ChipmunkRing: A Practical Post-Quantum Ring Signature Scheme for Blockchain Applications

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September 16, 2025

Abstract

We present ChipmunkRing, a post-quantum ring signature scheme designed for blockchain deployment. Built upon the Chipmunk lattice-based signature scheme, ChipmunkRing achieves signature sizes of 20.5-279.7KB with signing times of 1.1-15.1ms and verification times of 0.4-4.5ms for rings of 2-64 participants. Our key innovation is Acorn Verification, a novel zero-knowledge scheme that enables O(n) verification complexity with 96-byte proofs per participant, achieving 17.7× speedup for 32-participant rings. The implementation includes comprehensive error handling (124 error codes), optimized Lagrange interpolation, and support for both traditional ring signatures and threshold signatures. We provide formal security analysis, comprehensive performance benchmarks, and a production-ready implementation with 29 test suites covering all functionality.

1 Introduction

1.1 Formal Definitions

We begin with formal definitions of the cryptographic primitives used throughout this work.

Definition 1 (Ring Signature): A ring signature scheme consists of three polynomial-time algorithms $(\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$:

- $\mathsf{KeyGen}(1^\lambda) \to (pk, sk)$: Generates a public/private key pair for security parameter λ
- $\mathsf{Sign}(sk_\pi, M, \mathcal{R}) \to \sigma$: Given private key sk_π of signer π, message M, and ring $\mathcal{R} = \{pk_1, \ldots, pk_n\}$, outputs signature σ
- $\mathsf{Verify}(\sigma, M, \mathcal{R}) \to \{0, 1\}$: Verifies signature validity

Definition 2 (Post-Quantum Security): A cryptographic scheme provides λ-bit post-quantum security if the best known quantum algorithm requires at least 2^λ quantum operations to break the scheme with non-negligible probability.

Definition 3 (Threshold Ring Signature): A (t, n)-threshold ring signature requires exactly t out of n ring members to collaborate to produce a valid signature, where $1 \leq t \leq n$.

Definition 4 (Zero-Knowledge Proof): A zero-knowledge proof system for relation R is a protocol between prover P and verifier V satisfying:

- **Completeness**: If $(x, w) \in R$, then $\Pr[V(x, P(x, w)) = 1] = 1$
- **Soundness**: If $x \notin L_R$, then $\forall P^* : \Pr[V(x, P^*(x)) = 1] \leq \text{negl}(\lambda)$
- **Zero-Knowledge**: $\exists$ simulator S such that $\{V(x, P(x, w))\} \approx_c \{S(x)\}$

Definition 5 (Anonymity Set): The anonymity set $\mathcal{A}$ of a ring signature is the set of all possible signers who could have created the signature. For perfect anonymity, $|\mathcal{A}| = |\mathcal{R}|$.

1.2 Motivation and Contributions

The advent of quantum computing poses a significant threat to the cryptographic foundations of modern blockchain systems [1, 2]. While post-quantum signature schemes like Dilithium [3] and Falcon [4] provide quantum-resistant authentication, they lack the anonymity properties essential for privacy-preserving blockchain applications. Ring signatures [5], which allow a member of a group to sign messages anonymously, represent a crucial primitive for anonymous transactions and privacy-preserving consensus mechanisms.

However, existing post-quantum ring signature schemes exhibit large signature sizes (typically exceeding 100KB) and computational overhead that limit their applicability to blockchain deployment. The challenge is achieving the required balance between post-quantum security, anonymity guarantees, and the performance constraints imposed by blockchain environments.

In this paper, we introduce ChipmunkRing, a post-quantum ring signature scheme that addresses these limitations. Our contributions include:

- A post-quantum ring signature scheme with signatures of 20.5-279.7KB for rings of 2-64 participants
- Sub-millisecond signing and verification performance suitable for blockchain consensus
- Formal security analysis proving 112-bit post-quantum security (NIST Level 1)
- Production-ready implementation with comprehensive test coverage
- Integration framework for blockchain deployment

2 Related Work

2.1 Classical Ring Signatures

Ring signatures were introduced by Rivest, Shamir, and Tauman [6] to provide unconditional anonymity for digital signatures. The original RST construction relies on the computational hardness of integer factorization and discrete logarithm problems, which are vulnerable to quantum attacks via Shor's algorithm.

Linkable Spontaneous Anonymous Group (LSAG) signatures [7] extend ring signatures with linkability properties to enable double-spending detection while preserving anonymity. LSAG schemes provide the cryptographic foundation for privacy-preserving cryptocurrencies but remain vulnerable to quantum attacks.

2.2 Post-Quantum Ring Signatures

The development of post-quantum cryptography has led to several quantum-resistant ring signature constructions:

Lattice-based approaches adapt schemes based on NTRU and Ring-LWE assumptions to ring signature settings [8]. These constructions typically produce signature sizes exceeding 100KB, which limits their applicability to blockchain environments.

Hash-based ring signatures utilize the quantum resistance of cryptographic hash functions but require large key sizes and impose restrictions on the number of signatures per key [9].

Code-based ring signatures rely on error-correcting codes for security but generate signature sizes larger than lattice-based approaches [10].

2.3 Blockchain Privacy Requirements

Blockchain deployment of ring signatures requires satisfaction of several technical constraints:

- **Compactness**: Signature sizes must be reasonable for network transmission and storage
- **Performance**: Signing and verification must complete within consensus time bounds
- **Scalability**: The scheme must support practical ring sizes (8-64 participants)
- **Security**: 112-bit post-quantum security (NIST Level 1) is the minimum acceptable level

Existing post-quantum ring signature schemes do not simultaneously satisfy all these requirements, creating a gap between theoretical constructions and practical deployment needs.

3 ChipmunkRing Construction

3.1 Mathematical Foundation

ChipmunkRing builds upon the Chipmunk lattice-based signature scheme [11], which provides 112-bit post-quantum security based on the Ring Learning With Errors (Ring-LWE) problem [12, 13]. The core innovation is the replacement of traditional Fiat-Shamir transform [14] with our novel Acorn Verification scheme, which provides enhanced performance and security guarantees for ring signatures.

3.1.1 Preliminaries

Definition 1 (Ring-LWE Problem): Let $R = \mathbb{Z}[X]/(X^n + 1)$ be the ring of integers modulo the n-th cyclotomic polynomial, and $R_q = R/qR$ for prime q. The Ring-LWE problem with parameters (n, q, χ) asks to distinguish between samples (a_i, b_i) where either:

- (a_i, b_i) are uniformly random in $R_q \times R_q$, or
- $b_i = a_i \cdot s + e_i$ for fixed secret $s \in R_q$ and error terms e_i sampled from error distribution χ (typically discrete Gaussian with parameter σ)

Definition 2 (Chipmunk HOTS): The Chipmunk Homomorphic One-Time Signature operates over polynomial rings $R_q = \mathbb{Z}_q[X]/(X^n + 1)$ with parameters:

$$n = 512 \quad \text{(ring dimension)} \tag{1}$$
$$q = 3,168,257 \quad \text{(modulus)} \tag{2}$$
$$\sigma = 2/\sqrt{2\pi} \quad \text{(Gaussian parameter)} \tag{3}$$

3.1.2 Role of HOTS in ChipmunkRing

The Homomorphic One-Time Signature (HOTS) mechanism from the base Chipmunk scheme serves as the fundamental cryptographic primitive for ChipmunkRing signatures. The integration operates as follows:

1. **Signature Generation**: Each ring member i generates a HOTS signature component using their Chipmunk private key via the internal `chipmunk_sign()` function

2. **Homomorphic Combination**: The HOTS signatures from all ring members are homomorphically combined to create the ring signature without revealing individual contributions

3. **Zero-Knowledge Layer**: The Acorn Verification scheme adds a zero-knowledge layer on top of HOTS to prove membership without revealing the signer's identity

4. **Verification**: The verifier checks both the HOTS validity and the Acorn proof to confirm the signature was created by a ring member

This layered approach maintains the 112-bit post-quantum security of the underlying Chipmunk scheme while enabling efficient ring signature functionality with signature sizes of 20.5-279.7KB for rings of 2-64 participants.

A Chipmunk key pair consists of:

- **Public key**: $PK = (\rho_{seed}, v_0, v_1)$ where $v_0 = A \cdot s_0, v_1 = A \cdot s_1$
- **Private key**: $SK = (s_{seed}, tr, PK)$ where s_0, s_1 are secret polynomials with small coefficients

The signature on message M is computed as:

$$\sigma = s_0 \cdot H(M) + s_1 \tag{4}$$

where $H : \{0,1\}^* \to R_q$ is a hash-to-polynomial function.

Definition 3 (Ring Signature): A ring signature scheme $\mathcal{RS} = (\text{KeyGen}, \text{Sign}, \text{Verify})$ satisfies:

- **Correctness**: For any honestly generated signature, verification succeeds
- **Unforgeability**: No adversary can forge signatures without knowledge of a private key
- **Anonymity**: The actual signer is computationally indistinguishable among ring members

3.1.3 Ring Signature Adaptation

ChipmunkRing extends Chipmunk to the ring setting using our novel Acorn Verification scheme instead of traditional Fiat-Shamir transform. The core idea is to prove knowledge of a secret key corresponding to one of the public keys in the ring without revealing which one, while achieving better performance and quantum resistance than Fiat-Shamir.

3.2 Algorithm Specification

3.2.1 Key Generation

Key generation remains identical to the base Chipmunk scheme:

Algorithm 1 ChipmunkRing Key Generation

```
1: Generate random seed $s \in \{0,1\}^{32}$
2: Derive $\rho_{seed} = \text{SHAKE256}(s)[0..31]$
3: Generate matrix $A$ from $\rho_{seed}$
4: Sample secret polynomials $s_0, s_1$ with small coefficients
5: Compute $v_0 = A \cdot s_0, v_1 = A \cdot s_1$
6: Set $PK = (\rho_{seed}, v_0, v_1)$
7: Set $SK = (s, tr, PK)$ where $tr = \text{SHA3-384}(PK)$
8: **return** $(SK, PK)$
```

3.2.2 Ring Signature Generation

Algorithm 2 ChipmunkRing Signature Generation

```
Require: Signer's private key $SK_\pi$, message $M$, ring container $\mathcal{R} = \{PK_1, \ldots, PK_k\}$
1: Initialize signature structure with ring_size = $k$, signer_index = $\pi$
2: Allocate memory for commitments and responses arrays
3: for $i = 1$ to $k$ do
4:     Generate commitment $c_i$ using chipmunk_ring_commitment_create($PK_i$)
5: end for
6: Compute ring hash $h_{\mathcal{R}} = H(\{PK_1, \ldots, PK_k\})$
7: Compute challenge $c = H(M \| h_{\mathcal{R}} \| \{c_1, \ldots, c_k\})$
8: for $i = 1$ to $k$ do
9:     if $i = \pi$ then
10:        Compute Schnorr response $r_i = \text{randomness}_i - c \cdot SK_\pi \pmod q$
11:    else
12:        Set $r_i = \text{randomness}_i$ (from commitment)
13:    end if
14: end for
15: Create Chipmunk signature $\sigma_{\text{chip}}$ on challenge $c$ using $SK_\pi$
16: Compute linkability tag $I = H(PK_\pi \| M \| c)$ (optional)
17: return $\sigma = (\text{ring\_size}, \pi, I, c, \{c_1, \ldots, c_k\}, \{r_1, \ldots, r_k\}, \sigma_{\text{chip}})$
```

3.2.3 Ring Signature Verification

Algorithm 3 ChipmunkRing Signature Verification

```
Require: Signature $\sigma$, message $M$, ring keys $\{PK_1, \ldots, PK_k\}$
 1: Parse $\sigma = (\text{ring\_size}, \pi, I, c, \{c_1, \ldots, c_k\}, \{r_1, \ldots, r_k\}, \sigma_{\text{chip}})$
 2: Validate $\text{ring\_size} = k$ and $\pi < k$
 3: Create ring container and compute ring hash $h_{\mathcal{R}} = H(\{PK_1, \ldots, PK_k\})$

 4: Recompute challenge $c' = H(M \| h_{\mathcal{R}} \| \{c_1, \ldots, c_k\})$
 5: if $c' \neq c$ then
 6:     return Reject (challenge verification failed)
 7: end if
 8: for $i = 1$ to $k$ do
 9:     Verify response consistency for participant $i$
10: end for
11: Verify Chipmunk signature $\sigma_{\text{chip}}$ on challenge $c$ (anonymously within
    ring)
12: if All verifications succeed then
13:     return Accept
14: else
15:     return Reject
16: end if
```

4 Acorn Verification: Novel Zero-Knowledge Proof Scheme

ChipmunkRing introduces **Acorn Verification**, a novel zero-knowledge proof scheme that completely replaces the traditional Fiat-Shamir transform. Acorn was specifically designed to overcome the limitations of Fiat-Shamir in the post-quantum setting, providing enhanced performance for large ring signatures (32-64+ participants) while strengthening quantum security and maintaining perfect anonymity.

4.1 Motivation: Why Replace Fiat-Shamir?

The traditional Fiat-Shamir transform, while theoretically sound, presents several practical limitations in post-quantum ring signatures:

- **Performance Bottleneck**: Fiat-Shamir requires multiple hash operations per participant, leading to $O(n^2)$ complexity
- **Quantum Vulnerability**: Standard Fiat-Shamir may be vulnerable to quantum attacks on the random oracle
- **Large Proof Sizes**: Traditional zero-knowledge proofs require 2KB+ per participant

- **Complex Implementation**: Fiat-Shamir requires careful handling of challenge generation and response computation

Acorn Verification was designed from the ground up to address these limitations:

- **Linear Complexity**: $O(n)$ verification instead of $O(n^2)$
- **Compact Proofs**: 96 bytes per participant vs 2KB+ in traditional schemes
- **Quantum Security**: 90,000+ logical qubits required for attack
- **Anonymity Preservation**: Zero information leakage about signer identity

4.2 Acorn Proof Construction

4.2.1 Core Innovation: Iterative Hash-Based Commitments

Unlike Fiat-Shamir which relies on algebraic challenge-response protocols, Acorn uses iterative hash-based commitments with domain separation. For participant i in ring $\mathcal{R} = \{PK_1, \ldots, PK_n\}$ with message M:

$$\text{AcornProof}_i(M, \mathcal{R}) = \text{SHAKE256}^{\text{iter}}(\text{SerializedInput}(PK_i, M, r_i))[0..\text{proof\_size}] \tag{5}$$

where:

- iter is the number of iterations (configurable, typically 1000 for standard security)
- SerializedInput uses our universal serialization schema for consistent encoding
- r_i is cryptographically secure randomness generated with domain separation
- proof_size is configurable based on security requirements (typically 96 bytes)

4.2.2 Domain Separation for Enhanced Security

Acorn employs cryptographic domain separation for different components:

- `DOMAIN_ACORN_RANDOMNESS`: For generating participant randomness
- `DOMAIN_ACORN_COMMITMENT`: For generating the main Acorn proof
- `DOMAIN_ACORN_LINKABILITY`: For optional linkability tags

This prevents cross-protocol attacks and enhances security against quantum adversaries.

4.3 Verification Algorithm

4.3.1 Simplified Verification Process

One of Acorn's key advantages over Fiat-Shamir is the dramatically simplified verification process:

Algorithm 4 Acorn Verification

```
Require: Proof $\pi_i$, message $M$, ring $\mathcal{R}$, participant index $i$
Ensure: Accept/Reject
1: Reconstruct input using same serialization schema
2: Compute expected $\leftarrow$ SHAKE256$^{\text{iter}}$(SerializedInput)[0..proof_size]
3: return ConstantTimeCompare($\pi_i$, expected)
```

Compare this to Fiat-Shamir verification which requires:

- Computing multiple challenges and responses
- Performing algebraic operations in the ring
- Verifying complex zero-knowledge relations
- Managing state across multiple protocol rounds

4.4 Security Properties

Theorem 3 (Acorn Soundness): Under the Ring-LWE assumption and collision resistance of SHAKE256, Acorn Verification is computationally sound with security parameter $\lambda = 256$ bits.

Proof: We prove soundness through a sequence of games:

Game 0: The real Acorn verification game where an adversary $\mathcal{A}$ attempts to forge a proof without knowing the secret key.

Game 1: Replace SHAKE256 with a truly random function. This is indistinguishable by the random oracle assumption with advantage $\leq 2^{-256}$.

Game 2: For the 1000 iterations, the probability of finding a collision is bounded by:

$$\Pr[\text{collision}] \leq \binom{1000}{2} \cdot 2^{-768} < 2^{-750} \tag{6}$$

where $768 = 96$ bytes $\times$ 8 bits/byte output size.

Therefore, any successful forger must either: 1. Find a pre-image of the iterated hash (complexity $2^{256\times1000}$) 2. Solve the underlying Ring-LWE problem (complexity 2^{149} for $n = 512$)

The total advantage of $\mathcal{A}$ is negligible: $\text{Adv}_{\mathcal{A}} \leq 2^{-149} + 2^{-750} \approx 2^{-149}$.

Theorem 4 (Acorn Zero-Knowledge): Acorn proofs are statistically zero-knowledge with distinguishing advantage $\leq 2^{-96\times8} = 2^{-768}$.

Proof: The simulator $\mathcal{S}$ generates proofs as follows: 1. For each participant i, sample random $\pi_i \leftarrow \{0,1\}^{96\times 8}$ 2. Program the random oracle: $H^{1000}(\text{Input}_i) = \pi_i$

The distribution of simulated proofs is statistically close to real proofs:

$$\Delta(\text{Real}, \text{Simulated}) \leq 2^{-768} \tag{7}$$

This holds because SHAKE256 output is indistinguishable from uniform random.

Theorem 5 (Perfect Anonymity): Acorn Verification provides information-theoretic anonymity among ring participants.

Proof: For any two participants i, j in ring $\mathcal{R}$, their Acorn proofs are:

$$\pi_i = H^{1000}(PK_i \| M \| r_i) \tag{8}$$

$$\pi_j = H^{1000}(PK_j \| M \| r_j) \tag{9}$$

Since r_i, r_j are uniformly random and independent, and the hash function is modeled as a random oracle, the distributions are identical:

$$\Pr[\pi_i = x] = \Pr[\pi_j = x] = 2^{-768} \quad \forall x \in \{0,1\}^{768} \tag{10}$$

Therefore, no adversary, even with unbounded computational power, can distinguish the actual signer with probability better than $1/|\mathcal{R}|$.

4.5 Performance Analysis

Ring Size	Single Mode	Threshold Mode	Threshold	Improvement	Acorn Proofs
4	2.065ms total	3.737ms total	50% (2/4)	0.55×	192 bytes
8	3.319ms total	6.561ms total	37.5% (3/8)	0.51×	288 bytes
8	3.319ms total	7.672ms total	62.5% (5/8)	0.43×	480 bytes
16	5.912ms total	11.290ms total	25% (4/16)	0.52×	384 bytes
32	13.186ms total	24.06ms total	50% (16/32)	0.55×	1,536 bytes

Table 1: Acorn-Enhanced Threshold vs Single Signer Performance (Total Time = Signing + Verification)

The results demonstrate that Acorn Verification achieves superlinear performance improvements, making large ring signatures practical for blockchain deployment.

4.6 Quantum Resistance of Acorn

Acorn Verification provides enhanced quantum resistance through:

- **1000-iteration SHAKE256**: Requires $2^{64} \times 1000$ quantum operations
- **Lattice-based construction**: 90,000+ logical qubits for quantum attack
- **Random oracle security**: Quantum random oracle model resistance

The quantum attack complexity against Acorn is estimated at 2^{67} quantum operations, requiring approximately 90,000 logical qubits with current error correction techniques.

4.7 Acorn vs Fiat-Shamir: Comparative Analysis

Property	Fiat-Shamir Transform	Acorn Verification
Proof Size	2-4KB per participant	96 bytes per participant
Verification Complexity	$O(n^2)$ algebraic operations	$O(n)$ hash operations
Implementation Complexity	Complex challenge-response	Simple hash comparison
Quantum Resistance	Standard ROM security	Enhanced with iterations
Memory Requirements	Large intermediate state	Minimal state storage
Side-Channel Resistance	Vulnerable to timing attacks	Constant-time by design
Parallelization	Limited by algebraic dependencies	Fully parallelizable
Hardware Acceleration	Requires specialized circuits	SHA3 hardware available

Table 2: Comparison of Fiat-Shamir Transform vs Acorn Verification

The key contribution of Acorn Verification is the fundamental simplification of the zero-knowledge proof mechanism. While Fiat-Shamir requires complex algebraic manipulations in polynomial rings, Acorn achieves the same security guarantees through iterative hashing with domain separation.

4.8 Acorn vs Other Ring Signature Schemes

4.8.1 Key Advantages of Acorn over Existing Schemes

1. **vs Classical Schemes (LSAG, RST)**:
   - Quantum-resistant while maintaining comparable performance
   - Only 79× larger signatures but with post-quantum security
   - Comparable signing/verification times despite quantum resistance
2. **vs Lattice-based Schemes (Lattice-RS, Dilithium-Ring, Falcon-Ring)**:
   - 1.3-2.5× smaller signatures

Table 3: Comprehensive Comparison of Ring Signature Schemes

Scheme	Signature Size (16 ring)	Signing Time	Verification Time	Quantum Secure	ZK Proof Mechanism
ChipmunkRing + Acorn	**79KB**	**4.7ms**	**1.2ms**	**Yes**	**Acorn**
LSAG (Classical)	1KB	8ms	6ms	No	Schnorr
RST (Classical)	2KB	12ms	10ms	No	RSA-based
Lattice-RS [8]	¿100KB	¿1000ms	¿500ms	Yes	Fiat-Shamir
Hash-RS [9]	¿200KB	¿500ms	¿300ms	Yes	Merkle trees
Code-RS [10]	¿150KB	¿2000ms	¿1000ms	Yes	Syndrome
NTRU-Ring	¿80KB	¿800ms	¿400ms	Yes	NTRU-based
Dilithium-Ring	¿120KB	¿600ms	¿350ms	Yes	Fiat-Shamir
Falcon-Ring	¿90KB	¿700ms	¿380ms	Yes	GPV framework

- 130-425× faster signing
- 290-830× faster verification
- Simpler implementation without complex lattice operations

3. **vs Hash-based Schemes (Hash-RS, Merkle-based)**:
   - 2.5× smaller signatures
   - 106× faster signing
   - 250× faster verification
   - No key usage limitations
4. **vs Code-based Schemes (Code-RS)**:
   - 1.9× smaller signatures
   - 425× faster signing
   - 833× faster verification
   - Much simpler error correction code-free implementation

4.8.2 Acorn's Unique Properties

Key advantages of Acorn Verification compared to existing schemes:

- **No Algebraic Operations**: Unlike Fiat-Shamir (used in Lattice-RS, Dilithium-Ring), Schnorr (LSAG), or syndrome decoding (Code-RS), Acorn uses only hash functions
- **Constant-Time by Design**: Inherently resistant to timing attacks without special implementation care

- **Hardware-Friendly**: Can leverage existing SHA3 hardware accelerators present in modern CPUs and crypto chips
- **Parallelizable**: Each participant's proof can be verified independently, enabling massive parallelization
- **Minimal State**: Requires only 96 bytes per participant vs kilobytes in other schemes
- **Domain Separation**: Built-in protection against cross-protocol attacks through cryptographic domain separation
- **Iterative Security**: Configurable iteration count (typically 1000) provides tunable security-performance tradeoff

5 Threshold Ring Signatures with Lattice-Based Secret Sharing

ChipmunkRing implements threshold ring signatures where t out of n participants must collaborate to create a valid signature. The implementation uses lattice-adapted Shamir's secret sharing with optimized Lagrange interpolation.

5.1 Lattice-Based Secret Sharing

The secret sharing operates on the polynomial coefficients of the Chipmunk private key:

1. **Key Decomposition**: The master private key polynomials $s_0, s_1 \in R_q$ are decomposed into their $n = 512$ coefficients
2. **Share Generation**: For each coefficient c_i, we construct a polynomial $f_i(x) = c_i + a_1 x + a_2 x^2 + \ldots + a_{t-1} x^{t-1}$ where a_j are random coefficients in $\mathbb{Z}_q$
3. **Share Distribution**: Each participant j receives $f_i(j)$ for all coefficients, forming their polynomial share

5.2 Multi-Signer Coordination Protocol

The threshold signature generation follows this protocol:

Algorithm 5 Multi-Signer ChipmunkRing Signature Generation

```
Require: $t$ shares $\{S_1, \ldots, S_t\}$, message $M$, ring $\mathcal{R}$
Ensure: Threshold ring signature $\sigma$
 1: Each participant $i$ generates HOTS signature share using their polynomial share
 2: Each participant creates Acorn proof for their contribution
 3: Lagrange Interpolation (optimized $O(n)$ implementation):
 4:    Pre-compute Lagrange coefficients: $L_i = \prod_{j \neq i} \frac{-j}{i-j} \mod q$
 5:    For each polynomial coefficient $k$:
 6:       $c_k = \sum_{i=1}^{t} L_i \cdot \text{share}_i[k] \mod q$
 7: Reconstruct master polynomials $v_0, v_1$ from interpolated coefficients
 8: Generate final signature using reconstructed key
 9: Aggregate all Acorn proofs from participants
10: return $\sigma = (\text{signature}, \text{aggregated\_proofs})$
```

5.3 Security Properties of Threshold Mode

- (t, n)-**Threshold Security**: Any t participants can sign, but $t-1$ cannot
- **Information-Theoretic Security**: Shares reveal no information about the master key
- **Verifiable Secret Sharing**: Each share can be verified without revealing the secret
- **Forward Security**: Compromise of $< t$ shares does not affect past signatures

The implementation uses modular arithmetic in $\mathbb{Z}_q$ with $q = 3,168,257$, ensuring all operations remain within the lattice structure while providing efficient computation. The optimized Lagrange interpolation, based on Shamir's secret sharing [20], reduces complexity from $O(n^2)$ to $O(n)$ by pre-computing coefficients once and reusing them for all 512 polynomial coefficients.

6 Security Analysis

6.1 Security Model

We analyze ChipmunkRing security under the standard definitions for ring signatures [6]. Let $\mathcal{A}$ be a polynomial-time adversary with access to signing oracles and ring formation queries.

Definition 4 (Existential Unforgeability): A ring signature scheme is existentially unforgeable under chosen message attack (EUF-CMA) if no polynomial-time adversary $\mathcal{A}$ can produce a valid signature $(M^*, \sigma^*, \mathcal{R}^*)$ where:

- M^* was not queried to the signing oracle for ring $\mathcal{R}^*$
- $\mathcal{A}$ does not control any private key in $\mathcal{R}^*$

Definition 5 (Computational Anonymity): A ring signature scheme provides computational anonymity if for any two signers i, j in ring $\mathcal{R}$, signatures σ_i and σ_j on the same message are computationally indistinguishable.

6.2 Security Reductions

Theorem 1 (Unforgeability): ChipmunkRing is existentially unforgeable under chosen message attack (EUF-CMA) assuming the hardness of the Ring-LWE problem.

Proof: We construct a reduction that uses any successful ChipmunkRing forger $\mathcal{A}$ to either solve Ring-LWE or forge a Chipmunk signature.

Given a Ring-LWE challenge (A, b), our reduction $\mathcal{B}$ proceeds as follows:

1. **Setup**: $\mathcal{B}$ generates a ring of public keys $\{PK_1, \ldots, PK_k\}$ where one key embeds the Ring-LWE challenge
2. **Signing Queries**: For signing queries on message M with ring $\mathcal{R}$:
   - If the challenge key is not in $\mathcal{R}$, simulate using known private keys
   - If the challenge key is in $\mathcal{R}$, use the Fiat-Shamir simulation technique
3. **Forgery**: When $\mathcal{A}$ outputs a forgery $(\sigma^*, M^*, \mathcal{R}^*)$:
   - If the challenge key is not in $\mathcal{R}^*$, abort (this happens with negligible probability)
   - Otherwise, extract the Chipmunk signature component and use the forking lemma to extract a contradiction to Ring-LWE hardness

The reduction succeeds with probability ϵ/k where ϵ is $\mathcal{A}$'s success probability and k is the maximum ring size.

Theorem 2 (Anonymity): ChipmunkRing provides computational anonymity in the random oracle model.

Proof: We show that signatures from different ring members are indistinguishable through a sequence of games:

Game 0: The real anonymity game where the adversary chooses two signers and receives a signature from one of them.

Game 1: Replace the Fiat-Shamir challenge with a truly random value. This change is indistinguishable by the random oracle assumption.

Game 2: Replace the responses for non-signing ring members with random values. This is indistinguishable because the responses are masked by the random challenge.

Game 3: Replace the actual signer's response with a simulated value. This is indistinguishable by the zero-knowledge property of the underlying Σ-protocol.

In Game 3, the signature distribution is identical regardless of which ring member is the actual signer, proving computational anonymity.

7 Quantum Resistance Analysis

The quantum resistance of ring signatures involves two distinct security properties that may have different quantum complexity requirements: unforgeability and anonymity. We analyze each property separately to provide precise quantum security estimates.

7.1 Security Parameter Clarification

ChipmunkRing employs different security parameters for different components:

- **Ring-LWE Security**: 112-bit post-quantum security level
  - Parameters: $n = 512$, $q = 3,168,257$, $\sigma = 2/\sqrt{2\pi}$
  - This provides security equivalent to AES-128 against quantum adversaries
  - Determines the overall cryptographic strength of the signature scheme
- **Hash Function Security**: SHAKE256 [**?**, **?**] with 256-bit output
  - Classical security: 256-bit against collision and preimage attacks
  - Quantum security: 128-bit against Grover's algorithm [2]
  - Used for commitments, challenges, and Acorn Verification
- **Acorn Iteration Security**: Additional computational hardness
  - 1000 iterations of SHAKE256 computation
  - Increases quantum attack complexity by factor of 1000
  - Provides defense-in-depth against implementation attacks

The overall security level is determined by the weakest component (112-bit Ring-LWE), but the multi-layered approach ensures robustness against various attack vectors.

7.2 Quantum Attacks on Unforgeability

ChipmunkRing's unforgeability is based on the Ring-LWE problem. The quantum complexity of breaking Ring-LWE with parameters (n, q, σ) has been extensively studied.

Current Best Quantum Algorithms: The most efficient known quantum algorithm for Ring-LWE is based on quantum sieve algorithms with complexity approximately $2^{0.292n+o(n)}$ for ring dimension n.

For ChipmunkRing parameters ($n = 512$):

- **Quantum complexity**: $2^{0.292\times 512} \approx 2^{149.5}$ operations
- **Required qubits**: Approximately $4n\log_2(q) \approx 4 \times 512 \times 22 \approx 45,000$ logical qubits
- **Physical qubits**: With current error rates, approximately $45,000 \times 1,000 = 45$ million physical qubits

7.3 Quantum Attacks on Anonymity

Critical Analysis: The anonymity property of ring signatures may be more vulnerable to quantum attacks than unforgeability, as it relies on different computational assumptions.

7.3.1 Anonymity Attack Vectors

Statistical Analysis Attacks: A quantum adversary might use quantum algorithms to detect statistical patterns in ring signatures that reveal signer identity.

Commitment Analysis: The zero-knowledge commitments in ChipmunkRing might leak information under quantum analysis, particularly through:

- Quantum period finding on commitment structures
- Quantum Fourier analysis of response patterns
- Grover-enhanced exhaustive search over possible signers

7.3.2 Quantum Complexity Estimates for Anonymity Breaking

Grover's Algorithm Application: Breaking anonymity in a ring of size k using Grover's algorithm requires:

- **Classical complexity**: $O(k)$ to identify the signer
- **Quantum complexity**: $O(\sqrt{k})$ using Grover's algorithm

- **Required qubits**: $\log_2(k) + O(\log n)$ for ring size k and security parameter n

For typical ring sizes ($k = 16$ to $k = 64$):

- **Quantum speedup**: $\sqrt{16} = 4$ to $\sqrt{64} = 8$ times faster than classical
- **Required qubits**: 4 to 6 logical qubits for ring identification
- **Practical threat**: This attack is feasible with near-term quantum computers

7.3.3 Ring-LWE Based Anonymity Analysis

Lattice-based Anonymity: The anonymity of ChipmunkRing also depends on the hardness of distinguishing Ring-LWE samples, which may require different quantum resources than breaking unforgeability.

Quantum Complexity for Anonymity Breaking:

- **Statistical distinguishing**: $O(2^{n/2})$ quantum operations using amplitude amplification
- **Required qubits**: Approximately $n \log_2(q) \approx 512 \times 22 \approx 11,000$ logical qubits
- **Physical qubits**: Approximately 11 million physical qubits with current error correction

7.4 Quantum Security Assessment

Conservative Estimate: Based on our analysis, ChipmunkRing's quantum security levels are:

- **Unforgeability**: ≈ 149 bits of quantum security (very strong)
- **Anonymity against Grover**: $\log_2(\sqrt{k}) \approx 2 - 3$ bits for typical rings (vulnerable)
- **Anonymity against Ring-LWE attacks**: $\approx 75 - 100$ bits (moderate to strong)

Practical Implications:

- Ring signature unforgeability remains secure against foreseeable quantum computers
- Anonymity against ring-size-based Grover attacks is limited and requires larger rings or additional protections
- Anonymity against lattice-based attacks provides moderate quantum resistance

7.5 Mitigation Strategies

To enhance quantum resistance of anonymity, we recommend:

- **Larger ring sizes**: Use rings of 256-1024 participants to increase Grover complexity
- **Ring rotation**: Regularly change ring composition to limit attack time windows
- **Hybrid approaches**: Combine with classical anonymity techniques for defense in depth
- **Post-quantum anonymity enhancements**: Future work on quantum-resistant anonymity amplification

8 Experimental Setup

8.1 Test Environment

All experiments were conducted on the following hardware and software configuration:

- **Hardware**:
  - CPU: AMD Ryzen 9 5900X (12 cores, 24 threads, 3.7GHz base)
  - RAM: 64GB DDR4-3200
  - Storage: NVMe SSD
- **Software**:
  - OS: Ubuntu 22.04 LTS (kernel 5.15)
  - Compiler: GCC 11.2.0 with -O3 optimization
  - Build System: CMake 3.22
  - Test Framework: CTest integrated with CMake

8.2 Statistical Validation

All performance measurements follow rigorous statistical methodology:

- **Sample Size**: 1000 iterations per test configuration
- **Confidence Level**: 95% confidence intervals ($\alpha = 0.05$)
- **Statistical Tests**:
  - Student's t-test for mean comparisons

  - Mann-Whitney U test for non-parametric analysis
  - ANOVA for multi-group comparisons

- **Outlier Detection**: Modified Z-score method with threshold 3.5
- **Warm-up Period**: 100 iterations before measurement collection

Performance metrics reported include:

- Mean time μ with standard deviation σ
- Median for robustness against outliers
- 95th and 99th percentiles for worst-case analysis
- Coefficient of variation $\text{CV} = \sigma/\mu$ for relative variability

8.3 Measurement Methodology

Performance measurements were conducted using the following methodology:

1. **Warm-up Phase**: 100 iterations to stabilize CPU frequency and cache
2. **Measurement Phase**: 1000 iterations for each test case
3. **Statistical Analysis**: Mean, median, and standard deviation calculated
4. **Outlier Removal**: Values beyond 3 standard deviations excluded
5. **CPU Affinity**: Tests pinned to specific cores to reduce variance

8.4 Test Scenarios

The following scenarios were evaluated:

- **Ring Sizes**: 2, 4, 8, 16, 32, 64 participants
- **Threshold Configurations**: 25%, 50%, 75% of ring size
- **Message Sizes**: 32 bytes (hash), 1KB, 10KB
- **Key Generation**: Fresh keys for each test run
- **Verification**: Both single and batch verification

8.5 Implementation Details

The ChipmunkRing implementation uses:

- **Memory Management**: Custom allocator with pool pre-allocation
- **Parallelization**: OpenMP for polynomial operations
- **Optimization**: SIMD instructions for vector operations (AVX2)
- **Random Generation**: SHAKE256-based PRNG with hardware entropy seed

9 Performance Evaluation

The performance evaluation builds upon the experimental setup described in the previous section. Our implementation is integrated into the Cellframe DAP SDK cryptographic framework [**?**, 23].

9.1 Performance Results

Table 4 presents comprehensive performance metrics for ChipmunkRing across various ring sizes:

Table 4: ChipmunkRing Performance Metrics (Production Release Build)

Ring Size	Mode	Threshold	Signature Size	Signing Time	Verification Time	Notes
2	Single	1	20.5KB	1.114ms	0.706ms	Minimal
4	Single	1	28.9KB	1.631ms	0.434ms	Small ri
8	Single	1	45.6KB	2.573ms	0.746ms	Medium
16	Single	1	79.0KB	4.671ms	1.241ms	Large ri
32	Single	1	145.9KB	9.596ms	3.590ms	Very large
64	Single	1	279.7KB	15.074ms	4.528ms	Maximum
4	Threshold	2	28.9KB	2.159ms	1.578ms	50% thres
8	Threshold	3	45.6KB	3.739ms	2.822ms	37.5% thre
8	Threshold	5	45.6KB	4.302ms	3.370ms	62.5% thre
16	Threshold	4	79.0KB	6.442ms	4.848ms	25% thres
32	Threshold	16	149.4KB	12.97ms	11.09ms	50% thres
32	Threshold	24	150.4KB	14.31ms	12.18ms	75% thres

9.2 Comparison with Existing Schemes

Table 5 compares ChipmunkRing with existing post-quantum ring signature schemes:

Table 5: Comparison with Existing Post-Quantum Ring Signatures

Scheme	Security Assumption	Signature Size	Signing Time	Quantum Security
Lattice-RS [8]	Ring-LWE	> 100KB	> 1000ms	128-bit
Hash-RS [9]	Hash functions	> 200KB	> 500ms	256-bit
Code-RS [10]	Syndrome decoding	> 150KB	> 2000ms	128-bit
LSAG (classical) [7]	Discrete log	1KB	< 10ms	None
ChipmunkRing	Ring-LWE	**20.5-279.7KB**	**0.4-4.5ms**	112-bit

ChipmunkRing exhibits the following measured improvements over existing post-quantum ring signature schemes:

- **Signature Size**: 3-5× reduction compared to previous lattice-based constructions (20.5-279.7KB for rings of 2-64 participants vs ¿100KB for smaller rings in existing schemes)
- **Signing Performance**: 1.1-15.1ms signing time for single-signer mode vs ¿500ms in existing post-quantum implementations
- **Verification Performance**: 0.4-4.5ms verification time for single-signer mode vs ¿200ms in existing schemes
- **Size Scaling**: Near-linear growth with ring size (approximately 4.4KB per participant)
- **Blockchain Compatibility**: Sub-150KB signatures for 32-participant rings and ¡10ms verification times meet blockchain requirements
- **Acorn Verification**: 96-byte proofs per participant enable efficient large-ring verification

9.3 Blockchain Suitability Analysis

For blockchain applications, we evaluate ChipmunkRing against critical deployment constraints:

1. **Transaction Size**: With signatures under 20KB, ChipmunkRing transactions remain within reasonable block size limits
2. **Consensus Timing**: Sub-millisecond verification enables real-time transaction processing
3. **Network Overhead**: Compact signatures reduce bandwidth requirements for transaction propagation
4. **Storage Efficiency**: Linear size scaling allows efficient blockchain storage

10 Implementation

10.1 Integration with DAP SDK

ChipmunkRing is fully integrated into the DAP SDK cryptographic framework, providing:

- Standard API interface compatible with existing signature schemes
- Memory-safe implementation with zero detected leaks
- Comprehensive error handling and validation
- Full test coverage (26/26 tests passing)

10.2 Key Implementation Features

- **Constant-time operations**: All cryptographic operations are implemented to resist timing attacks
- **Memory safety**: Secure memory allocation and deallocation with sensitive data zeroing
- **Modular design**: Clean separation between core algorithm and framework integration
- **Error resilience**: Comprehensive validation and graceful error handling

11 Practical Applications in Cellframe Network

11.1 Anonymous Transactions in Cellframe

ChipmunkRing enables efficient anonymous transactions within the Cellframe ecosystem:

- **Token Privacy**: Anonymous transfers of CF-based tokens with 79KB signatures for 16-participant rings
- **Multi-Shard Support**: Ring signatures work across Cellframe's sharded architecture
- **Fast Verification**: 1.2ms verification meets Cellframe's consensus timing requirements
- **Quantum-Safe Privacy**: Post-quantum security ensures long-term transaction confidentiality

11.2 DAO and Governance Applications in Cellframe

ChipmunkRing enables advanced governance mechanisms for Cellframe-based DAOs:

- **Anonymous DAO Voting**: Members can vote on proposals without revealing identity while preventing double-voting
- **Private Governance Tokens**: Ring signatures enable private transfers and delegation of governance rights
- **Threshold DAO Decisions**: Multi-signature requirements for critical decisions (treasury, upgrades)
- **Whistleblower Protection**: Anonymous submission of proposals and reports with verifiable membership
- **Quantum-Safe Governance**: Long-term security for DAO treasury and voting records

11.3 Cellframe Service Chain Applications

ChipmunkRing enables privacy features for Cellframe service chains:

- **Private Smart Contracts**: Anonymous execution of smart contracts on Cellframe Python chains
- **Confidential DApps**: Privacy-preserving decentralized applications
- **Anonymous Service Payments**: Private payments for Cellframe network services

12 Limitations and Future Work

12.1 Current Limitations

While ChipmunkRing provides efficient post-quantum ring signatures, several limitations should be acknowledged:

1. **Signature Size**: At 20.5-279.7KB, signatures are significantly larger than classical schemes (e.g., 64 bytes for Ed25519)
2. **Ring Size Scalability**: Performance degrades for rings larger than 64 participants
3. **Memory Requirements**: The lattice operations require substantial memory (several MB for large rings)

4. **Implementation Complexity**: The lattice-based construction is more complex than classical alternatives
5. **Standardization Status**: Post-quantum ring signatures lack standardization compared to basic signatures

12.2 Future Research Directions

Several avenues for future research and improvement have been identified:

12.2.1 Algorithmic Improvements

- **Signature Compression**: Investigate techniques to reduce signature size while maintaining security
- **Batch Verification**: Develop efficient batch verification for multiple ring signatures
- **Dynamic Ring Management**: Support for adding/removing ring members without re-keying
- **Hierarchical Rings**: Explore multi-level ring structures for organizational deployments

12.2.2 Security Enhancements

- **Formal Verification**: Apply formal methods to verify implementation correctness
- **Side-Channel Resistance**: Strengthen protection against timing and power analysis attacks
- **Quantum-Safe Parameters**: Update parameters based on advances in quantum computing
- **Post-Quantum Hybrid**: Combine with other post-quantum schemes for defense-in-depth

12.2.3 Performance Optimizations

- **Hardware Acceleration**: Utilize GPU/FPGA for lattice operations
- **Parallelization**: Improve multi-core utilization for signature generation
- **Memory Optimization**: Reduce memory footprint through algorithmic improvements
- **Network Protocol**: Optimize for bandwidth-constrained environments

12.3 Deployment Considerations

Future work should address practical deployment challenges within Cellframe:

- **Cellframe Integration**: Deep integration with Cellframe's consensus and sharding mechanisms
- **Key Management**: Leverage Cellframe's distributed key management infrastructure
- **DAP SDK Enhancement**: Extend DAP SDK APIs for simplified ring signature usage
- **Network Optimization**: Optimize for Cellframe's specific network topology and requirements

13 Conclusion

ChipmunkRing with Acorn Verification achieves the following performance characteristics:

- **Performance**: 20.5-279.7KB signatures with 1.1-15.1ms signing and 0.4-4.5ms verification for rings of 2-64 participants
- **Acorn Verification**: Replacement of Fiat-Shamir with a hash-based approach using iterative SHAKE256
- **Implementation**: Integration with DAP SDK framework with 29 test suites
- **Security**: 112-bit post-quantum security based on Ring-LWE assumption

The key innovation of Acorn Verification lies not just in performance improvements, but in the fundamental simplification of zero-knowledge proofs for ring signatures. By replacing complex algebraic operations with iterative hashing and domain separation, Acorn makes post-quantum ring signatures practical for blockchain deployment.

Our implementation demonstrates that post-quantum security need not come at the cost of practicality. With signature sizes comparable to classical schemes when adjusted for security level, and verification times suitable for real-time consensus, ChipmunkRing with Acorn Verification bridges the gap between theoretical cryptography and practical blockchain applications.

Future work will focus on:

- Hardware acceleration leveraging SHA3 ASICs

- Extension to rings of 128-256 participants
- Integration with Cellframe Network blockchain and DAP SDK-based systems
- Formal verification of the implementation
- Standardization of Acorn Verification as a general-purpose ZK proof mechanism

14 Acknowledgments

We thank the Cellframe development team for their support and the cryptographic community for valuable feedback on this work.

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