Ejercicio 10

1- $T(n) \begin{cases} 2 & , \text{si } n = 1 \\ T(n-1) + n & , \text{si } n \geq 2 \end{cases}$

- paso 1: $T(n) = T(n-1) + n \quad , \text{si } n \geq 2$
- paso 2: $T(n) = ((T(n-1)+n)-1)+n$
  $= T(n-2) + (n-1) + n$
- paso 3: $T(n) = ((T(n-1)+n)-2) + (n-1) + n$
  $= (T(n-3) + (n-2) + (n-1)) + n$
- paso 4: $T(n) = ((T(n-1)+n)-3 + (n-2) + (n-1)) + n$
  $= (T(n-4) + (n-3) + (n-2) + (n-1)) + n$
- paso i: $T(n) = (T(n-i) + (n-i+1) + (n-i+2) + \dots + (n-i+i))$
  $T(n) = T(i) + \sum_{K=i+1}^{n} K$

- encontre la formula general, ahora busco para el valor donde termina la recursion, osea (1)

$$T(n) = T(1) + \sum_{K=1+1}^{n} K = 2 + \sum_{K=1}^{n} K-1 = 2 + \left(\sum_{K=1}^{n} K\right) - 1$$

$$T(n) = 2 + \left(\frac{n(n+1)}{2}\right) - 1 = 1 + \frac{n^2+n}{2} \quad \therefore O(n^2)$$

- Justificacion con Big-Oh

1er termino

$1 \leq c_1 . n^2$

$\frac{1}{c_1} \leq n^2$

$n_1 = 1 \quad c_1 = 1$

2do termino

$n^2 + n/2 \leq c_2 . n^2$

$\frac{n^2+n}{2 . c_2} \leq n^2$

$n_1 = 1 \quad c_1 = 1$

- $1 + \frac{n^2+n}{2} \leq (c_1 + c_2) . n^2$
  $T(n) \leq (1+1) n^2$
  $T(n) \leq c\, n^2$

$\boxed{\star\ T(n) \leq O(n^2), \text{ con } c = 2 \text{ para todo } n \geq n_0 \text{ con } n_0 = 1}$

2-

$$t(n)\begin{cases} 2 & , si\ n=1 \\ t(n-1)+\frac{n}{2} & , si\ n \geq 2 \end{cases}$$

- paso 1: $t(n) = t(n-1) + \frac{n}{2}$
- paso 2: $t(n) = \left(\left(t(n-1) + \frac{n}{2}\right) - 1\right) + \frac{n}{2}$
  $= t(n-2) + (n/2 - 1)) + n/2$
- paso 3: $t(n) = \left(t(n-1) + \frac{n}{2}\right) - 2 + (n/2 - 1) + n/2$
  $= t(n-3) + (n/2 - 2) + (n/2 - 1) + n/2$
- paso i: $t(n) = t(n-i) + (n/2 - i + 1) + (n/2 - i + 2) + \dots + (n/2 - i + i)$

- encontré la formula general, ahora busco para el valor donde termina la recursion, osea (1)
  $\Rightarrow T(n) = 1 \rightarrow$ cuando $n - i = 1$
  $i = n-1$

reemplazo i:

$$T(n) = T(1) + (n/2 - (n-1) + 1) + (n/2 - (n-1) + 2) + \dots + (n/2 - (n-1) + i)$$

$$T(n) = 2 + \left(\frac{n-(n-1)+1}{2}\right) + \left(\frac{n-(n-1)+2}{2}\right) + \dots + \left(\frac{n-(n-1)+i}{2}\right)$$

$$t(n) = 2 + \left(\frac{n-n+1+1}{2}\right) + \left(\frac{n-n+1+2}{2}\right) + \dots + \left(\frac{n-n+1+i}{2}\right)$$

$$t(n) = 2 + \frac{2}{2} + \frac{3}{2} + \dots + \frac{i}{2}$$

$$t(n) = 2 + \sum_{i=2}^{n} \frac{i}{2} = 2 + \sum_{i=2}^{n} i \cdot \frac{1}{2} = 2 + \frac{1}{2}\sum_{i=2}^{n} i$$

$$T(n) = 2 + \frac{1}{2}\left(\sum_{i=1}^{n} i - 1\right) = 2 + \frac{1}{2}\left(\frac{n(n+1)}{2} - 1\right) = 2 + \frac{1}{2}\left(\frac{n^2+n}{2} - 1\right)$$

$$T(n) = 2 + \frac{n^2}{4} + \frac{n}{4} - \frac{1}{2} = \frac{n^2}{4} + \frac{n}{4} + \frac{3}{2} \therefore O(n^2)$$

- Justificación con Big-oh

1er termino	2do termino	3er termino
$\frac{n^2}{4} \leq c_1 n^2$	$n/4 \leq c_2 n^2$	$3/2 \leq c_3 n^2$
$c_1 = 1$, $n_1 = 1$	$c_2 = 1$, $n_2 = 1$	$c_3 = 1$, $n_3 = 2$

- $n^2/4 + n/4 + 3/2 \leq (c_1 + c_2 + c_3) n^2$
  $t(n) \leq (1+1+1) n^2$
  $t(n) \leq c n^2$

$\boxed{\star\ T(n) \in O(n^2) \text{ con } c = 3 \text{ para todo } n \geq n_0, \text{ con } n_0 = 2}$

demian Banzi

3- $T(n) = \begin{cases} 1 & , si \ n = 1 \\ 2T\left(\frac{n}{4}\right) + \sqrt{n}, & si \ n \geq 2 \end{cases}$

- paso 1: $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ si $n \geq 2$
- paso 2: $T(n) = 2\left(\frac{2T\left(\frac{n}{4}\right) + \sqrt{n}}{4}\right) + \sqrt{n}$

$= 2\left(\frac{2T(n/4)}{4} + \frac{\sqrt{n}}{4}\right) + \sqrt{n} = 4T(n/16) + \cancel{2}\frac{\sqrt{n}}{\cancel{2}} + \sqrt{n}$

$= 4T(n/16) + \sqrt{n} + \sqrt{n} = 4T(n/16) + 2\sqrt{n}$

- paso 3: $T(n) = 4\left[\frac{2T(n/4) + \sqrt{n}}{16}\right] + 2\sqrt{n}$

$= 8T\left(2T(n/48) + \sqrt{(n/16)}\right) + 2\sqrt{n}$

$= 8T(n/48) + 4\sqrt{n/16} + 2\sqrt{n}$

$= 8T(n/48) + 4\frac{\sqrt{16}}{4} + 2\sqrt{n} = 8T(n/48) + 3\sqrt{n}$

- paso i: $T(n) = 2^i T(n/4^i) + i\sqrt{n}$
- encontre la formula general ahora busco para el valor donde termina la recursion, osea $T(n) = 1 \longrightarrow$ cuando $n/4^i = 1$

$n = 4^i \rightarrow \log_b(b^x) = x$

$\log_4(n) = \log_4(4^i)$

$\log_4(n) = i$

- reemplazo i:

$T(n) = \underbrace{2^{\log_4(n)}} + T(n/\underbrace{4^{\log_4(n)}}_{n}) + \log_4(n).\sqrt{n}$

$2^{\log_4(n)} = 2^{\frac{\log_2(n)}{2}} = n^{1/2} = \sqrt{n}$

$T(n) = \sqrt{n} \underbrace{T(1)}_{=1} + \log_4(n)\sqrt{n} = \sqrt{n} + \log_4(n).\sqrt{n}$ $\therefore O(\log_4(n).\sqrt{n})$

justificacion con Big-oh

1er termino

$\sqrt{n} \leq \log_4(n)\sqrt{n}\,c_1$

$\sqrt{n}/\sqrt{n} \leq \log_4(n).c_1$

$1 \leq \log_4(n)\,c_1$

$n_1 = 4 \quad c_1 = 1$

2do termino

$\log_4(n)\sqrt{n} \leq \log_4(n)\sqrt{n}\,c_2$

$\log_4(n)\sqrt{n}/\log_4(n)\sqrt{n} \leq c_2$

$1 \leq c_2$

$n_2 = 1, \ c_2 = 1$

- $\sqrt{n} + \log_4(n)\sqrt{n} \leq (c_1 + c_2)\log_4(n)\sqrt{n}$

$T(n) \leq (1+1)\log_4(n)\sqrt{n}$

$T(n) \leq c\log_4(n)\sqrt{n}$

$\boxed{* \ T(n) \leq O(\log_4(n)\sqrt{n}) \text{ para todo } n \geq n_0,\ n_0 = 4 \text{ y } c = 2}$

4.

Damian Bongi

$$T(n) = \begin{cases} 1 & , \text{si } n = 1 \\ 4T\left(\frac{n}{2}\right) + n^2 & , \text{si } n \geq 2 \end{cases}$$

- paso 1: $T(n) = 4T\left(\frac{n}{2}\right) + n^2$ si $n \geq 2$
- paso 2: $T(n) = 4\left[4T\left(\frac{n/2}{2}\right) + n^2\right] + n^2$

$$= 4\left[4T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2\right] + n^2 = 4^2T\left(\frac{n}{4}\right) + 4\left(\frac{n^2}{4}\right) + n^2$$

$$= 4^2T\left(\frac{n}{4}\right) + 2n^2$$

- paso 3: $T(n) = 4^2T\left(\frac{4T(n/2) + n^2}{4}\right) + 2n^2$

$$= 4^3T\left(\frac{n}{8}\right) + 3n^2$$

- paso i: $T(n) = 4^iT\left(\frac{n}{2^i}\right) + i n^2$
  - encontre la formula general ahora busco para el valor donde termina la recursion, osea (1)

$$T(n) = 1 \to \text{cuando} \to \frac{n}{2^i} = 1 \to n = 2^i = \log_2(n) = i$$

reemplazo i:

$$T(n) = 4^{\log_2(n)}T\left(\frac{n}{2^{\log_2(n)}}\right) + \log_2(n)\, n^2$$

$$T(n) = 4^{\log_2(n)}T\left(\frac{n}{n}\right) + \log_2(n)\, n^2 = 4^{\log_2(n)}T(1) + \log_2(n)\, n^2$$

$$(2^2)^{\log_2(n)} = 2^{2\log_2(n)} = n^2$$

$$T(n) = n^2 + \log_2(n) \cdot n^2 \quad \therefore \quad O(\log_2(n) \cdot n^2)$$

1er termino

$$n^2 \leq c_1 \log_2(n)\, n^2$$
$$n^2/n^2 \leq c_1 \log_2(n)$$
$$1 \leq c_1 \log_2(n)$$
$$c_1 = 1, \; n_1 = 2$$

2do termino

$$\log_2(n)\, n^2 \leq c_2 \log_2(n)\, n^2$$
$$\frac{\log_2(n)\, n^2}{\log_2(n) \cdot n^2} \leq c_2$$
$$1 \leq c_2$$
$$c_2 = 1, \; n_1 = 2$$

- $\log_2(n) \cdot n^2 \leq (\overset{1}{c_1} + \overset{1}{c_2}) \log_2(n) \cdot n^2$

$$T(n) \leq c \log_2(n)\, n^2$$

$\boxed{* \; T(n) \leq O(\log_2(n) \cdot n^2), \text{ con } c = 2, \text{ para todo } n \geq n_0 \text{ con } n_0 = 2}$