COMP3121 Assignment 4 - Q3

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Answer

We model this as a max flow problem, with vertex capacities. We first construct a flow network as a directed graph where square 1 is the source, square n is the sink. Each rest square i will be split into two vertices as v_{iin} and v_{iout}, each v_{iin} has only one outgoing edge which towards to v_{iout}, the capacity of the edge between v_{iin} and v_{iout} equals to $A[i]$. Each vertex v_{iout} and source has directed edges towards to $v_{(i+1)in}$, $v_{(i+2)in}$ ... $v_{(i+k)in}$, with capacity equal to infinity.
To find the largest number of children who can successfully complete the game is to find the max flow in our constructed graph. We now run Edmonds-Karp algorithm on our graph, in the final residual graph, the sum of the weight of the incoming edges towards source is the largest number we are looking for.

Time complexity: since the time complexity of Edmonds-Karp algorithm is $O(|V||E|^2)$, where $|E|$ is the number of edges, $|V|$ is the number of vertices, we have $kn + n$ edges, $2n + 2$ vertices in the graph, hence the time complexity is $O((2n + 2) \cdot (kn + n)^2) = O(k^2n^3)$.