Physics Math-Trig Review

Contents

This document serves as brief review of the mathematical concepts needed to be successful in a physics course. This is not a comprehensive resources but instead a quick check of mathematics vital to physics.

Algebra Review

In this chapter we will highlight some of the key algebra tools/tricks that are utilize throughout physics.

Order of Operations

Consider the following mathematical expression,

$$6 \div 2(1+2) \tag{1}$$

Problems like this one often go viral has people disgree on what the answer should be. Is it 1, is it 9, or is it some other number? It *feels* ambiguous on what the answer should be because depending on how you do your operations you will get different results.

To prevent these types of confusion mathematics has adopted the **order of operations**. The sequence of rules dictating how to evaluate expressions lke the one above. You can remember the steps with the *PEMDAS* acronym.

PEMDAS

- *P*(arentheses)
- *E*(xponents)
- *M*(ultiplication)
- *D*(ivision)
- *A*(ddition)
- *S*(ubtraction)

The order of operations using PEMDAS for any expression will be

1. Simplify anything inside a *parentheses*.
2. Simplify anything involving *exponents* (this includes squareroots or any other radical).
3. Evaluate all *multiplications* and *divisions*.
4. Evaluate all *additions* and *subtractions*

Note that steps three and four are evaluated from left to right.

Using PEMDAS we can calculate the equation (1) correctly.

$$6 \div 2\underbrace{(1+2)}$$

$$\underbrace{6 \div 2}(3)$$

$$3(3)$$

$$9$$

Quadratic Formula

Many problems in physics result in solving a quadratic polynomial. As an example how much will a spring compress if a weight is dropped on it.

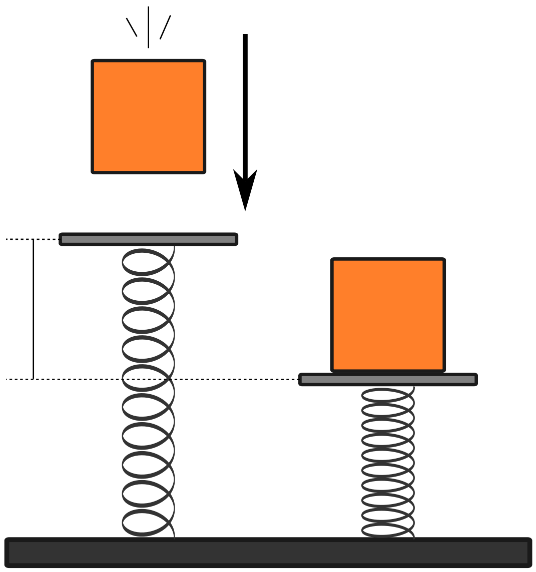


Fig. 1 Where you define the y-axis to be zero will change what potential energies you have initially and finally. However, your choice of coordinate system will not change the final answer.

Assume the block was released from rest at a height h above the spring. Let y represent the height of the spring and set it equal to zero when the spring is relaxed. Conservation of energy tells us that

$$mgh = \frac{1}{2}ky^2 + mgy \tag{2}$$

This is a quadratic equation in y and it has no nice simplifications to help us solve for y. This is where we can use the *quadratic formula* to tackle any problem.

Quadratic Formula

$$\begin{gathered} ax^2 + bx + c = 0 \\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{gathered} \tag{3}$$

Exercises

1. Solve for how much the spring in figure Fig. 1 will compress, given the following values. m=.40 kg, g=9.8 m/s^{2}m h=.60 m, k=340 N/m.

Solution

Angles and the Unit Circle

In this chapter we will define the *unit circle* and the various ways we make **angular** measurements on it.

Angle Measurements

In cartesian space (an xy-grid) we define a ray along the x axis to be at zero degrees. We can rotate that ray *counter-clockwise* to an angle ϕ and sweep around the xy plane.

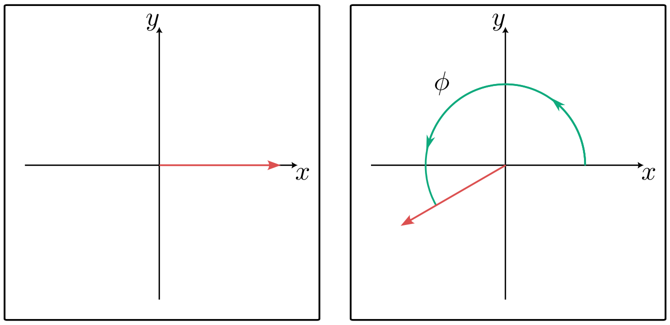
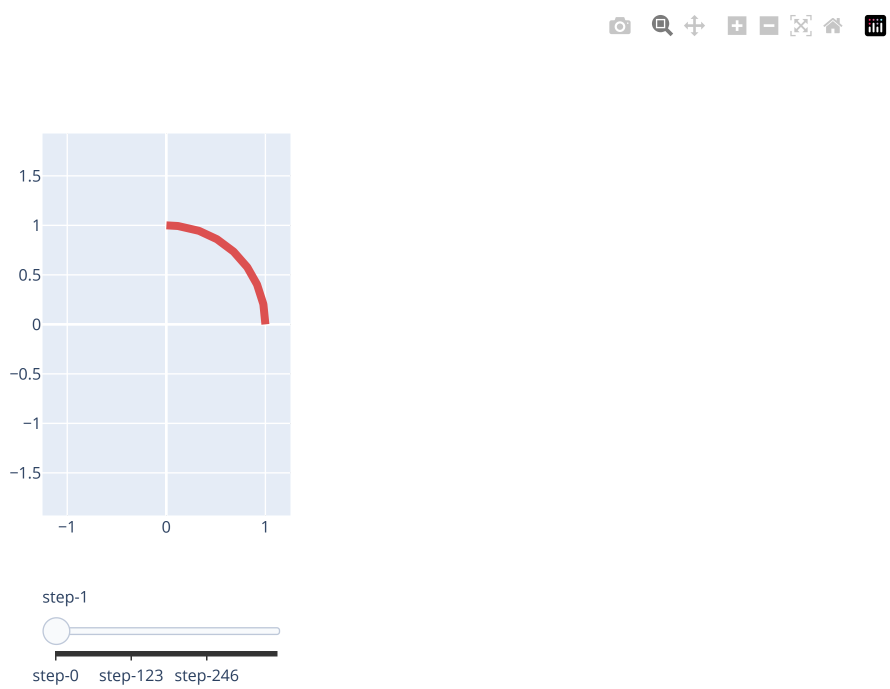


Fig. 2 Counterclockwise rotations are by definition positive (increasing in angle).

A complete circle is defined to be 360°. Manipulate the slider on the next plot to get an idea of where different angular measurements fall on the circle.



Arc Length and Radians

The circumference of a circle is

$$C = 2\pi r \tag{4}$$

We define a *radian* as the angle swept out by a line equal in length to the radius of the circle.

Fig. 3 One radian is about 57.3 degrees.

A radian is the “natural” unit for angles because it defines arcs around a circle in terms of its natural unit, π. We can quickly convert between degrees and radians with

$$360^\circ = 2\pi \,\text{radians}$$

Given an angle ϕ measured in radians, we also define *arc-length* s as the length on the circumference of a circle.

$$s = r\phi \tag{5}$$

Arc-length as a beautifl result that helps us remember it by recognizing that we complete one complete loop, $\phi = 2\pi$, we will “travel” an arc-length equal to the circumference.

$$\text{Arc-length once around}: s = r(2\pi) = \text{the circumference}$$

Exercises

1. A car makes 32° turn on a road with a radius of curvature of 115 meters. How far did the car travel on that turn?

   Solution

2. How many times will 5.5 meter long rope wrap around an axle with a diameter of 8 centimeters?

   Solution

Trigonometry

It is **impossible** to overstate how crucial right triangles and trigonometry are to our knowledge in physics. From analyzing vector motions to modelling the quantum mechanical interactions of atoms, trigonometry is **always** present. In this chapter you will find a brief overview of the crucial aspects of trigonometry.

Right Triangles

A right triangle is any triangle were one of the interior angles in 90°. We often label one of the angles of the triangle with a greek theta, θ, and discuss the sides of the triangle relative to θ.

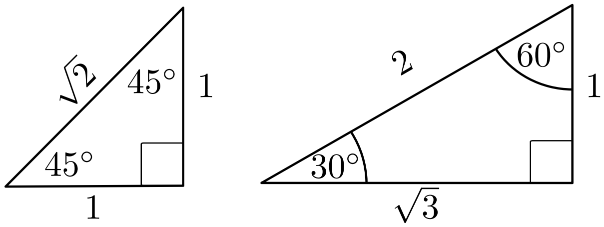


Fig. 4 The 45-45 and 30-60-90 triangles frequently occur in physics, so it is convenient to have them for quick reference.

In general we describe the edges of a right triangle as the side adjacent to the given angle, or opposite the given angle. The longest edge is called the hypotenuse.

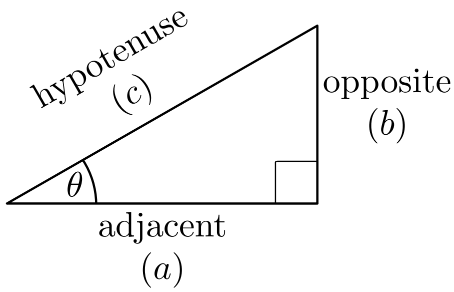


Fig. 5 The names of the right triangle sides are relative to the angle θ.

Figure Fig. 6 visual demonstration of the *Pythagorean theorem*.

Pythagorean theorem

$$a^2 + b^2 = c^2 \tag{6}$$

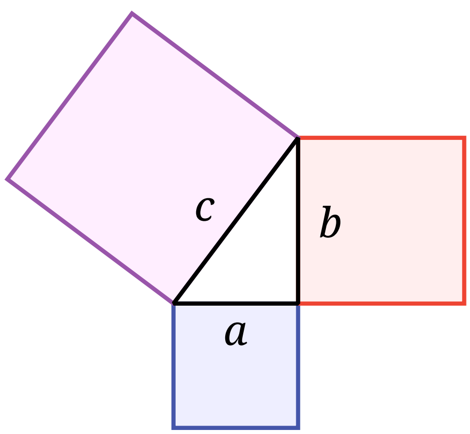


Fig. 6 With some clever rearranging, it can be shown the blue square and the red square are equal to the purple square.

SOH-CAH-TOA

Figure Fig. 7 shows a right triangle of arbitrary angle inscribed in a circle.

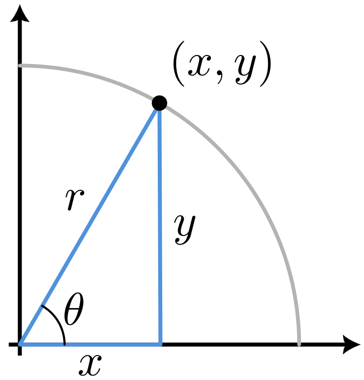


Fig. 7 If $\theta = 0^\circ$ than x will be the radius of the circle and $y = 0$.

The first three trigonometric functions are defined using this figure,

$$\begin{aligned} \text{Sine} \qquad & \sin\theta = \frac{y}{r} \\ \text{Cosine} \qquad & \cos\theta = \frac{x}{r} \\ \text{Tangent} \qquad & \tan\theta = \frac{y}{x} \end{aligned} \tag{7}$$

These definitions can be remembered with the mneumonic **SoH-CaH-TOA**. **S**ine is **o**pposite over **h**ypotenuse, **C**osine is **a**djacent over **h**ypotenuse, **T**angent is **o**pposite over **a**djacent.

Now apply the Pythagorean theorem to the trianlge in figure Fig. 7,

$$x^2 + y^2 = r^2$$

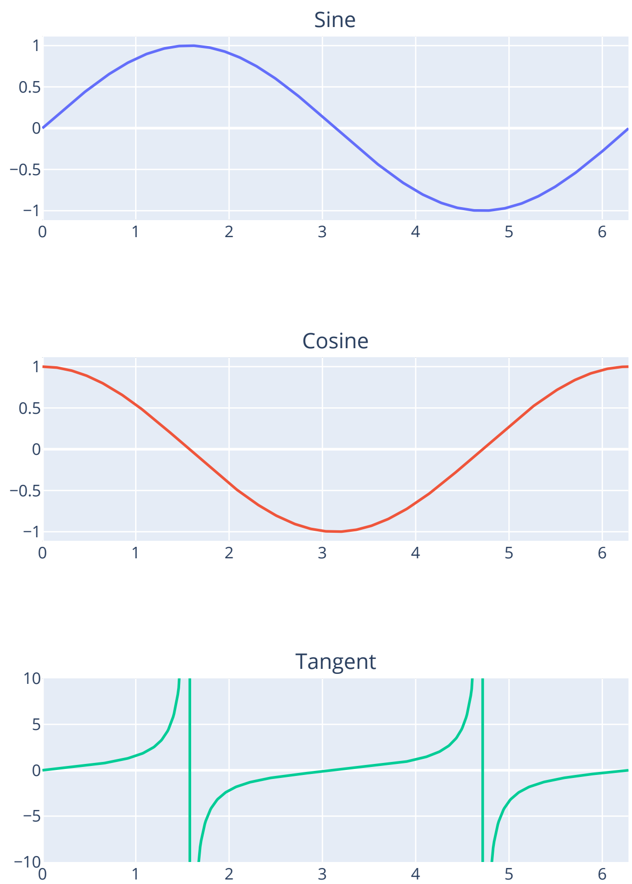
$$(r\cos\theta)^2 + (r\sin\theta)^2 = r^2$$

and we derived our first Trig. Identity!

$$\sin^2\theta + \cos^2\theta = 1 \tag{8}$$

Plotting Trig Functions

Here are plots of sine, cosine, and tangent along the circle (0 to 2π). They are periodic functions, meaning that they repeat every 2π radians. Notice that tangent diverges at $\pi/2$ and $3\pi/2$, angles corresponding to $x = 0$.



Exercises

1. An airplane travelled 500 km going 18° south of east. How far east and how far south did it go?

   Solution

2. A certain time of the day, a tall tree cast a shadow that is 6.35 meters long on the ground. At the same time, a meter stick cast a shadow that is 29 centimeters long. How tall is the tree?

   Solution

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