COMP3121 Assignment 4 - Q1

Demiao Chen z5289988

August 5, 2021

Answer

We model this as a Max Flow - Min Cut problem. We first construct a a flow network as a directed graph where computer 1 is the source, computer N is the sink, and the rest computers are vertices between source and sink. The capacity of each edge between two vertices will be equal the cost of removing its corresponding link between two computers. We now need to find a min cut in the graph, which produces two subsets of the graph S and T, where $S \cup T = V$, $S \cap T = \varnothing$ and $s \in S$, $t \in T$. Removing the edges in min cut will make computer 1 and computer N not connected, and the cost is minimum. So our goal is to find the min cut in the graph.
To find the minimum cut, we run Edmonds-Karp algorithm to our flow network graph to get the residual graph. Then we take source as start vertex, run breadth first search in the residual graph. Hence we get a set of vertices that is reachable from the source, those are the vertices belong to S, and the rest vertices will belong to T. Then we find the minimum cut, that is the edges with forward direction in the original flow network graph between set S and set T. Therefore, the links we need to remove are the links between S and T with forward direction.

Time complexity: since the time complexity of Edmonds-Karp is $O(|V||E|^2)$, where $|V|$ is number of vertices, $|E|$ is the number of edges, and time complexity of breadth first search algorithm is $O(|V| + |E|)$. It takes $O(|E|)$ to find all edges between two sets. We have M edges (links) and N vertices (computers). Therefore, the time complexity of our algorithm to find the links to remove is $O(NM^2 + N + 2M) = O(NM^2)$