COMP3121 Assignment1 - Q5

Demiao Chen z5289988

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Answer

(a) $f(n) = O(g(n))$.
We show that eventually $\log_2 n < \sqrt[10]{n}$. It is clear that both $\log_2 n$ and $\sqrt[10]{n}$ are monotonically increasing function, and $\sqrt[10]{n}$ eventually dominates $\log_2 n$. By applying L'Hôpital rule we get:

$$\lim_{n\to\infty} \frac{\sqrt[10]{n}}{\log n} = \lim_{n\to\infty} \frac{(\sqrt[10]{n})'}{(\log n)'} = \lim_{n\to\infty} \frac{\frac{1}{10} n^{-\frac{9}{10}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^{1/10}}{10} = \infty$$

This proves that eventually $\log_2 n < \sqrt[10]{n}$, hence $\log_2 n = O(\sqrt[10]{n})$.

(b) $f(n) = O(g(n))$.
By manipulating $g(n)$, we get

$$2^{n \log_2 n^2} = 2^{\log_2 n^{2n}} = n^{2n}.$$

Then we get

$$\frac{g(n)}{f(n)} = \frac{n^{2n}}{n^n} = n^n.$$

So it is clear that $f(n) < g(n)$ for $n > 0$, $g(n)$ dominates $f(n)$. Hence $f(n) = O(g(n))$.

(c) $f(n) = \Theta(g(n))$.
As we assume n is a positive integer, for any given n, $\sin \pi n = 0$.
Hence,

$$f(n) = n^{1+\sin \pi n} = n^{1+0} = n = g(n).$$

Therefore, $f(n)$ and $g(n)$ have the same growth rate, so $f(n) = O(g(n))$ and $g(n) = O(f(n))$ both hold, i.e. $f(n) = \Theta(g(n))$.