDash v\colon P}{\Gamma;\Phi \vDash \mathbf{return}\,v\colon {\uparrow}P}\ \textsc{ATReturn}$$

$$\frac{\Gamma;\Phi \vDash v\colon P \quad \Gamma;\Phi, x : P \vDash c\colon N}{\Gamma;\Phi \vDash \mathbf{let}\,x = v; c\colon N}\ \textsc{ATVarLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma;\Phi \vDash v\colon {\downarrow}M \\ \Gamma;\Phi;\cdot \vDash M \bullet \vec{v} \Rightarrow\!\!\!> {\uparrow}Q \dashv \Theta; SC_1 \quad \Gamma;\ \Theta \vDash {\uparrow}Q \leqslant {\uparrow}P \dashv SC_2 \\ \Theta \vdash SC_1 \& SC_2 = SC \quad \Gamma;\Phi, x : P \vDash c\colon N\end{array}}{\Gamma;\Phi \vDash \mathbf{let}\,x : P = v(\vec{v}); c\colon N}\ \textsc{ATAppLetAnn}$$

$$\frac{\begin{array}{l}\Gamma;\Phi \vDash v\colon {\downarrow}M \quad \Gamma;\Phi;\cdot \vDash M \bullet \vec{v} \Rightarrow\!\!\!> {\uparrow}Q \dashv \Theta; SC \\ \text{<<multiple parses>>} \\ \Gamma;\Phi, x : [\widehat{\sigma}]Q \vDash c\colon N\end{array}}{\Gamma;\Phi \vDash \mathbf{let}\,x = v(\vec{v}); c\colon N}\ \textsc{ATAppLet}$$

$$\frac{\Gamma;\Phi \vDash v\colon \exists\overrightarrow{\alpha^{-}}.P \quad \Gamma, \overrightarrow{\alpha^{-}};\Phi, x : P \vDash c\colon N \quad \Gamma \vdash N}{\Gamma;\Phi \vDash \mathbf{let}^{\exists}(\overrightarrow{\alpha^{-}}, x) = v; c\colon N}\ \textsc{ATUnpack}$$

$\boxed{\Gamma;\Phi;\Theta_1 \vDash N \bullet \vec{v} \Rightarrow\!\!\!> M \dashv \Theta_2; SC}$ Application type inference

$$\frac{}{\Gamma;\Phi;\Theta \vDash N \bullet \cdot \Rightarrow\!\!\!> \mathbf{nf}\,(N) \dashv \Theta; \cdot}\ \textsc{ATEmptyApp}$$

$$\frac{\begin{array}{l}\Gamma;\Phi \vDash v\colon P \quad \Gamma;\Theta \vDash Q \geqslant P \dashv SC_1 \quad \Gamma;\Phi;\Theta \vDash N \bullet \vec{v} \Rrightarrow M \dashv \Theta'; SC_2\\ \Theta \vdash SC_1 \& SC_2 = SC\end{array}}{\Gamma;\Phi;\Theta \vDash Q \to N \bullet v, \vec{v} \Rrightarrow M \dashv \Theta'; SC}\ \textsc{ATArrowApp}$$

<<multiple parses>>

$$\frac{\vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot}{\Gamma;\Phi;\Theta \vDash \forall\overrightarrow{\alpha^+}.N \bullet \vec{v} \Rrightarrow M \dashv \Theta'; SC}\ \textsc{ATForallApp}$$

$\boxed{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \vDash \alpha^+ \overset{a}{\simeq} \alpha^+ \dashv (\cdot, \alpha^+, \cdot, \cdot)}\ \textsc{AUPVar}$$

$$\frac{\Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash {\downarrow}N_1 \overset{a}{\simeq} {\downarrow}N_2 \dashv (\Xi, {\downarrow}M, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \Gamma = \varnothing \quad \Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \exists\overrightarrow{\alpha^-}.P_1 \overset{a}{\simeq} \exists\overrightarrow{\alpha^-}.P_2 \dashv (\Xi, \exists\overrightarrow{\alpha^-}.Q, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUExists}$$

$\boxed{\Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}$

$$\frac{}{\Gamma \vDash \alpha^- \overset{a}{\simeq} \alpha^- \dashv (\cdot, \alpha^-, \cdot, \cdot)}\ \textsc{AUNVar}$$

$$\frac{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi, Q, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash {\uparrow}P_1 \overset{a}{\simeq} {\uparrow}P_2 \dashv (\Xi, {\uparrow}Q, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUShiftU}$$

$$\frac{\overrightarrow{\alpha^+} \cap \Gamma = \varnothing \quad \Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi, M, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \forall\overrightarrow{\alpha^+}.N_1 \overset{a}{\simeq} \forall\overrightarrow{\alpha^+}.N_2 \dashv (\Xi, \forall\overrightarrow{\alpha^+}.M, \widehat{\tau}_1, \widehat{\tau}_2)}\ \textsc{AUForall}$$

$$\frac{\Gamma \vDash P_1 \overset{a}{\simeq} P_2 \dashv (\Xi_1, Q, \widehat{\tau}_1, \widehat{\tau}_2) \quad \Gamma \vDash N_1 \overset{a}{\simeq} N_2 \dashv (\Xi_2, M, \widehat{\tau}'_1, \widehat{\tau}'_2)}{\Gamma \vDash P_1 \to N_1 \overset{a}{\simeq} P_2 \to N_2 \dashv (\Xi_1 \cup \Xi_2, Q \to M, \widehat{\tau}_1 \cup \widehat{\tau}'_1, \widehat{\tau}_2 \cup \widehat{\tau}'_2)}\ \textsc{AUArrow}$$

$$\frac{\text{if any other rule is not applicable} \quad \Gamma \vdash N \quad \Gamma \vdash M}{\Gamma \vDash N \overset{a}{\simeq} M \dashv (\widehat{\alpha}^-_{\{N,M\}}, \widehat{\alpha}^-_{\{N,M\}}, (\widehat{\alpha}^-_{\{N,M\}} :\approx N), (\widehat{\alpha}^-_{\{N,M\}} :\approx M))}\ \textsc{AUAU}$$

$\boxed{\Gamma \vdash e_1 \ \&\ e_2 = e_3}$ Subtyping Constraint Entry Merge

$$\frac{\Gamma \vDash P_1 \vee P_2 = Q}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P_1) \ \&\ (\widehat{\alpha}^+ :\geqslant P_2) = (\widehat{\alpha}^+ :\geqslant Q)}\ \textsc{SCMESupSup}$$

$$\frac{\Gamma; \cdot \vDash P \geqslant Q \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \ \&\ (\widehat{\alpha}^+ :\geqslant Q) = (\widehat{\alpha}^+ :\approx P)}\ \textsc{SCMEEqSup}$$

$$\frac{\Gamma; \cdot \vDash Q \geqslant P \dashv \cdot}{\Gamma \vdash (\widehat{\alpha}^+ :\geqslant P) \ \&\ (\widehat{\alpha}^+ :\approx Q) = (\widehat{\alpha}^+ :\approx Q)}\ \textsc{SCMESupEq}$$

<<multiple parses>>

$$\frac{}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \ \&\ (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)}\ \textsc{SCMEPEqEq}$$

<<multiple parses>>

$$\frac{}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \ \&\ (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)}\ \textsc{SCMENEqEq}$$

$\boxed{\Theta \vdash SC_1 \& SC_2 = SC_3}$ Merge of subtyping constraints

$\boxed{\Gamma \vdash e_1 \& e_2 = e_3}$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^+ :\approx P) \& (\widehat{\alpha}^+ :\approx P') = (\widehat{\alpha}^+ :\approx P)}\quad \textsc{UCMEPEqEq}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash (\widehat{\alpha}^- :\approx N_1) \& (\widehat{\alpha}^- :\approx N') = (\widehat{\alpha}^- :\approx N)}\quad \textsc{UCMENEqEq}$$

$\boxed{\Theta \vdash UC_1 \& UC_2 = UC_3}$

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies with the subtyping constraint entry

$$\frac{\Gamma \vdash P \geqslant_1 Q}{\Gamma \vdash P : (\widehat{\alpha}^+ :\geqslant Q)}\quad \textsc{SATSCESup}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash P : (\widehat{\alpha}^+ :\approx Q)}\quad \textsc{SATSCEPEq}$$

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies with the subtyping constraint entry

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash N : (\widehat{\alpha}^- :\approx M)}\quad \textsc{SATSCENEq}$$

$\boxed{e_1\ \textbf{singular with}\ P}$ Positive Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^+ :\approx P\ \textbf{singular with}\ \textbf{nf}\,(P)}\quad \textsc{SINGPEq}$$

$$\frac{}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.\alpha^+\ \textbf{singular with}\ \alpha^+}\quad \textsc{SINGSupVar}$$

$$\frac{N \simeq_1^D \alpha_i^-}{\widehat{\alpha}^+ :\geqslant \exists \overrightarrow{\alpha^-}.{\downarrow}N\ \textbf{singular with}\ \exists \alpha^-.{\downarrow}\alpha^-}\quad \textsc{SINGSupShift}$$

$\boxed{e_1\ \textbf{singular with}\ N}$ Negative Subtyping Constraint Entry Is Singular

$$\frac{}{\widehat{\alpha}^- :\approx N\ \textbf{singular with}\ \textbf{nf}\,(N)}\quad \textsc{SINGNEq}$$

$\boxed{SC\ \textbf{singular with}\ \widehat{\sigma}}$ Subtyping Constraint Is Singular

$\boxed{N \simeq_1^D M}$ Negative multi-quantified type equivalence

$$\frac{}{\alpha^- \simeq_1^D \alpha^-}\quad \textsc{E1NVar}$$

$$\frac{P \simeq_1^D Q}{{\uparrow}P \simeq_1^D {\uparrow}Q}\quad \textsc{E1ShiftU}$$

$$\frac{P \simeq_1^D Q \qquad N \simeq_1^D M}{P \to N \simeq_1^D Q \to M}\quad \textsc{E1Arrow}$$

$$\frac{\overrightarrow{\alpha^+} \cap \textbf{fv}\,M = \varnothing \qquad \mu : (\overrightarrow{\beta^+} \cap \textbf{fv}\,M) \leftrightarrow (\overrightarrow{\alpha^+} \cap \textbf{fv}\,N) \qquad N \simeq_1^D [\mu]M}{\forall \overrightarrow{\alpha^+}.N \simeq_1^D \forall \overrightarrow{\beta^+}.M}\quad \textsc{E1Forall}$$

$\boxed{P \simeq_1^D Q}$ Positive multi-quantified type equivalence

$$\frac{}{\alpha^+ \simeq_1^D \alpha^+}\quad \textsc{E1PVar}$$

$$\frac{N \simeq_1^D M}{{\downarrow}N \simeq_1^D {\downarrow}M}\quad \textsc{E1ShiftD}$$

$$\frac{\overrightarrow{\alpha^-} \cap \mathbf{fv}\, Q = \varnothing \quad \mu : (\overrightarrow{\beta^-} \cap \mathbf{fv}\, Q) \leftrightarrow (\overrightarrow{\alpha^-} \cap \mathbf{fv}\, P) \quad P \simeq_1^D [\mu] Q}{\exists \overrightarrow{\alpha^-}. P \simeq_1^D \exists \overrightarrow{\beta^-}. Q} \quad \text{E1Exists}$$

$\boxed{P \simeq_1^D Q}$ Positive unification type equivalence

$\boxed{N \simeq_1^D M}$ Positive unification type equivalence

$\boxed{\Gamma \vdash N \simeq_1^{\leqslant} M}$ Negative equivalence on MQ types

$$\frac{\Gamma \vdash N \leqslant_1 M \quad \Gamma \vdash M \leqslant_1 N}{\Gamma \vdash N \simeq_1^{\leqslant} M} \quad \text{D1NDef}$$

$\boxed{\Gamma \vdash P \simeq_1^{\leqslant} Q}$ Positive equivalence on MQ types

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma \vdash P \simeq_1^{\leqslant} Q} \quad \text{D1PDef}$$

$\boxed{\Gamma \vdash N \leqslant_1 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_1 \alpha^-} \quad \text{D1NVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \uparrow P \leqslant_1 \uparrow Q} \quad \text{D1ShiftU}$$

$$\frac{\Gamma \vdash P \geqslant_1 Q \quad \Gamma \vdash N \leqslant_1 M}{\Gamma \vdash P \to N \leqslant_1 Q \to M} \quad \text{D1Arrow}$$

$$\frac{\Gamma, \overrightarrow{\beta^+} \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma, \overrightarrow{\beta^+} \vdash [\sigma] N \leqslant_1 M}{\Gamma \vdash \forall \overrightarrow{\alpha^+}. N \leqslant_1 \forall \overrightarrow{\beta^+}. M} \quad \text{D1Forall}$$

$\boxed{\Gamma \vdash P \geqslant_1 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_1 \alpha^+} \quad \text{D1PVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma \vdash \downarrow N \geqslant_1 \downarrow M} \quad \text{D1ShiftD}$$

$$\frac{\Gamma, \overrightarrow{\beta^-} \vdash \sigma : \overrightarrow{\alpha^-} \quad \Gamma, \overrightarrow{\beta^-} \vdash [\sigma] P \geqslant_1 Q}{\Gamma \vdash \exists \overrightarrow{\alpha^-}. P \geqslant_1 \exists \overrightarrow{\beta^-}. Q} \quad \text{D1Exists}$$

$\boxed{\Gamma_2 \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : \Gamma_1}$ Equivalence of substitutions

$\boxed{\Gamma \vdash \sigma_1 \simeq_1^{\leqslant} \sigma_2 : vars}$ Equivalence of substitutions

$\boxed{\Theta \vdash \widehat{\sigma}_1 \simeq_1^{\leqslant} \widehat{\sigma}_2 : vars}$ Equivalence of unification substitutions

$\boxed{\Gamma \vdash \Phi_1 \simeq_1^{\leqslant} \Phi_2}$ Equivalence of contexts

$\boxed{\Gamma \vdash N \simeq_0^{\leqslant} M}$ Negative equivalence

$$\frac{\Gamma \vdash N \leqslant_0 M \quad \Gamma \vdash M \leqslant_0 N}{\Gamma \vdash N \simeq_0^{\leqslant} M} \quad \text{D0NDef}$$

$\boxed{\Gamma \vdash P \simeq_0^{\leqslant} Q}$ Positive equivalence

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash Q \geqslant_0 P}{\Gamma \vdash P \simeq_0^{\leqslant} Q} \quad \text{D0PDef}$$

$\boxed{\Gamma \vdash N \leqslant_0 M}$ Negative subtyping

$$\frac{}{\Gamma \vdash \alpha^- \leqslant_0 \alpha^-} \quad \text{D0NVar}$$

$$\frac{\Gamma \vdash P \simeq^{\leqslant}_{0} Q}{\Gamma \vdash {\uparrow}P \leqslant_0 {\uparrow}Q}\;\textsc{D0ShiftU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash [P/\alpha^+]N \leqslant_0 M \quad M \neq \forall \beta^+.M'}{\Gamma \vdash \forall \alpha^+.N \leqslant_0 M}\;\textsc{D0ForallL}$$

$$\frac{\Gamma, \alpha^+ \vdash N \leqslant_0 M}{\Gamma \vdash N \leqslant_0 \forall \alpha^+.M}\;\textsc{D0ForallR}$$

$$\frac{\Gamma \vdash P \geqslant_0 Q \quad \Gamma \vdash N \leqslant_0 M}{\Gamma \vdash P \to N \leqslant_0 Q \to M}\;\textsc{D0Arrow}$$

$\boxed{\Gamma \vdash P \geqslant_0 Q}$ Positive supertyping

$$\frac{}{\Gamma \vdash \alpha^+ \geqslant_0 \alpha^+}\;\textsc{D0PVar}$$

$$\frac{\Gamma \vdash N \simeq^{\leqslant}_{0} M}{\Gamma \vdash {\downarrow}N \geqslant_0 {\downarrow}M}\;\textsc{D0ShiftD}$$

$$\frac{\Gamma \vdash N \quad \Gamma \vdash [N/\alpha^-]P \geqslant_0 Q \quad Q \neq \exists \alpha^-.Q'}{\Gamma \vdash \exists \alpha^-.P \geqslant_0 Q}\;\textsc{D0ExistsL}$$

$$\frac{\Gamma, \alpha^- \vdash P \geqslant_0 Q}{\Gamma \vdash P \geqslant_0 \exists \alpha^-.Q}\;\textsc{D0ExistsR}$$

$\boxed{\Gamma; \Phi \vdash v \colon P}$ Positive type inference

$$\frac{x : P \in \Phi}{\Gamma; \Phi \vdash x \colon P}\;\textsc{DTVar}$$

$$\frac{\Gamma; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \{c\} \colon {\downarrow}N}\;\textsc{DTThunk}$$

$$\frac{\Gamma \vdash Q \quad \Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P}{\Gamma; \Phi \vdash (v : Q) \colon Q}\;\textsc{DTPAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash v \colon P'}\;\textsc{DTPEquiv}$$

$\boxed{\Gamma; \Phi \vdash c \colon N}$ Negative type inference

$$\frac{\Gamma \vdash P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \lambda x : P.c \colon P \to N}\;\textsc{DTtLam}$$

$$\frac{\Gamma, \alpha^+; \Phi \vdash c \colon N}{\Gamma; \Phi \vdash \Lambda \alpha^+.c \colon \forall \alpha^+.N}\;\textsc{DTTLam}$$

$$\frac{\Gamma; \Phi \vdash v \colon P}{\Gamma; \Phi \vdash \mathbf{return}\, v \colon {\uparrow}P}\;\textsc{DTReturn}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma; \Phi, x : P \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v; c \colon N}\;\textsc{DTVarLet}$$

$$\frac{\Gamma; \Phi \vdash v \colon {\downarrow}M \quad \Gamma; \Phi \vdash M \bullet \vec{v} \Longrightarrow {\uparrow}Q \text{ unique} \quad \Gamma; \Phi, x : Q \vdash c \colon N}{\Gamma; \Phi \vdash \mathbf{let}\, x = v(\vec{v}); c \colon N}\;\textsc{DTAppLet}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \quad \Gamma; \Phi \vdash v \colon {\downarrow}M \\ \Gamma; \Phi \vdash M \bullet \vec{v} \Longrightarrow {\uparrow}Q \quad \Gamma \vdash {\uparrow}Q \leqslant_1 {\uparrow}P \quad \Gamma; \Phi, x : P \vdash c \colon N\end{array}}{\Gamma; \Phi \vdash \mathbf{let}\, x : P = v(\vec{v}); c \colon N}\;\textsc{DTAppLetAnn}$$

$$\frac{\texttt{<<multiple parses>>} \quad \Gamma, \overrightarrow{\alpha^-}; \Phi, x : P \vdash c \colon N \quad \Gamma \vdash N}{\Gamma; \Phi \vdash \mathbf{let}^{\exists}(\overrightarrow{\alpha^-}, x) = v; c \colon N} \quad \text{DTUnpack}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash (c : M) \colon M} \quad \text{DTNAnnot}$$

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash c \colon N'} \quad \text{DTNEquiv}$$

$\boxed{\Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M}$ Application type inference

$$\frac{\texttt{<<multiple parses>>}}{\Gamma; \Phi \vdash N \bullet \cdot \Rrightarrow N'} \quad \text{DTEmptyApp}$$

$$\frac{\Gamma; \Phi \vdash v \colon P \quad \Gamma \vdash Q \geqslant_1 P \quad \Gamma; \Phi \vdash N \bullet \vec{v} \Rrightarrow M}{\Gamma; \Phi \vdash Q \to N \bullet v, \vec{v} \Rrightarrow M} \quad \text{DTArrowApp}$$

$$\frac{\begin{array}{l}\Gamma \vdash \sigma : \overrightarrow{\alpha^+} \quad \Gamma; \Phi \vdash [\sigma]N \bullet \vec{v} \Rrightarrow M \\ \vec{v} \neq \cdot \quad \overrightarrow{\alpha^+} \neq \cdot\end{array}}{\Gamma; \Phi \vdash \forall \overrightarrow{\alpha^+}.N \bullet \vec{v} \Rrightarrow M} \quad \text{DTForallApp}$$

$\boxed{N = M}$ Negative type equality (alpha-equivalence)

$\boxed{P = Q}$ Positive type equuality (alphha-equivalence)

$\boxed{P \; = \; Q}$

$\boxed{P_1 \vee P_2}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ N}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ P}$

$\boxed{\mathbf{ord}\ vars\ \mathbf{in}\ N}$

$\boxed{\mathbf{nf}\ (N')}$

$\boxed{\mathbf{nf}\ (P')}$

$\boxed{\mathbf{nf}\ (N')}$

$\boxed{\mathbf{nf}\ (P')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{N}')}$

$\boxed{\mathbf{nf}\,(\overrightarrow{P}')}$

$\boxed{\mathbf{nf}\,(\sigma')}$

$\boxed{\mathbf{nf}\,(\widehat{\sigma}')}$

$\boxed{\mathbf{nf}\,(\mu')}$

$\boxed{\sigma'|_{vars}}$

$\boxed{\widehat{\sigma}'|_{vars}}$

$\boxed{\widehat{\tau}'|_{vars}}$

$\boxed{\Xi'|_{vars}}$

$\boxed{\mathbf{SC}|_{vars}}$

$\boxed{\mathbf{UC}|_{vars}}$

$\boxed{e_1\ \&\ e_2}$

$\boxed{e_1\ \&\ e_2}$

$\boxed{UC_1\ \&\ UC_2}$

$\boxed{UC_1 \cup UC_2}$

$\boxed{\Gamma_1 \cup \Gamma_2}$

$\boxed{SC_1 \,\&\, SC_2}$

$\boxed{\widehat{\tau}_1 \,\&\, \widehat{\tau}_2}$

$\boxed{\mathbf{dom}\,(UC)}$

$\boxed{\mathbf{dom}\,(SC)}$

$\boxed{\mathbf{dom}\,(\widehat{\sigma})}$

$\boxed{\mathbf{dom}\,(\widehat{\tau})}$

$\boxed{\mathbf{dom}\,(\Theta)}$

$\boxed{|SC|}$

$\boxed{\Gamma \vDash P_1 \vee P_2 = Q}$ Least Upper Bound (Least Common Supertype)

$$\frac{}{\Gamma \vDash \alpha^+ \vee \alpha^+ = \alpha^+} \quad \text{LUBVar}$$

$$\frac{\Gamma, \cdot \vDash \mathbf{nf}\,(\downarrow N) \overset{a}{\simeq} \mathbf{nf}\,(\downarrow M) \dashv (\Xi, P, \widehat{\tau}_1, \widehat{\tau}_2)}{\Gamma \vDash \downarrow N \vee \downarrow M = \exists \overrightarrow{\alpha^-}.[\overrightarrow{\alpha^-}/\Xi]\,P} \quad \text{LUBShift}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}, \overrightarrow{\beta^-} \vDash P_1 \vee P_2 = Q}{\Gamma \vDash \exists \overrightarrow{\alpha^-}.P_1 \vee \exists \overrightarrow{\beta^-}.P_2 = Q} \quad \text{LUBExists}$$

$\boxed{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q}$

$$\frac{\begin{array}{c}\Gamma = \Delta, \overrightarrow{\alpha^\pm} \quad \overrightarrow{\beta^\pm}\,\mathbf{is\,fresh} \quad \overrightarrow{\gamma^\pm}\,\mathbf{is\,fresh} \\ \Delta, \overrightarrow{\beta^\pm}, \overrightarrow{\gamma^\pm} \vDash [\overrightarrow{\beta^\pm}/\overrightarrow{\alpha^\pm}]P \vee [\overrightarrow{\gamma^\pm}/\overrightarrow{\alpha^\pm}]P = Q\end{array}}{\mathbf{upgrade}\,\Gamma \vdash P\,\mathbf{to}\,\Delta = Q} \quad \text{LUBUpgrade}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\alpha^-) = \alpha^-} \quad \text{NrmNVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\uparrow P) = \uparrow Q}\ \textsc{NrmShiftU}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(P \to N) = Q \to M}\ \textsc{NrmArrow}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\forall \overrightarrow{\alpha^+}.N) = \forall \overrightarrow{\alpha^+}'.N'}\ \textsc{NrmForall}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\alpha^+) = \alpha^+}\ \textsc{NrmPVar}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\downarrow N) = \downarrow M}\ \textsc{NrmShiftD}$$

$$\frac{\texttt{<<multiple parses>>}}{\mathbf{nf}\,(\exists \overrightarrow{\alpha^-}.P) = \exists \overrightarrow{\alpha^-}'.P'}\ \textsc{NrmExists}$$

$\boxed{\mathbf{nf}\,(N) = M}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^-) = \widehat{\alpha}^-}\ \textsc{NrmNUVar}$$

$\boxed{\mathbf{nf}\,(P) = Q}$

$$\frac{}{\mathbf{nf}\,(\widehat{\alpha}^+) = \widehat{\alpha}^+}\ \textsc{NrmPUVar}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}$

$$\frac{\alpha^- \in \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^- = \alpha^-}\ \textsc{ONVarIn}$$

$$\frac{\alpha^- \notin \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^- = \cdot}\ \textsc{ONVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\uparrow P = \vec{\alpha}}\ \textsc{OShiftU}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}_1 \quad \mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}_2}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P \to N = \vec{\alpha}_1, (\vec{\alpha}_2 \backslash \vec{\alpha}_1)}\ \textsc{OArrow}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^+} = \varnothing \quad \mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\forall \overrightarrow{\alpha^+}.N = \vec{\alpha}}\ \textsc{OForall}$$

$\boxed{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}$

$$\frac{\alpha^+ \in \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^+ = \alpha^+}\ \textsc{OPVarIn}$$

$$\frac{\alpha^+ \notin \mathit{vars}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\alpha^+ = \cdot}\ \textsc{OPVarNIn}$$

$$\frac{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,N = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\downarrow N = \vec{\alpha}}\ \textsc{OShiftD}$$

$$\frac{\mathit{vars} \cap \overrightarrow{\alpha^-} = \varnothing \quad \mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,P = \vec{\alpha}}{\mathbf{ord}\ \mathit{vars}\,\mathbf{in}\,\exists \overrightarrow{\alpha^-}.P = \vec{\alpha}}\ \textsc{OExists}$$

$\boxed{\textbf{ord}\ vars\ \textbf{in}\ N = \vec{\alpha}}$

$$\frac{}{\textbf{ord}\ vars\ \textbf{in}\ \widehat{\alpha}^- = \cdot}\ \text{ONUVAR}$$

$\boxed{\textbf{ord}\ vars\ \textbf{in}\ P = \vec{\alpha}}$

$$\frac{}{\textbf{ord}\ vars\ \textbf{in}\ \widehat{\alpha}^+ = \cdot}\ \text{OPUVAR}$$

$\boxed{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC}$ Negative unification

$$\frac{}{\Gamma; \Theta \models \alpha^- \overset{u}{\simeq} \alpha^- \dashv \cdot}\ \text{UNVAR}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \uparrow P \overset{u}{\simeq} \uparrow Q \dashv UC}\ \text{USHIFTU}$$

$$\frac{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC_1 \quad \Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC_2}{\Gamma; \Theta \models P \to N \overset{u}{\simeq} Q \to M \dashv UC_1\ \&\ UC_2}\ \text{UARROW}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \forall \overrightarrow{\alpha^+}. N \overset{u}{\simeq} \forall \overrightarrow{\alpha^+}. M \dashv UC}\ \text{UFORALL}$$

$$\frac{\widehat{\alpha}^-\{\Delta\} \in \Theta \quad \Delta \vdash N}{\Gamma; \Theta \models \widehat{\alpha}^- \overset{u}{\simeq} N \dashv (\widehat{\alpha}^- :\approx N)}\ \text{UNUVAR}$$

$\boxed{\Gamma; \Theta \models P \overset{u}{\simeq} Q \dashv UC}$ Positive unification

$$\frac{}{\Gamma; \Theta \models \alpha^+ \overset{u}{\simeq} \alpha^+ \dashv \cdot}\ \text{UPVAR}$$

$$\frac{\Gamma; \Theta \models N \overset{u}{\simeq} M \dashv UC}{\Gamma; \Theta \models \downarrow N \overset{u}{\simeq} \downarrow M \dashv UC}\ \text{USHIFTD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Theta \models P \overset{u}{\simeq} Q \dashv UC}{\Gamma; \Theta \models \exists \overrightarrow{\alpha^-}. P \overset{u}{\simeq} \exists \overrightarrow{\alpha^-}. Q \dashv UC}\ \text{UEXISTS}$$

$$\frac{\widehat{\alpha}^+\{\Delta\} \in \Theta \quad \Delta \vdash P}{\Gamma; \Theta \models \widehat{\alpha}^+ \overset{u}{\simeq} P \dashv (\widehat{\alpha}^+ :\approx P)}\ \text{UPUVAR}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma \vdash \alpha^-}\ \text{WFTNVAR}$$

$$\frac{\Gamma \vdash P}{\Gamma \vdash \uparrow P}\ \text{WFTSHIFTU}$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash N}{\Gamma \vdash P \to N}\ \text{WFTARROW}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+} \vdash N}{\Gamma \vdash \forall \overrightarrow{\alpha^+}. N}\ \text{WFTFORALL}$$

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma \vdash \alpha^+}\ \text{WFTPVAR}$$

$$\frac{\Gamma \vdash N}{\Gamma \vdash \downarrow N} \quad \text{WFTShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-} \vdash P}{\Gamma \vdash \exists \overrightarrow{\alpha^-}.P} \quad \text{WFTExists}$$

$\boxed{\Gamma \vdash N}$ Negative type well-formedness

$\boxed{\Gamma \vdash P}$ Positive type well-formedness

$\boxed{\Gamma \vdash \overrightarrow{N}}$ Negative type list well-formedness

$\boxed{\Gamma \vdash \overrightarrow{P}}$ Positive type list well-formedness

$\boxed{\Gamma; \Theta \vdash N}$ Negative unification type well-formedness

$$\frac{\alpha^- \in \Gamma}{\Gamma; \Theta \vdash \alpha^-} \quad \text{WFATNVar}$$

$$\frac{\widehat{\alpha}^- \in \Theta}{\Gamma; \Theta \vdash \widehat{\alpha}^-} \quad \text{WFATNUVar}$$

$$\frac{\Gamma; \Theta \vdash P}{\Gamma; \Theta \vdash \uparrow P} \quad \text{WFATShiftU}$$

$$\frac{\Gamma; \Theta \vdash P \quad \Gamma; \Theta \vdash N}{\Gamma; \Theta \vdash P \to N} \quad \text{WFATArrow}$$

$$\frac{\Gamma, \overrightarrow{\alpha^+}; \Theta \vdash N}{\Gamma; \Theta \vdash \forall \overrightarrow{\alpha^+}.N} \quad \text{WFATForall}$$

$\boxed{\Gamma; \Theta \vdash P}$ Positive unifiication type well-formedness

$$\frac{\alpha^+ \in \Gamma}{\Gamma; \Theta \vdash \alpha^+} \quad \text{WFATPVar}$$

$$\frac{\widehat{\alpha}^+ \in \Theta}{\Gamma; \Theta \vdash \widehat{\alpha}^+} \quad \text{WFATPUVar}$$

$$\frac{\Gamma; \Theta \vdash N}{\Gamma; \Theta \vdash \downarrow N} \quad \text{WFATShiftD}$$

$$\frac{\Gamma, \overrightarrow{\alpha^-}; \Theta \vdash P}{\Gamma; \Theta \vdash \exists \overrightarrow{\alpha^-}.P} \quad \text{WFATExists}$$

$\boxed{\Gamma; \Xi \vdash N}$ Negative anti-unification type well-formedness

$\boxed{\Gamma; \Xi \vdash P}$ Positive anti-unification type well-formedness

$\boxed{\Gamma; \Xi_2 \vdash \widehat{\tau} : \Xi_1}$ Antiunification substitution well-formedness

$\boxed{\Gamma \vdash^{\supseteq} \Theta}$ Unification context well-formedness

$\boxed{\Gamma_1 \vdash \sigma : \Gamma_2}$ Substitution well-formedness

$\boxed{\Theta \vdash \widehat{\sigma}}$ Unification substitution well-formedness

$\boxed{\Theta \vdash \widehat{\sigma} : UC}$ Unification substitution satisfies unification constraint

$\boxed{\Theta \vdash \widehat{\sigma} : SC}$ Unification substitution satisfies subtyping constraint

$\boxed{\Gamma \vdash e}$ Unification constraint entry well-formedness

$\boxed{\Gamma \vdash e}$ Subtyping constraint entry well-formedness

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies unification constraint

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies unification constraint

$\boxed{\Gamma \vdash P : e}$ Positive type satisfies subtyping constraint

$\boxed{\Gamma \vdash N : e}$ Negative type satisfies subtyping constraint

$\boxed{\Theta \vdash UC}$ Unification constraint well-formedness

$\boxed{\Theta \vdash SC}$ Subtyping constraint well-formedness
$\boxed{\Gamma \vdash \vec{v}}$ Argument List well-formedness
$\boxed{\Gamma \vdash \Phi}$ Context well-formedness
$\boxed{\Gamma \vdash v}$ Value well-formedness

$$\frac{}{\Gamma \vdash x} \quad \textsc{WFATVar}$$

$\boxed{\Gamma \vdash c}$ Computation well-formedness

$$\frac{\Gamma \vdash v \quad \Gamma \vdash c \quad \Gamma \vdash \vec{v}}{\Gamma \vdash \textbf{let}\, x = v(\vec{v}); c} \quad \textsc{WFATAppLet}$$

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Definition rules:        117 good    21 bad
Definition rule clauses: 241 good    21 bad
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