

03-Report

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1 Binary Heaps: Homework 2

1.1 Exercise 1

- By modifying the code written during the last lessons, provide an array-based implementation of binary heaps which avoids to swap the elements in the array A. (Hint: use two arrays, `key_pos` and `rev_pos`, of natural numbers reporting the position of the key of a node and the node corresponding to a given position, respectively)

Solution: By using the code and following the hint, `unsigned int *key_pos` and `unsigned int *rev_pos` added to struct of the `binary_heap`. `key_pos` is the position of the key of a node inside array A and `rev_pos` is node (in `key_pos`) corresponding to a given position in A. `swap_keys` function modified, this time swapping is done by using XOR operator. Other functions are adapted to these changes. All implementations can be found in `include/binheap.h` and `src/binheap.c`.

1.2 Exercise 2

- Consider the next algorithm:

```
def Ex2 (A)
  D ← build (A)
  while ¬ is_empty (D)
    extract_min (D)
  endwhile
enddef
```

where A is an array. Compute the time-complexity of the algorithm when:

- `build`, `is_empty` $\in \Theta(1)$, `extract_min` $\in \Theta(|D|)$;
- `build` $\in \Theta(|A|)$, `is_empty` $\in \Theta(1)$, `extract_min` $\in \mathcal{O}(\log n)$;

Solution: In the first case, `D ← build (A)` block will cost $\Theta(1)$ and while loop will be run $|D|$ times. Since complexity of `extract_min` is $\Theta(|D|)$, overall complexity will be $\Theta(|D|^2)$

In the second case, `D ← build (A)` block will cost $\Theta(|A|)$ and again while loop will be run $|D|$ times. However this time complexity of `extract_min` is $\mathcal{O}(\log n)$. So overall complexity will be upper bounded by the complexity of `extract_min` which is $\mathcal{O}(|D| \cdot \log n)$