Gravitational N-body Simulations

Abstract

Gravitational N-body simulations, that is numerical solutions of the equations of motions for N particles interacting gravitationally, are widely used tools in astrophysics, with applications from few bodies or solar system like systems all the way up to galactic and cosmological scales. In this article we present a summary review of the field highlighting the main methods for N-body simulations and the astrophysical context in which they are usually applied.

The gravitational force $\vec{F}$i acting on particle i of mass mi is:

$$\vec{F}_i = -\sum_{j \neq i} G \frac{m_i m_j (\vec{r}_i - \vec{r}_j)}{|\vec{r}_i - \vec{r}_j|^3} - \vec{\nabla} \cdot \phi_{ext}(\vec{r}_i),$$

A shared adaptive timestep scheme can correctly follow a close encounter between two charged particles of the same size and speed. Constant timesteps lead to unphysical accelerations during close encounters with each other, as all the other particles evolve on a smaller time scale.

A self-gravitating N-body system made of single particles has a negative specific heat, that is it increases its kinetic energy as a result of energy losses. This is analogous to the acceleration of a satellite in the presence of atmospheric drag. A single force evaluation through a direct method would require about 1 second for a system with N = 104 particles.

The Boltzmann equation for an N-body system is modified by introducing a collision operator C[f] on its left side. This allows us to pass from a 6N+1 to a 6+1 dimension phase space. Monte Carlo methods are available to solve the dynamics of the system.

The velocity moments of the Boltzmann Equation define a set of equations known as the Jeans Equations. These equations are formally identical to the Navier-Stokes equations for self-gravitating gas. Dense stellar systems made of components of roughly equal mass present a rich dynamic, with multiple close encounters of stars.

The Fast Multipole Method (Greengard & Rokhlin 1987) uses a multipole expansion to compute the force from a distant source cell within a sink cell. Particles do not interact directly between each other but only through a mean field. The price to pay is in terms of short-range accuracy as the force is a poor approximation of Newton's law.

Adaptive Mesh Refinement (AMR) is a variant of the Particle Mesh code. Grid elements are concentrated where a higher resolution is needed, for example around the highest density regions. AMR codes are well suited for cosmological simulations (e.g. see ENZO code).