Ctrl + S

Lab03: Logistic Regression.

- Student ID:
- Student name:

How to do your homework

You will work directly on this notebook; the word `TODO` indicate the parts you need to do.

You can discuss ideas with classmates as well as finding information from the internet, book, etc...; but *this homework must be your*.

How to submit your homework

- Before submitting, save this file as `<ID>.jl`. For example, if your ID is 123456, then your file will be `123456.jl`. And export to PDF with name `123456.pdf` then submit zipped source code and pdf into `123456.zip` onto Moodle.

Danger

Note that you will get 0 point for the wrong submit.

Contents:

- Logistic Regression.

1. Feature Extraction

Import Library

```
begin
    using Distributions, Plots, Images, LinearAlgebra, Random
end
```

```
MersenneTwister(2024)
```

```
Random.seed!(2024)
```

Load data

```
# I DON'T KNOW WHAT YOUR OS, SO I ATTACHED DATA FOR YOU
# YOU DON'T NEED TO DOWNLOAD AND EXTRACT BY YOURSELF

# IF YOU WANT TO USE JULIA TO DOWNLOAD DATA, LET'S USE THESE CODE
# FOR EXTRACTING MNIST .gz FILE, PLEASE USE gzip

# function download_dataset(save_path::String="data")
#   # setup directory
#   mkpath(joinpath(dirname(@__FILE__), save_path))
#   data_dir = joinpath(dirname(@__FILE__), save_path)
#   mkpath(joinpath(data_dir, "train"))
#   train_dir =  joinpath(data_dir, "train")
#   mkpath(joinpath(data_dir, "test"))
#   test_dir = joinpath(data_dir, "test")

#   # download dataset
#   mkpath(joinpath(train_dir, "images"))
#   download("http://yann.lecun.com/exdb/mnist/train-images-idx3-ubyte.gz",joinpath(train_dir, "images/train-images-idx3-ubyte.gz"))
#   train_images_file = joinpath(train_dir, "images/train-images-idx3-ubyte.gz")

#   mkpath(joinpath(train_dir, "labels"))
#   download("http://yann.lecun.com/exdb/mnist/train-labels-idx1-ubyte.gz",joinpath(train_dir, "labels/train-labels-idx1-ubyte.gz"))
#   train_labels_file = joinpath(train_dir, "labels/train-labels-idx1-ubyte.gz")

#   mkpath(joinpath(test_dir, "images"))
#   download("http://yann.lecun.com/exdb/mnist/t10k-images-idx3-ubyte.gz",joinpath(test_dir, "images/t10k-images-idx3-ubyte.gz"))
#   test_images_file = joinpath(test_dir, "images/t10k-images-idx3-ubyte.gz")

#   mkpath(joinpath(test_dir, "labels"))
#   download("http://yann.lecun.com/exdb/mnist/t10k-labels-idx1-ubyte.gz",joinpath(test_dir, "labels/t10k-labels-idx1-ubyte.gz"))
#   test_labels_file = joinpath(test_dir, "labels/t10k-labels-idx1-ubyte.gz")
# end
```

```
# If you have downloaded the dataset yet, please uncomment this line below and run this cell. Otherwise, keep it in uncomment state.
# download_dataset()
```

```
10000
```

```julia
begin
    data_dir = joinpath(dirname(@__FILE__), "data")
    train_x_dir = joinpath(data_dir, "train/images/train-images.idx3-ubyte")
    train_y_dir = joinpath(data_dir, "train/labels/train-labels.idx1-ubyte")

    test_x_dir = joinpath(data_dir, "test/images/t10k-images.idx3-ubyte")
    test_y_dir = joinpath(data_dir, "test/labels/t10k-labels.idx1-ubyte")

    NUMBER_TRAIN_SAMPLES = 60000
    NUMBER_TEST_SAMPLES = 10000
end
```

```
60000×784 adjoint(::Matrix{Float64}) with eltype Float64:
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 ⋮                        ⋮              ⋱                      ⋮
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
```

```julia
begin
    # Init arrays
    train_x = Array{Float64}(undef, 28^2, NUMBER_TRAIN_SAMPLES)
    train_y = Array{Int64}(undef, NUMBER_TRAIN_SAMPLES)

    # Init io streams
    io_images = open(train_x_dir)
    io_labels = open(train_y_dir)

    # Iterating through sample length
    for i ∈ 1:NUMBER_TRAIN_SAMPLES
        seek(io_images, (i-1)*28^2 + 16) # offset 16 to skip header
        seek(io_labels, (i-1)*1 + 8) # offset 8 to skip header
        train_x[:,i] = convert(Array{Float64}, read(io_images, 28^2))
        train_y[i] = convert(Int, read(io_labels, UInt8))
    end

    # Close io streams
    close(io_images)
    close(io_labels)

    # Transpose features
    train_x = train_x'
end
```

```
10000×784 adjoint(::Matrix{Float64}) with eltype Float64:
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 ⋮                        ⋮              ⋱                 ⋮
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
```

```julia
begin
    # Init arrays
    test_x = Array{Float64}(undef, 28^2, NUMBER_TEST_SAMPLES)
    test_y = Array{Int64}(undef, NUMBER_TEST_SAMPLES)

    # Init io streams
    io_images_test = open(test_x_dir)
    io_labels_test = open(test_y_dir)

    # Iterating through sample length
    for i ∈ 1:NUMBER_TEST_SAMPLES
        seek(io_images_test, (i-1)*28^2 + 16) # offset 16 to skip header
        seek(io_labels_test, (i-1)*1 + 8) # offset 8 to skip header
        test_x[:,i] = convert(Array{Float64}, read(io_images_test, 28^2))
        test_y[i] = convert(Int, read(io_labels_test, UInt8))
    end

    # Close io streams
    close(io_images)
    close(io_labels)

    # Transpose features
    test_x = test_x'
end
```

```
((60000, 784), (60000), (10000, 784), (10000))
```

```julia
size(train_x), size(train_y), size(test_x), size(test_y)
```

Extract Features

So we basically have 70000 samples with each sample having 784 features - pixels in this case and a label - the digit the image represent.

Let's play around and see if we can extract any features from the pixels that can be more informative. First I'd like to know more about average intensity - that is the average value of a pixel in an image for the different digits

```
compute_average_intensity (generic function with 1 method)
```

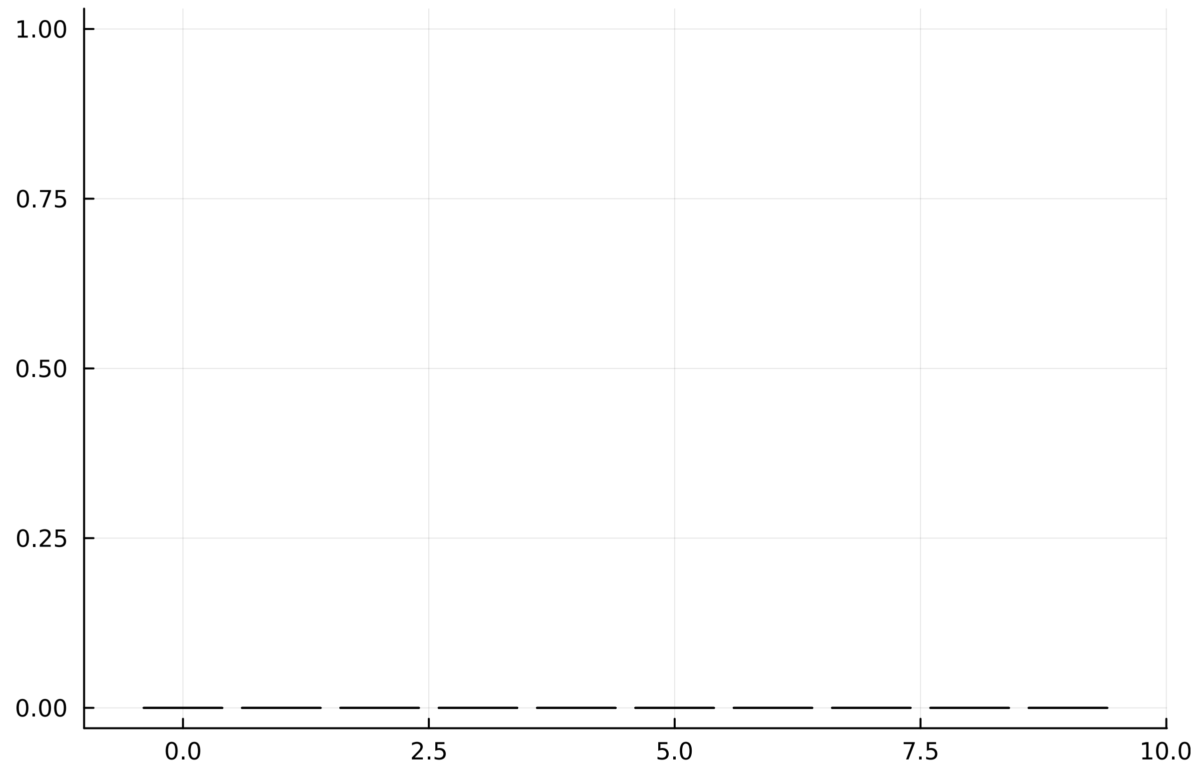
```
#TODO compute average intensity for each label
function compute_average_intensity(x, y)
    mean_ = zeros(10) # 10 is number of labels
    #TODO compute average intensity for each label

    return mean_
end
```

```
l_mean =  [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
```

```
l_mean = compute_average_intensity(train_x, train_y)
```

Plot the average intensity using matplotlib




```
bar(0:9, l_mean, legend=false)
```

UndefVarError: intensity not defined

1. **top-level scope** @ *Local: 3*

```
begin
    #TODO compute average intensity for each data sample
    intensity =
    size(intensity)
end
```

Some digits are symmetric (1, 3, 8, 0) some are not (2, 4, 5, 6, 9). Creating a new feature capturing this could be useful. Specifically, we calculate $s = -\frac{s_1+s_2}{2}$ for each image:

- s_1 : flip the image along y-axis and compute the mean value of result
- s_2 : flip the image along x-axis and compute the mean value of result

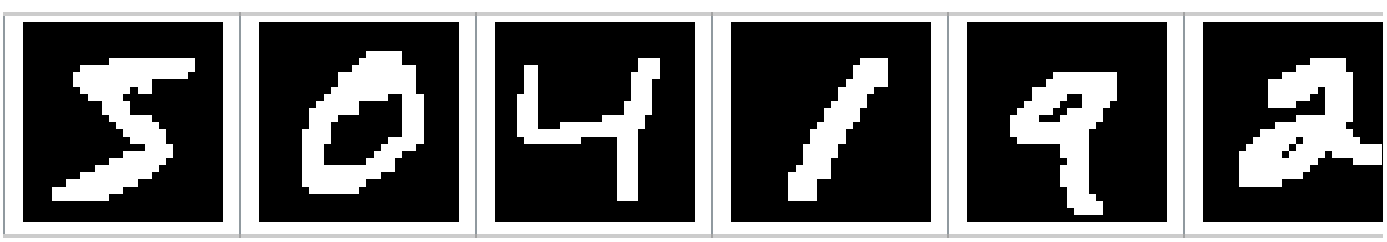
```
compute_symmetry (generic function with 1 method)
```

```
function compute_symmetry(train_x)
    symmetry = []
    for i in 1:size(train_x)[1]
        img = reshape(train_x[i,:], (28,28))
        s1 = mean(abs.(img - reverse(img, dims=1)))
        s2 = mean(abs.(img - reverse(img, dims=2)))
        s = -0.5 .* (s1 + s2)
        append!(symmetry, s)
    end
    return symmetry
end
```

```
(60000)
```

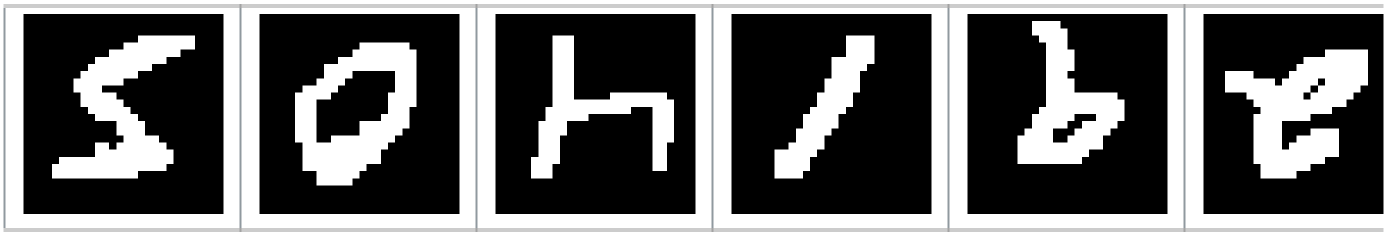
```
begin
    symmetry = compute_symmetry(train_x)
    size(symmetry)
end
```

Visualize 10 samples in order to illustrate symmetry



(a vector displayed as a row to save space)

```
begin
    num_img = 10
    img_flat = train_x[1:num_img,:]
    img = [reshape(img_flat[i,:], (28,28))' for i in 1:num_img]
    [colorview(Gray, Float32.(img[i])) for i in 1:num_img]
end
```



(a vector displayed as a row to save space)

```
begin
    img_reverse_flat = reverse(img_flat, dims=2)
    img_reverse = [reshape(img_reverse_flat[i,:], (28,28))' for i in 1:num_img]
    [colorview(Gray, Float32.(img_reverse[i])) for i in 1:num_img]
end
```

Our new data will have 70000 samples and 2 features: intensity, symmetry.

```
UndefVarError: train_x_new not defined

 1. top-level scope @ Local: 3
```

```
begin
    #TODO create X_new by horizontal stack intensity and symmetry
    train_x_new =
    size(train_x_new)
end
```

2. Training

Usually logistic regression is a good first choice for classification. In this homework we use logistic regression for classifying digit 1 images and not digit 1's images.

Normalize data

First normalize data using Z-score normalization

- **TODO: Study about Z-score normalization**
- **TODO: Why should we normalize data?**

```
normalize (generic function with 3 methods)
```

```julia
function normalize(x, mean_=nothing, std_=nothing)
   if mean_ == nothing && std_ == nothing
        #TODO normalize x_train
        #return 'normalized_train_x', 'mean_', 'std_'
        #mean_ and std_ will be re-used to pre-process test set
    end
    return #return 'normalized_train_x' calculated by using mean_ and std_ (both of them are passed and not null)
end


```

Another cell defining train_x_new contains errors.

```julia
begin
    normalized_train_x, mean_, std_ = normalize(train_x_new)

    s_mean = compute_average_intensity(normalized_train_x, train_y)
    bar(0:9, s_mean, legend=false)
end
```

Construct data

```
(60000, 1)
```

```julia
begin
    train_y_new = reshape(deepcopy(train_y), (size(train_y)[1], 1))
    train_y_new[train_y_new .!= 1] .= 0
    size(train_y_new)
end
```

Another cell defining train_x_new contains errors.

```julia
begin
    # contruct data by adding ones
    add_one_train_x = hcat(ones(size(normalized_train_x)[1],), normalized_train_x)
    size(add_one_train_x)
end
```

Sigmoid function and derivative of the sigmoid function

```
sigmoid_activation (generic function with 1 method)
```

```julia
function sigmoid_activation(x)
    #TODO
    """compute the sigmoid activation value for a given input"""
    #return?
    return
end
```

sigmoid_deriv (generic function with 1 method)

```
function sigmoid_deriv(x)
    #TODO
    """
    Compute the derivative of the sigmoid function ASSUMING
    that the input 'x' has already been passed through the sigmoid
    activation function
    """
    #return?
    return
end
```

Compute output

compute_h (generic function with 1 method)

```
function compute_h(W, X)
    #TODO
    """
    Compute output: Take the inner product between our features 'X' and the weight
    matrix 'W'
    """
    return
end
```

predict (generic function with 1 method)

```
function predict(W, X)
    #TODO
    """
    Take the inner product between our features and weight matrix,
    then pass this value through our sigmoid activation
    """
    preds =

    # apply a step function to threshold the outputs to binary
    # class labels
    preds[preds .<= 0.5] .= 0
    preds[preds .> 0] .= 1

    return preds
end
```

Compute gradient

Loss Function: Average negative log likelihood

$$\mathcal{L} = \frac{1}{N}\sum_{i=1}^{N} -\left(y^i \ln h_{\mathbf{w}}\left(\mathbf{x}^i\right) + \left(1-y^i\right)\ln\left(1-h_{\mathbf{w}}\left(x^i\right)\right)\right)$$

$$\text{Sigmoid Activation: } z = \sigma\left(h\right) = \frac{1}{1+e^{-h}}$$

$$\text{Cross-entropy: } J(w) = -\left(ylog(z) + (1-y)log(1-z)\right)$$

$$\text{Chain rule: } \frac{\partial J(w)}{\partial w} = \frac{\partial J(w)}{\partial z}\frac{\partial z}{\partial h}\frac{\partial h}{\partial w}$$

$$\frac{\partial J(w)}{\partial z} = -\left(\frac{y}{z} - \frac{1-y}{1-z}\right) = \frac{z-y}{z(1-z)}$$

$$\frac{\partial z}{\partial h} = z(1-z)$$

$$\frac{\partial h}{\partial w} = X$$

$$\frac{\partial J(w)}{\partial w} = X^T(z-y)$$

compute_gradient (generic function with 1 method)

```julia
function compute_gradient(error, train_x)
    #TODO
    """
    This is the gradient descent update of "average negative loglikelihood" loss function.
    In lab02 our loss function is "sum squared error".
    """
    return
end
```

```
train (generic function with 1 method)
```

```
function train(W, train_x, train_y, learning_rate, num_epochs)
    losses = []
    for epoch in 1:num_epochs
        y_hat = sigmoid_activation(compute_h(W, train_x))
        error = y_hat - train_y
        append!(losses, mean(-1 .* train_y .* log.(y_hat) .- (1 .- train_y) .* log.(1 .- y_hat)))
        grad = compute_gradient(error, train_x)
        W -= learning_rate * grad

        if epoch == 1 || epoch % 50 == 0
            print("Epoch=$epoch; Loss=$(losses[end])\n")
        end
    end
    return W, losses
end
```

Train our model

Another cell defining train_x_new contains errors.

```
begin
    W = rand(Normal(), (size(add_one_train_x)[2], 1))

    num_epochs=2000
    learning_rate=0.01
    W, losses = train(W, add_one_train_x, train_y_new, learning_rate, num_epochs)
end
```

Another cell defining train_x_new and num_epochs contains errors.

```
plot(1:num_epochs, losses, legend=false)
```

3. Evaluate our model

In this section, you will evaluate your model on train set and test set and make some comment about the result.

Evaluate model on training set

tpfptnfn_cal (generic function with 1 method)

```
function tpfptnfn_cal(y_test, y_pred, positive_class=1)
    true_positives = 0
    false_positives = 0
    true_negatives = 0
    false_negatives = 0

    # Calculate true positives, false positives, false negatives, and true negatives
    for (true_label, predicted_label) in zip(y_test, y_pred)
        if true_label == positive_class && predicted_label == positive_class
            true_positives += 1
        elseif true_label != positive_class && predicted_label == positive_class
            false_positives += 1
        elseif true_label == positive_class && predicted_label != positive_class
            false_negatives += 1
        elseif true_label != positive_class && predicted_label != positive_class
            true_negatives += 1
        end
    end

    return true_positives, false_positives, true_negatives, false_negatives
end
```

Another cell defining train_x_new contains errors.

```
begin
    preds_train = predict(W, add_one_train_x)
    train_y_n = reshape(train_y_new, length(train_y_new), 1)

    acc = 0
    precision = 0
    recall = 0
    f1 = 0

    for i ∈ 1:10
        # Calculate true positives, false positives, false negatives, and true negatives
        true_positives, false_positives, true_negatives, false_negatives = tpfptnfn_cal(train_y_n, preds_train)

        # Calculate precision, recall, and F1-score
        acc += (true_positives + true_negatives) / (true_positives + false_positives + true_negatives + false_negatives)
        precision += true_positives / (true_positives + false_positives)
        recall += true_positives / (true_positives + false_negatives)
    end

    acc = acc / 10
    precision = precision / 10
    recall = recall / 10
    f1 = 2 * precision * recall / (precision + recall)
    print(" acc: $acc\n precision: $precision\n recall: $recall\n f1_score: $f1\n")
end
```

Evaluate model on test set

In order to predict the result on test set, you have to perform data pre-process first. The pre-process is done exactly what we have done on train set. That means, you have to:

- Change the label in `test_y` to `0` and `1` and store in a new variable named `test_y_new`
- Calculate `test_intensity` and `test_symmetry` to form `test_x_new` (the shape should be `(10000,2)`)
- Normalized `test_x_new` by z-score. Note the you will re-use variable `mean_` and `std_` to calculate `test_x_new` instead of compute new ones. You will store the result in `normalized_test_x`
- Add a column that's full of one to `test_x_new` and store in `add_one_test_x` (the shape should be `(10000,3)`)

`UndefVarError: add_one_test_x not defined`

1. **`top-level scope`** @ *Local: 4*

```
begin
    #TODO
    # compute test_y_new
    test_y_new =


    # compute test_intensity and test_symmetry to form test_x_new
    test_intensity =
    test_symmetry =
    test_x_new =

    # normalize test_x_new to form normalized_test_x
    normalized_test_x =

    # add column 'ones' to test_x_new
    add_one_test_x =
    size(add_one_test_x)
end
```

After doing all these stuffs, you now can predict and evaluate your model

Another cell defining train_x_new, add_one_test_x and test_y_new contains errors.

```julia
begin
    preds_test = predict(W, add_one_test_x)

    test_y_n = reshape(test_y_new, length(test_y_new), 1)

    _acc = 0
    _p = 0
    _r = 0
    _f1 = 0

    for i ∈ 1:10
        # Calculate true positives, false positives, false negatives, and true negatives
        tp, fp, tn, fn = tpfptnfn_cal(test_y_n, preds_test)

        # Calculate precision, recall, and F1-score
        _acc += (tp + tn) / (tp + fp + tn + fn)
        _p += tp / (tp + fp)
        _r += tp / (tp + fn)
    end

    _acc = _acc / 10
    _p = _p / 10
    _r = _r / 10
    _f1 = 2 * _p * _r / (_p + _r)

    print(" acc: $_acc\n precision: $_p\n recall: $_r\n f1_score: $_f1\n")
end
```

TODO: Comment on the result