COMP3121 Assignment 4 - Q4

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Answer

We initialise a 2D array board with size $n \times n$, then for each white shop s_i, based on its position (a_i, b_i), we mark board$[a_i][b_i]$ and all entries on its diagonal directions as "attacked".
Model this as a max flow problem. We construct a bipartite graph with vertices on the left side representing n rows in the board, denoted as r_i, and vertices on the left side representing n columns of the board, denoted as l_i. Introduce a super source and a super sink. Connect each r_i with the super source by a directed edge of capacity equal to 1. Connect each l_i with the super sink with a directed edge of capacity equal to 1. For constructing the edges between r_i and l_i, we check the array we made, if board$[r_i][l_i]$ is not marked as "attacked" (means the position is not under attacked by white bishop), we construct a edge from r_i to l_i, with capacity equal to 1.
Now, the largest number of black rooks we can place is the max flow of our constructed graph. We run Edmonds-Karp algorithm on our graph to get the residual graph. In the final residual graph, the sum of the weight of the incoming edges towards source equals to the max flow, which is the largest number we are looking for.

Time complexity: constructing the 2D array board takes $O(kn)$, $k \leq n^2$. Since the time complexity of Edmonds-Karp algorithm is $O(|V||E|^2)$, where $|E|$ is the number of edges, $|V|$ is the number of vertices, we have maximum $2n + n^2$ edges, and $2n + 2$ vertices in the constructed graph, hence the time complexity to get the max flow is $O((2n + 2) \cdot (2n + n^2)^2) = O(n^5)$. Therefore the overall time complexity of our algorithm is $O(kn) + O(n^5) = O(n^5)$.