

COMP3121/9101: Assignment 1  
Due date: Tuesday 15 of June at Noon

In this assignment we review basic algorithms and data structures. You have **five problems**, marked out of a total of 100 marks.

**NOTE:** Your solutions must be typed, machine readable .pdf files. **All submissions will be checked for plagiarism!**

1. You are given an array  $A$  of  $n$  distinct positive integers.
  - (a) Design an algorithm which decides in time  $O(n^2 \log n)$  (in the worst case) if there exist four **distinct** integers  $m, s, k, p$  in  $A$  such that  $m^2 + s = k + p^2$  (10 points)
  - (b) Solve the same problem but with an algorithm which runs in the **expected time** of  $O(n^2)$ . (10 points)
2. You are given a set of  $n$  fractions of the form  $x_i/y_i$  ( $1 \leq i \leq n$ ), where  $x_i$  and  $y_i$  are positive integers. Unfortunately, all values  $y_i$  are incorrect; they are all of the form  $y_i = c_i + E$  where numbers  $c_i \geq 1$  are the correct values and  $E$  is a positive integer (equal for all  $y_i$ ). Fortunately, you are also given a number  $S$  which is equal to the correct sum  $S = \sum_{i=1}^n x_i/c_i$ . Design an algorithm which finds all the correct values of fractions  $x_i/c_i$  and which runs in time  $O(n \log \min\{y_i : 1 \leq i \leq n\})$ . (20 points)
3. You are given an array  $A$  consisting of  $n$  positive integers, **not** necessarily all distinct. You are also given  $n$  pairs of integers  $(L_i, U_i)$  and have to determine for all  $1 \leq i \leq n$  the number of elements of  $A$  which satisfy  $L_i \leq A[m] \leq U_i$  by an algorithm which runs in time  $O(n \log n)$ . (20 points)
4. You are given an array containing a sequence of  $2^n - 1$  consecutive positive integers starting with 1 except that one number was skipped; thus the sequence is of the form  $1, 2, 3, \dots, k-1, k+1, \dots, 2^n$ . You have to determine the missing term accessing at most  $O(n)$  many elements of  $A$ . (20 points)
5. Read about the asymptotic notation in the review material and determine if  $f(n) = O(g(n))$  or  $g(n) = O(f(n))$  or both (i.e.,  $f(n) = \Theta(g(n))$ ) or neither of the two, for the following pairs of functions
  - (a)  $f(n) = \log_2(n)$ ;  $g(n) = \sqrt[n]{n}$ ; (6 points)

(b)  $f(n) = n^n$ ;  $g(n) = 2^{n \log(n^2)}$ ; (6 points)

(c)  $f(n) = n^{1+\sin(\pi n)}$ ;  $g(n) = n$ . (8 points)

You might find useful L'Hôpital's rule: if  $f(x), g(x) \rightarrow \infty$  and they are differentiable, then  $\lim_{x \rightarrow \infty} f(x)/g(x) = \lim_{x \rightarrow \infty} f'(x)/g'(x)$