Métodos Matemáticos
Tarea 3

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Contents

Chapter 1

Dado que estas funciones dependían de n tanto como de x se considero que lo mejor era hacer una animación en la que se cambiaba n. Por lo tanto, se pondrá un link al vídeo de youtube que tiene esta animación.

1.1 $\frac{n}{\pi}\frac{1}{1+n^2x^2}$

1.1.1 Animación:

https://youtu.be/eP-jSiI97WQ?si=sAfjQWGOw0YCOSjl

1.1.2 Demostración:

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{n}{\pi}\frac{1}{1+n^2x^2}\phi(x)\,dx &= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{n}{1+n^2x^2} \\
nx &= y \\
ndx &= dy \\
x &= \frac{y}{n} \\
&= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+y^2}\phi\left(\frac{y}{n}\right)dy.
\end{aligned}$$

Con esto entonces podemos probar con el limite:

$$\begin{aligned}
\lim_{n\to\infty}\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+y^2}\phi\left(\frac{y}{n}\right)dy &= \lim_{n\to\infty}\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+y^2}\phi(0)\,dy \\
&= \frac{1}{\pi}\phi(0)\int_{-\infty}^{\infty}\frac{1}{1+y^2}dy \\
\int_{-\infty}^{\infty}\frac{1}{1+y^2}dy &= \int_{-\infty}^{\infty}\frac{\sec^2(u)}{\tan^2(u)+1} \\
\int_{-\infty}^{\infty}\frac{1}{1+y^2}dy &= \int_{-\infty}^{\infty}1du \\
\int_{-\infty}^{\infty}\frac{1}{1+y^2}dy &= [u]_{-\infty}^{\infty} \\
\int_{-\infty}^{\infty}\frac{1}{1+y^2}dy &= [\arctan(x)]_{-\infty}^{\infty} \\
\int_{-\infty}^{\infty}\frac{1}{1+y^2}dy &= \pi \\
&= \phi(0).
\end{aligned}$$

1.2 $\frac{n}{\sqrt{\pi}}e^{-n^2x^2}$

1.2.1 Animación:

https://youtu.be/gdPG-1n8PcA?si=7mDJtpv3Cvwjr_pb

1.2.2 Demostración:

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{n}{\sqrt{\pi}} e^{-n^2x^2} \phi(x)\, dx &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} n e^{-n^2x^2} \phi(x)\, dx \\
nx &= y \\
x &= \frac{y}{n} \\
ndx &= dy \\
&= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-y^2} \phi\left(\frac{y}{n}\right) dy.
\end{aligned}$$

Con esto entonces podemos buscar el limite

$$\begin{aligned}
\lim_{n\to\infty} \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-y^2} \phi\left(\frac{y}{n}\right) dy &= \lim_{n\to\infty} \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-y^2} \phi(0)\, dy \\
&= \frac{1}{\sqrt{\pi}} \phi(0) \int_{-\infty}^{\infty} e^{-y^2} dy \\
\int_{-\infty}^{\infty} e^{-y^2} dy &= \frac{\sqrt{\pi}}{2} \int_{-\infty}^{\infty} \frac{2e^{-y^2}}{\sqrt{\pi}} dy \\
\int_{-\infty}^{\infty} e^{-y^2} dy &= \left[\frac{\sqrt{\pi}\operatorname{erf}(y)}{2}\right]_{-\infty}^{\infty} \\
\int_{-\infty}^{\infty} e^{-y^2} dy &= \sqrt{\pi} \\
&= \phi(0).
\end{aligned}$$

1.3 $\frac{1}{n\pi}\frac{\sin^2(nx)}{x^2}$

1.3.1 Animación:

https://youtu.be/ljhxbuyKN64

1.3.2 Demostración:

Para comenzar

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{1}{n\pi} \frac{\sin^2(nx)}{x^2} \phi(x)\, dx &= \int_{-\infty}^{\infty} \frac{1}{n\pi} \frac{n^2 \sin^2(y)}{y^2} \phi\left(\frac{y}{n}\right) dx \\
nx &= y \\
x &= \frac{y}{n} \\
ndx &= dy \\
&= \int_{-\infty}^{\infty} \frac{1}{\pi} \frac{\sin^2(y)}{y^2} \phi\left(\frac{y}{n}\right) dy
\end{aligned}$$

.

Por lo tanto,

$$
\begin{aligned}
\lim_{n\to\infty}\int_{-\infty}^{\infty}\frac{1}{n\pi}\frac{\sin^2(nx)}{x^2}\phi(x)\,dx &= \lim_{n\to\infty}\int_{-\infty}^{\infty}\frac{1}{\pi}\frac{\sin^2(y)}{y^2}\phi\left(\frac{y}{n}\right)dy \\
&= \frac{1}{\pi}\phi(0)\int_{-\infty}^{\infty}\frac{\sin^2(y)}{y^2}dy \\
\int_{-\infty}^{\infty}\frac{\sin^2(y)}{y^2}dy &= \pi \\
&= \phi(0).
\end{aligned}
$$

Chapter 2

En este caso hacemos un desarrollo muy similar al punto anterior:

$$
\begin{aligned}
\frac{x}{\varepsilon} &= y \\
x &= y\varepsilon \\
\frac{1}{\varepsilon}dx &= dy \\
\lim_{\varepsilon\to\infty}\int_{-\infty}^{\infty}\frac{f\left(\frac{x}{\varepsilon}\right)}{\varepsilon}\phi\left(x\right)dx &= \lim_{\varepsilon\to\infty}\int_{-\infty}^{\infty}f\left(y\right)\phi\left(y\cdot\varepsilon\right)dy \\
&= \phi\left(0\right)\int_{-\infty}^{\infty}f\left(y\right)dy \\
&= \phi\left(0\right).
\end{aligned}
$$

Chapter 3

1. $xv.p.\frac{1}{x} = 1$

$$\left\langle xv.p\frac{1}{x}, \phi \right\rangle = \lim_{\varepsilon \to 0} \int_{\mathbb{R}\setminus(-\varepsilon,\varepsilon)} \frac{1}{x} x\phi(x)\, dx$$
$$\left\langle xv.p\frac{1}{x}, \phi \right\rangle = \lim_{\varepsilon \to 0} \int_{\mathbb{R}\setminus(-\varepsilon,\varepsilon)} \phi(x)\, dx$$
$$\left\langle xv.p\frac{1}{x}, \phi \right\rangle = \left\langle 1, \phi \right\rangle.$$

2. $xPf\frac{1}{|x|} = 2H - 1$

3. $x\delta = 0$

$$\begin{aligned} \left\langle x\delta, phi \right\rangle &= \int_{\mathbb{R}} \delta x\phi(x)\, dx \\ &= 0\phi(0) \\ &= 0 \end{aligned}$$
.

4. $x\delta' = -\delta$

$$\begin{aligned} \left\langle x\delta', \phi \right\rangle &= -\left\langle \delta, (x\phi)' \right\rangle \\ &= -\left\langle \delta, x'\phi \right\rangle - \left\langle \delta, x\phi' \right\rangle \\ &= -x'(0)\left\langle \delta, \phi \right\rangle - x(0)\left\langle x, \phi' \right\rangle \\ &= x(0)\delta' - x'(0)\delta \\ &= -\delta \end{aligned}$$
.

5. $x^n\delta^{(m)} = 0$ para $n > m$

6. $x^n\delta^{(n)} = (-1)^n n!\delta$

7. $x^n\delta^{(n+1)} = (-1)(n+1)!\delta'$