Lab04: Decision Tree and Naive Bayes

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How to do your homework

You will work directly on this notebook; the word `TODO` indicate the parts you need to do.

You can discuss ideas with classmates as well as finding information from the internet, book, etc...; but *this homework must be your*.

How to submit your homework

- Before submitting, save this file as `<ID>.jl`. For example, if your ID is 123456, then your file will be `123456.jl`. And export to PDF with name `123456.pdf` then submit zipped source code file to `123456.zip` onto Moodle.

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> **Danger**
>
> **Note that you will get 0 point for the wrong submit**.

Contents:

Import library

```
begin
    using  Distributions, Plots, LinearAlgebra, Random, Statistics
end
```

```
TaskLocalRNG()
```

```
Random.seed!(2022)
```

Load Iris dataset

```
download_dataset (generic function with 2 methods)
```

```
# If you use Linux, use this function to download Iris dataset
function download_dataset(save_path::String="data")
    # setup directory
    mkpath(joinpath(dirname(@__FILE__), save_path))
    data_dir = joinpath(dirname(@__FILE__), save_path)
    download("https://archive.ics.uci.edu/static/public/53/iris.zip",joinpath(data_d
    ir, "iris.zip"))
    iris_file = joinpath(data_dir, "iris.zip")
    cd(data_dir)
    run(`unzip $iris_file -d $data_dir`)
    rm(iris_file)
    cd("..")
end
```

```
# If you have downloaded the dataset yet, please uncomment this line below and run
this cell. Otherwise, keep it in uncomment state.
# download_dataset()
```

```
# If you don't use Linux, I have no solution for you. Please makedir data, and goto
https://archive.ics.uci.edu/dataset/53/iris for dowloading
# Then, you can extract data to this dir by yourself.

# Structure:
# ├── data
# │   ├── bezdekIris.data
# │   ├── Index
# │   ├── iris.data
# │   └── iris.names
# ├── Lab4.jl
```

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```
iris_dataloader (generic function with 2 methods)
```

```
function iris_dataloader(data_path::String="data/iris.data")
    # Initialize empty arrays to store data
    sepal_length = Float64[]
    sepal_width = Float64[]
    petal_length = Float64[]
    petal_width = Float64[]
    classes = Int64[]

    # open and read the data file
    open(data_path, "r") do file
        # read data each line
        for line in eachline(file)
            if line != ""
                parts = split(line, ",")
                push!(sepal_length, parse(Float64, parts[1]))
                push!(sepal_width, parse(Float64, parts[2]))
                push!(petal_length, parse(Float64, parts[3]))
                push!(petal_width, parse(Float64, parts[4]))

                if parts[5] == "Iris-setosa"
                    push!(classes, 0)
                elseif parts[5] == "Iris-versicolor"
                    push!(classes, 1)
                else
                    push!(classes, 2)
                end
            end
        end
    end

    # concat features
    features = [sepal_length, sepal_width, petal_length, petal_width]
    features = vcat(transpose.(features)...)
    return features, classes
end
```

Selection deleted

```
# function change_class_to_num(y)
#   class = Dict("setosa"=> 0,"versicolor"=> 1, "virginica" => 2)
#   classnums = [class[item] for item in y]
#   return classnums
# end
```

train_test_split (generic function with 2 methods)

```
function train_test_split(X, y, test_ratio=0.33)
    X = X'
    n = size(X)[1]
    idx = shuffle(1:n)
    train_size = 1 - test_ratio
    train_idx = view(idx, 1:floor(Int, train_size*n))
    test_idx = view(idx, (floor(Int, train_size*n)+1):n)

    X_train = X[train_idx,:]
    X_test = X[test_idx,:]

    y_train = y[train_idx]
    y_test = y[test_idx]

    return X_train, X_test, y_train, y_test
end
```

((100, 4), (50, 4), (100), (50))

```
begin
    # Load features, and labels for Iris dataset
    iris_features, iris_labels = iris_dataloader("data/iris.data")

    #split dataset into training data and testing data
    X_train, X_test, y_train, y_test = train_test_split(iris_features, iris_labels,
    0.33)
Selection deleted
    size(X_train), size(X_test), size(y_train), size(y_test)
end
```

1. Decision Tree: Iterative Dichotomiser 3 (ID3)

1.1 Information Gain

Expected value of the self-information (entropy):

$$Entropy = -\sum_{i}^{n} p_i log_2(p_i)$$

The entropy function gets the smallest value if there is a value of p_i equal to 1, reaches the maximum value if all p_i are equal. These properties of the entropy function make it is an expression of the disorder, or randomness of a system, ...

entropy

Parameters:

- `counts` : shape (n_classes): list number of samples in each class
- `n_samples:` number of data samples

Returns

- entropy

```
"""
Parameters:
- `counts`: shape (n_classes): list number of samples in each class
- `n_samples:` number of data samples

Returns
- entropy
"""
function entropy(counts, n_samples)
    #TODO
    if n_samples == 0
        return 0.0
    end

    probs = counts ./ n_samples
    entropy = -sum(probs .* log2.(probs))

    return entropy
end
```

Selection deleted

`entropy_of_one_division`

Returns entropy of a divided group of data

Data may have multiple classes

```
"""
Returns entropy of a divided group of data

Data may have multiple classes
"""
function entropy_of_one_division(division)

    n_samples = size(division, 1)
    n_classes = Set(division)

    counts=[]
    # println("DIVISION: ", division)
    # println("SAMPLES:", n_samples)
    # println("CLASSES:", n_classes)

    # count samples in each class then store it to list counts
    #TODO:
    for sample in n_classes
        push!(counts, count(i->(i==sample), division))
    end
    entropy_value = entropy(counts, n_samples)


    return entropy_value, n_samples

end
```

Selection deleted

get_entropy

Returns entropy of a split

y_predict is the split decision by cutoff, True/Fasle

```
"""
Returns entropy of a split

y_predict is the split decision by cutoff, True/Fasle
"""
function get_entropy(y_predict, y)
    n = size(y,1)
    # println("y_predict:", y_predict)
    # println("y:", y)
    # left hand side entropy
    entropy_true, n_true = entropy_of_one_division(y[y_predict])

    # right hand side entropy
    entropy_false, n_false = entropy_of_one_division(y[.~y_predict])

    # println("TRUE:", entropy_true, n_true)
    # println("FALSE:", entropy_false, n_false)
    # overall entropy
     #TODO s=?
    p_true = n_true / n
    p_false = n_false / n
    s = p_true * entropy_true + p_false * entropy_false

    return s
end
```

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The information gain of classifying information set D by attribute A:

$$Gain(A) = Entrophy(D) - Entrophy_A(D)$$

At each node in ID3, an attribute is chosen if its information gain is highest compare to others.

All attributes of the Iris set are represented by continuous values. Therefore we need to represent them with discrete values. The simple way is to use a `cutoff` threshold to separate values of the data on each attribute into two part: `<cutoff` and `> = cutoff`.

To find the best `cutoff` for an attribute, we replace `cutoff` with its values then compute the entropy, best `cutoff` achieved when value of entropy is smallest $(\arg\min Entrophy_A(D))$.

1.2 Decision tree

dtfit

Parameters:

- X: training data
- y: label of training data

Returns

- node

node: each node represented by cutoff value and column index, value and children.

- cutoff value is thresold where you divide your attribute.
- column index is your data attribute index.
- value of node is mean value of label indexes, if a node is leaf all data samples will have same label.

Note that: we divide each attribute into 2 part => each node will have 2 children: left, right.

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```
"""
Parameters:
- X: training data
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Returns
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node: each node represented by cutoff value and column index, value and children.
- cutoff value is thresold where you divide your attribute.
- column index is your data attribute index.
- value of node is mean value of label indexes, if a node is leaf all data samples will have same label.

Note that: we divide each attribute into 2 part => each node will have 2 children: left, right.
"""
function dtfit(X, y, node=Dict(), depth=0)
    #Stop conditions

    #if all value of y are the same
    if all(y.==y[1])
        return Dict("val"=>y[1])

    else
        # find one split given an information gain
        col_idx, cutoff, entropy = find_best_split_of_all(X, y)

        y_left = y[X[:,col_idx] .< cutoff]
        y_right = y[X[:,col_idx] .>= cutoff]

        node = Dict("index_col"=>col_idx,
                    "cutoff"=>cutoff,
```

```
                    "val"=> mean(y),
                    "left"=> Any,
                    "right"=> Any)

        left = dtfit(X[X[:,col_idx] .< cutoff, :], y_left, Dict(), depth+1)
        right= dtfit(X[X[:,col_idx] .>= cutoff, :], y_right, Dict(), depth+1)

        push!(node, "left" => left)
        push!(node, "right" => right)

        depth += 1
    end
    return node
end
```

find_best_split_of_all

Parameters:

- X: training data
- y: label of training data

Returns

- column index, cut-off value, and minimum entropy

Selection deleted

```
"""
Parameters:
- X: training data
- y: label of training data

Returns
- column index, cut-off value, and minimum entropy
"""
function find_best_split_of_all(X, y)
    col_idx = nothing
    min_entropy = 1
    cutoff = nothing

    for i in 1:size(X,2)
        col_data = X[:,i]
        entropy, cur_cutoff = find_best_split(col_data, y)

        # best entropy
        if entropy == 0
            return i, cur_cutoff, entropy
        elseif entropy <= min_entropy
            min_entropy = entropy
            col_idx = i
            cutoff = cur_cutoff
        end
    end
    return col_idx, cutoff, min_entropy
end
```

`find_best_split`

Parameters:

- col_data: data samples in column
- y: label of training data

Returns

- minimum entropy, and cut-off value

```
"""
Parameters:
- col_data: data samples in column
- y: label of training data

Returns
- minimum entropy, and cut-off value
"""
function find_best_split(col_data, y)
    min_entropy = 10
    cutoff = 0

    #loop through col_data find cutoff where entropy is minimum

    for value in Set(col_data)
        y_predict = col_data .< value
        my_entropy = get_entropy(y_predict, y)

        #TODO
        #min entropy=?, cutoff=?
        for entropy in my_entropy
            if entropy <= min_entropy
                min_entropy = entropy
                cutoff = value
            end
        end

    end
    return min_entropy, cutoff
end
```

Selection deleted

```
dtpredict (generic function with 1 method)
```

```
function dtpredict(tree, data)
    pred = []
    n_sample = size(data, 1)
    for i in 1:n_sample
        push!(pred, _dtpredict(tree, data[i,:]))
    end
    return pred
end
```

_dtpredict (generic function with 1 method)

```
function _dtpredict(tree, row)
    cur_layer = tree
    while haskey(cur_layer, "cutoff")
            if row[cur_layer["index_col"]] < cur_layer["cutoff"]
                cur_layer = cur_layer["left"]
            else
                cur_layer = cur_layer["right"]
            end
        end
    if !haskey(cur_layer, "cutoff")
        return get(cur_layer, "val", false)
    end
end
```

1.3 Classification on Iris Dataset

tpfptnfn_cal (generic function with 2 methods)

```
function tpfptnfn_cal(y_test, y_pred, positive_class=1)
    true_positives = 0
    false_positives = 0
    true_negatives = 0
    false_negatives = 0

    # Calculate true positives, false positives, false negatives, and true negatives
    for (true_label, predicted_label) in zip(y_test, y_pred)
        if true_label == positive_class && predicted_label == positive_class
            true_positives += 1
        elseif true_label != positive_class && predicted_label == positive_class
            false_positives += 1
        elseif true_label == positive_class && predicted_label != positive_class
            false_negatives += 1
        elseif true_label != positive_class && predicted_label != positive_class
            true_negatives += 1
        end
    end

    return true_positives, false_positives, true_negatives, false_negatives
end
```

tree =

```
Dict("left" => Dict("val" => 0), "cutoff" => 1.0, "right" => Dict("left" => Dict("left"
```

```
tree = dtfit(X_train, y_train)
```

```
begin
    pred = dtpredict(tree, X_test)

    acc = 0
    precision = 0
    recall = 0
    f1 = 0

    for i ∈ [0, 1, 2]
        # Calculate true positives, false positives, false negatives, and true negatives
        true_positives, false_positives, true_negatives, false_negatives = tpfptnfn_cal(y_test, pred, i)

        # Calculate precision, recall, and F1-score
        acc += (true_positives + true_negatives) / (true_positives + false_positives + true_negatives + false_negatives)
        precision += true_positives / (true_positives + false_positives)
        recall += true_positives / (true_positives + false_negatives)
    end

    acc = acc / 3
    precision = precision / 3
    recall = recall / 3
    f1 = 2 * precision * recall / (precision + recall)
    print(" acc: $acc\n precision: $precision\n recall: $recall\n f1_score: $f1\n")
end
```

```
acc: 0.9733333333333333
precision: 0.9649122807017544
recall: 0.9393939393939394
f1_score: 0.9519821328866556
```

2. Bayes Theorem

Bayes formulation $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

If B is our data $\mathcal{D}$, A and w are parameters we need to estimate:

$$\underbrace{P(w|\mathcal{D})}_{Posterior} = \frac{1}{\underbrace{P(\mathcal{D})}_{Normalization}} \overbrace{P(\mathcal{D}|w)P(w)}^{\text{Likelihood } Prior}$$

Naive Bayes

To make it simple, it is often assumed that the components of the D random variable (or the features of the D data) are independent with each other, if w is known. It mean:

$$P(\mathcal{D}|w) = \prod_{i=1}^{d} P(x_i|w)$$

- d: number of features

2.1. Probability Density Function

```
update (generic function with 1 method)
```

```
#update histogram for new data
function update(_hist, _mean, _std, data)
    """
    P(hypo/data)=P(data/hypo)*P(hypo)*(1/P(data))
    """
    hist = copy(_hist)
    #P(hypo/data)=P(data/hypo)*P(hypo)*(1/P(data))

    #Likelihood * Prior
    #TODO
    for hypo in keys(hist)
        hist[hypo] = likelihood(_mean, _std, data, hypo) * hist[hypo]
    end

    #Normalization

    #TODO: s=P(data)
    #s=?
    s = sum(values(hist))

    for hypo in keys(hist)
        hist[hypo] = hist[hypo]/s
    end
    return hist
end
```

maxHypo (generic function with 1 method)

```
function maxHypo(hist)
    #find the hypothesis with maximum probability from dictionary hist
    #TODO
    max_prob = maximum(values(hist))
    max_hypo = 0
    for (key, val) in hist
        if val == max_prob
            max_hypo = key
            break
        end
    end


    return max_hypo
end
```

2.2 Classification on Iris Dataset

Gaussian Naive Bayes

- Naive Bayes can be extended to use on continuous data, most commonly by using a normal distribution (Gaussian distribution).
- This extension called Gaussian Naive Bayes. Other functions can be used to estimate data distribution, but Gauss (or the normal distribution) is the easiest to work with since we only need to estimate the mean and standard deviation from the training data.

Define Gauss function

$$f\left(x;\mu,\sigma\right)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{\left(x-\mu\right)^2}{2\sigma^2}\right)$$

Gauss (generic function with 1 method)

```
function Gauss(std, mean, x)
    #Compute the Gaussian probability distribution function for x
    #TODO
    return (1 ./ (std * sqrt( 2 * π ))) * exp( -((x .- mean)^2) / (2 * std^2))
end
```

likelihood (generic function with 5 methods)

```
function likelihood(_mean=nothing, _std=nothing ,data=nothing, hypo=nothing)
    """
    Returns: res=P(data/hypo)
    -----------------
    Naive bayes:
        Atributes are assumed to be conditionally independent given the class value.
    """

    std=_std[hypo]
    mean=_mean[hypo]
    res=1
    #TODO
    #res=res*P(x1/hypo)*P(x2/hypo)...
    for i in 1:length(data)
        res *= pdf(Normal(mean[i], std[i]), data[i])
    end

    return res
end
```

custom_std (generic function with 1 method)

```
# Custom standard deviation function
function custom_std(data, mean_val)
    return sqrt(sum((data .- mean_val) .^ 2) / length(data))
end
```

custom_mean (generic function with 1 method)

```
# Custom mean function
function custom_mean(data)
    return sum(data) / length(data)
end
```

```
gfit (generic function with 4 methods)
```

```
function gfit(X, y, _std=nothing, _mean=nothing, _hist=nothing)
    """Parameters:
    X: training data
    y: labels of training data
    """
    n=size(X,1)
    #number of iris species
    #TODO
    #n_species=???
    n_species = length(Set(y))
    # println(n)
    # println(Set(y))

    hist=Dict()
    mean=Dict()
    std=Dict()

    #separate  dataset into rows by class
    for hypo in Set(y)
        #rows have hypo label
        #TODO rows=
        rows = X[y .== hypo, :]
        # println("hypo=", hypo, rows)
        #histogram for each hypo
        #TODO probability=?
        probability = size(rows, 1) / n
        hist[hypo]=probability
        # println(hist)

        #Each hypothesis represented by its mean and standard derivation
        # """mean and standard derivation should be calculated for each column (or each attribute)"""
        #TODO mean[hypo]=?, std[hypo]=?

        mean[hypo] = [custom_mean(rows[:, col]) for col in 1:size(rows, 2)]
        std[hypo] = [custom_std(rows[:, col], mean[hypo][col]) for col in 1:size(rows, 2)]

    end
    _mean=mean
    _std=std
    _hist=hist
    return _hist, _mean, _std
end
```

_gpredict (generic function with 2 methods)

```
function _gpredict(_hist, _mean, _std, data, plot=true)
    """
    Predict label for only 1 data sample
    ------------
    Parameters:
    data: data sample
    plot: True: draw histogram after update new record
    -----------
    return: label of data
    """
    hist = update(_hist, _mean, _std, data)
    # println(hist)
    if (plot == true)
        plt = bar(collect(keys(hist)), collect(values(hist)))
    end
    return maxHypo(hist)
end
```

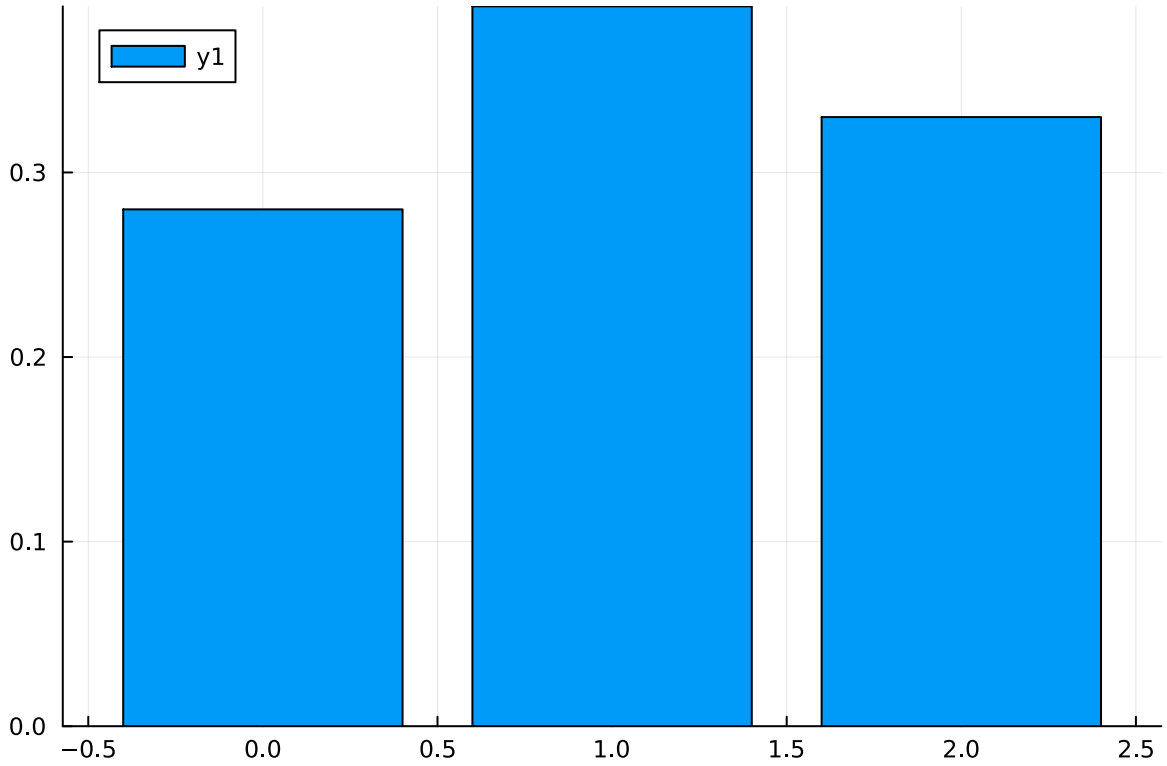
plot_pdf (generic function with 1 method)

```
function plot_pdf(_hist)
     bar(collect(keys(_hist)), collect(values(_hist)))
end
```

gpredict (generic function with 1 method)

```
function gpredict(_hist, _mean, _std, data)
    """Parameters:
    Data: test data
    ----------
    return labels of test data
    """
    pred=[]
    n_sample = size(data, 1)
    for i in 1:n_sample
        push!(pred, _gpredict(_hist, _mean, _std, data[i,:]))
    end
    return pred
end
```

Show histogram of training data




```
begin
    # print(X_train, y_train)
    _hist, _mean, _std = gfit(X_train, y_train)
    plt = plot_pdf(_hist)
end
```

Test wih 1 data record

```
begin
    #label of test_y[10]
    print("Label of X_test[10]: ", y_test[20])

    #update model and show histogram with X_test[10]:
    print("\nOur histogram after update X_test[10]: ", _gpredict(_hist, _mean,
    _std, X_test[20,:], true))

end
```

```
Label of X_test[10]: 1
Our histogram after update X_test[10]: 1
```

Evaluate your Gaussian Naive Bayes model

```
begin
    _pred = gpredict(_hist, _mean, _std, X_test)

    _acc = 0
    _p = 0
    _r = 0
    _f1 = 0

    #TODO: Self-define and calculate accuracy, precision, recall, and f1-score

    for i ∈ [0, 1, 2]
        # Calculate true positives, false positives, false negatives, and true
        negatives
        tp, fp, tn, fn = tpfptnfn_cal(y_test, _pred, i)

        # Calculate precision, recall, and F1-score
        _acc += (tp + tn) / (tp + fp + tn + fn)
        _p += tp / (tp + fp)
        _r += tp / (tp + fn)
    end

    _acc = _acc / 3
    _p = _p / 3
    _r = _r / 3
    _f1 = 2 * _p * _r / (_p + _r)

    print(" acc: $_acc\n precision: $_p\n recall: $_r\n f1_score: $_f1\n")
end
```

```
acc: 0.9866666666666667
precision: 0.9814814814814815
recall: 0.9696969696969697
f1_score: 0.9755536381938454
```

TODO: F1, Recall and Precision report

The Gaussian Naive Bayes model demonstrates strong performance across multiple evaluation metrics:

- **Accuracy (acc):** The model achieves an accuracy of approximately 98.67%, signifying the overall correctness of its predictions. This metric is calculated as the ratio of correctly predicted instances to the total number of instances.
- **Precision:** With a precision value of approximately 98.15%, the model excels in making accurate positive predictions. Precision measures the accuracy of the model's positive predictions, reflecting its ability to avoid false positives.
- **Recall:** The model achieves a recall, or sensitivity, of approximately 96.97%. This metric indicates the model's capability to capture a high percentage of actual positive instances. A higher recall value suggests that the model is effective at identifying positive cases.
- **F1-score:** The F1-score, computed as approximately 97.56%, represents the harmonic mean of precision and recall. This balanced metric considers both false positives and false negatives, providing a comprehensive assessment of the model's performance.

In summary, the Gaussian Naive Bayes model exhibits robust performance, demonstrating high accuracy, precision, recall, and F1-score across various aspects of its predictions.