Punto 1)

Con $\omega - \omega_0 >> \gamma$ el segundo termino del denominador es despreciable

$$n = 1 + \frac{Ne^2}{2\epsilon_0 m(\omega_0^2 - \omega^2)}$$

$$\frac{1}{\omega_0^2 - \omega^2} = \frac{1}{\omega_0^2} \cdot \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2}$$

$$= \frac{1}{\omega_0^2}\left[1 + \left(\frac{\omega}{\omega_0}\right)^2 + \left(\frac{\omega}{\omega_0}\right)^4 + \dots\right]$$

$$n = 1 + \frac{Ne^2}{2\epsilon_0 m\omega_0^2} + \frac{Ne^2}{2\epsilon_0 m\omega_0^4}\omega^2 + \dots$$

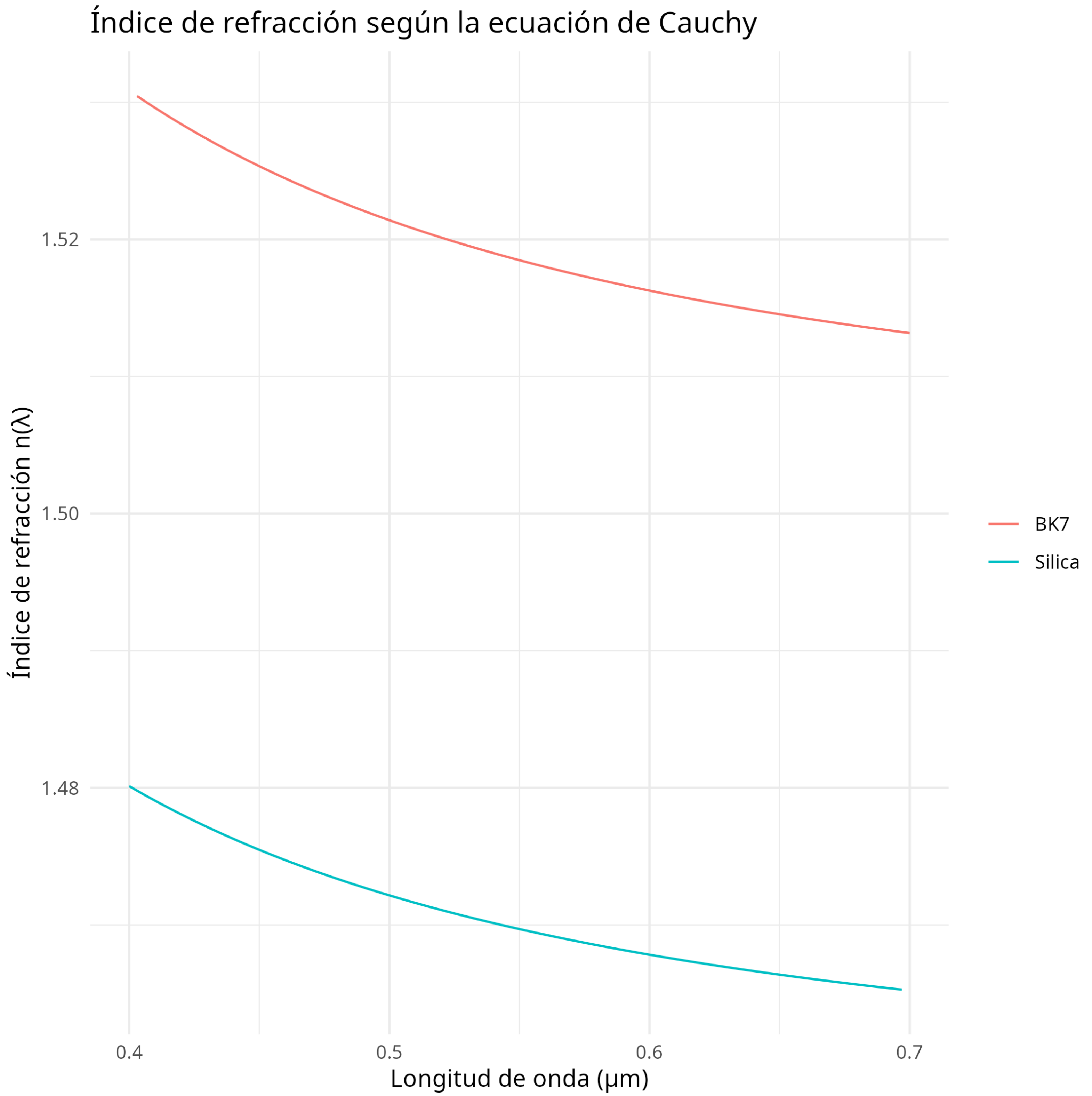
$$\omega = \frac{2\pi c}{\lambda}$$

$$n = \underbrace{\left[1 + \frac{Ne^2}{2\epsilon_0 m\omega_0}\right]}_{A} + \underbrace{\frac{2\pi^2 c^2 Ne^2}{2\epsilon_0 m\omega_0^4}}_{B}\frac{1}{\lambda^2}$$

$$n = A + \frac{B}{\lambda^2}$$

con el cool: o,o:

```r
library(ggplot2)

elements <- read.csv("./punto_1.csv")

calculate_n <- function(element) {
  space <- seq(0.4, 0.7, length.out = 100)
  n_values <- element$A + (element$B / space^2)
  data.frame(wavelength = space, n = n_values, element = element$name)
}


plot_data <- calculate_n(elements)

plot_data |>
ggplot(aes(x = wavelength, y = n, color = element)) +
  geom_line() +
  labs(
    x = "Longitud de onda (µm)",
    y = "Índice de refracción n(λ)",
    title = "Índice de refracción según la ecuación de Cauchy"
  ) +
  theme_minimal() +
  theme(legend.title = element_blank())

ggsave("punto_1.png")
```


Índice de refracción según la ecuación de Cauchy
Índice de refracción n(λ)
1.52
1.50
1.48
0.4
0.5
0.6
0.7
Longitud de onda (μm)
BK7
Silica

Punto 3)

A)

$$\langle S \rangle = \frac{1}{2}\Re\{E \times H^*\}$$

$$S_z = E_x H_y^* - E_y H_x^*$$

$$E_x H_y^* = \left(-j\frac{\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}\right)\left((-j)^*\frac{\beta}{k_c^2}\frac{\partial H_z^*}{\partial y}\right)$$

$$= \left(-j\frac{\omega\mu}{k_c^2}\right)\left(j\frac{\beta}{k_c^2}\right)\frac{\partial H_z}{\partial y}\frac{\partial H_z^*}{\partial y}$$

$$= \frac{\omega\mu\beta}{k_c^4}\frac{\partial H_z}{\partial y}\frac{\partial H_z^*}{\partial y}$$

$$E_y H_x^* = \left(j\frac{\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}\right)\left((-j)^*\frac{\beta}{k_c^2}\frac{\partial H_z^*}{\partial x}\right)$$

$$= -\frac{\omega\mu\beta}{k_c^4}\frac{\partial H_z}{\partial x}\frac{\partial H_z^*}{\partial x}$$

$$\langle S_z \rangle = \frac{1}{2}\Re\{S_z\}$$

$$= \frac{1}{2}\frac{\omega\mu\beta}{2k_c^4}\left(\left|\frac{\partial H_z}{\partial x}\right|^2 + \left|\frac{\partial H_z}{\partial y}\right|^2\right)$$

$$= \frac{1}{2}\frac{\omega\mu\beta}{2k_c^4}|\nabla_t H_z(x,y)|^2$$

B)

$$\langle S_z \rangle = \frac{1}{2}\frac{\omega\mu k_z}{h^4}H_0^2\left[\left(\frac{n\pi}{b}\right)^2\cos^2\left(\frac{m\pi x}{a}\right)\sin^2\left(\frac{n\pi y}{b}\right) + \left(\frac{m\pi}{a}\right)^2\sin\left(\frac{m\pi x}{a}\right)\cos^2\left(\frac{n\pi y}{b}\right)\right]$$

$$P = \int_0^a\int_0^b \langle S_z \rangle\, dx\, dy$$

$$= \int_0^a\int_0^b \frac{1}{2}\frac{\omega\mu k_z}{h^4}H_0^2\left[\left(\frac{n\pi}{b}\right)^2\cos^2\left(\frac{m\pi x}{a}\right)\sin^2\left(\frac{n\pi y}{b}\right) + \left(\frac{m\pi}{a}\right)^2\sin^2\left(\frac{m\pi x}{a}\right)\cos^2\left(\frac{n\pi y}{b}\right)\right] dx\, dy$$

$n, m \neq 0$

$$P = \frac{\omega\mu k_z ab}{8h^2}H_0^2$$

$n=0 \lor m=0$

$$P = \frac{\omega\mu k_z ab}{4h^2}H_0^2$$

$a = 2\,cm = 0.02\,m$
$b = 1\,cm = 0.01\,m$
$c = 3 \times 10^8\,m$
$f = 3 \times 10^{10}\,Hz = 30\,GHz$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= 1.5 \times 10^8\sqrt{\left(\frac{m}{0.02}\right)^2 + \left(\frac{n}{0.01}\right)^2}$$

$$= 7.5 \times 10^9\sqrt{m^2 + 4n^2}\,Hz$$

$$7.5 \times 10^9\sqrt{m^2+4n^2} < 3 \times 10^{10}$$

$$\sqrt{m^2+4n^2} < 4$$

$$m^2 + 4n^2 < 16$$

haciendo un pequeñisimo scipt encontramos

(0, 1) (2, 0) (3,1)

(1, 0) (2, 1)

(1, 1) (3, 0)

Punto 4)

A)

$$\nabla_t^2 E_z + k_c^2 E_z = 0$$

suponga

$$E_z = X(x) Y(y)$$

$$\frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} + k_c^2 = 0$$

$$\frac{X''(x)}{X(x)} = -k_x^2 \qquad \frac{Y''(y)}{Y(y)} = -k_y^2$$

$$k_x^2 + k_y^2 = k_c^2$$

$E_z(0,y)=0 \Rightarrow X(0)=0$ $\qquad$ $E_z(x,0)=0 \Rightarrow Y(0)=0$

$E_z(a,y)=0 \Rightarrow X(a)=0$ $\qquad$ $E_z(x,b)=0 \Rightarrow Y(b)=0$

$X(x)=\sin(k_x x)$ $\qquad$ $Y(y)=\sin(k_y y)$

$k_x = \frac{m\pi}{a}$ $\qquad$ $k_y = \frac{n\pi}{b}$

Con todas las soluciones

$$E_z(x,y) = E_0 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

B)

$$k_c^2 = k_x^2 + k_y^2$$

$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$k_c^2 = \pi^2\left(\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right)$$

$$k_c = \pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

C)

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

con valor mínimo

$$f_c = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

$$= \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$$

Punto 2)

A)

$\rho_f = 0$ tenemos

$$\nabla \cdot \bar{E} = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} + \mu\sigma \frac{\partial \vec{E}}{\partial t}$$

$$\tilde{\bar{E}} = \tilde{\bar{E}}_0 e^{i(kz - \omega t)}$$

$$\nabla^2 \bar{B} = \mu\epsilon \frac{\partial^2 B}{\partial t^2} + \mu\sigma \frac{\partial B}{\partial t}$$

$$\tilde{\bar{B}} = \tilde{B}_0 e^{i(kz - \omega t)}$$

$$\tilde{k}^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega$$

$$\tilde{k} = k + iK$$

$$\tilde{k}^2 = (k^2 - K^2) + 2kKi$$

$$k^2 - K^2 = \mu\epsilon\omega^2$$

$$K = \frac{\mu\sigma\omega}{2k}$$

$$\underbrace{k^2 - \left(\frac{\mu\sigma\omega}{2k}\right)^2 = \mu\epsilon\omega^2}_{\text{cuadratica}}$$

$$k^2 = \frac{\mu\epsilon\omega^2 \pm \sqrt{\mu^2\epsilon^2\omega^4 - 4 \cdot 1\left(-\frac{\mu^2\sigma^2\omega^2}{4}\right)}}{2}$$

B)

En un conductor $J_1 = \sigma \bar{E}$

$$\tilde{k}^2 = \mu \epsilon \omega^2 + i \mu \sigma \omega$$

$$\tilde{k} = k + i K$$

$$\frac{\mu \sigma \omega}{\mu \epsilon \omega^2} = \frac{\sigma E}{\epsilon \omega E} = \frac{J_{1b}}{J_{des}}$$

A corrientes altas J_{des} domina

$$k = \omega \sqrt{\frac{\epsilon \mu}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} + 1 \right)^{1/2}$$

$$K = \omega \sqrt{\frac{\epsilon \mu}{2}} \left[\underbrace{\sqrt{1 + \left(\frac{\sigma}{\epsilon \omega}\right)^2}}_{1} - 1 \right]^{1/2}$$

Esto va a 1 para grandes ω

$K \to 0$

C)

$$\omega_c = \frac{\sigma}{\epsilon}$$

$\sigma_{cu} \approx 59.6 \times 10^6$ S/m

$\sigma_{al} \approx 37.8 \times 10^6$ S/m

$\sigma_{acero} \approx 10 \times 10^6$ S/m

$\epsilon \approx \epsilon_0 = 8.85 \times 10^{12}$

$\omega_{cu} = 6.7 \times 10^{18}$ hz

$\omega_{al} = 4.3 \times 10^{18}$ hz

$\omega_{acero} = 1.12 \times 10^{18}$ hz