Linear Algebra
Lista 1
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1 P.1

1.1 e

$$S = \begin{cases} ix + z = 2i \\ 2x - iz = 4 \\ -ix + z = -i \end{cases} \implies \left(\begin{array}{cc|c} i & 1 & 2i \\ 2 & -i & 4 \\ -i & 1 & -i \end{array}\right)$$

$$-i(I) \implies \left(\begin{array}{cc|c} 1 & -i & 2 \\ 2 & -i & 4 \\ -i & 1 & -i \end{array}\right)$$

$$(-2)(I) + (II) \to (II) \implies \left(\begin{array}{cc|c} 1 & -i & 2 \\ 0 & i & 0 \\ -i & 1 & -i \end{array}\right)$$

$$(i)(I) + (III) \to (III) \implies \left(\begin{array}{cc|c} 1 & -i & 2 \\ 0 & i & 0 \\ 0 & 2 & i \end{array}\right)$$

$$(-i)(II) \implies \left(\begin{array}{cc|c} 1 & -i & 2 \\ 0 & 1 & 0 \\ 0 & 2 & i \end{array}\right)$$

$$(i)(II) + (I) \to (I) \implies \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & i \end{array}\right)$$

$$(-2)(II) + (III) \to (III) \implies \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{array}\right)$$

It can't exists a solution!

1.2 g

$$
S = \begin{cases} x - 2y + 3z = 0 \\ 2x - y + 2z = 0 \\ 3x + y + 2z = 0 \end{cases} \implies \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 2 & -1 & 2 & 0 \\ 3 & 1 & 2 & 0 \end{array}\right)
$$

$$
(-2)(I) + (II) \to (II) \implies \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 3 & -4 & 0 \\ 3 & 1 & 2 & 0 \end{array}\right)
$$

$$
(-3)(I) + (II) \to (II) \implies \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 3 & -4 & 0 \\ 0 & 7 & -7 & 0 \end{array}\right)
$$

$$
\left(\frac{1}{3}\right)(II) \implies \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & -\frac{4}{3} & 0 \\ 0 & 7 & -7 & 0 \end{array}\right)
$$

$$
(2)\,(II) + (I) \to (I) \implies \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{4}{3} & 0 \\ 0 & 7 & -7 & 0 \end{array}\right)
$$

$$
(-7)\,(II) + (III) \to (III) \implies \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{4}{3} & 0 \\ 0 & 0 & -\frac{7}{8} & 0 \end{array}\right)
$$

$$
\left(-\frac{8}{7}\right)(III) \implies \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{4}{3} & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)
$$

$$
\left(\frac{3}{4}\right)(III) + (II) \to (II) \implies \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)
$$

$$
\left(\frac{3}{1}\right)(III) + (II) \to (II) \implies \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)
$$

Trivial solution!

1.3 i

$$S = \begin{cases} x_1 + 3x_2 + 2x_3 + 3x_4 - 7x_5 = 14 \\ 2x_1 + 6x_2 + x_3 - 2x_4 + 5x_5 = -2 \\ x_1 + 3x_2 - x_3 + 2x_5 = -1 \end{cases} \implies \left(\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 2 & 6 & 1 & -2 & 5 & -2 \\ 1 & 3 & -1 & 0 & 2 & -1 \end{array}\right)$$

$$(-2)(I) + (II) \to (II) \implies \left(\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 0 & 0 & -3 & -8 & 19 & -30 \\ 1 & 3 & -1 & 0 & 2 & -1 \end{array}\right)$$

$$(-1)(I) + (III) \to (III) \implies \left(\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 0 & 0 & -3 & -8 & 19 & -30 \\ 0 & 0 & -3 & -3 & 9 & -15 \end{array}\right)$$

$$\left(-\frac{1}{3}\right)(II) \implies \left(\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 0 & 0 & 1 & \frac{8}{3} & -\frac{19}{3} & 10 \\ 0 & 0 & -3 & -3 & 9 & -15 \end{array}\right)$$

$$(3)\,(II) + (III) \to (III) \implies \left(\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 0 & 0 & 1 & \frac{8}{3} & -\frac{19}{3} & 10 \\ 0 & 0 & 0 & 5 & -10 & 15 \end{array}\right)$$

$$\left(\frac{1}{5}\right)(III) \implies \left(\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 0 & 0 & 1 & \frac{8}{3} & -\frac{19}{3} & 10 \\ 0 & 0 & 0 & 1 & -2 & 3 \end{array}\right)$$

$$\left(-\frac{8}{3}\right)(III) + (II) \to (II) \implies \left(\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & -2 & 3 \end{array}\right)$$

$$(-3)\,(III) + (I) \to (I) \implies \left(\begin{array}{ccccc|c} 1 & 3 & 2 & 0 & -1 & 5 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & -2 & 3 \end{array}\right)$$

$$(-2)\,(II) + (I) \to (I) \implies \left(\begin{array}{ccccc|c} 1 & 3 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & -2 & 3 \end{array}\right)$$

This left us with the sistem:

$$\begin{aligned} x_1 + 3x_2 + x_5 &= 1 \\ x_3 - x_5 &= 2 \\ x_4 - 2x_5 &= 2 \end{aligned}$$

Wich implies

$$\begin{aligned} &x_1 = 1 - 3x_2 - x_5 \\ &x_2 = x_2 \\ &x_3 = 2 + x_5 \\ &x_4 = 2 + 2x_5 \\ &x_5 = x_5 \end{aligned}$$

1.4 n

$$S = \begin{cases} x+y+z=4 \\ 2x+5y-2z=3 \\ x+7y-7z=5 \end{cases} \implies \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 5 & -2 & 3 \\ 1 & 7 & -7 & 5 \end{array}\right)$$

$$(-2)\,(I)+(II)\to(II) \implies \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 1 & 7 & -7 & 5 \end{array}\right)$$

$$(-1)\,(I)+(III)\to(III) \implies \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 6 & -8 & 1 \end{array}\right)$$

$$\left(\frac{1}{3}\right)(II) \implies \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} \\ 0 & 6 & -8 & 1 \end{array}\right)$$

$$(-1)\,(II)+(I)\to(I) \implies \left(\begin{array}{ccc|c} 1 & 0 & \frac{7}{3} & \frac{17}{3} \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} \\ 0 & 6 & -8 & 1 \end{array}\right)$$

$$(-6)\,(II)+(I)\to(I) \implies \left(\begin{array}{ccc|c} 1 & 0 & \frac{7}{3} & \frac{17}{3} \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} \\ 0 & 0 & 0 & 11 \end{array}\right)$$

Doesn't exists a solution!

2 P.6

Before any subsection lets get the reduce row echelon form.

$$S = \begin{cases} x+y+kz=2 \\ 3x+4y+2z=k \\ 2x+3y-z=1 \end{cases} \implies \left(\begin{array}{ccc|c} 1 & 1 & k & 2 \\ 3 & 4 & 2 & k \\ 2 & 3 & -1 & 1 \end{array}\right)$$

$$(-3)\,(I)+(II)\to(II) \implies \left(\begin{array}{ccc|c} 1 & 1 & k & 2 \\ 0 & 1 & 2-3k & k-6 \\ 2 & 3 & -1 & 1 \end{array}\right)$$

$$(-2)\,(I)+(III)\to(III) \implies \left(\begin{array}{ccc|c} 1 & 1 & k & 2 \\ 0 & 1 & 2-3k & k-6 \\ 0 & 2 & -1-2k & -3 \end{array}\right)$$

$$(-1)\,(II)+(I)\to(I) \implies \left(\begin{array}{ccc|c} 1 & 0 & -2+4k & 8-k \\ 0 & 1 & 2-3k & k-6 \\ 0 & 2 & -1-2k & -3 \end{array}\right)$$

$$(-2)\,(II)+(III)\to(III) \implies \left(\begin{array}{ccc|c} 1 & 0 & -2+4k & 8-k \\ 0 & 1 & 2-3k & k-6 \\ 0 & 0 & -3+4k & 9-2k \end{array}\right)$$

$$S = \begin{cases} x + y + kz = 2 \\ 3x + 4y + 2z = k \\ 2x + 3y - z = 1 \end{cases} \implies \left(\begin{array}{ccc|c} 1 & 1 & k & 2 \\ 3 & 4 & 2 & k \\ 2 & 3 & -1 & 1 \end{array}\right)$$

$$(-3)(I) + (II) \to (II) \implies \left(\begin{array}{ccc|c} 1 & 1 & k & 2 \\ 0 & 1 & 2-3k & k-6 \\ 2 & 3 & -1 & 1 \end{array}\right)$$

$$(-2)(I) + (II) \to (III) \implies \left(\begin{array}{ccc|c} 1 & 1 & k & 2 \\ 0 & 1 & 2-3k & k-6 \\ 0 & 1 & -1-2k & -3 \end{array}\right)$$

$$(-1)(II) + (I) \to (I) \implies \left(\begin{array}{ccc|c} 1 & 0 & 4k-2 & 8-k \\ 0 & 1 & 2-3k & k-6 \\ 0 & 1 & -1-2k & -3 \end{array}\right)$$

$$(-1)(II) + (III) \to (III) \implies \left(\begin{array}{ccc|c} 1 & 0 & 4k-2 & 8-k \\ 0 & 1 & 2-3k & k-6 \\ 0 & 0 & k-3 & 3-k \end{array}\right)$$

2.1 a

For an only solution we can try $k = 4$ as it get us in:

$$\left(\begin{array}{ccc|c} 1 & 0 & 16-2 & 8-4 \\ 0 & 1 & 2-12 & 4-6 \\ 0 & 0 & 4-3 & 3-4 \end{array}\right)$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 14 & 4 \\ 0 & 1 & -10 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$
$$(10)(III) + (II) \to (II) \implies \left(\begin{array}{ccc|c} 1 & 0 & 14 & 4 \\ 0 & 1 & 0 & -12 \\ 0 & 0 & 1 & -1 \end{array}\right)$$
$$(-14)(III) + (I) \to (I) \implies \left(\begin{array}{ccc|c} 1 & 0 & 0 & 18 \\ 0 & 1 & 0 & -12 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

And the solution is trivial (from there)

2.2 b

For infinite solutions take $k = 3$

$$\left(\begin{array}{ccc|c} 1 & 0 & 12-2 & 8-3 \\ 0 & 1 & 2-9 & 3-6 \\ 0 & 0 & 3-3 & 3-3 \end{array}\right)$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 10 & 5 \\ 0 & 1 & -7 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Therefore,

$$\begin{aligned} x + 10y &= 5 \\ y - 7z &= -3 \\ \implies x &= 5 - 10y \\ \implies y &= y \\ \implies z &= \frac{3+y}{7} \end{aligned}$$

2.3 c

There is no possible k values such that this sistem does not have any solution. Particularly because the last row is consistent (meaning that there is no way to make the row of variables 0 and the result different to 0)

$$\left(\begin{array}{ccc|c} 1 & 0 & 4k-2 & 8-k \\ 0 & 1 & 2-3k & k-6 \\ 0 & 0 & k-3 & 3-k \end{array}\right)$$

3 P.11

3.1 a

$$A = \begin{pmatrix} -2 & -1 & 1 \\ 5 & 3 & -1 \\ 3 & 1 & -3 \end{pmatrix}$$

$$(A|I) \implies \left(\begin{array}{ccc|ccc} -2 & -1 & 1 & 1 & 0 & 0 \\ 5 & 3 & -1 & 0 & 1 & 0 \\ 3 & 1 & -3 & 0 & 0 & 1 \end{array}\right)$$

$$(I) \to -\frac{1}{2}(I) \implies \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 5 & 3 & -1 & 0 & 1 & 0 \\ 3 & 1 & -3 & 0 & 0 & 1 \end{array}\right)$$

$$(II) \to (II) - 5(I) \implies \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{5}{2} & 1 & 0 \\ 3 & 1 & -3 & 0 & 0 & 1 \end{array}\right)$$

$$(III) \to (III) - 3(I) \implies \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{5}{2} & 1 & 0 \\ 0 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} & 0 & 1 \end{array}\right)$$

$$(I) \to (I) - (II) \implies \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & -3 & -1 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{5}{2} & 1 & 0 \\ 0 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} & 0 & 1 \end{array}\right)$$

$$(III) \to (III) + (II) \implies \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & -3 & -1 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{5}{2} & 1 & 0 \\ 0 & 0 & 0 & 4 & 1 & 1 \end{array}\right)$$

$$(II) \to 2(II) \implies \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & -3 & -1 & 0 \\ 0 & 1 & 3 & 5 & 2 & 0 \\ 0 & 0 & 0 & 4 & 1 & 1 \end{array}\right)$$

3.2 b

$$B = \begin{pmatrix} i & -1 \\ 1+2i & -3 \end{pmatrix}$$
$$(B|I) = \left(\begin{array}{cc|cc} i & -1 & 1 & 0 \\ 1+2i & -3 & 0 & 1 \end{array}\right)$$
$$(I) \to (-i)(I) \implies \left(\begin{array}{cc|cc} 1 & i & -i & 0 \\ 1+2i & -3 & 0 & 1 \end{array}\right)$$
$$(i) \to (-i)(i) \implies \left(\begin{array}{cc|cc} 1 & i & -i & 0 \\ 1+2i & -3 & 0 & 1 \end{array}\right)$$
$$(II) \to (-1-2i)(I) + (II) \implies \left(\begin{array}{cc|cc} 1 & i & -i & 0 \\ 0 & (-i-1) & (i-2) & 1 \end{array}\right)$$
$$(II) \to \left(\frac{-1+i}{2}\right)(II) \implies \left(\begin{array}{cc|cc} 1 & i & -i & 0 \\ 0 & 1 & (\frac{1}{2} - \frac{3}{2}i) & \frac{-1+i}{2} \end{array}\right)$$
$$(I) \to (-i)\,(II) + (I) \implies \left(\begin{array}{cc|cc} 1 & 0 & (-\frac{3}{2} - \frac{3i}{2}) & \frac{i+1}{2} \\ 0 & 1 & (\frac{1}{2} - \frac{3}{2}i) & \frac{-1+i}{2} \end{array}\right)$$

Check:

$$\begin{pmatrix} i & -1 \\ 1+2i & -3 \end{pmatrix} \begin{pmatrix} (-\frac{3}{2} - \frac{3i}{2}) & \frac{i+1}{2} \\ (\frac{1}{2} - \frac{3}{2}i) & \frac{-1+i}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

3.3 c

$$B = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$
$$(B|I) \implies \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$
$$(I) + (II) \implies \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$
$$(II) \leftrightarrow (III) \implies \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & 1 & 1 & 0 \end{array}\right)$$
$$(-3)(II) + (III) \implies \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 1 & -3 \end{array}\right)$$
$$(-1)(III) \implies \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right)$$
$$(-1)(III) + (II) \implies \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right)$$
$$(-1)(III) + (I) \implies \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & -3 \\ 0 & 1 & 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right)$$

Check:

$$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 1 & 1 & -2 \\ -1 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 2-1 & 1-1 & -3+3 \\ -2+3-1 & -1+3-1 & 3-6+3 \\ 1-1 & 1-1 & -2+3 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4 P.12

$$\begin{aligned}
\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} &\implies \left(\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right) \\
\frac{1}{2}(I) &\implies \left(\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 3 & 4 & 0 & 1 \end{array}\right) \\
(-3)(I)+(II) &\implies \left(\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & -\frac{3}{2} & 1 \end{array}\right) \\
(-2)(II) &\implies \left(\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 3 & -2 \end{array}\right) \\
(-\frac{3}{2})(II)+(I) &\implies \left(\begin{array}{cc|cc} 1 & 0 & -4 & 3 \\ 0 & 1 & 3 & -2 \end{array}\right)
\end{aligned}$$

Check:

$$\begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -8+9 & -12+12 \\ 6-6 & 9-8 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

5 P.17

$$
\begin{aligned}
A = \begin{pmatrix} -5 & 3 & 1 \\ 0 & -1 & 2 \\ 4 & -1 & -3 \end{pmatrix} &\implies \left(\begin{array}{ccc|cc} -5 & 3 & 1 & 4 & 11 \\ 0 & -1 & 2 & 4 & -2 \\ 4 & -1 & -3 & -7 & -6 \end{array}\right) \\
(-\frac{1}{5})(I) &\implies \left(\begin{array}{ccc|cc} 1 & -\frac{3}{5} & -\frac{1}{5} & -\frac{4}{5} & -\frac{11}{5} \\ 0 & -1 & 2 & 4 & -2 \\ 4 & -1 & -3 & -7 & -6 \end{array}\right) \\
(-4)(I) + (III) &\implies \left(\begin{array}{ccc|cc} 1 & -\frac{3}{5} & -\frac{1}{5} & -\frac{4}{5} & -\frac{11}{5} \\ 0 & -1 & 2 & 4 & -2 \\ 0 & \frac{7}{5} & -\frac{11}{5} & -\frac{19}{5} & \frac{14}{5} \end{array}\right) \\
(-1)(II) &\implies \left(\begin{array}{ccc|cc} 1 & -\frac{3}{5} & -\frac{1}{5} & -\frac{4}{5} & -\frac{11}{5} \\ 0 & 1 & -2 & -4 & 2 \\ 0 & \frac{7}{5} & -\frac{11}{5} & -\frac{19}{5} & \frac{14}{5} \end{array}\right) \\
(\frac{3}{5})(II) + (I) &\implies \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{7}{5} & -\frac{16}{5} & -1 & \\ 0 & 1 & -2 & -4 & 2 & \\ 0 & \frac{7}{5} & -\frac{11}{5} & -\frac{19}{5} & \frac{14}{5}(-\frac{7}{5})(II) + (III) & \implies \end{array}\right) \\
\left(\begin{array}{ccc|cc} 1 & 0 & -\frac{7}{5} & -\frac{16}{5} & -1 \\ 0 & 1 & -2 & -4 & 2 \\ 0 & 0 & \frac{3}{5} & \frac{9}{5} & 0 \end{array}\right) & \\
(\frac{5}{3})(III) &\implies \left(\begin{array}{ccc|cc} 1 & 0 & -\frac{7}{5} & -\frac{16}{5} & -1 \\ 0 & 1 & -2 & -4 & 2 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right) \\
(2)(III) + (II) &\implies \left(\begin{array}{ccc|cc} 1 & 0 & -\frac{7}{5} & -\frac{16}{5} & -1 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right) \\
(\frac{7}{5})(III) + (I) &\implies \left(\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right)
\end{aligned}
$$

Verification:

$$
\begin{aligned}
A \cdot B &= \begin{pmatrix} -5 & 3 & 1 \\ 0 & -1 & 2 \\ 4 & -1 & -3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 3 & 0 \end{pmatrix} \\
&= \begin{pmatrix} -5 + 6 + 3 & 5 + 6 \\ -2 + 6 & -2 \\ 4 + -2 - 9 & -4 - 2 \end{pmatrix} \\
&= \begin{pmatrix} 4 & 11 \\ 4 & -2 \\ -7 & -6 \end{pmatrix} \square
\end{aligned}
$$