

## Determinants

**Q1** For the following system of equations

$$x + y + z = 1$$

$$-x + 2y = 5$$

$$2y + z = -7$$

the coefficient matrix is

(A)  $\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$

(B)  $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

(C)  $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

(D)  $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix}$

**Q2** If  $A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$ ,  $X = \begin{bmatrix} n \\ 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$

and  $AX = B$ , then value of  $n$  is

(A) 0

(B) 2

(C) 4

(D) 6

**Q3** Solution of the following system of equations

$$x - y = 3;$$

$$2x + 3y + 4z = 17;$$

$$y + 2z = 7$$
 is

(A)  $x = 2, y = 4, z = -1$

(B)  $x = 4, y = -1, z = 2$

(C)  $x = 4, y = 2, z = -1$

(D)  $x = 2, y = -1, z = 4$

**Q4** Solution of the following system of equations

$$x + 3y + 4z = 8$$

$$2x + y + 2z = 5$$

$$\text{and } 5x + y + z = 7 \text{ is}$$

(A)  $x = 3, y = 2, z = 1$

(B)  $x = 1, y = 2, z = 3$

(C)  $x = 1, y = 1, z = 1$

(D)  $x = 2, y = 1, z = 3$

**Q5** Solution of the following system of equations

$$x - 2y = 10$$

$$2x - y - z = 8$$

$$-2y + z = 7$$
 is

(A)  $x = 0, y = -5, z = -3$

(B)  $x = -3, y = 0, z = -5$

(C)  $x = -5, y = 0, z = -3$

(D)  $x = 0, y = -3, z = -5$

**Q6** The sum of three numbers is  $-1$ . If we multiply second number by 2, third number by 3 and add them, we get 5. If we subtract third number from the sum of first and second number, we get  $-1$ .

If first number is  $x$ , second number is  $y$  and third number is  $z$  then the system of equations corresponding to above situation is

(A)  $x + y + z = 12y + 3z = 5x - y - z = -1$

(B)  $x + y + z = -12x + 3y = 5x - y - z = -1$

(C)  $x + y + z = 12x + 3y + z = 5x + y + z = -1$

(D)  $x + y + z = -12y + 3z = 5x + y - z = -1$

**Q7** An amount of ₹ 5000 is put into three investments of the rate of 6%, 7% and 8% per annum. The total income is ₹ 358. If the combined income from the first two investment is ₹ 70 more than the income from the third.

(where ₹  $x$ , ₹  $y$  and ₹  $z$  be the investment at the rates of interest of 6%, 7% and 8% per annum respectively)



The system of equations corresponding to above situation is

(A)  $x + y + z = 50006x + 7y + 8z = 358x + y - z = 70$

(B)  $x + y + z = 50006x + 7y + 8z = 358006x + 7y - 8z = 7000$

(C)  $x + y + z = 50006x + 7y + 8z = 35800x + y - z = 70$

(D)  $x + y + z = 50006x + 7y + 8z = 358x - y - z = 70$

- Q8** A factory produces three products everyday. The production on a certain day is 45 kg. It is found that the production of third product exceeds the production of first by 8 kg, while the total production of first and third production is twice the production of second product.

If first product production is  $x$ , second product production is  $y$  and third product production is  $z$  then the system of equations corresponding to above situation is

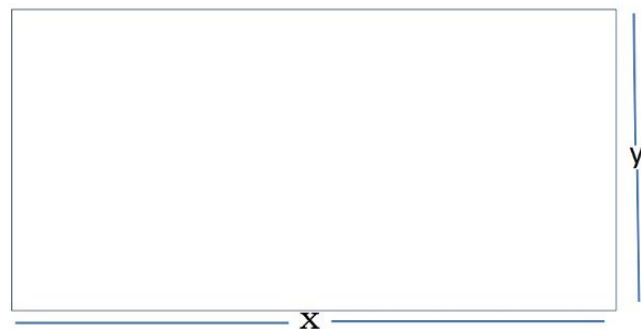
(A)  $x + y + z = 45x - z = 8x - 2y + z = 0$

(B)  $x + y + z = 45 - x + z = 8x + z = 2y$

(C)  $x + y + z = 45x - z = -8x + 2y - z = 0$

(D)  $x + y + z = 45x - z = -8x - 2y - z = 0$

- Q9** Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m<sup>2</sup>.



Based on the information given above, the equations in terms of  $x$  and  $y$  are

(A)  $x - y = 50, 2x - y = 550$

(B)  $x - y = 50, 2x + y = 550$

(C)  $x + y = 50, 2x + y = 550$

(D)  $x + y = 50, 2x - y = 550$

- Q10** The management committee of a residential colony decided to award some of its members (say  $x$ ) for honesty, some (say  $y$ ) for helping others and some others (say  $z$ ) for supervising the workers to kept the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. The sum of the number of awardees for honesty and supervision is twice the number of awardees for helping.

The system of linear equations corresponding to above situation is

(A)  $x + y + z = 332x + 3y + 3z = 12x - 2y + z = 0$

(B)  $x + y + z = 122x + 3y + 3z = 33x - 2y + z = 0$

(C)  $x + 2y + z = 122x + 3y + 3z = 332x - y + z = 0$

(D)  $x + y + z = 122x + 3y + 3z = 33x - 2y - z = 0$



# Answer Key

Q1 B  
Q2 B  
Q3 D  
Q4 C  
Q5 A

Q6 D  
Q7 B  
Q8 B  
Q9 B  
Q10 B



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# Hints & Solutions

Note: scan the QR code to watch video solution

## Q1 Text Solution:

The coefficient matrix is

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

Video Solution:



## Q2 Text Solution:

Since,  $AX = B$

$$\begin{aligned} \therefore \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 8 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x + 4y \\ 4x + 3y \end{bmatrix} \\ &= \begin{bmatrix} 8 \\ 11 \end{bmatrix} \\ \Rightarrow 2x + 4y &= 8 \quad \text{or} \quad 4x + 3y = 11 \\ \Rightarrow 2x &= 4 \quad \text{or} \quad 4x = 8 \\ \Rightarrow x &= 2 \quad \text{or} \quad x = 2 \end{aligned}$$

Video Solution:



## Q3 Text Solution:

System of equations can be written as  $AX = B$  where

$$\begin{aligned} A &= \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B \\ &= \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix} \end{aligned}$$

$$|A| = 2(-2) - 3(2 - 0) + 4(1 - 0) = -6 \neq 0$$

$\therefore A^{-1}$  exists

$$\text{Adj } A = \begin{bmatrix} -2 & -2 & 1 \\ -2 & 4 & -2 \\ 4 & 4 & -5 \end{bmatrix}^T$$

$$\text{Adj } A = \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{-6} \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$$

Now,  $AX = B$

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{-6} \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix} \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{-6} \begin{bmatrix} -34 - 6 + 28 \\ -34 + 12 + 28 \\ 17 - 6 - 35 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{-6} \begin{bmatrix} -12 \\ 6 \\ -24 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$\Rightarrow x = 2, y = -1, z = 4$$

Video Solution:



## Q4 Text Solution:



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$$|A| = 11; \text{Adj}(A) = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A$$

$$= \frac{1}{11} \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

$$\text{Taking } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$$

The system of equations in matrix form is

$$AX = B \therefore X = A^{-1}B$$

$\therefore$  Solution is :

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore x = 1, y = 1, z = 1$$

**Video Solution:**



#### Q5 Text Solution:

$$|A| = 1(-1 - 2) - 2(-2 - 0) \\ = -3 + 4 = 1 \neq 0$$

A is non-singular, therefore  $A^{-1}$  exists

$$\text{adj } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

The given equations can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

which is of the form  $A^T X = B$

$$\Rightarrow X = (A^T)^{-1} B = (A^{-1})^T B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow x = 0$$

$$y = -5$$

$$z = -3$$

**Video Solution:**



#### Q6 Text Solution:

The sum of three numbers is -1.

$$\Rightarrow x + y + z = -1$$

If we multiply the second number by 2, third number by 3 and add them, we get 5.

$$\Rightarrow 2y + 3z = 5$$

If we subtract the third number from the sum of first and second numbers, we get -1.

$$\Rightarrow x + y - z = -1$$

So, we get the three linear equation as

$$x + y + z = -1$$

$$2y + 3z = 5$$

$$x + y - z = -1$$

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**Q7 Text Solution:**

Let ₹  $x$ , ₹  $y$  and ₹  $z$  be the investment at the rates of interest of 6%, 7% and 8% per annum respectively.

Now, total investments = ₹ 5000

$$\Rightarrow x + y + z = 5000$$

Now, income from first investment of overline

$$\text{₹ } x = \text{₹ } \frac{6x}{100}$$

$$\text{Income from second investment of ₹ } y = \text{₹ } \frac{7y}{100}$$

$$\text{Income from third investment of ₹ } z = \text{₹ } \frac{8z}{100}$$

∴ Total annual income =

$$\left( \frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} \right)$$

$$\Rightarrow \frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 358 \Rightarrow 6x + 7y + 8z = 35800$$

Also, combined income from first two investments is ₹ 70 more than the income from the third

$$\text{i.e., } \frac{6x}{100} + \frac{7y}{100} = 70 + \frac{8z}{100} \Rightarrow 6x + 7y - 8z = 7000$$

Thus, we obtain the following system of simultaneous linear equations:

$$x + y + z = 5000, 6x + 7y + 8z = 35800$$

$$6x + 7y - 8z = 7000$$

**Video Solution:****Q8 Text Solution:**

$x + y + z = 45$  (Since the production on a certain day is 45 kg)

$-x + z = 8$  (Since the production of third

product exceeds the production of first by 8 kg)

$x + z = 2y$  (Since the production of first and third product is twice the production of second product)

**Video Solution:****Q9 Text Solution:**

Given that, if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same

$$(\text{Length} - 50) \times (\text{Breadth} + 50) = \text{Area}$$

$$(x - 50) \times (y + 50) = xy$$

$$x(y + 50) - 50(y + 50) = xy$$

$$xy + 50x - 50y - 2500 = xy$$

$$50x - 50y - 2500 = 0$$

$$50x - 50y = 2500$$

Dividing both sides by 50

$$x - y = 50 \quad \dots (1)$$

Also, if length is decreased by 10m & breadth is decreased by 20 m, then area will decrease by 5300 m<sup>2</sup>

$$(\text{Length} - 10) \times (\text{Breadth} - 20) = \text{Area} - 5300$$

$$(x - 10) \times (y - 20) = xy - 5300$$

$$x(y - 20) - 10(y - 20) = xy - 5300$$

$$xy - 20x - 10y + 200 = xy - 5300$$

$$-20x - 10y + 200 = -5300$$

$$-20x - 10y = -5300 - 200$$

$$-20x - 10y = -5500$$

$$20x + 10y = 5500$$

Dividing both sides by 10

$$2x + y = 550 \quad \dots (2)$$

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**Q10 Text Solution:**

Since the sum of all the awardees is 12

$$x + y + z = 12 \quad \dots (1)$$

Also,

Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33

$$3(y + z) + 2x = 33$$

$$\Rightarrow 2x + 3y + 3z = 33 \quad \dots (2)$$

The sum of the number of awardees for honesty and supervision is twice the number of awardees for helping.

$$x + z = 2y$$

$$\Rightarrow x - 2y + z = 0 \quad \dots (3)$$

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