

Homework 1 - EEB590C

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Homework 1: Lectures 1-4

This assignment is due prior to class in week 6. You are to self-select and work in groups: 2-3 in a group. For the assignment below submit one R-script. Annotations via comments are highly encouraged. The script should run!

Assignment Instructions:

1: Select some form of linear model containing a single dependent variable (continuous) and at least 1 independent variable. Next, simulate two datasets: the first with no relationship between X & Y, and the second with some positive association between X & Y. Perform 100 simulations under each condition. Run the linear models on all datasets to confirm that on average, the patterns for condition 1 (no relationship) and condition 2 (some relationship) are met. (HINT: this requires determining an appropriate summary measure extracted from the linear model).

2: Devise a permutation procedure to evaluate the above linear model. Write code for this permutation procedure. Next, devise a SECOND implementation of the same permutation procedure (ie, code the procedure in a different manner). For a single dataset compare the two implementations for their computational performance. Summarize your findings via comments in the code (e.g., which approach was faster? Which components of the slower approach could be improved, etc.).

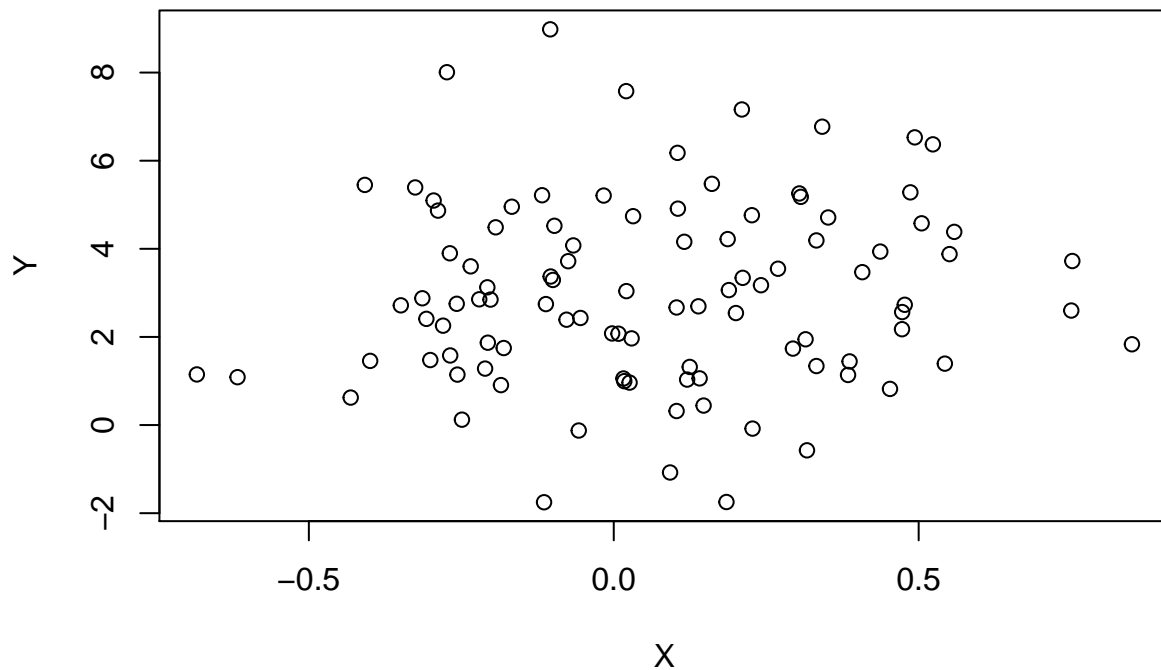
Homework Breakdown:

We selected lm as a linear model and we simulated two datasets: the first with no relationship between X and Y, and the second with positive association between X and Y. Here we have the random uncorrelated data, sampling 100 datapoints with y and x from normal distributions.

```
# data set with no relationship between x & Y
random_corr_gen <- function(elements) {
  x1 <- rnorm(elements, mean = 0, sd = 0.3)
  y1 <- rnorm(elements, mean = 3, sd = 2)
  uncor_dataframe <- data.frame(x1, y1)
}

uncor_data <- random_corr_gen(100)
plot(uncor_data, main = "Uncorrelated Data", xlab = "X", ylab = "Y")
```

Uncorrelated Data



Here, we have the positively correlated, normally distributed, Y and X data generated using a correlation of 0.9.

```
# data set with positive association between x & Y
library(mvtnorm)
library(tidyverse)

## -- Attaching packages ----- tidyverse 1.3.0 --

## v ggplot2 3.3.3      v purrr  0.3.4
## v tibble  3.0.6      v dplyr  1.0.4
## v tidyr   1.1.2      v stringr 1.4.0
## v readr   1.4.0      v forcats 0.5.1

## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()

corr.val <- 0.9
pos_cor_gen <- function(elements) {
  cor_data <- data.frame(rmvnorm(n = elements, mean = c(0,
    0), sigma = matrix(c(1, corr.val, corr.val, 1), 2, 2)))
  names(cor_data)[1] <- "x1"
  names(cor_data)[2] <- "y1"
  return(cor_data)
}

pos_cor_data <- pos_cor_gen(100)
```

We then perform 100 simulations under each condition using the replicate function.

First we simulate 100 uncorrelated x and y dataframes and put it a list called uncor_data_sim.

```
# Random Uncorrelated Data List
uncor_data_sim <- replicate(100, random_corr_gen(100), simplify = FALSE)
```

Then we simulate 100 lists of the correlated data, called cor_data_sim.

```
# Random Correlated Data List
cor_data_sim <- replicate(100, pos_cor_gen(100), simplify = FALSE)
```

We then run the linear model lm on the two datasets (uncorrelated and correlated) to confirm that on average, the patterns for condition 1 (no relationship) and condition 2 (some relationship) are met.

Below is the linear model lm function applied to each list of x and y in the uncorrelated dataset. The summary of the linear model is saved to summary_lm_uncor_tests. Of the 100 simulations, a grand total of X simulations had a pvalue more than 0.05, indicating that there is not a linear relationship between X and Y at a significance level of 0.05.

```
# model 1 - uncorrelated
lm_uncor_tests <- lapply(uncor_data_sim, lm, formula = y1 ~ x1)
summary_lm_uncor_tests <- lapply(lm_uncor_tests, summary)
anova_lm_uncor_tests <- lapply(lm_uncor_tests, anova)

sum_pvalue_uncor_nonsig <- 0
sum_pvalue_uncor_sig <- 0
count_pvalue_uncor_nonsig <- 0
count_pvalue_uncor_sig <- 0

for (i in 1:length(anova_lm_uncor_tests)) {
  if (anova_lm_uncor_tests[[i]][["Pr(>F)"]][1] > 0.05) {
    count_pvalue_uncor_nonsig <- count_pvalue_uncor_nonsig +
      1
    sum_pvalue_uncor_nonsig <- sum_pvalue_uncor_nonsig +
      anova_lm_uncor_tests[[i]][["Pr(>F)"]][1]
  } else {
    count_pvalue_uncor_sig <- count_pvalue_uncor_sig + 1
    print(i)
    sum_pvalue_uncor_sig <- sum_pvalue_uncor_sig + anova_lm_uncor_tests[[i]][["Pr(>F)"]][1]
  }
}
```

```
## [1] 2
## [1] 21
## [1] 27
## [1] 32
## [1] 37
## [1] 61
```

```
average_pvalue_uncor_tests_nonsig <- sum_pvalue_uncor_nonsig/count_pvalue_uncor_nonsig
average_pvalue_uncor_tests_sig <- sum_pvalue_uncor_sig/count_pvalue_uncor_sig
average_pvalue_uncor_tests <- (sum_pvalue_uncor_nonsig + sum_pvalue_uncor_sig)/(count_pvalue_uncor_sig +
  count_pvalue_uncor_nonsig)
```

Next is the linear model lm applied to each list of the positively correlated dataset. There is evidence of a linear relationship between X and Y at a significance level of 0.05. Of the 100 simulations, X simulations have a pvalue that is less than 0.05.

Next is the linear model `lm` applied to each list of the positively correlated dataset.

```
# model 2 - positively correlated model
lm_cor_tests <- lapply(cor_data_sim, lm, formula = y1 ~ x1)
summary_lm_cor_tests <- lapply(lm_cor_tests, summary)
anova_lm_cor_tests <- lapply(lm_cor_tests, anova)

sum_pvalue_cor_nonsig <- 0
sum_pvalue_cor_sig <- 0
count_pvalue_cor_nonsig <- 0
count_pvalue_cor_sig <- 0

for (i in 1:length(anova_lm_cor_tests)) {
  if (anova_lm_cor_tests[[i]][["Pr(>F)"]][1] > 0.05) {
    print(i)
    count_pvalue_cor_nonsig <- count_pvalue_cor_nonsig +
      1
    sum_pvalue_cor_nonsig <- sum_pvalue_cor_nonsig + anova_lm_cor_tests[[i]][["Pr(>F)"]][1]
  } else {
    count_pvalue_cor_sig <- count_pvalue_cor_sig + 1
    sum_pvalue_cor_sig <- sum_pvalue_cor_sig + anova_lm_cor_tests[[i]][["Pr(>F)"]][1]
  }
}

average_pvalue_cor_tests_nonsig <- sum_pvalue_cor_nonsig/count_pvalue_cor_nonsig
average_pvalue_cor_tests_sig <- sum_pvalue_cor_sig/count_pvalue_cor_sig
average_pvalue_cor_tests <- (sum_pvalue_cor_nonsig + sum_pvalue_cor_sig)/(count_pvalue_cor_sig +
  count_pvalue_cor_nonsig)
```

There is a significant linear relationship between `x` and `y` for the positively correlated model (model2) with a `p` value less than 0.05.

Question 2:

The first permutation procedure we used to evaluate the above linear model on the correlated data using the `RRPP` package.

```
# install.packages('RRPP')
library(RRPP)
ourdata <- rrpp.data.frame(pos_cor_data[, 2], pos_cor_data[,
  1])
model3 <- lm.rrpp(pos_cor_data[, 2] ~ pos_cor_data[, 1], iter = 999,
  print.progress = FALSE, data = ourdata)
anova(model3)

##
## Analysis of Variance, using Residual Randomization
## Permutation procedure: Randomization of null model residuals
## Number of permutations: 1000
## Estimation method: Ordinary Least Squares
## Sums of Squares and Cross-products: Type I
## Effect sizes (Z) based on F distributions
##
##              Df      SS      MS      Rsq      F      Z Pr(>F)
## pos_cor_data[, 1]  1 80.396 80.396 0.81999 446.41 8.2461 0.001 **
## Residuals         98 17.649  0.180 0.18001
```

```
## Total          99 98.045
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Call: lm.rrpp(f1 = pos_cor_data[, 2] ~ pos_cor_data[, 1], iter = 999,
##      data = ourdata, print.progress = FALSE)
```

Next, we devised a SECOND implementation of the same permutation procedure. This time we will write a permutation from scratch for 999 iterations using a for loop. We set the observed data's f value to be one of the iterations.

```
pos_cor_lm <- anova(lm(pos_cor_data$y1 ~ pos_cor_data$x1))
f.obs <- pos_cor_lm$`Pr(>F)`[1]
```

We then created a function for 999 permutations so that we could later time the operations.

```
# Randomization function

permute_correlated_data <- function(correlated_data, permute_num,
  f.obs.reported) {

  P.1tailed <- P.2tailed <- 1
  f.rand.vec <- array(NA, (permute_num + 1))
  f.rand.vec[permute_num + 1] <- f.obs.reported

  for (i in 1:permute_num) {
    ### Shuffle Data
    permuted_cor_data <- sample(correlated_data$y1)
    ### Run analysis on random data
    permuted_pos_cor_lm <- anova(lm(permuted_cor_data ~ correlated_data$x1))
    f.rand.vec[i] <- permuted_pos_cor_lm$`Pr(>F)`[1]
  } #end permute
  return(f.rand.vec)
}

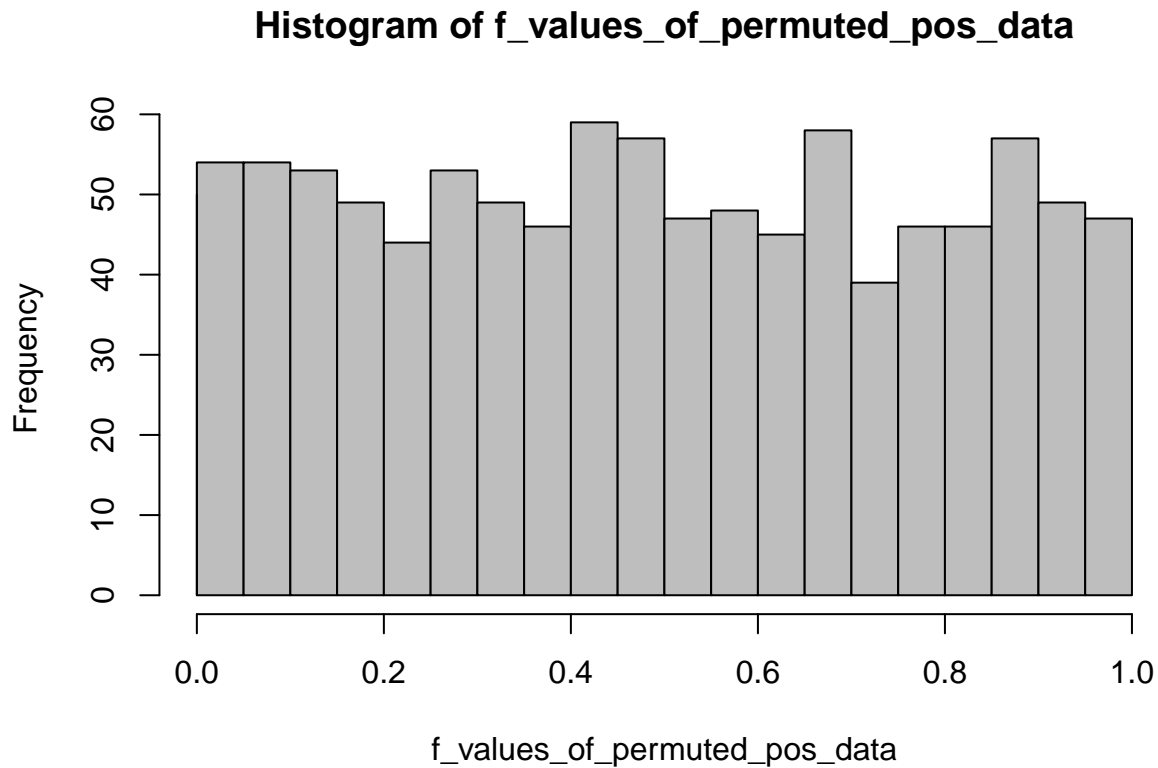
f_values_of_permuted_pos_data <- permute_correlated_data(pos_cor_data,
  999, f.obs)

permute <- 999

## Significance assessment
P.1tailed <- length(which(f_values_of_permuted_pos_data <= f_values_of_permuted_pos_data[permute +
  1]))/(permute + 1) #because observed is negative
P.2tailed <- length(which(abs(f_values_of_permuted_pos_data) >=
  abs(f_values_of_permuted_pos_data[permute + 1])))/(permute +
  1)
P.1tailed

## [1] 0.001
P.2tailed

## [1] 1
#### Plot
hist(f_values_of_permuted_pos_data, 20, freq = T, col = "gray")
segments(f.obs, 0, f.obs, 50) ##Plot Observed value
```



Below are the times of the two permutation methods. As you can see, RRPP is the superior method when simply comparing the time as well as ease of use. The time of RRPP permuting 999 iterations is reported in milliseconds (~ 156 milliseconds) while the permutation that was written out in a for loop yields results in terms of seconds (~ 1.866 seconds).

```
library(microbenchmark)
res <- microbenchmark(anova(lm.rrpp(pos_cor_data[, 2] ~ pos_cor_data[,
  1], iter = 999, print.progress = FALSE, data = ourdata)),
  times = 1) #using rrpp for permutation method
print(res)
```

```
## Unit: milliseconds
##
## anova(lm.rrpp(pos_cor_data[, 2] ~ pos_cor_data[, 1], iter = 999,      print.progress = FALSE, data = ourdata)
##   min      lq    mean  median      uq      max neval
## 91.4761 91.4761 91.4761 91.4761 91.4761 91.4761      1
```

```
res2 <- microbenchmark(permute_correlated_data(pos_cor_data,
  999, f.obs), times = 1)
print(res2) #second permutation method
```

```
## Unit: seconds
##
## expr      min      lq      mean
## permute_correlated_data(pos_cor_data, 999, f.obs) 3.025305 3.025305 3.025305
##   median      uq      max neval
## 3.025305 3.025305 3.025305      1
```