Mobile Game Marketing Budget Allocation Decision Support System

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Abstract

This paper presents an application of a framework aimed at finding the optimal budget allocation policy for a mobile game advertising campaign deployed across various social network platforms, such as Facebook and Ironsource. The method takes into account the distinctive characteristics of each social marketing platform as well as customer behaviors to achieve a daily budget allocation policy for multiple advertising campaigns spanning different platforms and customer segments. The primary objective is to minimize the campaign budget while attaining a predetermined level of exposure for each customer segment within a specific campaign duration. By employing this approach, the paper offers an algorithmic decision support framework for efficiently allocating budgets in multi-platform, multi-audience advertising campaigns, while also providing a revenue estimate to find the most profitable campaigns.

1 Introduction

Game publisher companies are responsible for a range of decision-making processes, including the design and selection of appropriate advertisements, identification of the target audience, allocation of budget to various social media platforms, and determination of the optimal budgeting strategy to maximize profit from a given game across different platforms. However, currently used methods for determining these strategies rely on past campaign experience of decision makers and are far from a rule-based and reliable budget allocation decision-making mechanism in most companies to our knowledge.

A critical component of every firm's marketing strategy is choosing how to spend an advertising budget over time in order to use a limited budget to bring as many users to the product as possible. Over time, various approaches to allocating advertising budgets have been developed. However, with the recent developments in social media platforms, it is crucial to adapt the budget allocation problem to social media platforms as they increasingly become key marketing platforms. Luzon, Pinchover, and Khmelnitsky (2022) suggested a method for dynamic budget allocation for social media advertising campaigns which specifically aims to optimize the advertiser's expenditures over time for specific target audiences, in addition to the duration of the ad campaign to reach a required level of installs across these audiences for a given total budget.

This research concentrates on the implementation of the proposed method outlined in the article within the context of a mobile game publisher's advertisement campaigns conducted on multiple social media platforms, along with some minor modifications. The algorithm is customized to align with the specific objectives of the firm. The primary goal is to maximize the number of game installs, a key performance indicator closely associated with the concept of customer Life Time Value (LTV). Customer LTV refers to the total revenue generated by a customer, throughout their lifespan within the game. The research aims to optimize the firm's marketing expenditures by considering factors such as campaign length, desired number of installs, customer segmentation, and advertising budget. The study also presents estimates for the revenue generated by various candidate campaigns. The aim is to provide decision-makers with rule-based tools to assess the potential revenues of different potential campaigns and distribute the available budget effectively. By employing this approach, the study offers valuable insights for decision-making processes in the realm of multi-platform marketing campaigns.

2 Problem Setting

2.1 Campaign Cost

In the context of mobile game marketing, apart from the LTV, another important metric that determines whether an advertisement will stay on display is the cost per install (CPI). It refers to the average cost incurred by a marketer to acquire a new user for their product. This cost is the expense related to marketing campaigns aimed at driving the installation of the game. CPI is a KPI that helps assess the efficiency of user acquisition efforts in the gaming industry. Throughout the research, we will consider the CPI as the bidding value assigned to an ad shown to a specific user. The bidding values in ad auctions on social media platforms are determined by real-time algorithms. These algorithms calculate the optimal bid amount for each ad space based on factors such as ad relevance and competition. When an ad successfully wins the auction, the bid value converts into the actual amount spent by the advertiser for that particular ad placement. The real-time nature of the bidding process allows advertisers to adjust their bids dynamically to maximize their campaign's effectiveness and achieve their desired marketing objectives. This study does not aim to determine these bid values but rather determine the daily budget, which is then converted into bid values by social media platforms automatically. It would be possible to change the timescale of the budget arbitrarily, for instance, an hourly budget, with the proposed algorithm however a daily budget was preferred as it is the most common and sometimes the only choice in most platforms.

Game companies often face the challenge of low LTV for their customers, especially in the case of hypercasual games. To increase LTV and decrease CPI, it is important to strike a balance between in-app advertisements and in-app purchases. Engaging gameplay, targeting the right audience, and incorporating user feedback are crucial for enhancing LTV. By offering appealing premium features and minimizing intrusive ads, game companies can maximize monetization opportunities and create a positive player experience.

The study does not propose making any changes to the game based on user feedback obtained after ad impressions. The project utilizes historical ad data from the company's previously marketed games for analysis and decision-making. The LTV and CPI targets are determined using data from the campaigns in the United States.

2.2 Target Audience

The selection of the target audience for a mobile game advertisement encompasses a wide range of user criteria, including geographical location, device and operating system, age range and gender, game preferences and behaviors, and ad click histories. Initially, target audience selection was in the scope of this project and the characteristics of the top 1000 players with the most playtime, which is the most important determinant of LTV, were identified in terms of their income and mobile devices in order to find what separates them from the rest of the crowd, which can be seen below. Income and devices were chosen because they were readily available in the data and could be used to directly create target audiences in Meta.

Phone Manufacturer	Percentage of Devices	Global Market Share
Samsung	0.34	0.34
Xiaomi	0.20	0.14
Oppo	0.14	0.11
Vivo	0.7	0.10
Huawei	0.05	0.06

Table 1: Percentage of devices in the top 1000 players with the most playtime vs the actual global market share of device manufacturers

As the manufacturers' global market shares were so close to the percentage of devices in the top 1000 players, it was determined to be a non-discriminative characteristic.

The household income data for the players were not readily available, instead the ZIP codes of US players were used to find the average household incomes for the players. Once again this was determined to be a non-discriminative characteristic as the top 1000 US players in red, all US-based players in blue on the figure on the left, and the income distribution in the whole of the US were similar. However, this was a very promising approach as Meta allowed for targeting of US individuals based on the percentile of income distribution they are in.

As a result of the findings above, rather than directly specifying individual criteria, this study involves selecting the target audience as a group of potential customers who exhibit the closest resemblance to the existing customer base of the company.

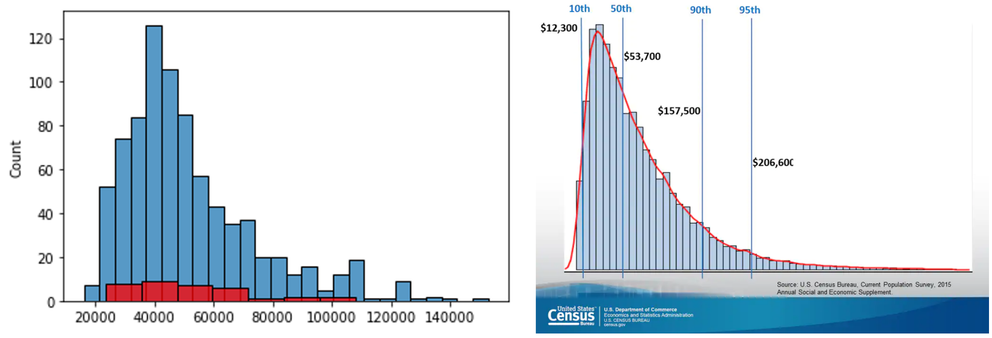

Figure 1: Comparison of the average household income for US players vs household income in the whole of the US

This selection, facilitated by the marketing platform, enables the identification of a certain percentage of individuals who share similar characteristics with the current customer base. By employing this approach, the marketing campaigns are built upon a foundation of individuals who are more likely to be receptive to the mobile game advertisement, resulting in a more effective and targeted promotional strategy.

This study focuses on Meta and Ironsource as advertising platforms, the first automatically determines the targeted audiences based on who is most likely to interact with ads and the latter has no fine grained audience targeting; so target audience selection is out of the question. Meta ads are shown on Facebook and Instagram and Ironsource ads are shown in other mobile apps such as mobile games.

2.3 Data

The data utilized in this project is sourced from three key platforms: Meta, Adjust, and AdMob. Each platform contributes valuable insights and metrics to inform the decision-making process.

a) Meta

Within the Meta environment, companies have the opportunity to publish their advertising campaigns on Facebook and Instagram. Meta provides comprehensive feedback and detailed data on user interactions with these campaigns, enabling companies to analyze the effectiveness of their marketing efforts. The collected data from Meta is typically organized in regular intervals, often on a daily basis. Metrics such as the number of users who watched an advertised video for more than 3 seconds, the count of unique users who clicked on the ad, and the daily Cost Per Click (CPC) values are among the data that can be retrieved from Meta.

b) Adjust

Adjust serves as a data aggregation platform, capturing various user activities such as ad views, interactions, game downloads, and in-game progress. It sources data from multiple channels, including advertisers such as Meta and Google, app platforms like Google Play and Apple App Store, as well as directly from the games themselves. By collecting data from these diverse sources, Adjust provides a comprehensive overview of user engagement and behavior.

c) AdMob

AdMob, a mobile application advertising platform, plays a pivotal role in displaying relevant ads to users within mobile applications. In the context of this project, AdMob collects distinct types of data that are not provided by other platforms. Instead of focusing solely on game and advertisement data, AdMob gathers information specifically from the in-app ads displayed within the games. This data offers valuable insights, such as the revenue generated from a published game or the profitability levels associated with different countries.

By leveraging the data from these platforms, the decision-making process is enriched with a comprehensive understanding of user interactions, campaign performance, and revenue generation. The model used to synthesize all this data and our solution approach are described in the following section.

3 Solution Approach

The problem is tackled in three parts: daily budgeting for different audiences in a platform, the budgeting between the platforms, and revenue estimation. The daily budgeting first requires the platform budget to be determined and revenue estimation is independent of the former parts.

This is a modular approach and is useful in a number of scenarios:

- Single platform single audience: The daily budgeting model can be used to determine the daily budget of a single campaign.
- Single platform multiple audiences: The daily budgeting model can be used to determine the daily budget of multiple campaigns across different audiences.
- Multiple platforms multiple audiences: The multiplatform and daily budgeting models can be used to determine the daily budgets of campaigns across different platforms and different audiences.

Different platforms can also be considered as different audiences in a single platform if the LTV of users from the platforms are similar, which allows for the daily budgeting model to be used instead of multiplatform budgeting.

The flowchart for our solution approach considering the aforementioned scenarios can be seen in Figure 2.

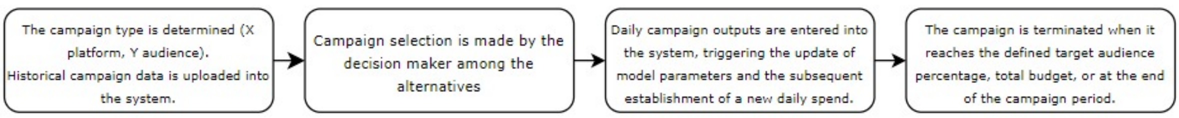

Figure 2: Solution Approach

Before introducing the model, it may be beneficial to take a glance at previous studies on advertising response models to provide a deeper understanding of our approach. Therefore, in Section 4, the equations which lay the foundation for the model used in our paper are provided.

4 Literature Review of Advertising Models

Aggregate advertising response models in the literature show the relationship between product sales and many factors such as advertising activities and the number of potential customers over a period of time. In this section, three different equations which substantially contribute to the model used in our study are given below.

a) Vidale-Wolfe Advertising Response Model

One of the advertising response models showing the relationship between the rate of sales and the rate of response was proposed by Vidale and Wolfe (1957). They used the following equation in their model:

$$\frac{d}{dt}S(t) = rA(t)\frac{M - S(t)}{M} - \lambda S(t)$$

where $\frac{d}{dt}S(t)$ is defined as the rate of sales at time t, $A(t)$ is the rate of advertising expenditure at time t, M is the saturation level, λ is the exponential sales decay constant, and r is the response constant.

The response constant (r) represents the sales behavior of the product when there is a promotion. Since $\frac{M-S(t)}{M}$ is the fraction of potential customers, $r\frac{M-S(t)}{M}$ can be interpreted as the sales generated per advertising unit cost at time t. Multiplying this component by the rate of advertising expenditure $(A(t))$, we obtain the component $rA(t)\frac{M-S(t)}{M}$, which is proportional to $\frac{d}{dt}S(t)$.

The sales decay constant (λ) shows the sales behavior of the product in the absence of promotion. In other words, it represents the falling trend of sales due to competing products and advertising. Thus, multiplying the term by $S(t)$, we get $\lambda S(t)$, which is the number of customers that are being lost at time t.

b) Nerlove-Arrow Advertising Response Model

Another advertising response model was proposed by Nerlove and Arrow (1962). One of the equations they used in their model is the following:

$$\frac{d}{dt}A(t) = qu(t) - \delta A(t)$$

where $\frac{d}{dt}A(t)$ is the rate of customers' awareness of the product at time t, q is a nonnegative constant representing advertising effectiveness, $u(t)$ is advertising expenditure at time t, and δ denotes the rate of decay in customers' awareness.

Since customers' interest toward a product may decrease as time passes due to forgetting, the component $\delta A(t)$ is inversely proportional to the awareness growth rate $(\frac{d}{dt}A(t))$. On the other hand, each expenditure on advertising results in an increase in the awareness growth rate. Thus, $qu(t)$ is directly proportional to $\frac{d}{dt}A(t)$.

c) Luzon-Pinchover-Khmelnitsky Advertising Response Model

Finally, the model we follow in this study is put forth by Luzon, Pinchover, and Khmelnitsky (2022). They proposed the following equation:

$$\frac{d}{dt}g_s(t) = (h_s - g_s(t)) \cdot r(b_s(t), g_s(t), t)$$

where $\frac{d}{dt}g_s(t)$ is the rate of change in the number of people in audience s who are exposed to the advertisement at time t, $b_s(t)$ denotes daily budget proposed for audience s at time t, h_s represents the population size of audience s, and $r(b_s(t), g_s(t), t)$ is effectiveness function including both direct and indirect of daily budget value at time t.

In our model, we redefine $\frac{d}{dt}g_s(t)$ as the rate of change in the number of people in audience s who downloaded the game up to time t since it is more important to companies that people download the game and provide long-term profit rather than just watching the ad. This equation is explained in detail in Section 5 where our daily budgeting model is proposed.

5 The Daily Budgeting Model

5.1 Problem Formulation

The budgeting strategy plays a crucial role in determining the outcome of an advertising campaign and is a crucial part of the project. Before delving into the specifics of how the budgeting strategy will be incorporated into the project, it is beneficial to stress the fact that an increase in LTV is aimed in order to obtain long-term profit. It is of great importance that users are not only exposed to advertisements but also click on the link in advertisements and install the game. Moreover, if they come back to play the game after the day they downloaded the game, there is a greater chance for users to be exposed to in-app advertisements and make in-app purchases, which brings a great deal of income to mobile game companies. Therefore, in order to ensure achieving long-term profit, the relationship between budget values and daily downloads should be taken into account when creating the model.

It is preferred that the function used in modeling the relationship between daily budgets and daily downloads is concave. The reason for this is that as the daily budget value increases, it is observed that a smaller number of new users are exposed to advertisements, and accordingly fewer new downloads are obtained. Indeed, concavity comes from the phenomenon of diminishing returns which is seen in online auction mechanisms. This effectiveness function will be denoted as $f(b_s(t), t)$, where $b_s(t)$ is the daily ad budget for audience s at time t. The output of this function is the number of downloads at time t.

Moreover, Hartl, Sethi and Vickson (1995) applied the Maximum Principle to the problem and put forth that the partial derivative of $f(b_s(t), t)$ with respect to $b_s(t)$ is constant at all t as shown in equation (1). This constant will be denoted as β_s.

$$\frac{\partial f(b_s(t), t)}{\partial b_s(t)} = \beta_s \tag{1}$$

Apart from the direct effect of the daily bid budget on the cumulative number of users who installed the game, we also need to consider the indirect effect of the daily budget. For example, there can be users who were not exposed to advertisements yet; however, they may decide to download the game by interacting with the people around them. Therefore, we need to include a coefficient representing the word-of-mouth effect in the model. Companies calculate their word-of-mouth coefficient as shown below.

$$a_s = \frac{\text{total number of downloads} - \text{number of downloads by exposed users}}{\text{number of downloads by exposed users}} \tag{2}$$

While calculating the rate of change in the total number of users who installed the game in audience s up to time t, which is denoted by $\frac{d}{dt}g_s(t)$, the effectiveness function must be multiplied by $(1 + a_s \cdot g_s(t))$ to capture both the direct effect of bid and the user-interaction effect. Furthermore, it should also be noted that the rate of change and the number of remaining unexposed users are proportional to each other. In other words, the rate of change in total users having installed the game will decrease as the number of remaining unexposed people decreases. We expect that this rate will be at its highest when $t = 0$, and gradually decrease as the number of unexposed users in the population decreases. Therefore, the term $(h_s - g_s(t))$, where h_s is population size of audience s, is included while calculating $\frac{d}{dt}g_s(t)$. As a result, we obtain:

$$\frac{d}{dt}g_s(t) = (h_s - g_s(t)) \cdot f(b_s(t), t) \cdot (1 + a_s g_s(t))$$

Furthermore, normalizing the dataset results in the following equation:

$$\frac{d}{dt}g_s(t) = (1 - g_s(t)) \cdot f(b_s(t), t) \cdot (1 + a_s g_s(t))$$

Rewriting in closed form, $g_s(t)$ is found as:

$$g_s(t) = 1 - \frac{(1 + a_s)(1 - q_s)}{(1 + a_s q_s)e^{(1+a_s)\int_0^t f(b_s(t),t)} + a_s(1 - q_s)}$$

With a limited budget, one can employ a model to minimize the campaign length in order to reach the target audience in short time as follows:

Parameters

B: total budget, determined by the problem owner
h_s: number of people in audience s, determined by Facebook
a_s: coefficient of interaction for audience s, determined by the problem owner
q_s: initial exposure ratio for audience s, determined by the problem owner
p_s: target exposure ratio for audience s, determined by the problem owner
$f(b_s(t), t)$: daily-budget effect on the number of downloads for audience s at time t
S: the set of target audiences, determined by the problem owner

Decision Variables

$g_s(t)$: number of people who downloaded the game in audience s at time t
$b_s(t)$: daily ad budget for audience s at time t, decision variable
$z(t)$: total budget spent up to time t
T: total length of the campaign

Daily Budgeting Strategy Model for a Single Platform with Multiple Audiences

Minimize T
subject to

$$\frac{d}{dt}g_s(t) = (h_s - g_s(t)) \cdot f(b_s(t), t) \cdot (1 + a_s g_s(t)) \tag{3}$$

$$g_s(0) = q_s h_s \tag{4}$$

$$g_s(T) \geq p_s h_s \tag{5}$$

$$\frac{d}{dt}z(t) = \sum_{s \in S} b_s(t) \tag{6}$$

$$z(0) = 0 \tag{7}$$

$$z(T) \leq B \tag{8}$$

$$b_s(t) \geq 0 \tag{9}$$

Equation (3) incorporates the coefficient of interaction (a_s) which is how many installs an exposed person leads to by swaying other people. Moreover, in equation (3), the rate of change in the total number of exposed people at time t is also proportional to unexposed people left and the effectiveness function constructed by the company's past data. Equation (4) models the initial exposure of the audience, which is zero when an ad campaign for a new game is launched and is a known quantity for the subsequent campaigns. Constraint (3) ensures that the exposure target is met. Equation (6) relates the bid caps on different audiences to the total money spent up to time t. Equation (7) sets the initial condition on the total money spent up to time t. Constraint (8) ensures that the money spent does not exceed the total budget. Constraint (9) ensures that all bid cap values are non-negative. To examine the solution steps, our algorithm for bidding strategy for single platform is shown in Figure 3 below.

Algorithm 1: Daily Budgeting Strategy for Single Platform

Step 1: For each type of audience, estimate the word-of-mouth constant and the parameters of the effectiveness function using historical data.
Step 2: By keeping β_s as an unknown constant, find a closed form solution of $b_s^*(t)$ as shown in equation (1).
Step 3: Substitute $b_s^*(t)$ into equation (3) to obtain $g_s(t)$.
Step 4: By keeping T as an unknown constant, solve $g_s(T) = p_s$ and rewrite β_s in terms of problem parameters.
Step 5: Solve $\int_0^T \sum_{s \in S} b_s^*(t)\, dt = B$ by substituting β_s in $b_s^*(t)$ to obtain the value of T.

Figure 3: Algorithm for Daily Budgeting Strategy for Single Platform

In Step 1, one can derive its word-of-mouth constant (a_s) considering similar past campaign's dataset, using equation (2). To find the parameters of the effectiveness function, its shape must be considered first. For instance, if the daily ad budget vs install count graph of the previous campaign shows stationary concave behavior, where equation (1) is assumed to be independent of time t, the structure of the effectiveness function can be defined as:

$$f(b_s(t)) = m_s(1 - e^{-\mu_s b_s(t)})$$

Fitting the function to the graph, the shape parameters m_s and μ_s are estimated. In Step 2, taking the partial derivative of $f(b_s(t))$ with respect to $b_s(t)$ and using equation (1) result in the following:

$$m_s \mu_s e^{-\mu_s b_s^*(t)} = \beta_s$$

Thus, $b_s^*(t)$ is found as:

$$b_s^*(t) = \frac{1}{\mu_s} \ln\left(\frac{m_s \mu_s}{\beta_s}\right)$$

Moving onto Step 3, substituting $b_s^*(t)$ into equation (3) gives:

$$\frac{d}{dt} g_s(t) = (h_s - g_s(t)) \cdot f(b_s^*(t)) \cdot (1 + a_s g_s(t))$$

After normalization, $h_s = 1$ and $0 \leq g_s(t) \leq 1$ are hold. Then, derivative of $g_s(t)$ with respect to time t is:

$$\frac{d}{dt} g_s(t) = (1 - g_s(t)) \cdot f(b_s^*(t)) \cdot (1 + a_s g_s(t))$$

To obtain $g_s(t)$, the equation above must be rewritten in closed form, which is:

$$g_s(t) = 1 - \frac{(1 + a_s)(1 - q_s)}{(1 + a_s q_s) e^{(1+a_s) t f(b_s^*(t))} + a_s(1 - q_s)}$$

In Step 4, using $g_s(T) = p_s$ and rewriting $f(b_s^*(t))$ in terms of problem parameters, β_s is found as:

$$\beta_s = \mu_s \left(m_s - \frac{1}{(1 + a_s) \cdot T} \cdot \ln\left(\frac{(1 + a_s)(1 - q_s)/(1 - p_s) - a_s(1 - q_s)}{1 + a_s q_s}\right)\right)$$

Finally, in Step 5, solving $\int_0^T \sum_{s \in S} b_s^*(t)\, dt = B$ results in the following:

$$B = \sum_{s \in S} \frac{T}{\mu_s} \cdot \ln\left(\frac{m_s \mu_s}{\mu_s \cdot \left(m_s - \frac{1}{(1+a_s) \cdot T} \cdot \ln\left(\frac{((1+a_s) \cdot (1-q_s)/(1-p_s)) - a_s \cdot (1-q_s)}{1+a_s \cdot q_s}\right)\right)}\right)$$

Using the relationship between B and T, Pareto frontiers for different target exposure ratio values (p_s) can be found in order to obtain the set of all efficient solutions. In this solution, the parameters of the effectiveness functions are constant throughout the ad campaign. However, user behavior may not be stable, and it is prone to change. Therefore, a method to update the effectiveness function using new data is proposed in Section 5.2.

5.2 Online Learning

Since we obtain new data every day during the campaign, updating the effectiveness function used in the model will make a great contribution to increasing the consistency of the following predictions. Since the last observations are more important than the old observations, a larger weight is assigned to the new data. The effect function parameters are updated daily by applying weighted curve fitting to the dataset. The algorithm used for updating effectiveness function is shown in Figure 4 below.

Algorithm 2: Updating Effectiveness Function

Step 1: Obtain new daily data from the campaign (number of downloads, budget spent).
Step 2: Select last n data from previous data.
Step 3: Combine them and obtain $n + 1$ data in total.
Step 4: Apply weighted curve fitting in order to update parameters (m_s, μ_s) of the effectiveness function (past data with the lowest weight, new data with the highest weight).

Figure 4: Algorithm for Updating Effectiveness Function

In Step 1, the data of the actual daily expenditure and the number of people newly installed the game is added to the dataset. Then, in Step 2 and 3, it is recommended to choose n as the length of the dataset in order to prevent any loss of information. In Step 4, exponential weighting is used where the weight of each data point is given by $0.99^{(n+1-t)}$ where n is the number of data points in the dataset and t is the index of data where $t = n + 1$ is the latest sample. Applying weighted regression, m_s and μ_s are reestimated and the problem parameters such as q_s, p_s, etc. are updated. Thus, with the new parameters, the model is employed in order to find the new daily ad budget.

6 Multiplatform Advertising

In the context of this particular study, the decision-maker responsible for the marketing campaign has multiple options when it comes to selecting the platform for the campaign. Consequently, the decision maker is faced with the task of determining how to allocate the initial marketing budget between these platforms, which introduces the concept of budget allocation for multiplatform advertising.

6.1 Problem Formulation

We formulated the problem as a knapsack problem where the decision maker has the option to maximize one of two variables: estimated total installs or estimated total revenue. The total budget for marketing is B, and it serves as an upper bound for the optimization model. Let N be the set of marketing platforms. The model that will maximize the estimated total revenue for a fixed campaign length T can be given as follows:

Parameters

G: Given total budget
B_n: Allocated budget for platform n

h_n: Targeted population at platform n
ltv_n: Estimated LTV per user at platform n

Decision Variable

p_n: Exposure percentage for platform n

Multiplatform Budgeting Model

Maximize $\sum_n p_n \cdot h_n \cdot ltv_n$
s.t.

$$B_n = f(p_n, ...) \tag{10}$$

$$\sum_n B_n \leq G \tag{11}$$

The function in equation 10 represents the relation between exposure percentage p and the corresponding B values from the Pareto Frontier explained by algorithm 1 in section 5.1. The provided model can be modified to maximize the estimated total exposure by making a minor adjustment to the objective function, namely $\sum_n p_n \cdot h_n$. It is worth noting that using this version, which focuses on maximizing estimated total exposure, aligns the objectives with the previously described model that determines the daily budget for the campaigns, with both models sharing the common objective of maximizing the number of installs. This alteration allows for a consistent approach in optimizing campaign performance and achieving desired outcomes related to user engagement and downloads.

6.2 Initial Assumptions

One of the complexities in this approach lies in the estimation of LTV parameters. When the decision maker aims to maximize the estimated total exposure, the parameters utilized in the calculations tend to be deterministic in nature. The percentage of exposure serves as a core component of the model as explained earlier, and the number of users in the target audience is directly provided by the marketing platform, allowing for observable accuracy during the campaign process. However, estimating LTV is not readily available from the marketing platform, and it poses a challenge in terms of accurate estimation. Numerous academic and industrial studies have explored this area, highlighting the complexity of LTV estimation. In this study, we adopted a relatively simple estimation approach for LTV, acknowledging its limitations, and placed emphasis on the overall methodology rather than precise LTV estimates.

6.3 Solution Algorithm

In the initial model, we aimed to find the minimum campaign budget B based on input parameters such as the desired exposure percentage (p) and campaign length (T), among others. However, in the context of the Multi-platform Budget Allocation Model, the exposure percentage for each platform is defined as a decision variable rather than an input parameter. As a result, to implement the Multi-platform Budget Allocation Model, we need to work with the inverse of the function used in the initial model.

Working directly with the inverse function poses significant challenges due to the complexity detailed in Chapter 4. Defining the inverse function is beyond the scope of this research. Therefore, we opted for an alternative heuristic method. We discretized the range of possible exposure percentage values (p) within the interval $[0, 1]$ into a predetermined number of values as k, which we set $k = 1000$ in our study. Subsequently, we recorded corresponding campaign budget values (B) for these alternative p values while keeping the other input parameters fixed. This collection of (p, B) pairs serves as an approximation of the inverse function for our calculations. While working in a discrete space introduces limitations compared to a continuous one, this methodology adequately serves its purpose when a sufficiently large number of discretized values is employed.

After the set of (p, B) pairs is obtained, we used an algorithm that is simply iterating over every possible set of p_n combinations in the feasible region to solve the model, which corresponds to a sample set of n^k, and finds the set of p_n that gives the maximum estimated total revenue, and returns solutions for the sets p_n, B_n, and optimum estimated total revenue, R'. Note that set of p_n corresponds to a set of p values for each platform, and set of B_n is the B values corresponding to p values in the set p_n, while R is calculated as $R = \sum_n p_n \cdot h_n \cdot ltv_n$ for each p_n. The algorithm used for daily budgeting strategy for multiplatform is given in Figure 5 below.

Algorithm 3: Daily Budgeting for Multiplatform

Step 1: Initialize the sets of B'_n and p'_n, and R' as zero.
Step 2: Select the next set of p_n in the sample space.
Step 3: Check whether the constraint $\sum_n B_n \leq G$ holds. If it does, continue to Step 4. Otherwise, return to Step 2.
Step 4: Calculate R for the selected set of p_n. If $R \geq R'$, set $R' = R$, $p'_n = p_n$, and $B'_n = b_n$.
Step 5: If there is a set left unchecked, go back to Step 2. Otherwise, stop.

Figure 5: Algorithm for Daily Budgeting Strategy for Multiplatform

Upon the completion of the algorithm's evaluation of feasible sets of p_n, the resulting values represent the optimal solution for the estimated total revenue, estimated exposure percentage for each platform, and the allocated budget for each platform. These values provide the most favorable combination that maximizes the estimated revenue while adhering to the given constraints.

As was the case in the model, the algorithm can be alternated to maximize estimated total exposure by calculating the exposure $E = \sum_n p_n \cdot h_n$ for each p_n instead of R in Step 4.

6.4 Further Improvements

While the proposed heuristic solution algorithm, which utilizes pre-calculated (p, B) combinations, yields near-optimal solutions for sufficiently large values of k, it can also be applied to calculate the inverse of the function $B = f(p)$ used in the initial model if a more accurate solution is desired. The solution provided in this study is suitable for the scope of the research; however, it is worth noting that exploring the calculation of the inverse function may be a valuable consideration for future studies seeking even greater accuracy and precision.

7 Revenue Estimation

Throughout the decision-making process for the marketing campaign, an advertiser is presented with a set of distinct campaign proposals, each offering varying exposure percentages (p), budgets (B), campaign duration (T), and daily bid budget (b). While this assortment provides valuable insights to the advertisers, it is essential to offer an estimated success metric for each proposed campaign. Consequently, we have introduced an estimated revenue as the proposed success metric for each campaign.

7.1 Problem Formulation

Estimating the revenue of a campaign is a complex process and a hard task to achieve accurately. However, the data that we worked on allowed us to obtain an estimation, useful for comparing the alternative campaigns at the very least. Marketing platforms offer advertisers a range of metrics to gauge campaign outcomes. Among these metrics, we have identified three specific ones that prove instrumental in estimating revenue: Daily Installs, Daily Campaign Expenditure, and 7-day Retention Rate. The latter refers to the percentage of users who actively engage with the application after installation on a daily basis. It is noteworthy that the users who actively engage with the application are the ones who contribute to revenue generation through activities such as watching ads or making in-game purchases. To account for this, we have combined two key metrics, namely Daily Installs and 7-day Retention Rate, to calculate a new metric called Daily Users. In our estimation process, we employ two key metrics, namely Daily Users and Daily Campaign Expenditure, to predict the outcome of Estimated Revenue. In this scenario, regression analysis proves to be a suitable approach. Although forecasting methods may appear appealing, it is important to acknowledge that in an environment where decision variables, such as daily campaign expenditure, directly influence the outcome, it is not reasonable to solely project daily revenue based on previous revenue data. Thus, regression analysis allows us to account for the interplay between these variables and provide a more accurate estimation of revenue.

We considered different regression methods as viable applications: Multi-linear and Polynomial regression.

7.2 Regression Analysis

In order to mitigate the potential impact of outliers on the accuracy of the regression models, a decision was made to exclude a small percentile of extreme values from the dataset during the analysis. Specifically, the upper and lower 0.05 percentiles of the data in terms of the number of installs were excluded. Removing these outliers, which are the instances with the highest and lowest number of installs, allowed for a more reliable estimation capability of the regression models.

We considered two regression methods for the problem: Multi-linear and Polynomial regression. Although we observed that a polynomial regression yields better results for the datasets available, it was determined that multi-linear regression would be employed as the chosen method for analysis in the context of this study. This decision was based on the widespread use and familiarity of multi-linear regression in the field of research. The summary of the analysis can be examined in Table 2.

Analysis Summary				
Regression Method	R^2	MAE	MSE	RMSE
Multi-linear	0.5219	0.0837	0.0142	0.1191
Polynomial	0.6195	0.0673	0.0113	0.1062

Table 2: Regression analysis results

7.3 Assumptions

Preliminary observations from the initial trials revealed that the duration of the campaign significantly and unreasonably impacts the estimated revenue. As a result, we reconfigured the estimation model to project the revenue that would be generated after a specific time frame for each campaign, with a standardized period of 30 days being selected. In addition, the previously discussed model that proposes a daily marketing budget works in a way that offers the same value on each day of the campaign if the online learning application is excluded. As we will show in the following section, we will work with estimated daily campaign expenditure values. Since the revenue estimation process is established to give the decision maker an initial evaluation between alternative campaigns, it is not possible to get updated daily expenditure suggestions from the model, resulting in using the initial expenditure suggestion for the entire 30-day projection period.

Furthermore, it is important to note that the previously discussed model, which proposes the daily marketing budget for the selected campaign, will give an equal amount for each day of the campaign if the application of online learning is excluded. As we will discuss in the following section, we will work with estimated daily campaign expenditure values. Since the revenue estimation process is established to give the decision maker an initial evaluation between alternative campaigns, it is not possible to get updated daily expenditure suggestions from the model. Hence, the initial expenditure suggestion will be used for the entire 30-day projection period.

7.4 Solution Algorithm

To conduct a regression analysis, it is crucial to format the test data in a manner that aligns with the format of the training data. During the training phase, the model utilizes historical data for daily installs, daily campaign expenditure, and 7-day retention rate. Therefore, when preparing the test data, it is essential to ensure that the data includes the same variables and follows the same structure as the training data. To ensure the test data aligns with the training data, the following steps can be followed to estimate each of the variables:

- **Estimated daily number of installs** D_i': To estimate the number of daily installs, we use the targeted reach percentage (p), targeted audience size (h), and campaign duration (T) of the campaign of interest.

$$R_i' = \frac{p \cdot h}{T} \tag{12}$$

It is important to note that this estimation includes the assumption that we will reach the targeted number of installs at the end of the campaign.

- **Estimated daily campaign expenditure** b'_i: For the daily campaign expenditure, we will utilize the daily expenditure value proposed by the MODEL. As mentioned earlier, the online learning aspect will be excluded for this specific part since it requires observing the campaign's effect. Therefore, we will employ the same initial daily spending value for the entire duration of the campaign.

- **Estimated 7-day retention rate** $ret'_i j$: To estimate the retention rate for each day, a simple approach was employed where the arithmetic mean of the retention rates for each day was calculated and utilized.

$$ret'_{ij} = \frac{\sum_n ret_{nj}}{N} \tag{13}$$

Parameters

p: Targeted reach percentage for the selected campaign
h: Audience size for the selected campaign
T: Campaign duration
b_i: Daily campaign expenditure in day i
D_i: Number of installs in day i
U_i: Number of active users in day i
R_i: Amount of revenue estimated for day i
Z: Selected revenue estimation period

The steps we follow in revenue estimation are shown in Figure 6 below.

Algorithm 4: Revenue Estimation

Step 1: Initialize the values for D'_i, b'_i, and $ret'_i j$ by using the proposed estimation methods.
Step 2: Calculate U_i using past data (D_i and $ret_i j$), and U'_i using estimated campaign data (D'_i and $ret'_i j$)
Step 3: The combined dataset consisting of past data and campaign data, including the newly calculated parameters U_i and U'_i, was subjected to a normalization process.
Step 4: Apply multi-linear regression over the past data to obtain regression parameters $w_{expenditure}$, w_{users}, and $bias$.
Step 5: Estimate the revenue R'_i for each day i of the campaign by $R'_i = w_{expenditure} \cdot b'_i + w_{users} \cdot U'_i + bias$.
Step 6: Calculate the cumulative estimated revenue over the campaign period, and then divide it to calculate the estimated daily average revenue. Multiply the result with the previously selected revenue estimation period Z using $R'_{cum} = \frac{\sum_i R'_i}{T} \cdot Z$.

Figure 6: Algorithm for Revenue Estimation

Upon running the algorithm for each feasible campaign proposed by Model 2, the estimated revenues for a specified period of Z days are obtained for each campaign. This analysis offers valuable insights into the potential revenue generation of each campaign during the specified timeframe. Such insights empower the decision maker to make informed choices and select the most suitable campaign option.

7.5 Limitations and Further Improvements

It is crucial to acknowledge that estimating the revenue generated from a marketing campaign is a challenging endeavor. The purpose of this revenue estimation is solely to provide the decision maker with an initial evaluation among the multiple marketing campaigns proposed by Model 2.

Moreover, it is important to emphasize that the focus of this study revolves around the business model of hypercasual mobile games, where aggressive marketing plays a pivotal role in the marketing strategy. In this context, it is commonly observed that campaigns published on marketing platforms exhibit significantly higher effectiveness compared to organic downloads.

The proposed solution takes into consideration the timeframe during which marketing campaigns remain active and effective, without explicitly addressing the long-term revenue potential of the marketed application. The emphasis lies on optimizing the performance and returns within the span of time when the campaigns are actively running.

It is important to acknowledge that the experiment phase of this study was conducted using a relatively small dataset. This limitation could potentially impact the generalizability of the results and may introduce some degree of uncertainty. The findings of this study provide valuable insights and demonstrate the potential effectiveness of the proposed approach, even within the limitations of the dataset. Further research with datasets with longer campaigns and more platforms could help validate and enhance the findings, offering a more comprehensive understanding of the proposed method's performance and applicability.

8 Experiment Results

In this research, we conducted a comprehensive study in collaboration with **The Game Circle**, a mobile game marketing company, to address the issue of developing a systematic approach for the daily spending strategy in their marketing campaigns. Prior to this study, the decision-making process for budget allocation relied heavily on the experience and intuition of the decision makers, lacking a standardized and methodological framework.

Through an iterative and collaborative process, we worked closely with the decision makers at The Game Circle for a duration of 8 months to develop and refine the proposed methods. We engaged in discussions to assess the feasibility and effectiveness of the approaches, taking into account the specific requirements and objectives of the company.

To test the finalized approach, we planned to conduct an experiment involving three marketing campaigns for a period of 5 days. The campaigns were implemented for a mobile hyper casual game called **Toy Shop**, which was developed by The Game Circle. During the experiment, data was collected from various platforms, including Meta, Adjust, AdMob, and ironSource, to evaluate the performance and effectiveness of the campaigns. During the course of this study, we were able to finalize the experiment for the single platform - single audience scenario

In terms of target audience selection, the decision maker at The Game Circle employed a strategy of choosing the 1st, 2nd, and 5th percentiles of the general audience that closely matched their existing customer base. This approach ensured that the campaigns were targeted towards potential customers who exhibited similar characteristics to their current user base.

8.1 Experiment - Single Platform and Single Audience

Multi-platform Budget Allocation Model, explained in Section 5, is not utilized for this experiment since the selected scenario is only applied over a single marketing platform. Daily Budgeting Model that is explained in Section 4 is utilized over the past marketing dataset to determine the feasible candidate campaigns. Then, revenue estimation methodology explained in Section 6 is applied over a dataset with 98 entries, obtained from the previous campaigns. The dataset is shuffled and split as 0.8 train and 0.2 test data, and multi-linear regression is performed over the test dataset. Resulting regression model is utilized to estimate the revenues for the feasible set of campaigns. Estimated revenues are used to display projected profit and return of investment (ROI) for each candidate campaign during the campaign selection phase.

Decision maker is offered a set of candidate campaigns ranging in duration, exposure, and total expenditure. After evaluating the campaigns, the decision maker selected the specific campaign that aims to reach 640 installs in 6 days, and the corresponding budget for the campaign is \$249, with the daily expenditure being \$41.45. Utilizing the revenue estimation model, it has been projected that this campaign will potentially yield a profit of \$95, after deducting the associated expenditure from the projected revenue. This indicates to a potential return of investment (ROI) of 0.38.

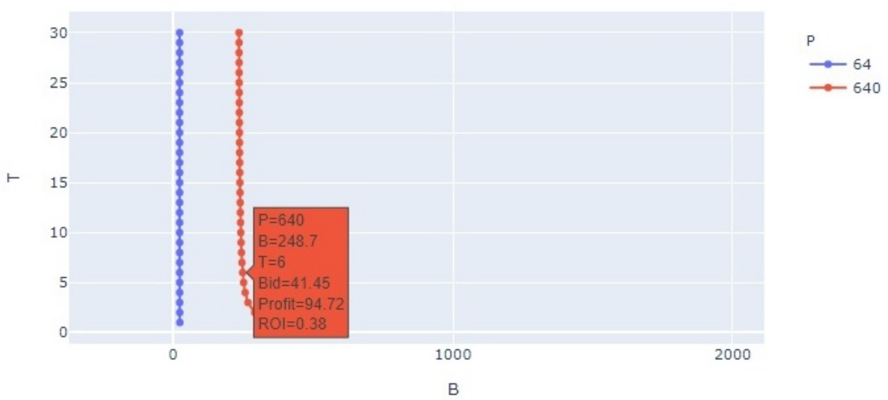

Figure 7: Selection screen from the demo

The daily expenditure is updated on a daily basis throughout the campaign based on the number of installs and the daily expenditure values obtained from the previous day. The online learning process, explained in Section 4.2 is employed to dynamically update the daily expenditure based on the observed campaign performance.

8.2 Discussing the Results

Table 3 summarizes the campaign results. An important metric to remember from Section 4 is a_s, the coefficient of interaction for audience s, determined by the problem owner. For this experiment, it is determined as 3, and the corresponding number of downloads for this campaign is found as 942. Results show that the initially determined budget is expired one day earlier, leading to a 5-day campaign instead of 6. Note that the daily expenditure observed during the campaign is slightly lower than the allocated daily budget. This discrepancy is a result of the operational characteristics of the selected marketing platform.

Campaign Day	Daily Budget	Daily Expenditure	Observed Downloads
1	42	42	58
2	44	43	53
3	48	44	64
4	55	54	66
5	65	55	73
6	-	-	-
Total	255	238	314

Table 3: Daily results for the experiment

Results showed that the total number of installs is 47% higher than the targeted number of installs. Although it is a preferable deviation, it is important to acknowledge that it still shows an inaccuracy. One potential factor contributing to this inaccuracy is the continuous updates and improvements made by The Game Circle in its ad designs. The datasets used in this research are from previous campaigns that utilized older versions of advertisements. The observed higher number of installs may show the effectiveness of the updated ad designs, which are not captured in the analyzed datasets.

9 Conclusion

In this research, our focus was on developing a decision support system tailored for companies that engage in multi-platform online advertising and seek a systematic approach to determine daily budgets for their marketing campaigns across various platforms and target audiences. Throughout the application, we addressed multiple aspects of the problem and devised methodological solutions to address each of them.

Our research encompasses key components such as determining the campaign budget for a specified duration and desired exposure percentage, providing the decision maker with alternative campaign options, leveraging online learning techniques to continuously update model parameters with new data, estimating campaign revenue, and allocating the initial budget effectively among multiple platforms. By utilizing our proposed method, the decision maker can determine the daily budgets for each campaign that maximize the number of installs. This is achieved by providing only the initial marketing budget and utilizing previous campaign data obtained from the marketing platforms as inputs to the model. Our approach aims to streamline the decision-making process, enhance campaign performance, and optimize the allocation of resources across multiple platforms.

In order to validate and evaluate the effectiveness of our approach, we conducted a case study in collaboration with a mobile-game company. The case study involved implementing our decision support system and comparing its results with those obtained through the company's current non-systematic approach to campaign budget allocation. By conducting this comparative analysis, we were able to assess the impact of our approach on campaign performance and the overall effectiveness of the decision-making process. We measured key performance indicators such as the number of installs, revenue generated, and cost-efficiency metrics to evaluate the outcomes achieved through our approach. We observed that our decision support system resulted in significant improvements over the company's unsystematic approach. Daily budgets determined by our system lead to higher installs, showing an improvement in campaign performance and reach. In addition, the revenue generated from campaigns using our method showed a significant increase, indicating a better return on investment. In addition, our decision support system has proven cost-effective by optimizing resource allocation across multiple platforms. By considering factors such as platform effectiveness and target audience engagement, our systems are able to efficiently allocate initial marketing budgets, ensuring maximum impact and reducing unnecessary spending.

In summary, our research contributes to the field of multi-platform online advertising by providing a comprehensive decision support system that addresses various aspects of campaign budget allocation. Our model validated the effectiveness of our approach and demonstrated its potential for streamlining decision-making and improving campaign performance for companies engaged in online advertising across multiple platforms.

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