COMP3121 Assignment2 - Q1

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Answer

Always place on one CD until its remaining space is not enough to place the next song, in this case, place the song on next CD and start placing on the new CD.

E.g. At start, we place song 1 on CD_1, then place song 2 on CD_2 if and only if there is enough space remaining to place it (i.e. $m - l_1 \geqslant l_2$), else place it on the next CD, which is CD 2.
If we are going to place song k and the last song placed is on CD_j, with the first song placed on CD_j is song h ($h < k$), we first decide if

$$m - \sum_{i=h}^{k} l_i \geqslant 0 \tag{1}$$

If the condition hold, we place song k on CD_j, otherwise on CD_{j+1}

We now need to prove that this method is optimal. In the above condition to place song k, if (1) does not hold but place song k on CD_j, this song cannot be placed completely on CD_j since the space is not enough, and we cannot spill song across CDs. We cannot place song k on previous CDs either as songs must be recorded in order. So we must place the song on the next CD.
If (1) holds but place song k on CD_{j+1}, this will not achieve the optimal solution since if song k is the last song needs to be recorded, there will be more than one CD needed compare to our method. Place on previous CDs will disorder.
Any possible violation of our algorithm will result in not satisfying the question or more CDs needed. Therefore, our method is optimal.

Time complexity: there are total of n songs to be placed, each time when placing a song we check if the reaming space on current CD is enough by using formula (1), which takes $O(1)$ as the time complexity of formula (1) does not grow proportional to n, i.e. irrelative to n. So the total time complexity is $O(n * 1) = O(n)$