COMP3121 Assignment 4 - Q2

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Answer

We model this as a max flow problem. Make a bipartite graph with vertices on the left side representing warehouse and vertices on the right side representing shops. Introduce a super source and a super sink. Connect each warehouse with the super source by a directed edge of capacity equal to 1 to represent if a truck has been sent in the shop. Connect each shop with the super sink with a directed edge of capacity equal to 1 to represent if a shop has been supplied.
We now sort all roads r_i in terms of its time taken to drive d_i to form an increasing sequence by merge sort. Then we take an i between 1 and the number of roads m in a binary search way, that is we first take $i = \frac{m}{2}$, then we construct edges in our graph by adding directed edges point to shop vertices to represent roads from r_1 to r_i in the graph to connect corresponding warehouse and shop vertices, with capacity equal to 1. Then we run Edmonds-Karp algorithm on our graph to get the residual graph, and get the max flow from the final residual graph. The max flow in our graph represent the maximum shops we can supply in the current constructed graph. If max flow equals number of shops n, means all shops can be supplied, we take a smaller i in binary search way and remove all edges to repeat above process, else we take a bigger i in binary search way and remove all edges to repeat above process, until max flow of the graph equals n and d_i is minimised.
After finishing the above process, we have get the smallest d_i and its corresponding flow network graph, hence we run Edmonds-Karp algorithm in the final constructed graph to get the residual graph. In the final residual graph, we output each warehouse vertex and its connected edge (road) with direction towards the warehouse and its connected shop vertex, which means this arrangement: send truck from the vertex represented warehouse to the vertex represented shop through the edge represented road.

Time complexity: We used binary search to get i from $i = 1$ to $i = m$, and each constructed graph we run Edmonds-Karp algorithm on it. As we have n ware houses, m roads, there are $2n + 2$ vertices in the graph in total, $m + 2n$ edges (including edges between super source and super sink), and time complexity of Edmonds-Karp algorithm is $O(|V||E|^2)$, hence the time complexity of our algorithm is $O((2n+2) \cdot (m+2n)^2 \cdot \log m) = O((m^2n + n^3 + mn^2) \cdot \log m)$. It is obvious that $m \geq n$, so $O((m^2n + n^3 + mn^2) \cdot \log m) = O(m^2n \cdot \log m)$. The final result output process takes $O(n)$.

Therefore, the overall time complexity of our algorithm is $O(m^2 n \cdot \log m) + O(n) = O(m^2 n \cdot \log m)$.