COMP3121 Assignment1 - Q2

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Answer

First loop the set to find the minimum y_{min} in O(n) as there are n elements in the set. Since y_i, c_i and E are positive integers, and $y_i = c_i + E$, the maximum value for E is $y_{min} - 1$. We know the value of y_i and x_i, and $S = \sum_{n=1}^{\infty} \frac{x_i}{y_i - E}$. Hence, we try $E = y_{min} - 1$, put it in $\sum_{n=1}^{\infty} \frac{x_i}{y_i - E}$ and calculate the sum, which takes $O(n)$, then do comparison with S. If it equals to S, we get the correct E and then use E to get all the correct values of fractions x_i/c_i in O(n). If it does not equal to S, we try another value of E in a binary search way. We first try half of it, i.e. $\frac{y_{min}-1}{2}$ (round it to an integer if it is not an integer), then compare it with S. If it equals to S, we get the correct E; if it is smaller than S, it means the value of E we try is smaller than the correct one ($\sum_{n=1}^{\infty} \frac{x_i}{y_i - E}$ and E are in positive correlation), so choose a higher value of E in binary search way; the same approach also applies when the sum is bigger than E, in that case we try a smaller E (binary search in lower value direction). We keep doing the process until we find the correct E, then easily get all x_i/c_i in time $O(n)$. As each search query we calculate a sum in $O(n)$, and the maximum times of query is $\log y_{min}$, the whole binary search process takes $O(n \log y_{min})$. So it can find all the correct values of fractions x_i/c_i by this method in time $O(n \log y_{min})$.