20.2 A university computer service conducted an experiment in which one coin-operated computer graphics terminal was placed at each of four different locations on the campus last semester during the midterm week and again during the final week of classes. The data that follow show the number of hours each terminal was not in use during the week at the four locations (factor A) and for the two different weeks (factor B).

(a) Plot the data in format of Figure 20.1. Does it appear that interaction effects are present? Does it appear that factor A and factor B main effects are present? Discuss.

Answer:

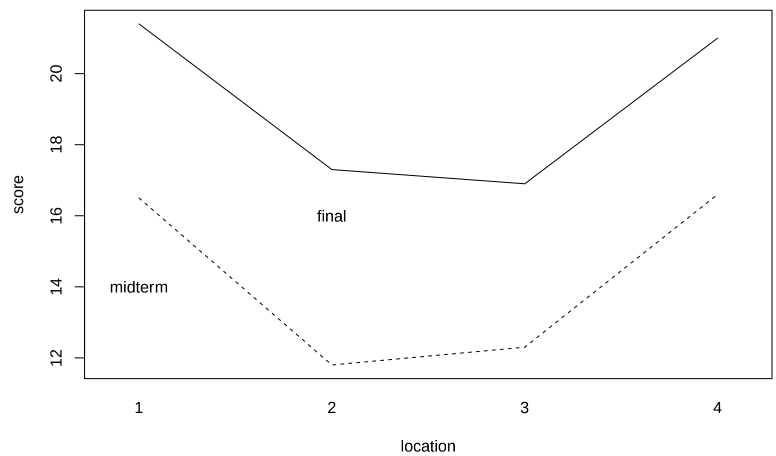

Figure 1

Yes there are main effects for both location and week, because the lines are not flat nor do they overlap.

(b) Conduct separate tests for location and week main effects. In each test, use level of significance $\alpha = 0.05$ and state the alternatives, decision rule, and conclusion. Give an upper bound for the family level of significance; use the Kimball inequality (19.53). What is the P-value for each test.

Answer: We conclude the alternative for location and week effects. The p-values were all above essentially 0 according to our ANOVA table.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
as.factor(location)	3	37.00	12.33	107.26	0.0015
as.factor(week)	1	47.04	47.04	409.09	0.0003
Residuals	3	0.35	0.12		

Recall a and b are the number of levels per factor For the test of location effects:

$$F^* = \frac{\text{MSA}}{\text{MSE}}$$

and, with $a = 4$ and $b = 2$, If $F^* \leq F[1-\alpha; a-1, (a-1)(b-1)] = F(.95;3,3) = 9.28$, conclude H_0.

For week effects:

$$F^* = \frac{\text{MSB}}{\text{MSE}}$$

and $F^* \leq F[1-\alpha; b-1, (a-l)(b-1)] = F(.95;1,3) = 10.12$, conclude H_0.

Our test for location (α factor) is (noting we just swap α and β to do the test for week effects)

$$H_0 : \text{all } (\alpha)_i = 0$$
$$H_a : \text{not all } (\alpha)_i \text{ equal zero}$$

We conclude the alternative because of high F-statistics from the ANOVA table.
The Kimball inequality says

$$\alpha \leq (1-\alpha_1)(1-\alpha_2) = 1-(0.95)\cdot(0.95) = 0.0975$$

(c) Make all pairwise comparisons among location means and estimate the difference between the means for the two weeks; use the Bonferroni procedure with a 90 percent family confidence coefficient. State your findings.

Answer: We have 4 locations and want to compare them, which we means we have $\binom{4}{2} = 6$ combinations.

$$\hat{D}_1 = \overline{Y}_{1\cdot} - \overline{Y}_{\cdot 2} = 18.95 - 14.55 = 4.40$$
$$\hat{D}_2 = \overline{Y}_{1\cdot} - \overline{Y}_{\cdot 3} = 18.95 - 14.6 = 4.35$$
$$\hat{D}_3 = \overline{Y}_{1\cdot} - \overline{Y}_{\cdot 4} = 18.95 - 18.8 = 0.15$$
$$\hat{D}_4 = \overline{Y}_{2\cdot} - \overline{Y}_{\cdot 3} = 14.55 - 14.6 = -0.05$$
$$\hat{D}_5 = \overline{Y}_{2\cdot} - \overline{Y}_{\cdot 4} = 14.55 - 18.8 = -4.25$$
$$\hat{D}_6 = \overline{Y}_{3\cdot} - \overline{Y}_{\cdot 2} = 14.6 - 18.8 = -4.20$$

We have $\binom{2}{2} = 1$ combination for the weeks. And for the case the weeks is

$$\hat{D}_7 = \overline{Y}_{\cdot 1} - \overline{Y}_{\cdot 2} = 14.3 - 19.15 = -4.85$$

To do the Bonferroni method, we need to find $s(\hat{D}_i) = \text{MSE}\frac{(a+b-1)}{ab}$, where $a = 4$ and $b = 2$, the number of levels of the location and week respectively. Therefore, we find for the first 6 cases, (noting all $c_i = 1$ or -1)

$$s(\hat{D}_i = \sqrt{\sum c_i^2 \cdot \text{MSE}/2} = \sqrt{\text{MSE}} = 0.34$$

because we have 2 levels of week for each location. For the week comparisons, we have 4 levels for each week, one for each location. So

$$s(\hat{D}_7) = \sqrt{\text{MSE}/2} = 0.240$$

The Bonferroni value is, noting $g = 7$ $t(1-\alpha/(2*7), (a-1)\cdot(b-1)) = 5.14$.

$$4.40 \pm (5.14)\cdot 0.34 \longrightarrow 2.65 \leq \mu_{1\cdot} \leq 6.15$$
$$4.35 \pm (5.14)\cdot 0.34 \longrightarrow 2.60 \leq \mu_{2\cdot} \leq 6.097$$
$$0.15 \pm (5.14)\cdot 0.34 \longrightarrow -1.59 \leq \mu_{3\cdot} \leq 1.89$$
$$-0.05 \pm (5.14)\cdot 0.34 \longrightarrow -1.79 \leq \mu_{4\cdot} \leq 1.69$$
$$-4.25 \pm (5.14)\cdot 0.34 \longrightarrow -5.99 \leq \mu_{5\cdot} \leq -2.50$$
$$-4.20 \pm (5.14)\cdot 0.34 \longrightarrow -5.95 \leq \mu_{6\cdot} \leq -2.45$$
$$-4.85 \pm (5.14)\cdot 0.24 \longrightarrow -6.08 \leq \mu_{7\cdot} \leq -3.62$$

20.3 Refer to problem 20.2. It is desired to estimate μ_{32}.

(a) Obtain a point estimate of μ_{32} using 20.5.

Answer: (20.5) says

$$\hat{\mu}_{ij} = \overline{Y}_{i\cdot} + \overline{Y}_{\cdot j} - \overline{Y}_{\cdot\cdot}$$

Therefore, In this case, i=3 and j=2. Therefore, we want the mean given location 3 plus the mean of week 2 (finals) minus the mean for all locations and weeks, i.e. all the responses. We find:

$$\hat{\mu}_{32} = 19.15 + 14.6 - 16.725 = 17.025$$

R-code:

```
m32=mean(data20part2$response[data20part2$week==2])+
mean(data20part2$response[data20part2$location==3])-mean(data20part2$response)
```

(b) Obtain the estimated variance of $\hat{\mu}_{32}$ by fitting the equivalent regression model.

Answer: ANOVA model

$$Y_{ij} = \mu_{..} + \alpha_i + \beta_j + \varepsilon_{ij}$$

The third subscript has been dropped from the Y and the ε terms bc there is now only one case per treatment. The equivalent regression model

$$Y_{ij} = \mu_{..} + \alpha_1 X_{ij1} + \alpha_2 X_{ij2} + \beta_1 X_{ij3} + \beta_2 X_{ij4} + \varepsilon_{ij}$$

where the $X_1 \dots X_4$ take on values of 1,2,3, or 4 depending on the location.
The fitted value for observation Y_{ij} will be:

$$\hat{Y}_{ij} = \overline{Y}_{..} + \hat{\alpha}_j + \hat{\beta}_j$$

which is equivalently:

$$\hat{Y}_{ij} = \overline{Y}_{..} + (\overline{Y}_{i.} - \overline{Y}_{..}) + (\overline{Y}_{i.} - \overline{Y}_{..}) = \overline{Y}_{i.} + \overline{Y}_{.j} - \overline{Y}_{..} = \hat{\mu}_{ij}$$

This is found using software, or analytically

$$s^2(\hat{Y}_h) = \text{MSE}(X_h^{'}(X^T X)^{-1} X_h) = X_h^{'} s^2(\hat{\boldsymbol{\beta}}) X_h$$

where X_h is defined such that

$$E(Y_h) = X_h^{'} \boldsymbol{\beta}$$

Another method is to note

$$\text{Var}(\hat{\mu}_{ij}) = \sigma^2 (a + b - 1)/ab$$

which means, replacing σ^2 with the MSE as always,

$$s^2(\hat{\mu}_{ij}) = \text{MSE}\frac{a + b - 1}{ab}$$

This is convenient bc we know the MSE, a, and b. We find from the ANOVA table and bc a=4 and b=2 (the respective number of levels per factor)

$$s^2(\hat{\mu}_{32}) = 0.115 * (2 + 4 - 1)/(2 * 4)$$

(c) Construct a 95 % confidence interval for μ_{32}. Interpret your interval estimate. Is your interval estimate applicable if next year two graphics terminals will be placed at location 3? Explain.

Answer: We use the usual method:

$$\hat{\mu}_{32} \pm s(\hat{\mu}_{32}) t(1 - \alpha/2, (a-1)(b-1)) = 17.025 \pm 0.268 \cdot 3.182 \longrightarrow 16.172 \le \mu_{32} \le 17.877$$

20.4 Refer to problem 20.2. Conduct the Tukey test for additivity; use $\alpha = 0.025$. State the alternatives, decision rule, and conclusion. If the additive model is not appropriate, what might you do?

Answer: Assume

$$(\alpha\beta)_{ij} = D\alpha_i\beta_j$$

Where D is some constant. A regular two-factor ANOVA model with interactions for the case $n = 1$ is

$$Y_{ij} = \mu_{..} + \alpha_i + \beta_j + D\alpha_i\beta_j + \varepsilon_{ij}$$

where each term has usual meaning. No third subscript because $n = 1$. Least squares and MLE of of D turns out to be

$$\hat{D} = \frac{\sum_i\sum_j \alpha_i\beta_j Y_{ij}}{\sum_i \alpha_i^2 \sum_j \beta_j^2}$$

The usual estimator of α_i is $\overline{Y}_{i\cdot} - \overline{Y}_{..}$ and that of β_j is $\overline{Y}_{\cdot j} - \overline{Y}_{....}$ Replacing the parameters in $\hat{D}$ with these estimators, we obtain:

$$\hat{D} = \frac{\sum_i\sum_j(\overline{Y}_{i\cdot} - \overline{Y}_{..})(\overline{Y}_{\cdot j} - \overline{Y}_{..})Y_{ij}}{\sum_i(\overline{Y}_{i\cdot} - \overline{Y}_{..})^2\sum_j(\overline{Y}_{\cdot j} - \overline{Y}_{..})^2}$$

If we substitute the sample estimates into $\sum\sum D^2\alpha_i^2\beta_j^2$, we find

$$\begin{aligned}
\text{SSAB}^* &= \sum_i\sum_j \hat{D}^2(\overline{Y}_{i\cdot} - \overline{Y}_{..})^2(\overline{Y}_{\cdot j} - \overline{Y}_{..})^2 \\
&= \frac{\left[\sum_i\sum_j(\overline{Y}_{i\cdot} - \overline{Y}_{..})(\overline{Y}_{\cdot j} - \overline{Y}_{..})\right]^2}{\sum_i(\overline{Y}_{i\cdot} - \overline{Y}_{..})^2\sum_j(\overline{Y}_{\cdot j} - \overline{Y}_{..})^2}
\end{aligned}$$

The analysis of variance decomposition for the special interaction model is

$$\text{SSTO} = \text{SSA} + \text{SSB} + \text{SSAB}^* + \text{SSRem}^*$$

where SSRem* is the *remainder sum of squares*:

$$\text{SSRem}^* = \text{SSTO} - \text{SSA} - \text{SSB} - \text{SSAB}^*$$

It can be shown that if $D = 0$, i.e. no interactions of type $D\alpha_i\beta_j$ exist, then SSAB* and SSRem* are independently distributed as chi-square random variables with 1 and $ab - a - b$ dof, respectively. Hence, if $D = 0$, the test statistic

$$F^* = \frac{\text{SSAB}^*}{1}\nabla\cdot\frac{\text{SSRem}^*}{ab - a - b}$$

is distributed as $F(1, ab - a - b)$, so for testing:

$$\begin{aligned}
&H_0 : D = 0 \quad \text{no interations present} \\
&H_a : D \neq 0 \quad \text{interactions } D\alpha_i\beta_j \text{ present}
\end{aligned}$$

where we control type I error at α with the following decision rule, if $F^* \leq F(1 - \alpha; 1, ab - a - b)$, we conclude H_0, otherwise conclude H_a.

$F^0.5897 < 30.8$, so we conclude H_0, that $D = 0$, and the p-value was 0.523. The numerator had 1 degree of freedom and denominator had 2 degrees of freedom.

```
tukeys.add.test(data20part2$response,data20part2$location , data20part2$week)
```

20.8 Soybean sausage. A food technologist, testing storage capabilities for a newly developed type of imitation sausage made from soybeans, conducted an experiment to test the effects of humidity level (factor A) and temperature level (factor B) in the freezer compartment on color change in the sausage. Three humidity levels and four temperature levels were considered. Five hundred sausages were stored at each of the 12 humidity-temperature combinations for 90 days. At the end of the storage period, the researcher determined the proportion of sausages for each humidity-temperature combination that exhibited color changes. The researcher transformed the data by means of the arcsine transformation to stabilize the variances. The transformed data $Y' = 2\arcsin\sqrt{Y}$ follow. Assume that no-interaction ANOVA model is appropriate.

(a) Plot the data in the format of Figure 20.1. Does it appear that interaction effects are present? Does it appear the factor A and factor B main effects are present? Discuss.

Answer:

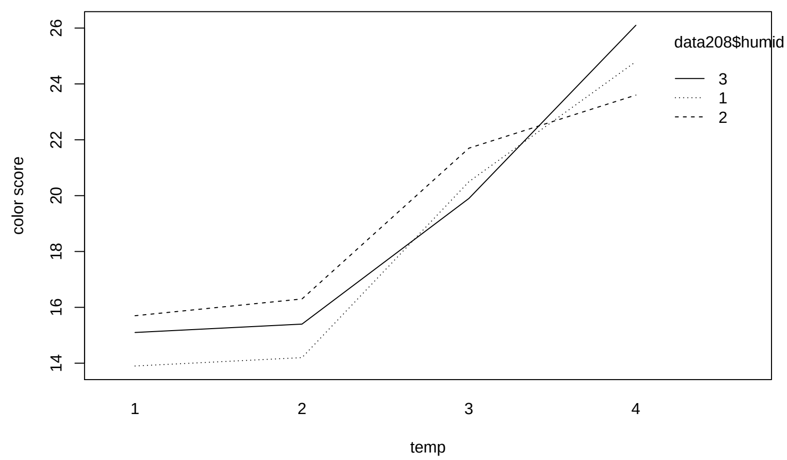

Figure 2: They are not super parallel, so there may be interaction effects. There is definitely a temperature effect, as the color changes dramatically at "higher temp" level. Not sure there is any humidity effect.

(b) Conduct separate tests for humidity and temperature main effects. In each test, use level of significance $\alpha = 0.025$ and state the alternatives, decision rule, and conclusion. What is the p-value for each test?

Answer: Recall a and b are the number of levels per factor For the test of temperature effects:

$$F^* = \frac{\text{MSA}}{\text{MSE}}$$

and If $F^* \leq F[1-\alpha; a-1, (a-1)(b-1)] = F(.95;3,6) = 6.599$, conclude H_0.

For humidity effects:

$$F^* = \frac{\text{MSB}}{\text{MSE}}$$

and $F^* \leq F[1-\alpha; b-1, (a-l)(b-1)] = F(.95;2,6) = 7.26$, conclude H_0.

Our test for temperature (α factor) is (noting we just swap α and β to do the test for humidity effects)

$$H_0 : \text{all } (\alpha)_i = 0$$
$$H_a : \text{not all } (\alpha)_i \text{ equal zero}$$

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
as.factor(temp)	3	202.20	67.40	61.41	0.0001
as.factor(humid)	2	2.12	1.06	0.97	0.4326
Residuals	6	6.58	1.10		

The ANOVA table: As expected, we reject the null hypothesis for temperature effects not being present, and conclude null for humidity effects. We use the p-values from the ANOVA table.

(c) Obtain the confidence intervals for $D_1 = \mu_{\cdot 2} - \mu_{\cdot 1}$, $D_2 = \mu_{\cdot 3} - \mu_{\cdot 2}$, and $D_3 = \mu_{\cdot 4} - \mu_{\cdot 3}$; use the Bonferroni procedure with a 95% family confidence coefficient. State your findings.

Answer: We take the mean of the different temperature levels in order to find the comparisons:

$$D_1 = \mu_{\cdot 2} - \mu_{\cdot 1} = 15.3 - 14.9 = 0.4$$
$$D_2 = \mu_{\cdot 3} - \mu_{\cdot 2} = 20.7 - 15.3 = 5.4$$
$$D_3 = \mu_{\cdot 4} - \mu_{\cdot 3} = 24.83 - 20.7 = 4.13$$

For the case where $\hat{D}_j = \sum_j c_j \mu_{\cdot j}$, We note, $s^2(\hat{D}_j) = \frac{\text{MSE} \sum c_j^2}{a}$, and since $a = 3$ (the number of humidity levels), we just use the ANOVA table, where we found MSE=1.098. Note then,

$$s^2(\hat{D}_j) = 0.5488 \longrightarrow s(\hat{D}) = 0.855$$

Had we been aggregating over the humidity means, $s^2(\hat{D}_i) = \frac{\text{MSE} \sum c_i^2}{b}$ for the case where $\hat{D}_i = \sum_i c_i \mu_{i\cdot}$, where b equals 4 in that case, the number of humidity levels.

The Bonferroni is here, for $(1-\alpha)100\%$ confidence intervals for μ_{ij} is $t\big(1 - \alpha/2g, (a-1)(b-1)\big) = 3.29$

$$0.4 \pm 3.29 * 0.855 \longrightarrow -2.41 \le D_1 \le 3.21$$
$$5.4 \pm 3.29 * 0.855 \longrightarrow 2.589 \le D_2 \le 8.21$$
$$4.133.29 * 0.855 \pm \longrightarrow 1.319 \le D_3 \le 6.941$$

(d) Is the Bonferroni procedure the most efficient one here?

Answer: Yes. The Tukey multiple is $T = \frac{1}{\sqrt{2}} q(1-\alpha; r, (a-1)\cdot(b-1)) = 3.46$ and for Scheffe $S = \sqrt{(r-1)F(1-\alpha, r-1, (a-1)\cdot(b-1)} = 3.78$. The Bonferroni has a value of 3.29, so its the most efficient.

20.9 Refer to 20.8. It is desired to estimate μ_{23}.

(a) Obtain a point estimate of μ_{23} using (20.5).

Answer: (20.5) says

$$\hat{\mu}_{ij} = \overline{Y}_{i\cdot} + \overline{Y}_{\cdot j} - \overline{Y}_{\cdot\cdot}$$

Therefore, In this case, i=2 and j=3, i.e. we sum over the means of humidity=2, temperature=3, and the total mean.

We find $\hat{\mu}_{32} = 21.09$.

```
m23=mean(data208$color[data208$humid==2])+
mean(data208$color[data208$temp==3])-mean(data208$color)
```

(b) Obtain the estimated variance of $\hat{\mu}_{23}$ by fitting the equivalent regression model.

Answer: We found in an earlier question:

$$s^2(\hat{\mu}_{ij}) = \text{MSE}\frac{a+b-1}{ab}$$

This is convenient bc we know the MSE, a, and b. We find from the ANOVA table and bc a=3 and b=4 (the respective number of levels per factor)

$$s^2(\hat{\mu}_{23}) = 1.10 * (3+4-1)/(3*4) = 0.549 \longrightarrow s(\hat{\mu}_{23}) = 0.741$$

(c) Construct a 98 percent confidence interval for μ_{23} and transform it back to the original units. Interpret your interval estimate. Is your interval estimate applicable if the two factors interact?

Answer: From part (b), we know $s(\hat{\mu}_{23}) = 0.741$, and the $(1-\alpha)100\%$ confidence intervals for μ_{ij} is:

$$\hat{\mu}_{ij} \pm t\big(1-\alpha/2;(a-1)(b-1)\big)s(\hat{\mu}_{ij})$$

which we find to be

$$21.09 \pm 3.14 \cdot 0.741 \longrightarrow 19.10 \le \mu_{23} \le 23.08$$

20.10 Refer to soybean sausage problem 20.8. Conduct the Tukey test for additivity; use $\alpha = 0.005$. State the alternatives, decision rule, and conclusion. If the additive model is not appropriate, what might you do?

Answer:

$$H_0 : D = 0 \quad \text{no interations present}$$
$$H_a : D \neq 0 \quad \text{interactions } D\alpha_i\beta_j \text{ present}$$

where we control type I error at α with the following decision rule, if $F^* \le F(1-\alpha;1,ab-a-b)$, we conclude H_0, otherwise conclude H_a.

We find $F^* = 2.048$ with p-value of 0.2118

Here is a convenient R-package for this

```
tukeys.add.test <- function(y,A,B){
## Y is the response vector
## A and B are factors used to predict the mean of y
## Note the ORDER of arguments: Y first, then A and B
dname <- paste(deparse(substitute(A)), "and", deparse(substitute(B)),
"on",deparse(substitute(y)) )
A <- factor(A); B <- factor(B)
ybar.. <- mean(y)
ybari. <- tapply(y,A,mean)
ybar.j <- tapply(y,B,mean)
len.means <- c(length(levels(A)), length(levels(B)))
SSAB <- sum( rep(ybari. - ybar.., len.means[2]) *
rep(ybar.j - ybar.., rep(len.means[1], len.means[2])) *
tapply(y, interaction(A,B), mean))^2 /
( sum((ybari. - ybar..)^2) * sum((ybar.j - ybar..)^2))
aovm <- anova(lm(y ~ A+B))
SSrem <- aovm[3,2] - SSAB
dfdenom <- aovm[3,1] - 1
STATISTIC <- SSAB/SSrem*dfdenom
names(STATISTIC) <- "F"
PARAMETER <- c(1, dfdenom)
names(PARAMETER) <- c("num df", "denom df")
D <- sqrt(SSAB/  ( sum((ybari. - ybar..)^2) * sum((ybar.j - ybar..)^2)))
names(D) <- "D estimate"
RVAL <- list(statistic = STATISTIC, parameter = PARAMETER,
p.value = 1 - pf(STATISTIC, 1,dfdenom), estimate = D,
method = "Tukey's one df F test for Additivity",
data.name = dname)
attr(RVAL, "class") <- "htest"
return(RVAL)
}

tukeys.add.test(data208$color,data208$humid, data208$temp)
```

21.9 Dental plan. An anesthesiologist made a comparative study of the effects of acupuncture and codeine on postoperative dental pain in male subjects. The four treatments were: (1) placebo

treatment-a sugar capsule and two inactive acupuncture points (A_1B_1), (2) codeine treatment only- a codeine capsule and two inactive acupuncture points (A_2B_1), (3) acupuncture treatment only- a sugar capsule and two active acupuncture points (A_1B_2), and (4) codeine and acupuncture treatment-a codeine capsule and two active acupuncture points (A_2B_2). Thirty-two subjects were grouped into eight blocks of four according to an initial evaluation of their level of pain tolerance. The subjects in each block were then randomly assigned to the four treatments. Pain relief scores were obtained for all subjects two hours after dental treatment. Data were collected on a double-blind basis. The data on pain relief scores follow (the higher the pain relief score, the more effective the treatment).

(a) Why do you think that pain tolerance of the subjects was used as a blocking variable.

Answer: Pain tolerance is good because it varies by person, so we get rid of subjectiveness.

(b) Which of the assumptions involved in randomized block model (21.11) are you most concerned with here?

Answer: (21.11) says

$$Y_{ijk} = \mu_{...} + \rho_i + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \varepsilon_{ijk}$$

Probably interaction effects.

(c) Obtain the residuals for randomized block model (21.11) and plot them against the fitted values. Also, prepare a normal probability plot of the residuals. What are your findings?

Answer: In this example, A_1B_1 is equivalent to no codeine, no acupuncture, i.e. the placebo, A_2B_1 is codeine and no acupuncture, etc.

```
data219 = read.table("http://users.stat.ufl.edu/~rrandles/sta4210/Rclassnotes/data/
    textdatasets/
KutnerData/Chapter%2021%20Data%20Sets/CH21PR09.txt")
data219$scores=data219$V1
data219$block=as.factor(data219$V2)
data219$codeine=as.factor(data219$V3)
data219$acupuncture=as.factor(data219$V4)
data219
#A1=no codeine
#A2=codeine
#B1=no acupuncture
#b2=acupuncture
data219=data219%>%
select(scores, block, codeine, acupuncture)
data219

residuals(aov(scores~block+codeine+acupuncture, data=data219))
```

	Block1	Block2	Block3	Block4	Block5	Block6	Block7	Block8
A_1B_1	0.01875	-0.03125	-0.01875	0.03125	0.16875	-0.08125	-0.06875	-0.01875
A_1B_2	0.09375	-0.15625	-0.04375	0.10625	0.11875	-0.03125	-0.11875	0.03125
A_2B_1	-0.05625	0.19375	-0.19375	0.05625	-0.05625	-0.00625	-0.09375	0.15625
A_2B_2	-0.05625	-0.00625	0.20625	-0.14375	0.06875	-0.18125	0.03125	0.08125

The normal probability plot and residuals against fitted values: The normality plot tells us the normality of errors appear to be satisfied. The residuals vs fitted also appears to vary over fitted value, so the errors are likely not constant.

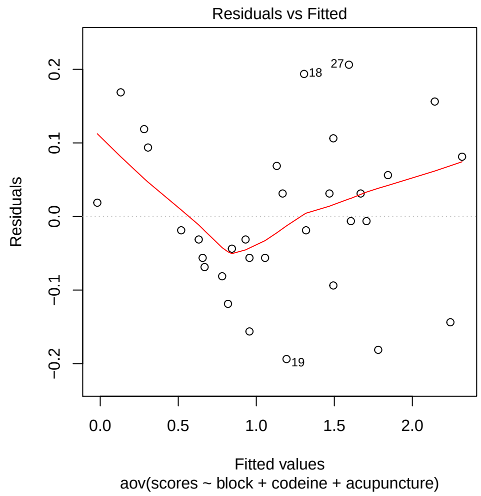

Figure 3: Residuals against fitted values.

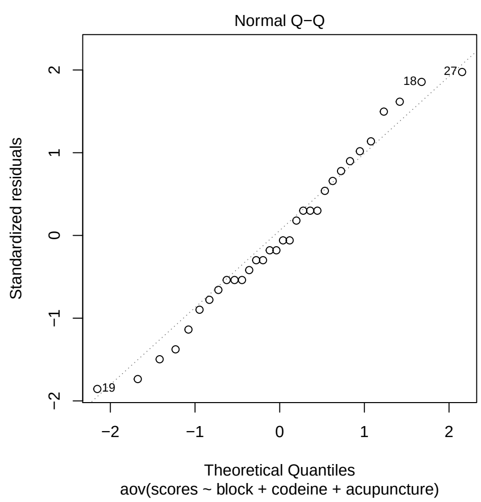

Figure 4: Normal probability plot.

(d) Plot the responses Y_{ijk} by blocks in the format of figure 21.2, ignoring the factorial structure of the treatments. What does this plot suggest about the appropriateness of the no-interaction assumption here?

Answer: There seem to be some interaction effects by block. This is seen because the blocks “intersect”. There are certainly treatment main effects and block main effects.

```
data219fix=data219%>%
mutate(ignorefactorial=ifelse(codeine==1&acupuncture==1,1,
ifelse(codeine==1&acupuncture==2,2,
ifelse(codeine==2&acupuncture==1,3,4))))
interaction.plot( data219fix$ignorefactorial , data219fix$block ,response=data219fix$scores , fun
    =mean , xlab='',ylab='score', legend=T)
```

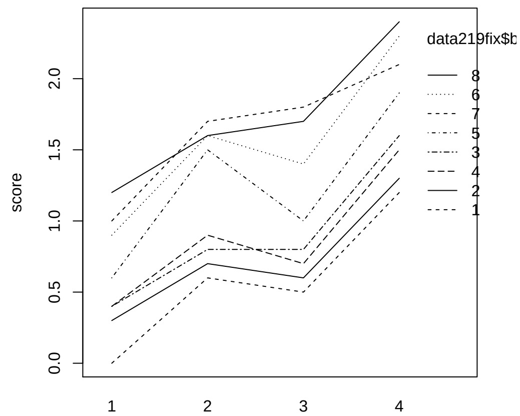

Figure 5: Interaction plot ignoring the factorial structure, i.e. merging the 4 treatments onto the x-axis, with 1 being placebo, 2 being only codeine, 3 only accupuncture, and 4 being both treatments.

(e) Conduct the Tukey test for additivity of block and treatment effects, ignoring the factorial structure of the treatments, use $\alpha = 0.01$. State the alternatives, decision rule, and conclusion. What is the P-value of the test?

Answer:

$$H_0 : D = 0 \quad \text{no interations present}$$
$$H_a : D \neq 0 \quad \text{interactions } D\alpha_i\beta_j \text{ present}$$

where we control type I error at α with the following decision rule, if $F^* \le F(1-\alpha;1,ab-a-b)$, we conclude H_0, otherwise conclude H_a. The critical value is 8.0959, calculated using $a = 8$, the number of blocks, and $b = 4$, the number of treatments ignoring factorial level. We find $F^* = 0.334$ with p-value of 0.5682, so we conclude the null hypothesis.

The R-code:

```
tukeys.add.test(data219fix$scores,data219fix$block,data219fix$ignorefactorial)
a219=length(unique(data219fix$block)) #really n_b
b219=length(unique(data219fix$ignorefactorial))  #really a*b
critF219=qf(1-.01,1,a219*b219-a219-b219)
```

21.10 Refer to dental pain problem 21.9. Assume that randomized block model (21.11) is appropriate.

(a) Obtain the analysis of variance table.

Answer:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
block	7	5.60	0.80	55.30	0.0000
codeine	1	2.31	2.31	159.79	0.0000
acupuncture	1	3.38	3.38	233.68	0.0000
codeine:acupuncture	1	0.05	0.05	3.11	0.0923
Residuals	21	0.30	0.01		

(b) Test whether or not the two factors interact; use $\alpha = 0.01$. State the alternatives, decision rule, and conclusion. What is the P-value of the test?

Answer: The test here is

$$H_0 : \text{all } (\alpha\beta)_{ij} = 0$$
$$H_a : \text{not all } (\alpha\beta)_{ij} \text{ equal zero}$$

The test statistic is

$$F^* = \frac{\text{MSAB}}{\text{MSBLTR}}$$

(MSBLTR is essentially MSE for this type of ANOVA) The critical value is (from ANOVA table 21.5 in textbook)

$$F[1-\alpha;(a-1)(b-1),(n_B-1)(ab-1)]$$

where in this a=b=2, and $n_b = 8$, the number of blocking. This equals 8.01. From our ANOVA table in part (a), the F-value is 3.11 with p-value of 0.0923. Therefore, we conclude the null, that there are no interaction effects.

(c) Prepare separate bar-interval graphs for each set of estimated factor level means using 95% confidence intervals. Does that substantial main effects are present here?

Answer: The 95% confidence interval comes from multiplying by $t[1-\alpha/2;(n_b-1)(r-1)]\cdot \text{MSBLTR}\frac{1}{2n_b}$. We multiply by 2 in the denominator since we have 8 measurements for the two factors, acupuncture and codeine (each with two levels). $r = ab = 4$ because we have 4 levels. So, our 95% condfidence widths are ± 0.120. The means are the 4 columns from the table in the book, i.e. A_1, A_2, B_2, and B_1.

$$\text{mean}(A_1) = 0.83125$$
$$\text{mean}(A_2) = 1.48125$$
$$\text{mean}(B_1) = 0.8875$$
$$\text{mean}(B_2) = 1.425$$

```
anova(aov(scores~block+codeine+acupuncture+codeine*acupuncture, data=data219))*1
mse219=0.01446

sd=(mse219/2*a219)
sd
tval=qt(1-.05/2,(a219-1)*(4-1))
rang=tval*sd
means=data219%>%
summarize(a1=mean(scores[acupuncture==1]),
a2=mean(scores[acupuncture==2]),
b1=mean(scores[codeine==1]),
b2=mean(scores[codeine==2]))
```

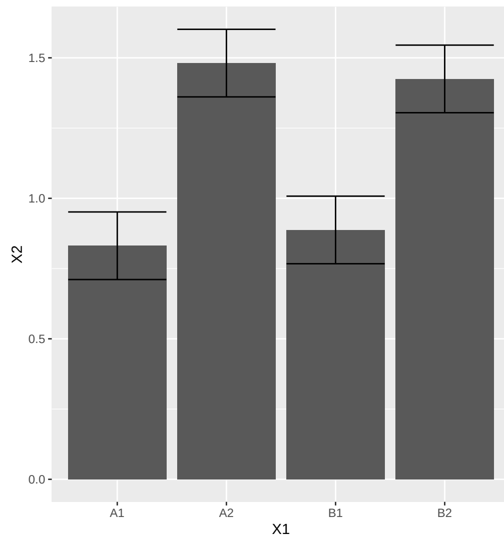

Figure 6: There are appears to be main effects because the means even with the confidence intervals there is no overlap.

```
means2=data.frame(c('A1','A2', 'B1','B2'),c(as.numeric(means$a1),
as.numeric(means$a2),as.numeric(means$b1), as.numeric(means$b2)))
colnames(means2)=c('X1','X2')
means2
means2%>%
ggplot(aes(x=X1, y=X2),stat='identity')+geom_col(position = "dodge")
+geom_errorbar(aes(x=X1,ymin=X2-rang, ymax=X2+rang))
```

(d) Test separately whether main effects are present for each of the factors; use $\alpha = 0.01$ for each test. State the alternatives, decision rule, and conclusion for each test. What is the P-value of each test?

Answer: We have two hypotheseses, whether the codeine means are the same and the acupuncture means are the same. i.e. whether or $A_1 = A_2$ with alternative $A_1 \neq A_2$ and for the codeine case $B_1 = B_2$ vs alternative $B_1 \neq B_2$. The tests are (swapping B for A otherwise)

$$F^* = \frac{\text{MSA}}{\text{MSBLTR}} \leq F(1-\alpha, (a-1)\cdot(b-1), (n_b-1)\cdot(ab-1)$$

The F-value here is 8.02 as we saw in part (b).

From our ANOVA table in part (a), the F-value's are 159.79 and 233.68, which p-values essentially at zero. We conclude the alternative for both cases.

(e) Estimate

$$L_1 = \mu_{\cdot 1 \cdot} - \mu_{\cdot 2 \cdot} = \alpha_1 - \alpha_2$$
$$L_2 = \mu_{\cdot\cdot 1} - \mu_{\cdot\cdot 2} = \beta_1 - \beta_2$$

Use the Bonferroni procedure with a 95 percent family confidence coefficient. State your findings.

Answer: We use our answers from (c) as an estimate (there we called α A and β B). We find:

$$\hat{L}_1 = .83125 - 1.48125 = -0.65$$
$$\hat{L}_2 = 0.8875 - 1.425 = -0.5375$$

Using the usual method of summing over the square of constants and dividing by sample for each factor, we find $s^2(\hat{L}) = 2\text{MSE}/2n_b$, which is $\text{MSE}/n_b = 0.0018$, which means $s(\hat{L}) = 0.04254$. . This works out nicely . The only thing that changes for the Bonferroni multiple is we have to account for 2 linear combination, so we divide α by 2g, so our multiple becomes $\pm t(1-.05/4, (n_b-1)\cdot(r-1)$. Therefore, our intervals are

$$\hat{L}_1 = -0.65 \pm 0.0725 \longrightarrow -0.722 \le L_1 \le -0.578$$
$$\hat{L}_2 = -0.5375 \pm 0.0725 \longrightarrow -0.610 \le L_2 \le -0.465$$

(f) Test whether or not blocking effects are present. Use $\alpha = 0.01$. State the alternatives, decision rule, and conclusion. What is the P-value of the test?

Answer:

The test is H_0 : all $\rho_i = 0$ vs the alternative that there are not all equal to 0. From the ANOVA table in part (a), $F^* = 55.296$, with p-value nearly 0. We test MSBL/MSBLTR, where

$$\text{MSBL} = r\sum_i (\overline{Y}_{i\cdot} - \overline{Y}_{\cdot\cdot})^2/(n_b - 1)$$

$$\text{MSBLTR} = \sum_i \sum_j (Y_i j - \overline{Y}_{i\cdot} - \overline{Y}_{\cdot j} + \overline{Y}_{\cdot\cdot})^2/(n_b r - 1) = \sum_i \sum_j e_{ij}^2/(n_b r - 1)$$

We compare this to $F(1-\alpha, n_b - 1, (n_b-1)(r-1)) = F(.99, 7, 21) = 4.87$. (perhaps easier to just find the df from the ANOVA table). Therefore, we easily conclude H_a, that

21.20 Refer to dental pain problems 21.9 and 21.10. According to the estimated efficiency measure (21.13). How effective was the use of a the blocking variable as compared to a completely randomized design?

Answer: We estimate E as follows: (using our ANOVA table)

$$\hat{E} = \frac{s_r^2}{\text{MSBLTR}} = \frac{(n_b-1)\cdot\text{MSBL} + n_b(r-1)\text{MSBLTR}}{(n_b r - 1)\cdot\text{MSBLTR}} = \frac{7\cdot 0.8 + 24\cdot 0.0145}{(32-1)\cdot 0.0145} = 13.27$$

We would require over thirteen times as many replications per treatment with a completely randomized block design to achieve the same variance of any estimated contrast as is obtained by blocking by pain score.