Quantitative Asset Management 431 Final Project Investment Strategy Proposal: Volatility Managed Portfolio Based on GARCH Model

Group 7: Bingqing Wu, Hongyi(Demi) He, Qiyang Li, Mingyue Qiu, Jessica Wu

We motivate our analysis from the vantage point of a mean-variance investor, who prescribes a 60/40 equity/bonds allocation since equity has much higher volatility compared to bonds. Constructively, most of the variation in returns is driven by the minority component. This dilemma introduced us to the risk parity strategy by exploiting the fact that high-volatility assets have tended to underperform low-volatility assets. We further looked into the time series of volatility by factors and found that there exists a common comovement in volatility across factors and the volatility generally increases for all factors in recessions (Figure 1). When reading *Volatility-Managed Portfolios, Moreira and Muir (2017)*, we noticed that this strategy is analogous to risk parity strategy with leverage.

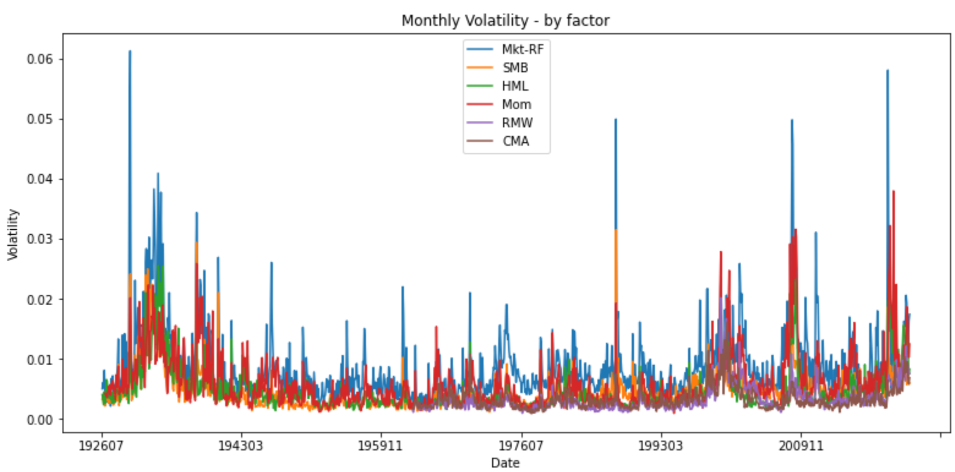

Figure 1: Time series of volatility by factors

The strategy constructs portfolios with scaled returns by the inverse of the conditional variance. According to **Moreira and Muir (2017)**, volatility managed portfolios produce large alphas, increase Sharpe ratios, and produce large utility gains for mean-variance investors. Encouraged by these promising findings, we believe replicating this strategy would not only be innovative but also valuable for the following reasons:

1. It challenges conventional wisdom by reducing risk during recessions and increasing it during more stable periods. This is based on the belief that variance can be reliably

forecasted in the short term, and variance forecasts have limited correlation with future returns within these horizons;

2. It presents a trading methodology that can be synthetically implemented alongside traditional factor investments, offering risk-adjusted returns and the potential to generate alpha;

3. We have the opportunity to incorporate our understanding of volatility and utilize forecasting models (GARCH model) to further adapt this strategy, making it easily implementable in real-time.

4. Many strategies work differently within different volatility environments. For example, momentum tends to "crashes" following market declines, when market volatility is high. Through implementing the VMP strategy, investors will cash out during this period and avoid large momentum crashes.

We will continue the proposal following this thread:

Part I - Economic Intuition: elaborate on and substantiate the strategy intuitions with lemmas;

Part II - Replication of Paper: introduce our replication of **Moreira and Muir (2017)** to show our understanding of strategy implementation;

Part III - GARCH Model VMP: add in GARCH model forecasted volatility into VMP and present in-sample and out-sample results;

Part IV - Strategy Analysis: address and quantify key results to justify our strategy regarding alpha, risk premia, Sharpe ratio, etc. We will also provide further improvement suggestions to the model developed in this proposal;

Part V - Appendix: present the empirical result summary.

Economic Intuition

1. **Investor Mindset:**The strategy stands by the vantage point of a mean-variance investor, who adjusts the allocation according to the attractiveness of the mean-variance

trade-off, $\frac{\mu_t}{\sigma_t^2}$. Here the mean-variance investor doesn't just consider the mean-variance trade-off between assets, they also consider the mean-variance trade-off across time.

The optimization target function for the mean-variance investor is

$$maxU(w_t) = r_f + w_t\mu_t - \frac{A}{2}w_t^2\sigma_t^2$$

Under F.O.C. we get

$$w_t = \frac{\mu_t}{A\sigma_t^2}$$

2. **Empirical Results:** Empirically, there is little relation between lagged volatility and average returns but there is a strong relationship between lagged volatility and current volatility. This means that the mean-variance trade-off weakens in periods of high volatility. From a portfolio choice perspective, we believe that the risk-adjusted return of the portfolio at this time point isn't enough for us to give them a disproportionate part of the risk budget compared with other time points. this pattern implies that a standard mean-variance investor should time volatility, that is, take more risk when the mean-variance trade-off is attractive, and take less risk when the mean-variance trade-off is unattractive.

3. **Testing of the Results:** To test the result, we use the monthly time series of realized volatility to sort the following month's return into five deciles, analogous to the book-to-market cross-sectional sorts but instead done in time series using lagged realized volatility. Then we calculated the average return and mean-variance tradeoff of each decile. From the two tables(figure2), we can see the result line up well with the stratey assumption on the correlation between conditional volatility and return and rationale.

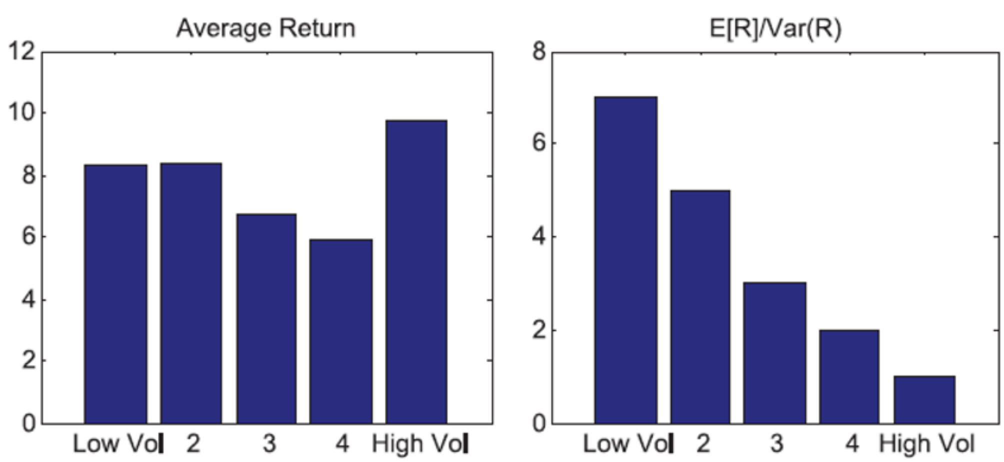

Figure 2: Results of Volatility Sorts

Replication of the paper

We started our strategy development and implementation from the replication of part I of **Moreira and Muir (2017)** which included data cleaning, portfolio formation, and empirical implementation. We will be explaining the process below:

1. **Data Cleaning:**

   (a) **Raw Data:** Since the benchmark portfolio is the efficient portfolio for mean-variance investors. We downloaded daily and monthly data directly from Kenneth French's website on the excess market return (Mkt), size factor (SMB), value factor (HML), momentum factor (Mom), profitability factor (RMW), and investment factor (CMA).

   (b) **Merging and indexing:** Since realized variance is calculated using lagged 22 days of return and monthly return is used as performance measurement data, we merged 5 factor, 3 factor and Mom data accordingly with cleaned and matched data index.

   (c) **Missing values:** We will deal with missing values when subsetting for analysis purposes.

2. **Portfolio Formation:**

   (a) **volatility-managed portfolios(VMP):** We construct VMPs by scaling an excess return by the inverse of its conditional variance. Each month our strategy increases or decreases risk exposure to the portfolio according to variation in our measure of conditional variance. The managed portfolio is then

   $$f^{\sigma}_{t+1} = \frac{c}{\hat{\sigma}^2_t} f_{t+1}$$

   where f_{t+1} is the buy-and-hold portfolio excess return for each factor and $\frac{1}{\hat{\sigma}^2_t}$ is the volatility weight assigned to that factor

   (b) **Volatility weight:** According to **Moreira and Muir (2017)** we used the previous month's realized variance as a proxy for the conditional variance,

   $$\hat{\sigma}^2_t(f) = RV^2_t(f) = \sum_{d=\frac{1}{22}}^{1} \left(f_{t+d} - \frac{1}{22} \sum_{d=\frac{1}{22}}^{1} f_{t+d} \right)^2$$

(c) **Leverage exposure c:** According to **Moreira and Muir (2015)**, we chose c so that the managed portfolio has the same unconditional standard deviation as the buy-and-hold portfolio.

3. **Empirical Implementation:**

(a) **Single-Factor Portfolios Univariate Regressions:** We ran a time-series regression of the volatility-managed portfolio on the original factors:

$$f^{\sigma}_{t+1} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$$

where a positive intercept implies that volatility timing increases Sharpe ratios relative to the original factors and a positive alpha implies that our volatility-managed strategy expands the mean-variance frontier.

The data are monthly and the sample period is 1926 to 2015 for Mkt, SMB, HML, and Mom; 1963 to 2015 for RMW and CMA. All factors are annualized in percent per year by multiplying monthly factors by 12.

For metrics associated with the univariate regression such as alphas, betas, etc., please refer to Appendix Table 2 as results from **Moreira and Muir (2017)** and Table 3 as our replicated results.

(b) **Cumulative returns:** We plotted the cumulative monthly returns to a buy-and-hold strategy versus a volatility-managed strategy for the market portfolio from 1926 to 2015. We can see that VMP performs better during the data period(Figure3).

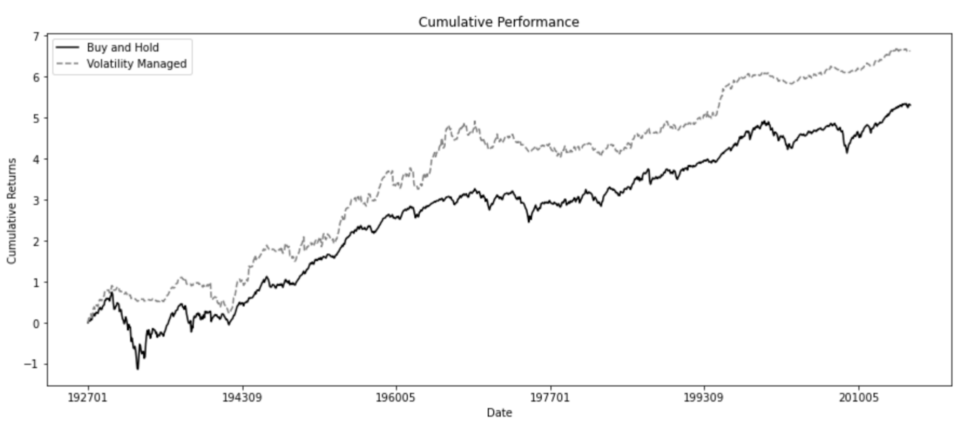

Figure 3: Cumulative returns

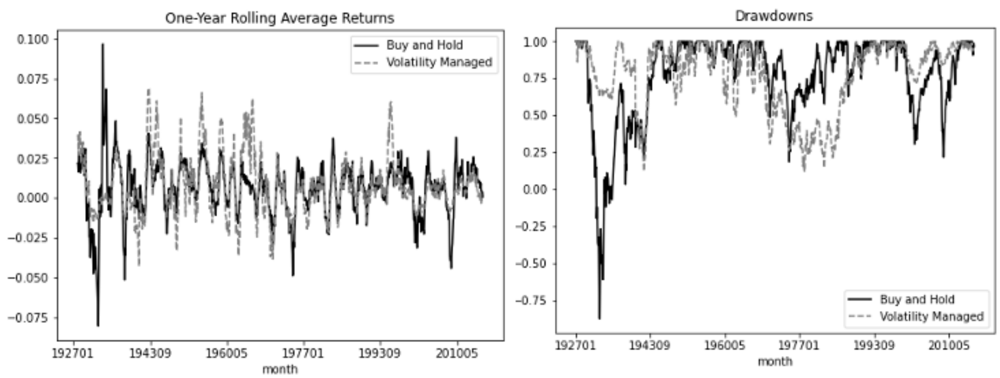

Figure 4: Rolling one-year returns and Drawdowns

(c) **Rolling one-year returns and Drawdowns:** We Also plotted the rolling one-year returns and drawdowns for buy-and-hold strategy and VMP for the market portfolio. (Figure4)

These graphs show that not surprisingly, VMPs take relatively more risk when volatility is low (e.g., the 1960s) and less risk (as less loss) when volatility is high (e.g., the Great Depression or recent financial crisis) since we bet negatively according to market volatility and thus avoiding crash periods. This result illustrates that our strategy works by shifting when it takes market risk and not by loading on extreme market realizations as profitable option strategies typically do.

Volatility Managed Portfolio with GARCH-modeled Volatility

Then we started our Volatility Managed Portfolio based on GARCH-modeled Volatility. There are two main innovations in our strategy: firstly, we use the GARCH model to predict conditional volatility rather than calculating the realized variance directly; secondly, we use 240 months of rolling data to get an estimated leverage scale (c) and update it each month, instead of calculating only one leverage scale for each factor in the entire period. We will be explaining the process (which is different from Part II - Replication of Paper) below:

1. **Data Cleaning:**

   (a) **Raw Data:** We downloaded daily data directly from Kenneth French's website on the excess market return (Mkt), size factor (SMB), value factor (HML), momentum factor (Mom), profitability factor (RMW), and investment factor (CMA).

(b) **Time Period:** In the sample - from 1963-07-01 to 2015-12-31; Out of the sample - from 2016-01-04 to 2023-04-28.

2. **Portfolio Formation:**

(a) **Volatility weight:** We used the previous month's volatility estimated by GARCH model as a proxy for the conditional variance (Figure5):

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

$$\epsilon_t = \sigma_t z_t$$

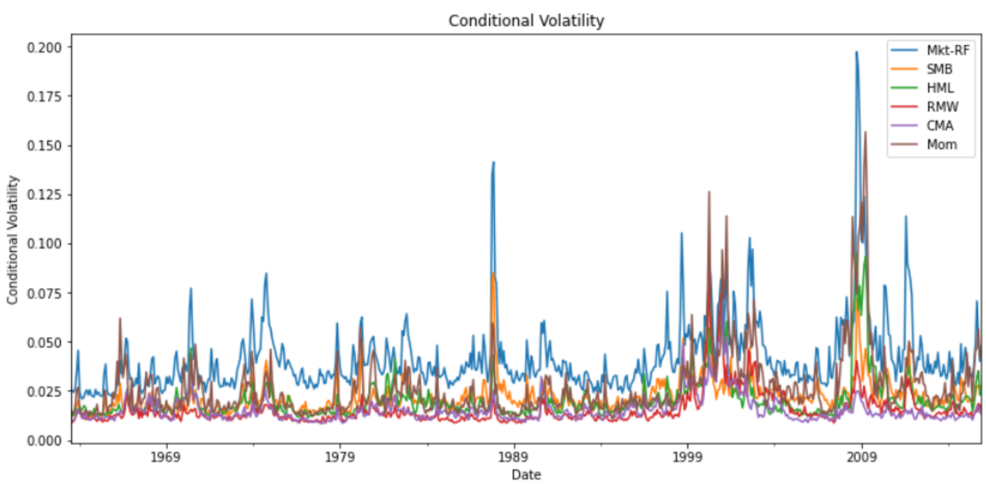

Figure 5: Conditional Volatility - In the Sample

(b) **Leverage exposure c:** We use method A and method B to calculate the leverage scale (c): **Method A:** We use the entire period to get the predicted volatility and calculate the c (the same way as the paper) **Method B:** In the real market, we cannot get the data in the entire period in the beginning, so we roll 240 months of data before each month to predict volatility and then calculate the c. For the strategy in the sample, we use method A; For the strategy out of the sample, we use both method A and method B and compare the result of the two methods.

3. **Empirical Implementation:**

(a) **Cumulative returns, Rolling one-year returns and Drawdowns:** We plotted the cumulative monthly returns (Figure6), rolling one-year returns and drawdowns (Figure7) to a buy-and-hold strategy versus a volatility-managed strategy for the market portfolio from 1963 to 2015. We can see that VMP performs better during the period.

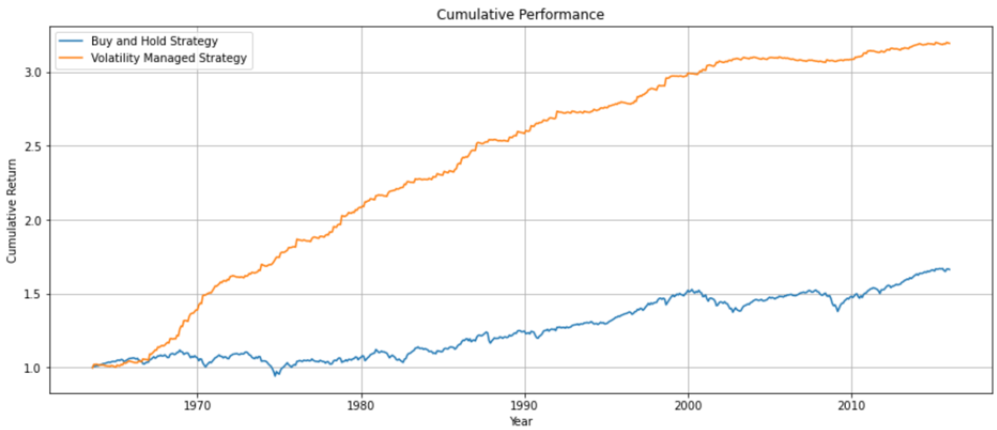

Figure 6: Cumulative returns - In the Sample

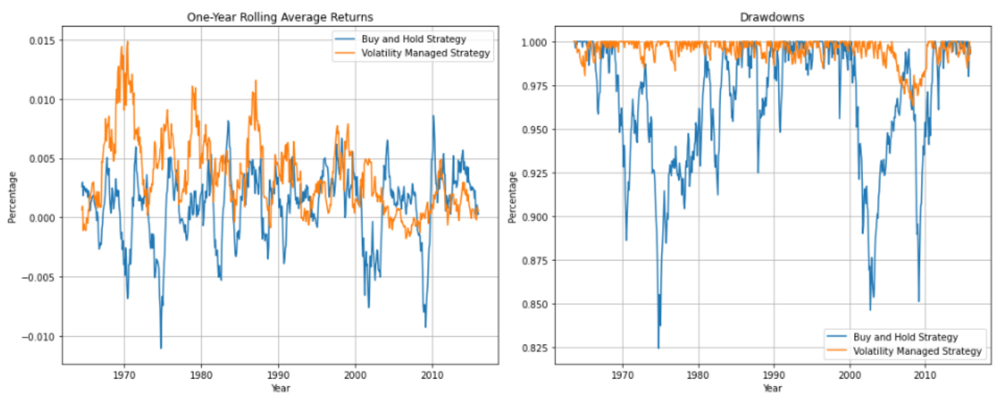

Figure 7: Rolling one-year returns and Drawdowns - In the Sample

(b) **Sharpe Ratios:** We construct the managed factor excess Sharpe ratio given by $\frac{\alpha}{\sigma_\epsilon}$, thus giving a measure of the extent to which dynamic trading expands the slope of the mean-variance efficient (MVE) frontier spanned by the original factors. The new Sharpe Ratio in the Sample is:

$$\text{Sharpe Ratio} : \text{SR}_{\text{new}} = \sqrt{\text{SR}_{\text{old}}^2 + (\frac{\alpha}{\sigma_\epsilon})^2}$$

(c) **Utility Gains:** We also compute the Utility Gains from the perspective of a simple mean-variance investor to quantify the economic relevance of our results.The

Utility Gains in the Sample is (Table 1):

$$\text{Utility Gains} : \mathrm{U}_{MV}(\%) = \frac{\mathrm{SR}^2_{\text{new}} - \mathrm{SR}^2_{\text{old}}}{\mathrm{SR}^2_{\text{old}}}$$

Table 1: Sharpe Ratios and Utility Gains in the Sample

	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) Mom^{σ}	(5) RMW^{σ}	(6) CMA^{σ}
Sharpe Ratio (old)	0.11	0.06	0.13	0.12	0.15	0.16
Sharpe Ratio (new)	0.77	0.68	0.74	0.71	0.79	0.77
Utility Gains	46.44	121.17	30.58	32.79	25.58	20.79

Strategy Analysis

Results Analysis of Two Methods

1. **Higher GARCH volatility in Method B (rolling c):** Compared to Method A (Figure 8), rolling c in Method B (Figure9) can be more practical which shows us the real and most recent volatility in the financial market.

2. **Better Cumulative Returns in Method A (constant c):** Compared to Method A (Figure10), higher volatility in Method B (Figure11) leads to fewer weights to more risky asset and lower cumulative returns.

3. **Higher rolling Returns in Method A (constant c):** Compared to Method A (Figure12), higher volatility in Method B (Figure13) leads to fewer weights to more risky asset and lower cumulative returns.

4. **Better Sharpe Ratios and Utility Gains in Method A (constant c):** Compared to Method A(Table7), new Shape Ratios and Utility Gains in Method B (Table8) show that rolling c lead to higher gains.

Pros and cons of our strategy

Pros:

1. Compared to the conventional view to add positions in risky assets in times of increasing volatility, our strategy works better to avoid crashes during the general recession or market crisis period. The weight which is negatively correlated with volatility also incorporates investors' willingness to take risk which is empirically shown to be negatively related to volatility.

2. The most important weight factor in the model is based on the belief that variance can be reliably forecasted in the short term, and variance forecasts have limited correlation with future returns within these horizons. We have shown in part III how the GARCH model is effectively used to predict volatility for strategy implication in real-time.

3. Our strategy can be synthetically implemented alongside traditional factor investments, offering risk-adjusted returns and the potential to generate alpha. It can also be applicable to betting-against-beta factors, as well as the currency carry trade.

4. Our strategy can effectively introduce diversification into traditional portfolios. For example, executing volatility-managed strategy may potentially avoid momentum crashes during sudden and significant market fluctuations.

Cons:

1. According to **Cederburg, O'Doherty, Wang, and Yan (2020)**, VMP it fails to accommodate the recent findings from some machine learning methods that a larger set of anomalies (firm-level characteristics) is needed to be jointly studied

2. Strategy may not be broadly applicable to other markets such as Chinese stock market according to **Liu, Tang, and Zhou, 2019**

3. Since we introduced a leverage ratio c in the strategy for risk exposure control purposes. This may trigger leverage and margin regulation requirements in actual implementation, which will affect the full execution of strategy in actual markets.

Appendix

Univariate Regressions:

Table 2: **Univariate Regressions - Moreira and Muir (2017)**

	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) Mom^{σ}	(5) RMW^{σ}	(6) CMA^{σ}
MktRF	0.61					
	(0.05)					
SMB		0.62				
		(0.08)				
HML			0.57			
			(0.07)			
Mom				0.47		
				(0.07)		
RMW					0.62	
					(0.08)	
CMA						0.68
						(0.05)
Alpha (α)	4.86	-0.58	1.97	12.51	2.44	0.38
	(1.56)	(0.91)	(1.02)	(1.71)	(0.83)	(0.67)
N	1,065	1,065	1,065	1,060	621	621
R^2	0.37	0.38	0.32	0.22	0.38	0.46
RMSE	51.39	30.44	34.92	50.37	20.16	17.55

Table 3: **Univariate Regressions - Replication**

	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) Mom^{σ}	(5) RMW^{σ}	(6) CMA^{σ}
MktRF	0.60					
SMB		0.61				
HML			0.57			
Mom				0.46		
RMW					0.59	
CMA						0.69
Alpha (α)	4.59**	-0.68	1.24	13.48****	3.02***	0.18
N	1,068	1,068	1,068	1,068	629	629
R^2	0.36	0.38	0.32	0.21	0.35	0.49
RMSE	51.79	30.46	34.95	50.75	21.60	17.09

Table 4: **Univariate Regressions - GARCH in sample (constant c)**

	(1) Mkt$^\sigma$	(2) SMB$^\sigma$	(3) HML$^\sigma$	(4) Mom$^\sigma$	(5) RMW$^\sigma$	(6) CMA$^\sigma$
MktRF	0.05					
SMB		0.17				
HML			0.22			
Mom				0.11		
RMW					0.26	
CMA						0.28
Alpha (α)	0.75	0.45	0.44	0.35	0.34	0.68
R^2	0.0022	0.0291	0.0493	0.0129	0.0652	0.0763
σ_ε	0.0098	0.0066	0.0061	0.0050	0.0044	0.0091

Table 5: **Univariate Regressions - GARCH out of sample - constant c**

	(1) Mkt$^\sigma$	(2) SMB$^\sigma$	(3) HML$^\sigma$	(4) Mom$^\sigma$	(5) RMW$^\sigma$	(6) CMA$^\sigma$
MktRF	0.33					
SMB		-0.02				
HML			-0.08			
Mom				0.13		
RMW					-0.02	
CMA						0.30
Alpha (α)	0.69	0.37	0.45	0.29	0.36	0.56
R^2	0.1062	0.0004	0.0060	0.0182	0.0004	0.0920
σ_ε	0.0101	0.0064	0.0090	0.0047	0.0055	0.0091

Table 6: **Univariate Regressions - GARCH out of sample - rolling c**

	(1) Mkt$^\sigma$	(2) SMB$^\sigma$	(3) HML$^\sigma$	(4) Mom$^\sigma$	(5) RMW$^\sigma$	(6) CMA$^\sigma$
MktRF	0.21					
SMB		-0.05				
HML			0.07			
Mom				0.41		
RMW					0.19	
CMA						0.13
Alpha (α)	0.79	0.47	0.58	0.40	0.40	1.00
R^2	0.0540	0.0026	0.0067	0.1183	0.0468	0.0075
σ_ε	0.0094	0.0060	0.0075	0.0053	0.0047	0.0138

GARCH Volatility:

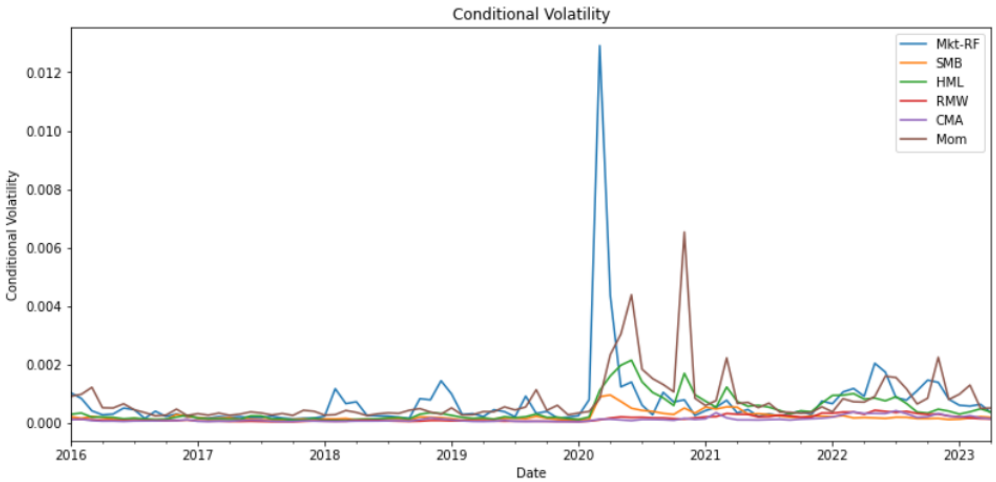

Figure 8: Conditional Volatility - constant c

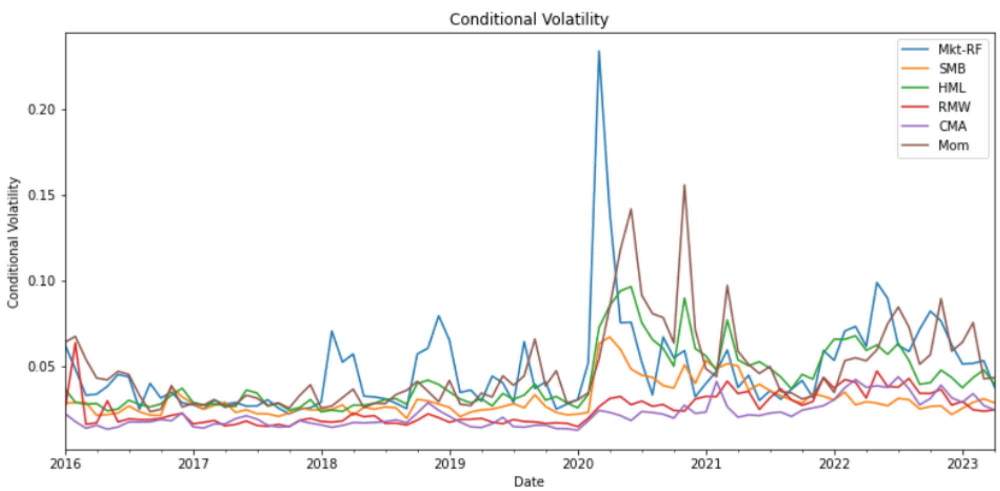

Figure 9: Conditional Volatility - rolling c

Performances:

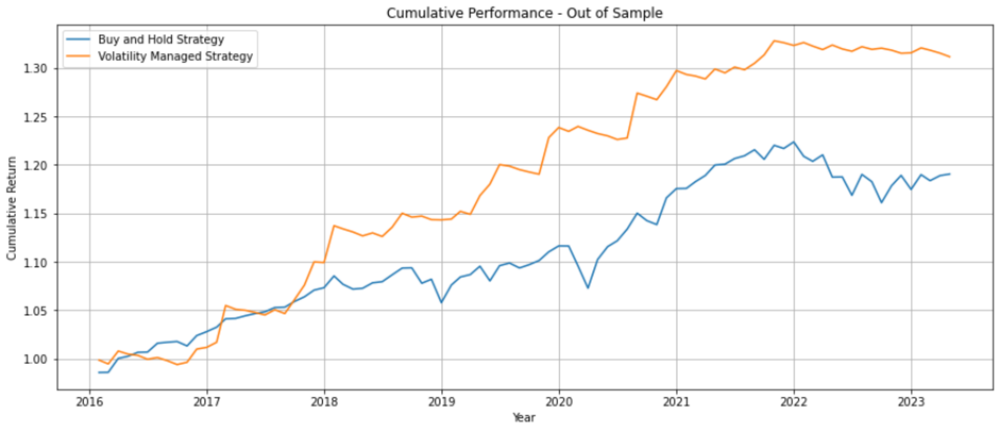

Figure 10: Cumulative returns - constant c

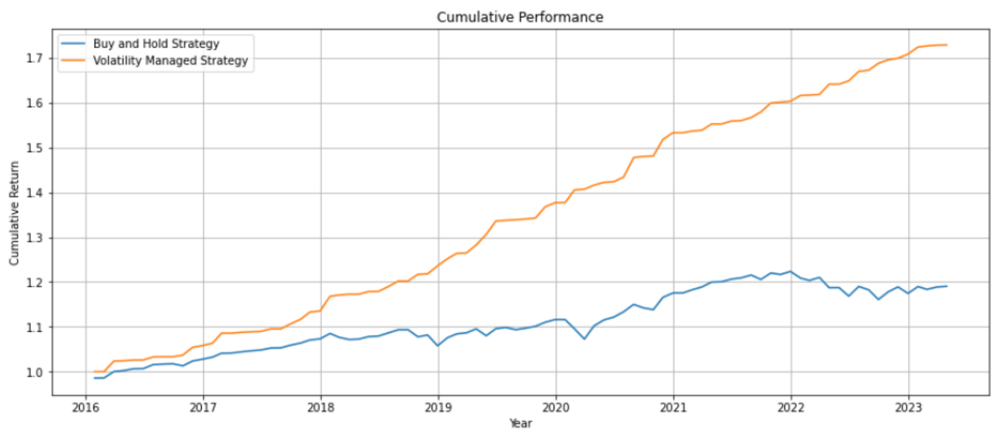

Figure 11: Cumulative returns - rolling c

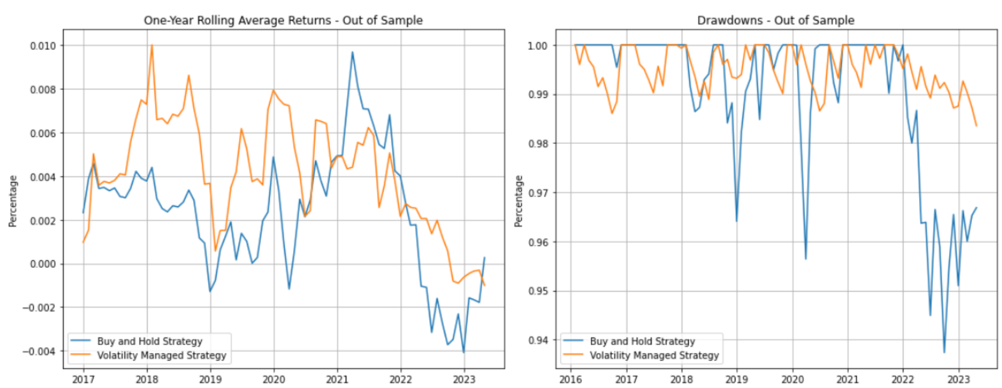

Figure 12: Rolling one-year returns and Drawdowns - constant c

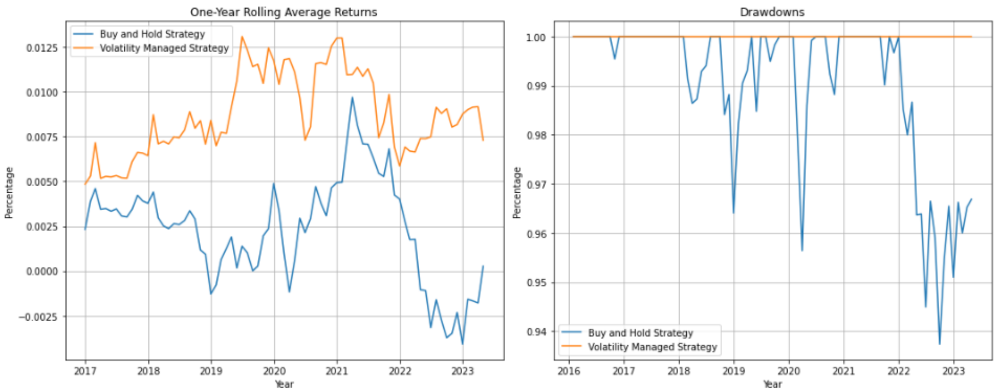

Figure 13: Rolling one-year returns and Drawdowns - rolling c

Table 7: **Performance - GARCH out of sample - constant c**

	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) Mom^{σ}	(5) RMW^{σ}	(6) CMA^{σ}
Sharpe Ratio (old)	0.21	-0.01	-0.02	0.19	0.08	-0.04
Sharpe Ratio (new)	0.71	0.58	0.50	0.65	0.66	0.62
Utility Gains	10.83	1894.06	819.07	10.73	64.06	288.86

Table 8: **Performance - GARCH out of sample - rolling c**

	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) Mom^{σ}	(5) RMW^{σ}	(6) CMA^{σ}
Sharpe Ratio (old)	0.21	-0.01	-0.02	0.19	0.08	-0.04
Sharpe Ratio (new)	0.86	0.79	0.78	0.77	0.85	0.72
Utility Gains	16.26	3512.17	1952.91	15.38	108.16	392.09