

COMP3121 Assignment1 - Q1

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Answer

(a) It is clear that there are ${}^nP_2 = \frac{n!}{(n-2)!} = n(n-1)$ distinct pairs in A, denoting them as $(A^2[k], A[m]), k \neq m$. Compute the sum of $A^2[k] + A[m]$ of pairs for all $1 \leq k \leq n, 1 \leq m \leq n$ (computing sum takes $O(n^2)$) and sort the sum by merge sort, putting the sorted sum with its associated pairs in an array of size $n(n-1)$, which takes $O(n^2 \log n^2) = O(2n^2 \log n) = O(n^2 \log n)$ time. Then go through the sorted array and determine if there are duplicated values (such case would be consecutive repeated numbers in the array). For each duplicated value case, we check if their associated four numbers in the two pairs are distinct, which takes $O(1)$. If they are distinct, we found a case that satisfies the question. This process takes $O(n^2 * 1) = O(n^2)$. In the end, the time taken is $O(2n^2 + n^2 \log n) = O(n^2 \log n)$.

(b) Use a hash table of size $n(n-1)$ (because there are maximum $n(n-1)$ possible values of $A^2[k] + A[m], k \neq m$). Hash all the results of $A^2[k] + A[m]$ for all $1 \leq k \leq n, 1 \leq m \leq n$ with their associated pairs to the hash table, then go through all the slots in the hash table to check if there is a value appears twice; if there is a value appears twice, we check if their associated two pairs are four distinct numbers, which takes time $O(1)$; if they are distinct, it satisfies the question. This whole process takes time $O(n^2)$.