OPTICS
Fourth Edition

INSTRUCTOR'S SOLUTIONS MANUAL

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Addison Wesley

San Francisco Boston New York
Cape Town Hong Kong London Madrid Mexico City
Montreal Munich Paris Singapore Sydney Tokyo Toronto

ISBN 0-8053-8578-9

4 5 6 7 8 9 10 —BTP— 05 04 03 02 01

Addison Wesley

www.aw.com/physics

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Chapter 2 Solutions

2.1 $(0.003)(2.54 \times 10^{-2}/580 \times 10^{-9}) =$ number of waves $= 131$, $c = \nu\lambda$, $\lambda = c/\nu = 3 \times 10^8/10^{10}$, $\lambda = 3$ cm. Waves extend 3.9 m.

2.2 $\lambda = c/\nu = 3 \times 10^8/5 \times 10^{14} = 6 \times 10^{-7}$ m $= 0.6\mu$ m.
$\lambda = 3 \times 10^8/60 = 5 \times 10^6$ m $= 5 \times 10^3$ km.

2.3 $v = \lambda\nu = 5 \times 10^{-7} \times 6 \times 10^8 = 300$ m/s.

2.4 The time between the crests is the period, so $\tau = 1/2$·s; hence $\nu = 1/\tau = 2.0$ Hz. As for the speed $v = L/t = 4.5$ m/1.5 s $= 3.0$ m/s. We now know τ, ν, and v and must determine λ. Thus, $\lambda = v/\nu = 3.0$ m/s/2.0 Hz $= 1.5$ m.

2.5 $v = \nu\lambda = 3.5 \times 10^3$ m/s $= \nu(4.3$ m); $\nu = 0.81$ kHz.

2.6 $v = \nu\lambda = 1498$ m/s $= (440$ Hz$)\lambda$; $\lambda = 3.40$ m.

2.7 $v = (10$ m$)/2.0$ s$) = 5.0$ m/s; $\nu = v/\lambda = (5.0$ m/s$)/(0.50$ m$) = 10$ Hz.

2.8 $v = \nu\lambda = (\omega/2\pi)\lambda$ and so $\omega = (2\pi/\lambda)v$.

2.9

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$
$\sin\theta$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$
$\cos\theta$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$
$\sin(\theta - \pi/4)$	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1
$\sin(\theta - \pi/2)$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$
$\sin(\theta - 3\pi/4)$	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0
$\sin(\theta + \pi/2)$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$

θ	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin\theta$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0
$\cos\theta$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1
$\sin(\theta-\pi/4)$	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$
$\sin(\theta-\pi/2)$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1
$\sin(\theta-3\pi/4)$	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$
$\sin(\theta+\pi/2)$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1

$\sin\theta$ leads $\sin(\theta-\pi/2)$.

2.10

x	$-\lambda/2$	$-\lambda/4$	0	$\lambda/4$	$\lambda/2$	$3\lambda/4$	λ
$\kappa x = 2\pi/\lambda x$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\cos(\kappa x-\pi/2)$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$
$\cos(\kappa x+3\pi/4)$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$

2.11

t	$-\tau/2$	$-\tau/4$	0	$\tau/4$	$\tau/2$	$3\tau/4$	τ
$\omega t = 2\pi/\tau$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	π
$\sin(\omega t+\pi/4)$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$
$\sin(\pi/4-\omega t)$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$

2.12 Comparing y with Eq. (2.13) tells us that $A = 0.02$ m. Moreover, $2\pi/\lambda = 157\ \mathrm{m}^{-1}$ and so $\lambda = 2\pi/(157\mathrm{m}^{-1}) = 0.0400$ m. The relationship between frequency and wavelength is $v = \nu\lambda$, and so $\nu = v/\lambda = 1.2$ m/s/0.0400 m $= 30$ Hz. The period is the inverse of the frequency, and therefore $\tau = 1/\nu = 0.033$ s.

2.13 (a) $\lambda = (4.0 - 0.0)$ m $= 4.0$ m. (b) $v = \nu\lambda$, so $\nu = v/\lambda = (20.0\ \mathrm{m/s})/(4.0\ \mathrm{m}) = 5.0$ Hz. (c) Eq. (2.28) $\psi(x,t) = A\sin(kx - \omega t + \epsilon)$. From the figure, $A = 0.020$ m; $k = 2\pi/\lambda = 2\pi/(4.0\ \mathrm{m}) = 0.5\pi\ \mathrm{m}^{-1}$; $w = 2\pi\nu = 2\pi(5.0\ \mathrm{Hz}) = 10.0\pi$ rad/s

At $t = 0$, $x = 0$, $\psi(0,0) = -0.020$ m;
$\psi(0,0) = (0.020\ \mathrm{m})\sin(0.5\pi(0) - 10.0\pi(0) + \epsilon) = (0.020\ \mathrm{m})\sin(\epsilon)$;
$\sin(\epsilon) = -1$; $\epsilon = -\pi/2$. $\psi(x,t) = (0.020\ \mathrm{m})\sin(0.5\pi x - 10.0\pi t - \pi/2)$

2.14 (a) $\lambda = (30.0 - 0.0)$ cm $= 30.0$ cm. (c) $v = \nu\lambda$, so $\nu = v/\lambda = (100\ \mathrm{cm/s})/(30.0\ \mathrm{cm}) = 3.33$ Hz

2.15 (a) $\tau = (0.20 - 0.00)$ s $= 0.20$ s. (b) $\nu = 1/\tau = 1/(0.20s) = 5.00$ Hz. (c) $v = \nu\lambda$, so $\lambda = v/\nu = (40.0 \text{ cm/s})/(5.00 \text{ s}^{-1}) = 8.00$ cm.

2.16 $\psi = A\sin 2\pi(\kappa x - \nu t)$, $\psi_1 = 4\sin 2\pi(0.2x - 3t)$. (a) $\nu = 3$, (b) $\lambda = 1/0.2$, (c) $\tau = 1/3$, (d) $A = 4$, (e) $v = 15$, (f) positive x
$\psi = A\sin(kx + \omega t)$, $\psi_2 = (1/2.5)\sin(7x + 3.5t)$. (a) $\nu = 3.5/2\pi$, (b) $\lambda = 2\pi/7$, (c) $\tau = 2\pi/3.5$, (d) $A = 1/2.5$, (e) $v = 1/2$, (f) negative x

2.17 Form of Eq. (2.26) $\psi(x,t) = A\sin(kx - \omega t)$ (a) $w = 2\pi\nu$, so $\nu = \omega/2\pi = (20.0 \text{ rad/s})/2\pi$, (b) $k = 2\pi/\lambda$, so $\lambda = 2\pi/k = 2\pi/(6.28 \text{ rad/m}) = 1.00$ m, (c) $\nu = 1/\tau$, so $\tau = 1/\nu = 1/(10.0/\pi \text{ Hz}) = 0.10\pi s$, (d) From the form of ψ, $A = 30.0$ cm, (e) $v = \omega/k = (20.0 \text{ rad/s})/(6.28 \text{ rad/m}) = 3.18$ m/s, (f) Negative sign indicates motion in $+x$ direction.

2.18 $\partial^2\psi/\partial x^2 = -k^2\psi$ and $\partial^2\psi/\partial t^2 = -k^2v^2\psi$. Therefore $\partial^2\psi/\partial x^2 - (1/v^2)\partial^2\psi/\partial t^2 = (-k^2 + k^2)\psi = 0$.

2.19 $\partial^2\psi/\partial x^2 = -k^2\psi$; $\partial^2\psi/\partial t^2 = -w^2\psi$; $w^2/v^2 = (2\pi\nu)^2/v^2 = (2\pi/\lambda)^2 = k^2$; therefore, $\partial^2\psi/\partial x^2 - (1/v^2)\partial^2\psi/\partial t^2 = (-k^2 + k^2)\psi = 0$

2.20 $\psi(x,t) = A\cos(hx - \omega t - (\pi/2)) =$
$A\{\cos(kx - \omega t)\cos(-\pi/2) - \sin(kx - \omega t)\sin(-\pi/2)\} = A\sin(kx - \omega t)$

2.21 $v_y = -\omega A\cos(kx - \omega t + \epsilon)$, $a_y = -\omega^2 y$. Simple harmonic motion since $a_y \propto y$.

2.22 $\tau = 2.2 \times 10^{-15}$ s; therefore $\nu = 1/\tau = 4.5 \times 10^{14}$ Hz; $v = \nu\lambda$, $\lambda = v/\nu = 6.6 \times 10^{-7}$ m and $k = 2\pi/\lambda = 9.5 \times 10^6$ m^{-1}.
$\psi(x,t) = (10^3 V/m)\cos[9.5 \times 10^6 m^{-1}(x + 3 \times 10^8 (m/s)t)]$. It's cosine because $\cos 0 = 1$.

2.23 $y(x,t) = C/[2 + (x + vt)^2]$.

2.24 $\psi(0,t) = A\cos(kvt+\pi) = -A\cos(kvt) = -A\cos(\omega t)$, then
$\psi(0,\tau/2) = -A\cos(\omega\tau/2) = -A\cos(\pi) = A$,
$\psi(0,3\tau/4) = -A\cos(3\omega\tau/4) = -A\cos(3\pi/2) = 0$.

2.25 Since $\psi(y,t) = (y-vt)A$ is only a function of $(y-vt)$, it does satisfy the conditions set down for a wave. Since $\partial^2\psi/\partial y^2 = \partial^2\psi/\partial t^2 = 0$, this function is a solution of the wave equation. However, $\psi(y,0) = Ay$ is unbounded, so cannot represent a localized wave profile.

2.26 $k = \pi 3\times 10^6\ \text{m}^{-1}$, $\omega = \pi 9\times 10^{14}$ Hz, $v = \omega/k = 3\times 10^8$ m/s.

2.27 $d\psi/dt = \partial\psi/\partial x dx/dt + (\partial\psi/\partial y)(dy/dt)$ and let $y = t$ whereupon $d\psi/dt = \partial\psi/\partial x(\pm v) + \partial\psi/\partial t = 0$ and the desired result follows immediately.

2.28 $\varphi/dt = (\partial\varphi/\partial x)(dx/dt) + \partial\varphi/\partial t = 0 = k(dx/dt) - kv$ and this is zero provided $dx/dt = \pm v$, as it should be. For the particular wave of Problem 2.20, $\varphi/dt = \partial\varphi/\partial y(\pm v) + \partial\varphi/\partial t = \pi 3\times 10^6(\pm v) + \pi 9\times 10^{14} = 0$ and the speed is -3×10^8 m/s.

2.29 $-a(bx+ct)^2 = -ab^2(x+ct/b)^2 = g(x+vt)$ and so $v = c/b$ and the wave travels in the negative x-direction. Using Eq. (2.34) $(\partial\psi/\partial t)_x/(\partial\psi/\partial x)_t =$ $-[A(-2a)(bx+ct)ce^{-a(bx+ct)^2}]/[A(-2a)(bx+ct)be{-}a(bx+ct)^2] = -c/b$; the minus sign tells us that the motion is in the negative x-direction.

2.30 $\psi(z,0) = A\sin(kz+\epsilon)$; $\psi(-\lambda/12,0) = A\sin(-\pi/6+\epsilon) = 0.866$;
$\psi(\lambda/6,0) = A\sin(\pi/3+\epsilon) = 1/2$; $\psi(\lambda/4,0) = A\sin(\pi/2+\epsilon) = 0$.
$A\sin(\pi/2+\epsilon) = A(\sin\pi/2\cos\epsilon + \cos\pi/2\sin\epsilon) = A\cos\epsilon = 0$, $\epsilon = \pi/2$.
$A\sin(\pi/3+\pi/2) = A\sin(5\pi/6) = 1/2$; therefore $A = 1$, hence
$\psi(z,0) = \sin(kz+\pi/2)$.

2.31 Both (a) and (b) are waves since they are twice differentiable functions of $z - vt$ and $x + vt$, respectively. Thus for (a) $\psi = a^2(z-bt/a)^2$ and the velocity is b/a in the positive z-direction. For (b) $\psi = a^2(x+bt/a+c/a)^2$ and the velocity is b/a in the negative x-direction.

2.32 (a) $\psi(y,t) = \exp -(ay - bt)^2$, a traveling wave in the $+y$ direction, with speed $v = \omega/k = b/a$. (b) not a traveling wave. (c) traveling wave in the $-x$ direction, $v = a/b$, (d) traveling wave in the $+x$ direction, $v = 1$.

2.33 $\psi(x,t) = 5.0 \exp[-a(x + \sqrt{b/a}t)^2]$, the propagation direction is negative x; $v = \sqrt{b/a} = 0.6$ m/s. $\psi(x,0) = 5.0 \exp(-25x^2)$.

2.34 $\lambda = v/\nu = 0.300$ m; 10.0 cm is a fraction of a wavelength viz. $(0.100 \text{ m})/(0.300 \text{ m}) = 1/3$; hence $2\pi/3 = 2.09$ rad.

2.35 $30°$ corresponds to $\lambda/12$ or $(1/12)3 \times 10^8/6 \times 10^{14} = 42$ nm.

2.36 $\psi(x,t) = A \sin 2\pi(x/\lambda \pm t/\tau)$, $\psi = 60 \sin 2\pi(x/400 \times 10^{-9} - t/1.33 \times 10^{-15})$, $\lambda = 400$ nm, $v = 400 \times 10^{-9}/1.33 \times 10^{-15} = 3 \times 10^8$ m/s. $\nu = (1/1.33) \times 10^{15}$ Hz, $\tau = 1.33 \times 10^{-15}$ s.

2.37 $e^{i\alpha}e^{i\beta} = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) = (\cos\alpha\cos\beta - \sin\alpha\sin\beta) + i(\sin\alpha\cos\beta + \cos\alpha\sin\beta) = \cos(\alpha+\beta) + i\sin(\alpha+\beta) = e^{i(\alpha+\beta)}$
$\psi\psi^* = Ae^{i\omega t}Ae^{-i\omega t} = A^2$; $\sqrt{\psi\psi^*} = A$. In terms of Euler's formula
$\psi\psi^* = A^2(\cos\omega t + i\sin\omega t)(\cos\omega t - i\sin\omega t) = A^2(\cos^2\omega t + \sin^2\omega t) = A^2$.

2.38 If $z = x + iy$, then $z^* = x - iy$ and $z - z^* = 2yi$.

2.39 $\psi = A \exp i(k_x x + k_y y + k_z z)$, $k_x = k\alpha$, $k_y = k\beta$, $k_z = k\gamma$, $|\vec{k}| = [(k\alpha)^2 + (k\beta)^2 + (k\gamma)^2]^{1/2} = k(\alpha^2 + \beta^2 + \gamma^2)^{1/2}$.

2.40 Consider Eq. (2.64), with $\partial^2\psi/\partial x^2 = \alpha^2 f''$, $\partial^2\psi/\partial y^2 = \beta^2 f''$, $\partial^2\psi/\partial z^2 = \gamma^2 f''$, $\partial^2\psi/\partial t^2 = v^2 f''$. Then
$\nabla^2\psi - (1/v^2)\partial^2\psi/\partial t^2 = (\alpha^2 + \beta^2 + \gamma^2 - 1)f'' = 0$ whenever
$\alpha^2 + \beta^2 + \gamma^2 = 1$.

2.41 $\lambda = h/mv = 6.6 \times 10^{-34}/6(1) = 1.1 \times 10^{-34}$ m.

2.42 $\vec{k}$ can be constructed by forming a unit vector in the proper direction and multiplying it by k. The unit vector is

$[(4-0)\hat{\imath}+(2-0)\hat{\jmath}+(1-0)\hat{k}]/\sqrt{4^2+2^2+1^2}=(4\hat{\imath}+2\hat{\jmath}+\hat{k})/\sqrt{21}$ and $\vec{k}=k(4\hat{\imath}+2\hat{\jmath}+\hat{k})/\sqrt{21}$. $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$, hence $\psi(x,y,z,t)=A\sin[(4k/\sqrt{21})x+(2k/\sqrt{21})y+(k/\sqrt{21})z-\omega t]$.

2.43 $\vec{k}=(1\hat{\imath}+0\hat{\jmath}+0\hat{k})$, $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$, so, $\psi=A\sin(\vec{k}\cdot\vec{r}-\omega t+\epsilon)=A\sin(kx-\omega t+\epsilon)$ where $k=2\pi/\lambda$ (could use cos instead of sin).

2.44 $\psi(\vec{r}_1,t)=\psi[\vec{r}_2-(\vec{r}_2-\vec{r}_1),t]=\psi(\vec{k}\cdot\vec{r}_1,t)=\psi[\vec{k}\cdot\vec{r}_2-\vec{k}\cdot(\vec{r}_2-\vec{r}_1),t]=\psi(\vec{k}\cdot\vec{r}_2,t)=\psi(\vec{r}_2,t)$ since $\vec{k}\cdot(\vec{r}_2-\vec{r}_1)=0$.

2.45

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin\theta$	-1	$-1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0
$2\sin\theta$	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$	0
$3\sin\theta$	-3	$-3/\sqrt{2}$	0	$3/\sqrt{2}$	3	$3/\sqrt{2}$	0	$-3/\sqrt{2}$	-3	$-3/\sqrt{2}$	0

2.46

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin\theta$	-1	$-1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0
$\sin(\theta-\pi/2)$	0	$-1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	-1
$\sin\theta+\sin(\theta-\pi/2)$	-1	$-\sqrt{2}$	-1	0	1	$\sqrt{2}$	1	0	-1	$-\sqrt{2}$	-1

2.47 Note that the amplitude of $\{\sin(\theta)+\sin(\theta-\pi/2)\}$ is greater than 1, while the amplitude of $\{\sin(\theta)+\sin(\theta-4\pi/4)$ is less than 1. The phase difference is $\pi/8$.

2.48

x	$-\lambda/2$	$-\lambda/4$	0	$\lambda/4$	$\lambda/2$	$3\lambda/4$	λ
kx	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\cos kx$	-1	0	1	0	-1	0	1
$\cos(kx+\pi)$	1	0	-1	0	1	0	-1

Chapter 3 Solutions

3.1 Compare $E_y = 2\cos[2\pi \times 10^{14}(t - x/c) + \pi/2]$ to $E_y = A\cos[2\pi\nu(t - x/v) + \pi/2]$. (a) $\nu = 10^{14}$ Hz, $v = c$, and $\lambda = c/\nu = 3 \times 10^8/10^{14} = 3 \times 10^{-6}$ m, moves in positive x-direction, $A = 2$ V/m, $\epsilon = \pi/2$ linearly polarized in the y-direction. (b) $B_x = 0$, $B_y = 0$, $B_z = E_y/c$.

3.2 $E_z = 0$, $E_y = E_x = E_0 \sin(kz - \omega t)$ or cosine; $B_z = 0$, $B_y = -B_x = E_y/c$, or if you like,

$$\vec{E} = \frac{E_0}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})\sin(kz - \omega t), \qquad \vec{B} = \frac{E_0}{c\sqrt{2}}(\hat{\jmath} - \hat{\imath})\sin(kz - \omega t).$$

3.3 First, by the right-hand rule, the directions of the vectors are right. Then $kE = \omega B$ and so $(2\pi/\lambda)E = \omega B = 2\pi\nu B$, hence $E = \lambda\nu B = c$.

3.4 $\partial E/\partial x = -kE_0 \sin(kx - \omega t)$; $-\partial B/\partial t = -\omega B_0 \sin(kx - \omega t)$; $-kE_0 = -\omega B_0$; $E_0 = (\omega/k)B_0$ and Eq. (2.33) $\omega/k = c$.

3.5 (a) The electric field oscillates along the line specified by the vector $-\hat{\imath} + \sqrt{3}\hat{\jmath}$. (b) To find E_0, dot $\vec{E}_0$ with itself and take the square root, thus $E_0 = \sqrt{9 + 27}10^4$V/m $= 6 \times 10^4$ V/m. (c) From the exponential $\vec{k} \cdot \vec{r} = (\sqrt{5}x + 2y)(\pi/3) \times 10^7$, hence $\vec{k} = (\sqrt{5}\hat{\imath} + 2\hat{\jmath})(\pi/3) \times 10^7$ and because the phase is $\vec{k} \cdot \vec{r} - \omega t$ rather than $\vec{k} \cdot \vec{r} + \omega t$ the wave moves in the direction of $\vec{k}$. (d) $\vec{k} \cdot \vec{k} = (\pi \times 10^7)^2$, $k = \pi \times 10^7$ m^{-1} and $\lambda = 2\pi/k = 200$ nm. (e) $\omega = 9.42 \times 10^{15}$ rad/s and $\nu = \omega/2\pi = 1.5 \times 10^{15}$ Hz. (f) $v = \nu\lambda = 3.00 \times 10^8$ m/s.

3.6 (a) The field is linearly polarized in the y-direction and varies sinusoidally from zero at $z = 0$ to zero at $z = z_0$. (b) Using the wave equation

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0,$$

$$\left[-k^2 - \frac{\pi^2}{z_0^2} + \frac{\omega^2}{c^2}\right] E_0 \sin\frac{\pi z}{z_0}\cos(kx - \omega t) = 0$$

and since this is true for all x, z, and t each term must equal zero and so $k = (\omega/c)\sqrt{1-(c\pi/\omega z_0)^2}$. (c) Moreover, $v = \omega/k = c/\sqrt{1-(c\pi/\omega z_0)^2}$.

3.7 (a) $c = \nu\lambda$, so $\nu = c/\lambda = (3\times10^8\ \text{m/s})/(550\times10^{-9}\ \text{m}) = 5.45\times10^{14}$ Hz. (b) $\omega = 2\pi\nu = 2\pi(5.45\times10^{14}\ \text{Hz}) = 3.43\times10^{15}$ rad/s; $k = 2\pi/\lambda = 2\pi/(550\times10^{-9}\ \text{m}) = 1.14\times10^{-7}\ \text{m}^{-1}$. (c) $E_0 = cB_0$, so $B_0 = E_0/c = (600\ \text{V/m})/(3\times10^8\ \text{m/s}) = 2\times10^{-6}\ \text{V-s/m}^2 = 2\times10^{-6}$ T. (d) $E(y,t) = E_0\sin(ky - \omega t + \epsilon)$; $E(0,0) = 0 = E_0\sin(\epsilon)$, $\epsilon = 0$; $B(y,t) = B_0\sin(ky - \omega t + \epsilon)$; $B(0,0) = 0 = B_0\sin(\epsilon)$, $\epsilon = 0$; $E(y,t) = (600\ \text{V/m})\sin((1.14\times10^{-7}\ \text{m}^{-1})y - (3.43\times10^{15}\ \text{rad/s})t)$; $B(y,t) = (2\times10^6 T)\sin((1.14\times10^{-7}\ \text{m}^{-1})y - (3.43\times10^{15}\ \text{rad/s})t)$.

3.8 By Gauss' law, $E = \sigma/\epsilon_0$, where $\sigma = q/A$ is the surface charge density. Putting the average value of this electric field into $u_E = \epsilon_0 E^2/2$ gives $u_E = \sigma^2/8\epsilon_0$.

3.9 $u_B = B^2/2\mu_0$; $c = 1/\sqrt{\epsilon_0\mu_0}$, so $c^2\epsilon_0 = 1/\mu_0$. $u_B = c^2\epsilon_0 B^2/2$; $E = cB$, so $u_B = \epsilon_0(cB)^2/2 = \epsilon_0 E^2/2 = u_E$.

3.10 $\langle\cos^2(\vec{k}\cdot\vec{r} - \omega t)\rangle = (1/T)\int_t^{t+T}\cos^2(\vec{k}\cdot\vec{r} - \omega t')\,dt'$. Let $\vec{k}\cdot\vec{r} - \omega t' = x$; then $\langle\cos^2(\vec{k}\cdot\vec{r} - \omega t)\rangle = -(1/\omega T)\int\cos^2 x\,dx = -(1/2\omega T)\int(1+\cos 2x)\,dx = -(1/2\omega T)[x + 0.5\sin 2x]_{\vec{k}\cdot\vec{r}-\omega t}^{\vec{k}\cdot\vec{r}-\omega(t+T)}$. Similarly use $\langle\sin^2(\vec{k}\cdot\vec{r} - \omega t)\rangle = (1/2)\langle 1 - \cos 2(\vec{k}\cdot\vec{r} - \omega t)\rangle$ and $\langle\sin(\vec{k}\cdot\vec{r} - \omega t)\cos(\vec{k}\cdot\vec{r} - \omega t)\rangle = (1/2)\langle\sin 2(\vec{k}\cdot\vec{r} - \omega t)\rangle$.

3.11 Using the identity $\cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha)$ we have
$\langle\cos^2\omega t\rangle_T = \langle\frac{1}{2}[1+\cos 2\omega t]\rangle_T = \frac{1}{2}[1+\langle\cos 2\omega t\rangle_T] = \frac{1}{2}[1+(\operatorname{sinc}\omega T)\cos 2\omega t]$.

3.12 Using the identity $\sin^2\alpha = \frac{1}{2}(1-\cos 2\alpha)$ we have
$\langle\sin^2\omega t\rangle_T = \langle\frac{1}{2}[1-\cos 2\omega t]\rangle_T = \frac{1}{2}[1-\langle\cos 2\omega t\rangle_T] = \frac{1}{2}[1-(\operatorname{sinc}\omega T)\cos 2\omega t]$.

3.13 $I = \langle S\rangle_T = \langle c^2\epsilon_0\vec{E}_0\times\vec{B}_0\cos^2(\vec{k}\cdot\vec{r}-\omega T)\rangle =$
$c^2\epsilon_0|\vec{E}_0\times\vec{B}_0|\langle\cos^2(\vec{k}\cdot\vec{r}-\omega t)\rangle = c^2\epsilon_0 E_0 B_0/2$; $E_0 = cB_0$, $c = 1/\sqrt{\mu_0\epsilon_0}$, so
$\epsilon_0 c = 1/\mu_0 c$. $I = E_0^2/2c\mu_0$. If $E_0 = 15.0$ V/m,
$I = (15.0\text{ V/m})^2/2(3\times10^8\text{ m/s})(4\pi\times10^{-7}\text{ m-kg/C}^2) = .9375/\pi\text{ W/m}^2 =$
$.298\text{ W/m}^2$.

3.14 Total Power = 20 W; Total Area at 1.0 m $= 4\pi(10.0\text{ m})^2 = 4\pi\text{ m}^2$;
$I = \text{Power/Area} = (20\text{ W})/(4\pi\text{ m}^2) = 5/\pi\text{ W/m}^2 = 1.6\text{ W/m}^2$.

3.15 (a) $\tau = 1/\nu = 10^{-7}$ s, $v = c$, $\lambda = c/\nu = c\tau = 30$ m.
(b) $E_y = 0.08\cos(2\pi\nu(t-x/)$, $B_z = E_y/c$. (c) By Eq. (3.44),
$\langle S\rangle = c\epsilon_0 E_0^2/2$.

3.16 Will find I, then E_0 using Eq. (3.44). Total Power $= L = 3.9\times10^{26}$ W;
Total Area at
$1.5\times10^{11}\text{ m} = 4\pi(1.5\times10^{11}\text{ m})^2 = 9.0\pi\times10^{22}\text{ m}^2 = 2.8\times10^{23}\text{ m}^2$.
$I = \text{Power/Area} = (3.9\times10^{26}\text{ W})/(2.8\times10^{23}\text{ m}^2) = 1.4\times10^3\text{ W/m}^2$.
From Eq. (3.44) $I = (c\epsilon_0/2)E_0^2$, so $E_0 = \sqrt{2I/c\epsilon_0}$;

$$E_0 = \sqrt{\frac{2(1.4\times10^3\text{ W/m}^2)}{(3\times10^8\text{ m/s})(8.85\times10^{-12}\text{ s}^2\text{-C}^2/\text{m-kg})}}$$

$$E_0 = 1.0\times10^3\text{ V/m}$$

3.17 $\vec{E}_0 = (E_0/\sqrt{2})(-\hat{\imath}+\hat{\jmath})$, $\vec{k} = (2\pi/\sqrt{2}\lambda)(\hat{\imath}+\hat{\jmath})$, hence
$\vec{E} = (10/\sqrt{2})(-\hat{\imath}+\hat{\jmath})\cos[(\sqrt{2}\pi/\lambda)(x+y)-\omega t]$ and
$I = c\epsilon_0 E_0^2/2 = 0.13\text{ W/m}^2$.

3.18 (a) $l = c\Delta t = (3.00 \times 10^8\ \text{m/s})(2.00 \times 10^{-9}\ \text{s}) = 0.600$ m. (b) The volume of one pulse is $V = l\pi R^2 = 2.9 \times 10^{-6}\ \text{m}^3$; therefore $(6.0\ \text{J})/V = 2.0 \times 10^6\ \text{J/m}^3$.

3.19 From Eq. (3.44), $I = c\varepsilon_0 E_0^2/2$ and so $E_0 = \sqrt{2I/c\varepsilon_0} = \sqrt{7.535 \times 10^{22}} = 2.7 \times 10^{11}$ V/m.

3.20 $u = (\text{power})(t)/(\text{volume}) = (10^{-3}\text{W}(t)/(\pi r^2)(ct) =$ $10^{-3}\text{W}/\pi(10^{-3})^2(3 \times 10^8)$, $u = 1.1 \times 10^{-6}\ \text{J/m}^3$.

3.21 $V = Al = Avt$ so that $N/At = nV/At = nv = 100\ \text{m}^{-3}6\ \text{m/60s} = 10\ \text{m}^{-2}\text{s}^{-1}$.

3.22 $I/E = I/h\nu = (19.88 \times 10^{-2})/(6.63 \times 10^{-34})(100 \times 10^6) =$ 3×10^{24} photons/m^{2}s. $n = (1/c)(I/E) = 10^{16}$ photons/m^3.

3.23 $N/t = P/h\nu = P\lambda/hc = 2.8 \times 10^{20}\ \text{s}^{-1}$.

3.24 $P_e = iV = (0.25)(3.0) = 0.75$ W. This is the electrical power dissipated. The power available as light is $P_l = (0.01)P_e = 75 \times 10^{-4}$ W. (a) The photon flux is $P_l/h\nu = P_l\lambda/hc = 2.1 \times 10^{16}$ photons/s. (b) There are 2.1×10^{16} in volume $(3 \times 10^8)(1s)(10^{-3})\ \text{m}^2$. Therefore $2.1 \times 10^{16}/3 \times 10^5 = 0.69 \times 10^{11}$ is the number of photons per cubic meter. (c) $I = 75 \times 10^{-4}\ \text{W}/10 \times 10^{-4}\ \text{m}^2 = 7.5\ \text{W/m}^2$.

3.25 $I = P/4\pi r^2$, $E_0 = \sqrt{2I/\epsilon_0 c}$, and $B_0 = E_0/c$.

3.26 Imagine two concentric cylinders of radii r_1 and r_2 surrounding the wave. The energy flowing per second through the first cylinder must pass through the second; that is, $\langle S_1\rangle 2\pi r_1 = \langle S_2\rangle 2\pi r_2$, and so $\langle S\rangle 2\pi r =$constant and $\langle S\rangle$ varies inversely with r. Therefore, since $\langle S\rangle \propto E_0^2$, E_0 varies as $\sqrt{1/r}$.

3.27 $p = E/c = h\nu/c = 2.2 \times 10^{-23}\ \text{kg m s}^{-1}$.

3.28 $\langle dp/dt\rangle = \langle dW/dt\rangle/c$, with area A, $\langle \mathcal{P}\rangle = \langle dp/dt\rangle/A = \langle dW/dt\rangle/Ac = I/c$.

3.29 From Eq. (3.52), the force exerted by the beam of light, $A\mathcal{P} = \Delta p/\Delta t$, where $p(\text{incident}) = \mathcal{E}/c$. For reflected light at normal incidence, $\Delta p =$ twice the incident momentum $= 2(\mathcal{E}/c)$
$A\mathcal{P} = 2(\mathcal{E}/c)/\Delta t$, but, $I = \mathcal{E}/\text{Area/time}$, so $\mathcal{P} = 2I/c$.
At an angle θ with respect to the normal, only the component of momentum normal to the surface changes, so $p(\text{normal}) = p\cos\theta$, so, $\mathcal{P}(\theta) = 2I\cos\theta/c$.

3.30 $E = 300 \cdot 100 = 3 \times 10^4$ J, $p = E/c = 10^{-4}$ kg m/s.

3.31 (a) $\langle \mathcal{P} = 2\langle S\rangle/c = 2(1.4 \times 10^3\ \text{W/m}^2)/(3 \times 10^8\ \text{m/s}) = 9 \times 10^{-6}\ \text{N/m}^2$.
(b) S, and therefore $\mathcal{P}$, drops off with the inverse square of the distance, and hence
$\langle S\rangle = [(0.7\times 10^9\ \text{m})^{-2}/(1.5\times 10^{11}\ \text{m})^{-2}](1.4\times 10^3\ \text{W/m}^2) = 6.4\times 10^7\ \text{W/m}^2$, and $\langle \mathcal{P}\rangle = 0.21\ \text{N/m}^2$.

3.32 $I(\text{absorbed}) = \alpha I$ and $I(\text{scattered}) = (1-\alpha)I$; the pressure arises from both contributions, viz. $\mathcal{P} = \alpha I/c + 2(1-\alpha)I/c = (2-\alpha)I/c$.

3.33 The reflected component has a momentum change, and thus a pressure, of twice the incident momentum, while the absorbed component has a momentum change of the incident momentum.

$$\mathcal{P}\ (\text{reflected}) = 2(70.0\%)I/c = 2(0.700)(2.00 \times 10^6\ \text{W/m}^2)/(3 \times 10^8\ \text{m/s})$$
$$= .93 \times 10^{-2}\ \text{N/m}^2.$$
$$\mathcal{P}\ (\text{absorbed}) = 2(30.0\%)I/c = 2(0.300)(2.00 \times 10^6\ \text{W/m}^2)/(3 \times 10^8\ \text{m/s})$$
$$= .20 \times 10^{-2}\ \text{N/m}^2.$$
$$\mathcal{P} = \mathcal{P}\ (\text{reflected}) + \mathcal{P}\ (\text{absorbed}) = 1.13 \times 10^{-2}\ \text{N/m}^2.$$

3.34 $\langle S\rangle = 1400\ \text{W/m}^2$, $\langle \mathcal{P}\rangle = 2(1400\ \text{W/m}^2/3 \times 10^8\ \text{m/s}) = 9.3 \times 10^{-6}\ \text{N/m}^2$.

3.35 $\langle S\rangle = (200 \times 10^3 \text{ W})500 \times 2 \times 10^{-6} \text{ s})/A(1s)$,
$\langle F\rangle = A\langle \mathcal{P}\rangle = A\langle S\rangle/c = 6.7 \times 10^{-7}$ N.

3.36 $\langle F\rangle = A\langle \mathcal{P}\rangle = A\langle S\rangle/c = 10 \text{ W}/3 \times 10^8 = 3.3 \times 10^{-8}$ N,
$a = 3.3 \times 10^{-8}/100 \text{ kg} = 3.3 \times 10^{-10} \text{ m/s}^2$,
$v = at = 3.3 \times 10^{-10}t = 10$ m/s. Therefore $t = 3 \times 10^{10}$ s or $t = 940$ years.

3.37 $\vec{B}$ surrounds $\vec{v}$ in circles, and $\vec{E}$ is radial, hence $\vec{E} \times \vec{B}$ is tangent to the sphere, and no energy radiates outward from it.

3.38 (a) $\nu = 5 \times 10^{14}$ Hz, (b) $\lambda = v/\nu = 0.65c/\nu = 3.9 \times 10^{-7}$ m,
(c) $n = c/v = 1.5$.

3.39 $c/v = 2.42$; $v = 1.24 \times 10^8$ m/s.

3.40 $\lambda_0 = 540$ nm; $n = \nu\lambda_0/\nu\lambda$; $\lambda_0/n = \lambda = 406$ nm.

3.41 $n = c/v = 1/0.90 = 1.11 = 1.1$.

3.42 $n = c/v = (3 \times 10^8 \text{ m/s})(1.245 \times 10^8 \text{ m/s}) = 2.410$

3.43 $l = vt = (c/n)t = (3.00 \times 10^8 \text{ m/s})(1.00 \text{ s})/1.333 = 2.25 \times 10^8$ m.
$n = 1.36 = c/v$; $v = c/n = 2.21 \times 10^8$ m/s.

3.44 $\lambda = \lambda_0/n = (500 \text{ nm})/1.60 = 3.12.5$ nm;
$(1.00 \times 10^{-2} \text{ m})/(312.5 \times 10^{-9} \text{ m}) = 3.2 \times 10^4$ waves.

3.45 $t_1 = (20.0 \text{ m})/(c/1.47)$ and $t_2 = (20.0 \text{ m})/(c/1.63)$, hence
$t_2 - t_1 = 3.2/c = 1.07 \times 10^{-8}$ s.

3.46 The number of waves in vacuum is $\overline{AB}/\lambda_0$. With the glass in place, there are $(\overline{AB} - L)/\lambda_0$ waves in vacuum and an additional L/λ waves in glass for a total of $(\overline{AB}/\lambda_0) + L(1/\lambda - 1/\lambda_0)$. The difference in number is $L(1/\lambda - 1/\lambda_0)$, giving a phase shift of $\Delta\phi$ of 2π for each wave; hence,
$2\pi L(1/\lambda - 1/\lambda_0) = 2\pi L(n/\lambda_0 - 1/\lambda_0) = 2\pi L/2\lambda_0 = 2000\pi$.

3.47 Thermal agitation of the molecular dipoles causes a marked reduction in K_e but has little effect on n. At optical frequencies n is predominantly due to electronic polarization, rotations of the molecular dipoles having ceased to be effective at much lower frequencies.

3.48 From Eq. (3.70), for a single resonant frequency we have

$$n = \left[1 + \frac{Nq_e^2}{\epsilon_0 m_e}\left(\frac{1}{\omega_0^2 - \omega^2}\right)\right]^{1/2};$$

since for low-density materials $n \approx 1$, the second term is $\ll 1$, and we need only retain the first two terms of the binomial expansion of n. Thus $\sqrt{1+x} \approx 1 + x/2$ and $n \approx 1 + Nq_e^2/2\epsilon_0 m_e(\omega_0^2 - \omega^2)$.

3.49 (a) The polar molecule, water, in the liquid state, is relatively free to move in response to the incident radiation. In the solid state, the molecules are not free to move. (b) The radar (microwave) interacts strongly with the liquid qater in the droplets.

3.50 The normal order of the spectrum for a glass prism is R, O, Y, G, B, V, with red (R) deviated the least and violet (V) deviated the most. For a fuchsin prism, there is an absorption band in the green, and so the indices for yellow and blue on either side (n_y and n_B) of it are extremes, that is, n_Y is the maximum, n_B the minimum, and $n_Y > n_O > n_R > n_V > n_B$. Thus the spectrum in order of increasing deviation is B, V, black band, R, O, Y.

3.51 Since $(Nq_e^2/\epsilon_0 m_e)^{1/2}$ has dimensions of frequency, the right-hand side is dimensionless and the units check.

3.52 With ω in the visible, $\omega_0^2 - \omega^2$ is smaller for lead glass and larger for fused silica. Hence $n(\omega)$ is larger for the former and smaller for the latter.

3.53 Subtract 1 from each side of Eq. (3.70) and then invert both sides: $1/(n^2 - 1) = (\varepsilon_0 m_e/Nq_e^2)(\omega_0^2 - \omega^2)$; since $\omega = 2\pi c/\lambda$ the desired result follows.

3.54 C_1 is the value that n approaches as λ gets larger.

3.55 The horizontal values of $n(\omega)$ approached in each region between absorption bands increase as ω decreases.

3.56 Subtracting the two equations $1.557 = n_1 = C_1 + C_2/\lambda_1^2$ and $1.547 = n_2 = C_1 + C_2/\lambda_2^2$ gives $\Delta n = 0.01 = n_1 - n_2 = C_2(1/\lambda_1^2 - 1/\lambda_2^2)$ so that $C_2 = \Delta n \lambda_1^2 \lambda_2^2/(\lambda_2^2 - \lambda_1^2) = 3.78 \times 10^3$ nm^2. Then $C_1 = n_1 - C_2/\lambda_1^2 = 1.5345$ and $n(610nm) = C_1 + C_2/\lambda_3^2 = 1.545$.

3.57 Binomially expanding $n^2 \approx 1 + A/(1 - \lambda_0^2/\lambda^2)$ gives $n^2 \approx 1 + A(1 + \lambda_0^2/\lambda^2)$ or $n^2 = (1 + A)[1 + A\lambda_0^2/(1 + A)\lambda^2]$. Taking the square root and expanding again gives $n \approx (1 + A)^{1/2}[1 + A\lambda_0^2/2(1 + A)\lambda^2$. This has the Cauchy form with $C_1 = (1 + A)^{1/2}$ and $C_2 = A\lambda_0^2/2(1 + A)^{1/2}$.

3.58 $\nu = E/h = 2.7 \times 10^{15}$ Hz.

Chapter 4 Solutions

4.1 $E_{os} \propto VE_{oi}/r = KVE_{oi}/r$; thus VK/r must be unitless, and so K has units of $(\text{length})^{-2}$. The only quantity unaccounted for is λ and so we conclude that $K = \lambda^{-2}$, and $I_i/I_s \propto K^2 \propto \lambda^{-4}$.

4.2 The degree of Rayleigh scattering is proportional to $1/\lambda^4$. But $\lambda_y = 1.45\lambda_v$ and so $1/\lambda_y^4 = (1/1.45\lambda_v)^4$ hence violet is scattered $(1.45)^4 = 4.42$ times more intensely than yellow. The ratio of yellow to violet is 22.6%.

4.3 The sinusoids represent the field, in this case the E-field of the disturbance. The wavefront is a surface of constant phase and it meets each sinusoid at the same point (same phase) in its development. The outward radial lines are rays and they are everywhere perpendicular to the wavefronts.

4.4 (a) On the left-hand side are the inertial, drag force, and elastic force terms; on the right-hand side is the electric driving force. (b) $x_0(-\omega^2 + \omega_0^2 + i\gamma\omega) = (q_eE_0/m_e)\exp(i\alpha)$, forming the absolute square of both sides yields $x_0^2[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2] = (q_eE_0/m_e)^2$ and x_0 follows by division and taking the square root. (c) As for α, divide the imaginary parts of both sides of the first equation above, namely $x_0\gamma\omega = (q_eE_0/m_e)\sin\alpha$, by the real parts, $x_0(\omega_0^2 - \omega^2) = (q_eE_0/m_e)\cos\alpha$ to obtain $\alpha = \tan^{-1}[\gamma\omega/(\omega_0^2 - \omega^2)]$. α ranges continuously from 0 to $\pi/2$ to π.

4.5 (a) The phase angle is retarded by an amount $(n\Delta y 2\pi/\lambda) - \Delta y 2\pi/\lambda$ or $(n-1)\Delta y\omega/c$. Thus $E_p = E_0 \exp i\omega[t - (n-1)\Delta y/c - y/c]$ or $E_p = E_0 \exp[-i\omega(n-1)\Delta y/c] \exp i\omega(t - y/c)$ (b) Since $e^x \approx 1 + x$ for small x, if $n \approx 1$ or $\Delta y \ll 1$, $\exp[-i\omega(n-1)\Delta y/c] \approx 1 - i\omega(n-1)\Delta y/c$ and since $\exp(-i\pi/2) = -i$, $E_p = E_u + \omega(n-1)\Delta y(E_u/c)\exp(-i\pi/2)$.

4.6 $\sin 58° = x/(5.0 \text{ m})$, $x = 4.2$ m.

4.7 The statue is 16 m from the point of incidence, and since the ray-triangles are similar, 4 m : 16 m as 3 m : Y and $Y = 12$ m.

4.8 At the first mirror, $\theta_r = \theta_i$. For the second, $\theta_i' = 90 - \theta_r = 90 - \theta_i$ and $\theta_r' = \theta_i'$, so $\theta_r' = 90 - \theta_i$.

4.9 $n_i \sin\theta_i = n_t \sin\theta_t$, $\sin 30° = 1.52 \sin\theta_t$, $\theta_t = \sin^{-1}(1/3.04)$, so $\theta_t = 19°13'$.

4.10
$$\begin{aligned} P_{\text{transverse}} &= mv_i \sin\theta_i \\ &= mv_i \sin\theta_t \end{aligned}$$
where "m" is the presumed mass. But $v_i = \frac{s_0}{t}, v_f = \frac{BP}{t}$. So

$$\begin{aligned} (s_0)\sin\theta_i &= (BP)\sin\theta_t \\ \sin\theta_i &= \frac{BP}{s_0}\sin\theta_t \end{aligned}$$

The factor $\frac{BP}{s_0}$ corresponds to r_{ti}.

4.11 The slope of the curve is $n_{it} = n_i/n_t$. Slope $\sim 0.75/1.00$, so that $n_t \simeq 1.33$. This suggests that the dense medium is water.

4.12 $\theta_t = \sin^{-1}[(\sin 45°)/2.42] = 17°$, the angular deviation is $45° - 17° = 28°$.

4.13 $\theta_t = \sin^{-1}[(n_w/n_g)\sin\theta_i] = \sin^{-1}[(8/9)\sin 45°] = 39°$. For a ray incident in the glass at this angle,
$\theta_t = \sin^{-1}[(n_g/n_w)\sin 39°] = \sin^{-1}[(9/8)\sin 39°] = 45°$.

4.14 (a) $n_{ti} = n_t/n_i = (c/v_t)/(c/v_i) = v_i/v_t = \nu\lambda_i/\nu\lambda_t = \lambda_i/\lambda_t$. Therefore $\lambda_t = \lambda_i 3/4 = 9$ cm. (b) $\sin\theta_i = n_{ti}\sin\theta_t$, $\theta_t = \sin^{-1}[(3/4)\sin 45°] = 32°$.

4.15 $\lambda_t = \lambda_i/n_{ti} = 600/1.5 = 400$ nm, violet light.

4.16 $1.00 \sin 55° = n \sin 40°$; $n = 1.27$ or 1.3.

4.17 $1.33 \sin 35° = 1.00 \sin\theta_t$; $\theta_t = 50°$.

4.18 For $\theta_i = 0$, 10, 20, 30, 40, 50, 60, 70, 80,90 degrees, $\theta_t = 0$, 6.7, 13.3, 19.6, 25.2, 30.7, 35.1, 38.6, 40.6, 41.8 degrees respectively.

4.19 Consider one ray on each side of the beam, with a perpendicular separation D. The width of the beam on the interface is $D\cos\theta_i$. Likewise, the width of the beam at the interface is $D'\cos\theta_t$, where D' is the perpendicular separation (width) of the rays in the glass, and $D\cos\theta_i = D'\cos\theta_t$. (4.4) $n_i \sin\theta_i = n_t \sin\theta_t$ so

$$\begin{aligned}\cos\theta_t &= (1-\sin^2\theta_t)^{1/2}\\ &= (1-\sin^2\theta_i/n_g^2)^{1/2}\end{aligned}$$

so

$$D' = \frac{D\cos\theta_i}{\left(1-\frac{\sin^2\theta_i}{n_g^2}\right)^{1/2}}$$

4.20 (4.4) $n_i \sin\theta_i = n_t \sin\theta_t$ so $\sin(60.0°) = n_t \sin G_\epsilon$. Diameter of emerging beam (D) is related to the difference in horizontal displacement of red and violet light (h) by $D\cos(60.0°) = h$ (See Problem 4.19). Red: $\sin\theta_{\text{red}} = \sin(60.0°)/n_{\text{red}} = (\sqrt{3}/2)/(1.505), \theta_{\text{red}} = 35.1°$; $\tan\theta_{\text{red}} = h_{\text{red}}/10.0$ cm so $h_{\text{red}} = (10.0\text{ cm})\tan(35.1°) = 7.04$ cm. Violet: $\sin\theta_{\text{violet}} = \sin(60.0°)/n_{\text{violet}} = (\sqrt{3}/2)/(1.545)$; $\theta_{\text{violet}} = 34.1°$; $h_{\text{violet}} = (10.0\text{ cm})\tan(34.1°) = 6.77$ cm. $D = h/\cos(60.0°) = (h_{\text{red}} - h_{\text{violet}})/\cos(60.0°) = (7.04 - 6.77)/(0.5) = 0.54$ cm.

4.21 $n_a/n_w = d_A/d_R = 1/1.333 = 0.750 = 3/4$.

4.22 Using Figure S.4.17, $1.00\sin 35° = 1.50\sin\theta_{t1}$; $\theta_{t1} = 22.48°$ and $\cos 22.48° = (2.00\text{ cm})/L$; $L = 2.16$ cm or 2.2 cm.

4.23 $\sin\theta_i = n\sin\theta_i/2$; since $\sin 2\alpha = 2\sin\alpha\cos\alpha$, $\sin\theta_i = 2\sin(\theta_i/2)\cos(\theta_i/2)$ and so setting these two expressions equal we get

$$1.70\sin(\theta_i/2) = 2\sin(\theta_i/2)\cos(\theta_i/2); \quad \cos\theta_i/2 = 0.85;$$
$$31.79° = \theta_i/2; \quad \theta_i = 63.6°.$$

4.24 The glass will change the depth of the object from d_R to d_A, where $d_A/d_R = 1.00/1.55$; but $d_R = 1.00$ mm; hence, $d_A = 0.645$ mm and the camera must be raised 1.00 mm − 0.645 mm = 0.355 mm.

4.25 $d_{A1}/d_{R1} = 1.50/1.33$; $d_{R1} = 1.00$ m; $d_{A1} = 1.1278$ m; $d_{R2} = d_{A1} + 0.02$ m; $d_{A2}/d_{R2} = 1.00/1.50$; $d_{A2} = 1.3278(1.00/1.50) = 0.885$ m.

4.26 The number of waves per unit length along $\overline{AC}$ on the interface equals $(\overline{BC}/\lambda_i)/(\overline{BC}\sin\theta_i) = (\overline{AD}/\lambda_t)/(\overline{AD}\sin\theta_t)$. Snell's law follows on multiplying both sides by c/ν.

4.27 With the origin in the plane of incidence, $z = 0$; with the origin on the interface $y = 0$ so $(\vec{k}_i \cdot \vec{r}) \rightarrow k_{ix}x$

$$(\vec{k}_r \cdot \vec{r} + \epsilon_r) \rightarrow k_{rx}x + \epsilon_r$$

$$(\vec{k}_t \cdot \vec{t} + \epsilon_t) \rightarrow k_{tx}x + \epsilon_t$$

and as $\epsilon_r = \epsilon_t = 0$, Eq. (4.19) becomes $k_{ix} = k_{rx} = k_{tx}$ or $k_i \sin\theta_i = k_r \sin\theta_r = k_t \sin\theta_t$. Since $k = 2\pi/\lambda$

$$\frac{\sin\theta_i}{\lambda_i} = \frac{\sin\theta_t}{\lambda_t},$$

which is the condition derived in Problem (4.26) for wave front continuity.

4.28 Let τ be the time for the wave to move along a ray from b_1 to b_2, from a_1 to a_2, and from a_1 to a_3. Thus $\overline{a_1a_2} = \overline{b_1b_2} = v_i\tau$ and $\overline{a_1a_3} = v_i\tau$.
$\sin\theta_i = \overline{b_1b_2}/\overline{a_1b_2} = v_i/\overline{a_1b_2}$, $\sin\theta_t = \overline{a_1a_3}/\overline{a_1b_2} = v_t/\overline{a_1b_2}$,
$\sin\theta_r = \overline{a_1a_2}/\overline{a_1b_2} = v_i/\overline{a_1b_2}$, $\sin\theta_i/\sin\theta_t = v_i/v_t = n_t/n_i = n_{ti}$ and $\theta_i = \theta_r$.

4.29 $n_i \sin\theta_i = n_t \sin\theta_t$, $n_i(\hat{k}_i \times \hat{u}_n) = n_t(\hat{k}_t \times \hat{u}_n)$, where $\hat{k}_i$, $\hat{k}_t$ are unit propagation vectors. Thus $n_t(\hat{k}_t \times \hat{u}_n) - n_i(\hat{k}_i \times \hat{u}_n) = 0$,
$(n_t\hat{k}_t - n_i\hat{k}_i) \times \hat{u}_n = 0$. Let $n_t\hat{k}_t - n_i\hat{k}_i = \vec{\Gamma} = \Gamma\hat{u}_n$. Γ is often referred to as the *astigmatic constant*; Γ is the difference between the projections of $n_t\hat{k}_t$ and $n_i\hat{k}_i$ on $\hat{u}_n$; in other words, take the dot product $\vec{\Gamma} \cdot \hat{u}_n$:
$\Gamma = n_t \cos\theta_t - n_i \cos\theta_i$.

4.30 Since $\theta_i = \theta_r$, $\hat{k}_{ix} = \hat{k}_{rx}$ and $\hat{k}_{iy} = -\hat{k}_{ry}$, and since $(\hat{k}_i \cdot \hat{u}_n)\hat{u}_n = \hat{k}_{iy}$, $\hat{k}_i - \hat{k}_r = 2(\hat{k}_i \cdot \hat{u}_n)\hat{u}_n$.

4.31 Since $\overline{SB'} > \overline{SB}$ and $\overline{B'P} > \overline{BP}$, the shortest path corresponds to B' coincident with B in the plane of incidence.

4.32 (Refer to Figure 4.28.) Let $\overline{SP} = a$, distance along interface $(S \rightarrow B) = x$, distance of S and P from interface $= h$.

$$t = \frac{\overline{SB}}{v_i} + \frac{\overline{BP}}{v_i}$$

$$= \frac{(h^2 + x^2)^{1/2}}{v_i} + \frac{(h^2 + (a-x)^2)^{1/2}}{v_i}$$

Minimize $t(x)$ *w.r.t.* x.

$$\frac{dt}{dx} = \frac{x}{v_i(h^2 + x^2)^{1/2}} + \frac{-(a-x)}{v_i(h^2 + (a-x)^2)^{1/2}} = 0$$

$$\sin\theta_i = \sin\theta_r \quad \text{or} \quad \theta_i = \theta_r$$

4.33 The mirrors are set as two sides of the acute triangle. The front of the laser is placed along the third side. The inscribed triangle is found by adjusting the position and the angle of the laser bean until the incoming and reflected beams meet on the triangle.

4.34 $n_1 \sin\theta_i = n_2 \sin\theta_t$, $\theta_t = \theta_i'$, $n_2 \sin\theta_i' = n_1 \sin\theta_t'$, $n_1 \sin\theta_i = n_1 \sin\theta_t'$ and $\theta_i = \theta_t'$. $\cos\theta_t = d/\overline{AB}$, $\sin(\theta_i - \theta_t) = a/\overline{AB}$, $\sin(\theta_i - \theta_t) = (a/d)\cos\theta_t$, $d\sin(\theta_i - \theta_t)/\cos\theta_t = a$.

4.35 The left and right beams will be parallel if $\theta_t(\text{Left}) = \theta_t(\text{Right})$ in the final medium (a). Since all interfaces are parallel, the transmitted angle into a medium equals the incident angle at the next medium.
At each interface (4.5) $\sin\theta_i = n_{ti}\sin\theta_t$.

$$\text{Left: } \sin\theta_i = n_{1a}\sin\theta_{t1} = n_{1a}(n_{a1}\sin\theta_{ta}) = n_{1a}n_{1a}(n_{2a}\sin\theta_{t2})$$

$$= n_{1a}n_{1a}n_{2a}(n_{a2}\sin\theta_{ta}) = \sin\theta_{ta}.$$

$$\text{Right: } \sin\theta_i = n_{1a}\sin\theta_{t1} = n_{1a}(n_{21}\sin\theta_{t2}) = n_{1a}(n_{21}\sin\theta_{ta})$$
$$= n_{1a}n_{21}(n_{a2}\sin\theta_{ta}) = \sin\theta_{ta}.$$

For each beam, $\theta_{ta} = \theta_i$.

4.36 Rather than propagating from point S to point P in a straight line, the ray traverses a path that crosses the plate at a sharper angle. Although in so doing the path lengths in air are slightly increased, the decrease in time spent within the plate more than compensates. This being the case, we might expect the displacement a to increase with n_{21}. As n_{21} gets larger for a given θ_i, θ_t decreases, $\theta_i - \theta_t$ increases, and from the results of Problem 4.30, a clearly increases.

4.37 $\int_C \vec{E}\cdot d\vec{l} = -\int\int_A (\partial\vec{B}/\partial t)\cdot d\vec{S}$. This reduces in the limit to $E_{2x}(\overline{BC}) - E_{1x}(\overline{AD}) = 0$, since the area goes to zero and $\partial\vec{B}/\partial t$ is finite. Thus $E_{2x} = E_{1x}$.

4.38 From Eq. (4.40), $r_\parallel = (1.52\cos 30° - \cos 19°13')/(\cos 19°13' + 1.52\cos 30°)$, where from Problem 4.8, $\theta_t = 19°13'$. Similarly, $t_\parallel = 2\cos 30°/(\cos 19°13' + 1.52\cos 30°)$, $r_\parallel = 0.165$, $t_\parallel = 0.766$.

4.39 Starting with Eq. (4.34), divide top and bottom by n_i and replace n_{ti} with $\sin\theta_i/\sin\theta_t$ to get

$$r_\perp = \frac{\sin\theta_t\cos\theta_i - \sin\theta_i\cos\theta_t}{\sin\theta_t\cos\theta_i + \sin\theta_i\cos\theta_t},$$

which is equivalent to Eq. (4.42). Equation (4.44) follows in exactly the same way. To find $r_\parallel$ start the same way with Eq. (4.40) and get

$$r_\parallel = \frac{\sin\theta_i\cos\theta_i - \sin\theta_t\cos\theta_t}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i}.$$

There are several routes that can be taken now; one is to rewrite $r_\parallel$ as

$$r_\parallel = \frac{\sin\theta_i\cos\theta_t - \sin\theta_t\cos\theta_i}{\sin\theta_i\cos\theta_t + \sin\theta_t\cos\theta_i}\,\frac{\cos\theta_i\cos\theta_t - \sin\theta_i\sin\theta_t}{\cos\theta_i\cos\theta_t + \sin\theta_i\sin\theta_t}$$

and so

$$r_\parallel = \frac{\sin(\theta_i - \theta_t)\cos(\theta_i + \theta_t)}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}.$$

We can find $t_\parallel$, which has the same denominator, in a similar way.

4.40 From Snell's law $\theta_t = 12.748°$; from Eq. (4.43),

$$r_\parallel = \tan 7.252° / \tan 32.748° = 0.1978;$$

using Eq. (4.42),

$$r_\perp = -\sin 7.252° / \sin 32.748° = -0.2352;$$

$$[E_{or}]_\parallel = r_\parallel [E_{oi}]_\parallel = 1.98 \text{V/m};$$

$$[E_{or}]_\perp = r_\perp [E_{oi}]_\perp = -4.70 \text{ V/m}.$$

4.41 For small angles Snell's Law becomes $1\theta_i = n\theta_t$: from Eq. (4.42) using the identity $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ and $r_\perp = -(\theta_i 1 - 1\theta_t)/(\theta_i 1 + 1\theta_t) = -(n-1)/(n+1)$.

4.42 From (4.47), $R = r^2 = (n - 1/n + 1)^2 = (1.522 - 1/1.522 + 1)^2 = 0.43$. $T = 1 - R = 0.957$.

4.43 $T = 1 - R = 1 - r^2 = 1 - (n - 1/n + 1)^2 = 1 - (1.33 - 1/1.33 + 1)^2 = 0.98$. From (4.55), $I_t = TI_i = (0.98)(500 \text{ W/m}^2) = 490 \text{ W/m}^2$.

4.44 From (4.47), $R = r^2 = (n_t - n_i/n_t + n_i)^2 = (1.376 - 1.33/1.376 + 1.33)^2 = 0.000289$. $T = 1 - R = 0.999711$. From (4.55) $I_t = TI_i = (0.999711)(400 \text{ W/m}^2) = 399.884 \simeq 400 \text{ W/m}^2$.

4.45 $r \simeq n_t - n_i/n_t + n_i$. Air-water: $r = \frac{4/3-1}{4/3+1} = 1/7 = 0.14$. Air-crown glass: $r = \frac{3/2-1}{3/2+1} = 1/5 = 0.20$. More reflectance for glass. From (4.54) and (4.56). $I_r/I_i = R = r^2$. Air-water: $R = (1/7)^2 = 0.02$. Air-crown glass: $R = (1/5)^2 = 0.04$.

4.46 $\sin x = x - x^3/3! + x^5/5! - \ldots$ and so $\sin(\alpha \pm \beta) = (\alpha \pm \beta)[1 - (\alpha \pm \beta)^2/6]$ using Snell's Law $\theta_t(1 - \theta_t^2/6 + \ldots) = (\theta_i/n)(1 - \theta_i^2/6 + \ldots)$. Use $1\theta_i = n\theta_t$ and the fact that when x is very small $(1+x)^{-1} \approx 1 - x$ we have $\theta_t = (\theta_i/n)(1 - \theta_i^2/6)(1 + \theta_i^2/6n^2)$ dropping terms higher than the third power in θ_i we get $\theta_t = (\theta_i/n)[1 - (n^2-1)\theta_i^2/6n^2]$ and so

$$\theta_i \pm \theta_t = \theta_i \left[1 \pm \frac{1}{n}\left(1 - \frac{n^2-1}{6n^2}\theta_i^2\right)\right].$$

Using Eq. (4.42) and the power series representation of the sine, where terms higher than the third power in θ_i are dropped,

$$-r_\perp = \frac{n - 1 + \frac{\theta_i^2}{6n^2}[n^2 - 1 - (n-1)^3]}{n + 1 - \frac{\theta_i^2}{6n^2}[n^2 - 1 + (n-1)^3]} = \left(\frac{n-1}{n+1}\right)\left(1 + \frac{\theta_i^2}{n}\right)$$

4.47 $\cos(\theta_i + \theta_t)/\cos(\theta_i - \theta_t) = 1 - 2\theta_i^2/n$ multiplying by the ratio of the sines from the previous problem, viz., $[(n-1)/(n+1)](1 - \theta_i^2/n)$ and dropping higher order terms yields the desired equation.

4.48 From Snell's Law $n \sin\theta_t = 1 \sin 90° = 1$ and so with Eq. (4.42) in mind,

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

and

$$\sin(90° \pm \theta_t) = \sin 90° \cos\theta_t \pm \cos 90° \sin\theta_t;$$

then

$$\sin(90° \pm \theta_t) = 1\cos\theta_t,$$

using $\sin^2\theta + \cos^2\theta = 1$ and Snell's Law

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sin(90° \pm \theta_t) = \sqrt{1 - (1/n)^2}$$

and so $r_\perp \to -1$ at glancing incidence.

4.49 Compute $dr_\perp/d\theta_i$ at $\theta_i = 90°$; we'll use $d\theta_t/d\theta_i = 0$ and then prove it; taking the derivative of Eq. (4.42) we get

$$dr_\perp/d\theta_i = -\cos(\theta_i - \theta_t)/\sin(\theta_i + \theta_t) + \sin(\theta_i - \theta_t)\cos(\theta_i + \theta_t)/\sin^2(\theta_i + \theta_t)$$

and for $\theta_i = 90°$ this becomes

$$dr_\perp/d\theta_i = -\sin\theta_t/\cos\theta_t - \sin\theta_t\cos\theta_t/\cos^2\theta_t = 2\tan\theta_t$$

and using Snell's Law, i.e, $\sin\theta_t = 1/n$ when $\theta_i \approx 90°$, and the fact that

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t},$$
$$dr_\perp/d\theta_i = 2\tan\theta_t = 2\sin\theta_t/\cos\theta_t = 2/n\cos\theta_t = 2/\sqrt{n^2 - 1}:$$

this is the rise over the run at the end of the curve where $\theta_i \approx 90°$. Thus if $\alpha_\perp$ is the angle made ith the vertical $\tan\alpha_\perp = \sqrt{n^2 - 1}/2$.

4.50 $[E_{or}]_\perp + [E_{oi}]_\perp = [E_{ot}]_\perp$; tangential field in incident medium equals that in transmitting medium, $[E_{ot}/E_{oi}]_\perp - [E_{or}/E_{oi}]_\perp = 1$, $t_\perp - r_\perp = 1$.
Alternatively, from Eqs. (4.42) and (4.44),

$$\frac{\sin(\theta_i - \theta_t) + 2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)} = 1$$
$$\frac{\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t + 2\sin\theta_t\cos\theta_i}{\sin\theta_i\cos\theta_t + \cos\theta_i\sin\theta_t} = 1.$$

4.51 $n_i\sin\theta_i = n_t\sin\theta_t$ so, $\sin\theta_t = (n_i/n_t)\sin\theta_i = (1.00/1.52)\sin(30°) = 0.33$.
$\theta_t = \sin^{-1}(0.33) = 19.2°$.
(Eq. 4.44) $t_\perp = 2\sin\theta_t\cos\theta_i/\sin(\theta_i + \theta_t) = 2\sin(19.2°)\cos(30°)/\sin(49.2°)$ $= 0.75$.
(Eq. 4.42) $r_\perp = -\sin(\theta_i - \theta_t)/\sin(\theta_i + \theta_t) = -\sin(10.8°)/\sin(49.2°) =$ -0.25. $t_\perp + (-r_\perp) = 0.75 + (0.25) = 1.00$.

4.52 $\theta_c = \sin^{-1}(1/1.5) = 42°$.

4.53 Light incident from air to glass. θ_t increases as θ_i increases, so Maximum θ_t should correspond to Maximum θ_i.
(4.4) $n_i \sin\theta_i = n_t \sin\theta_t$ so, $\sin\theta_t = (n_i/n_t)\sin\theta_i$.
Maximum $\theta_i \leq 90°$ as $\theta_i \to 90°$, $\sin\theta_i \to 1$ so, $\sin\theta_t = n_i/n_t = \sin\theta_c$.

4.54 $1.00/2.417 = \sin\theta_c$; $\theta_c = 24°$ diamond refracts light back out and so looks brilliant.

4.55 $\sin 48.0° = (1.00/n)$; $n = 1.35$.

4.56
$$\theta_i = 45° \to \theta_c$$
$$\sin\theta_i = \frac{n_t}{n_i}, \quad \text{where} \quad n_t = 1$$
$$n = \frac{1}{\sin\ 45°} = 1.41$$

4.57 Light entering at glancing incidence is transmitted at the critical angle and those rays limit the cone of light reaching the fish; $\sin\theta_c = 1/1.333$; $\theta_c = 49°$ and the cone-angle is twice this or 98°.

4.58 $\sin\theta_c = n_t/n_i$; $\theta_c = 59.1°$.

4.59 From Eq. (4.73) we see that the exponential will be in the form $k(x - vt)$, provided that we factor out $k_t \sin\theta_i/n_{ti}$, leaving the second term as $\omega n_{ti} t/k_t \sin\theta_i$, which must be $v_t t$. Hence $\omega n_t/(2\pi/\lambda_t) n_i \sin\theta_i = v_t$, and so $v_t = c/n_i \sin\theta_i = v_i/\sin\theta_i$.

4.60 From the defining equation, $\beta = k_t[(\sin^2\theta_i/n_{ti}^2) - 1]^{1/2} = 3.702 \times 10^6\ \mathrm{m}^{-1}$, and since $y\beta = 1$, $y = 2.7 \times 10^{-7}$ cm.

4.61 The beam scatters off the wet paper and is mostly transmitted until the critical angle is attained, at which point the light is reflected back toward the source. $\tan\theta_c = (R/2)/d$, and so $n_{ti} = 1/n_i = \sin[\tan^{-1}(R/2d)]$.

4.62 $1.00029 \sin 88.7° = n \sin 90°$, $n = 1.00003$.

4.63 Let $\theta_i = \theta_p = \pi/2 - \theta_t$. Reflected beam is polarized if $r_\perp$ or $r_\parallel$ equal zero.
(4.43)

$$r_\parallel = \tan(\theta_i - \theta_t)/\tan(\theta_i + \theta_t) = \tan(\pi/2 - \theta_t - \theta_t)/\tan(\pi/2 - \theta_t + \theta_t)$$
$$= \tan(\pi/2 - 2\theta_t)/\tan(\pi/2).$$

But $\tan(\pi/2)$ is infinite, so $r_\parallel = 0$.

4.64 $\theta_i + \theta_t = 90°$ when $\theta_i = \theta_p$, $n_i \sin\theta_p = n_t \sin\theta_t = n_t \cos\theta_p$, $\tan\theta_p = n_t/n_i = 1.52$, $\theta_p = 56°40'$.

4.65 At θ_p, $r_\parallel = 0$. So from (4.38) $\frac{n_t}{\mu_t}\cos\theta_i - \frac{n_i}{\mu_i}\cos\theta_t = 0$. Recall (4.4) $n_i \sin\theta_i = n_t \sin\theta_t$. (3.59) $n = \sqrt{\epsilon\mu/\epsilon_0\mu_0}$ and $\cos^2\theta = 1 - \sin^2\theta$. Approach: solve for $\tan\theta_p = \sin\theta_p/\cos\theta_p$ where $\theta_i = \theta_p$.

4.66 $\tan\theta_p = n_t/n_i = n_2/n_1$, $\tan\theta_p' = n_1/n_2$, $\tan\theta_p = 1/\tan\theta_p'$. $\sin\theta_p/\cos\theta_p = \cos\theta_p'/\sin\theta_p'$. Therefore $\sin\theta_p \sin\theta_p' - \cos\theta_p\cos\theta_p' = 0$, $\cos(\theta_p + \theta_p') = 0$, so $\theta_p + \theta_p' = 90°$.

4.67 From Eq. (4.94), $\tan\gamma_r = r_\perp[E_{oi}]_\perp/r_\parallel[E_{oi}]_\parallel = (r_\perp/r_\parallel)\tan\gamma_i$ and from Eqs. (4.42) and (4.43)

$$\tan\gamma_r = -\frac{\cos(\theta_i - \theta_t)}{\cos(\theta_i + \theta_r)}\tan\gamma_i.$$

4.68 (4.56) $R = \left(\frac{E_{or}}{E_{oi}}\right)^2$ $E_{or}^2 = E_{or\parallel}^2 + E_{or\perp}^2$. $E_{oi}^2 = E_{oi\parallel}^2 + E_{oi\perp}^2$. (4.34) $r_\perp \equiv \left(\frac{E_{or}}{E_{oi}}\right)_\perp$. (4.38) $r_\parallel \equiv \left(\frac{E_{or}}{E_{oi}}\right)_\parallel$.

$$R = \frac{E_{or\perp}^2 + E_{or\parallel}^2}{E_{oi\perp}^2 + E_{oi\parallel}^2}$$
$$= \frac{(E_{or\perp}/E_{oi\perp})^2}{1 + (E_{oi\parallel}/E_{oi\perp})^2}$$
$$+ \frac{(E_{or\parallel}/E_{oi\parallel})^2}{(E_{oi\perp}/E_{oi\parallel})^2} + 1$$

$$= \frac{r_\perp^2}{1+\cot^2\gamma_i} + \frac{r_\parallel^2}{\tan^2\gamma_i + 1}$$
$$= R_\perp \sin^2\gamma_i + R_\parallel \cos^2\gamma_i$$

$$(4.57) T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{\dot{E}_{ot}}{E_{oi}}\right)^2$$

as above, $\left(\frac{E_{ot}}{E_{oi}}\right)^2 = t_\perp^2 \sin^2\gamma_i + t_\parallel \cos^2\gamma_i$, and using (4.63, 4.64),

$$T_{\perp,\parallel} = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t_{\perp,\parallel}^2.$$

$T = T_\perp \sin^2\gamma_i + T_\parallel \cos^2\gamma_i$.

4.69 Note that $\theta_e = 41.8°$. Note that $R_\perp$ increases steadily, while $R_\parallel$ has a minimum at $\theta_i \neq 0$.

4.70 $T_\perp = n_t t_\perp^2 \cos\theta_t / n_i \cos\theta_i$. From Eq. (4.44) and Snell's law,

$$T_\perp = \left(\frac{\sin\theta_i \cos\theta_t}{\sin\theta_t \cos\theta_i}\right)\left(\frac{4\sin^2\theta_t \cos^2\theta_i}{\sin^2(\theta_i+\theta_t)}\right) = \frac{\sin 2\theta_i \sin 2\theta_t}{\sin^2(\theta_i+\theta_t)}.$$

4.71 Use (4.62) and (4.43). $R_\parallel = r_\parallel^2 = \tan^2(\theta_i - \theta_t)/\tan^2(\theta_i+\theta_t) = [\sin^2(\theta_i-\theta_t)/\cos^2(\theta_i-\theta_t)] \times [\cos^2(\theta_i+\theta_t)/\sin^2(\theta_i+\theta_t)]$. Note that $R_\parallel$ and $T_\parallel$ have now the same denominator.

Use (4.61) and (4.42). $R_\perp = r_\perp^2 = \sin^2(\theta_i - \theta)/\sin^2(\theta_i+\theta_t)$. Note that $R_\perp$ and $T_\perp$ have the same denominator.

4.72 If Φ_i is the incident radiant flux or power and T is the transmittance across the first air-glass boundary, the transmitted flux is then $T\Phi_i$. From Eq. (4.68) at normal incidence the transmittance from glass to air is also

T. Thus a flux $T\Phi_i T$ emerges from the first slide, and $\Phi_i T^{2N}$ from the last one. Since $T = 1 - R$, $T_t = (1-R)^{2N}$ from Eq. (4.67).

$$R = (0.5/2.5)^2 = 4\%, \quad T = 96\%, \quad T_t = (0.96)^6 \approx 78.3\%.$$

4.73 $T = I(y)/I_0 = e^{-\alpha y}$, $T_1 = e^{-\alpha}$, $T = (T_1)^y$. $T_t = (1-R)^{2N}(T_1)^d$.

4.74 At $\theta_i = 0$, $R = R_\parallel = R_\perp = [(n_t - n_i)/(n_t + n_i)]^2$. As $n_{ti} \to 1$, $n_t \to n_i$ and clearly $R \to 0$. At $\theta_i = 0$, $T = T_\parallel = T_\perp 4n_t n_i/(n_t + n_i)^2$ and since $n_t \to n_i$, $\lim_{n_{ti}\to 1} T = 4n_i^2/(2n_i)^2 = 1$. From Problem 4.61 and the fact that as $n_t \to n_i$ Snell's law says that $\theta_t \to \theta_i$, we have

$$\lim_{n_{ti}\to 1} T_\parallel = \sin^2 2\theta_i / \sin^2 2\theta_i = 1, \quad \lim_{n_{ti}\to 1} T_\perp = 1.$$

From Eq. (4.43) and the fact that $R_\parallel = r_\parallel^2$ and $\theta_t \to \theta_i$, $\lim_{n_{ti}\to 1} R_\parallel = 0$.

4.75 (4.34)
$$\begin{aligned}
r_\perp &= \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \\
&= \frac{\cos\theta_i - n_{ti}\cos\theta_t}{\cos\theta_i + n_{ti}\cos\theta_t} \\
&= \frac{\cos\theta_i - n_{ti}\sqrt{1-\sin^2\theta_t}}{\cos\theta_i + n_{ti}\sqrt{1-\sin^2\theta_t}} \\
&= \frac{\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n_{ti}^2 + \sin^2\theta_i}} \\
r_\parallel &= \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t} \\
&= \frac{n_{ti}\cos\theta_i - \sqrt{1-\sin^2\theta_t}}{n_{ti}\cos\theta_i + \sqrt{1-\sin^2\theta_t}} \\
&= \frac{n_{ti}^2\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_t}}{n_{ti}^2\cos\theta_i + \sqrt{n_{ti}^2 - \sin^2\theta_t}}
\end{aligned}$$

4.76 For $\theta_i > \theta_c$, Eq. (4.70) can be written

$$r_\perp = \frac{\cos\theta_i - i(\sin^2\theta_i - n_{ti}^2)^{1/2}}{\cos\theta_i + i(\sin^2\theta_i - n_{ti}^2)^{1/2}},$$

$$r_\perp r_\perp^* = \frac{\cos^2\theta_i + \sin^2\theta_i - n_{ti}^2)}{\cos^2\theta_i + \sin^2\theta_i - n_{ti}^2} = 1.$$

Similarly $r_\parallel r_\parallel^* = 1$.

4.77

$$t_\parallel = \frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)}$$

$$t'_\parallel = \frac{2\sin\theta_1\cos\theta_2}{\sin(\theta_1+\theta_2)\cos(\theta_2-\theta_1)}$$

$$t_\parallel t'_\parallel = \frac{\sin 2\theta_1\cos 2\theta_2}{\sin^2(\theta_1+\theta_2)\cos^2(\theta_1-\theta_2)} = T_\parallel$$

from Eq. (4.100). Similarly $t_\perp t'_\perp = T_\perp$.

$$r_\parallel^2 = \left[\frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)}\right]^2 = \left[\frac{-\tan(\theta_2-\theta_1)}{\tan(\theta_1+\theta_2)}\right]^2$$

$$r_\parallel'^2 = \left[\frac{\tan(\theta_2-\theta_1)}{\tan(\theta_1+\theta_2)}\right]^2 = r_\parallel^2 = R_\parallel.$$

4.78 (4.84) $E_{oi}t_\parallel(\theta_1)t'_\parallel(\theta_2) + E_{oi}r_\parallel(\theta_p)r_\parallel(\theta_p) = E_{oi}$.
(4.85) $E_{oi}r_\parallel(\theta_1)t_\parallel(\theta_1) + E_{oi}t_\parallel(\theta_1)r'_\parallel(\theta_2) = 0$ where $\theta_2 = \theta_i = \theta'_p$ and $r'_\parallel(\theta'_p) = 0$. From Problem (4.66), $\theta_p = \theta_1$. From (4.84), $t_\parallel(\theta_p)t'_\parallel(\theta'_p) + 0 = 1$; $t_\parallel(\theta_p)t'_\parallel(\theta'_p) = 1$. From (4.85), $r_\parallel(\theta_p)t_\parallel(\theta_p) + 0 = 0$ since $t_\parallel(\theta_p) \neq 0$, $r_\parallel(\theta_p) = 0$. From (4.100), $T_\parallel = t_\parallel t'_\parallel$, when $T_\parallel = 1$, there is no reflected wave, as $T + R = 1$.

4.79 From Eq. (4.45)

$$t'_{\parallel}(\theta'_p)t_{\parallel}(\theta_p) = \left[\frac{2\sin\theta_p\cos\theta'_p}{\sin(\theta_p+\theta'_p)\cos(\theta'_p-\theta_p)}\right]\left[\frac{2\sin\theta'_p\cos\theta_p}{\sin(\theta_p+\theta'_p)\cos(\theta_p-\theta'_p)}\right]$$

$$= \frac{\sin 2\theta'_p \sin 2\theta_p}{\cos^2(\theta_p-\theta'_p)} \text{ since } \theta_p+\theta'_p = 90°$$

$$= \frac{\sin^2 2\theta_p}{\cos^2(\theta_p-\theta'_p)} \text{ since } \sin 2\theta'_p = \sin 2\theta_p \frac{\sin^2 2\theta_p}{\cos^2(2\theta_p-90°)} = 1.$$

4.80 Can be used as a mixer to get various proportions of the two incident waves in the emitted beams. This could be done by adjusting the gaps. [For some further remarks, see H. A. Daw and J. R. Izatt, *J. Opt. Soc. Am.* **55**, 201 (1965).]

4.81 From Fig. 4.62 the obvious choice is silver. Note that in the vicinity of 300 nm, $n_I \approx n_R \approx 0.6$, in which case Eq. (4.83) yields $R \approx 0.18$. Just above 300 nm, n_I increases rapidly, while n_R decreases quite strongly, with the result that $R \approx 1$ across the visible and then some.

4.82 Light traverses the base of the prism as an evanescent wave, which propagates along the adjustable coupling gap. Energy moves into the dielectric film when the evanescent wave meets certain requirements. The film acts like a waveguide, which will support characteristic vibration configurations or modes. Each mode has associated with it a given speed and polarization. The evanescent wave will couple into the film when it matches a mode configuration.

Chapter 5 Solutions

5.1 All OPLs from S to P must be equal, therefore $\ell_0 n_1 + \ell_i n_2 = s_0 n_1 + s_i n_2 =$ constant; drop a perpendicular from A to the optical axis, the point where it touches is B. $\overline{BP} = s_0 + s_i - x$ and the rest follows from the Pythagorean Theorem.

5.2 $\ell_0 + \ell_i 3/2 =$ constant, $5 + (6)3/2 = 14$. Therefore $2\ell_0 + 3\ell_i = 28$ when $\ell_0 = 6$, $\ell_i = 5.3$, $\ell_0 = 7$, $\ell_i = 4.66$. Note that the arcs centered on S and P have to intercept for physically meaningful values of ℓ_0 and ℓ_i.

5.3 The OPD from F_1 to any point D on Σ must be constant: $(\overline{F_1A})n_2 + (\overline{AD})n_1 = C$ and $(\overline{F_1A}) + (\overline{AD})n_{12} = C/n_2 = C'$; if Σ corresponds to the directrix of the ellipse, $(\overline{F_2A}) = e(\overline{AD})$ where e is the eccentricity; if $n_{12} = e$ we get $(\overline{F_1A}) + (\overline{F_2A}) = C'$.

5.4 A plane wave impinging on a concave elliptical surface becomes spherical. If the second spherical surface has that same curvature, the wave will have all rays normal to it and emerge unaltered.

5.5 Recall that the angles of a triangle sum to 180°.

$$\theta_2 + (180° - \varphi) + \beta = 180°;$$

$$\begin{aligned} \theta_2 &= \varphi - \beta. \\ \sin\theta_2 &= \sin(\varphi - \beta) \\ &= \sin\varphi\cos(-\beta) + \cos\varphi\sin(-\beta) \\ &\simeq \sin\varphi - \sin\beta \\ &= h/R - h/s_i \end{aligned}$$

$$(180° - \theta_1) + \varphi - \alpha = 180°$$

$$\begin{aligned}\theta_1 &= \varphi + \alpha \\ \sin\theta_1 &= \sin(\varphi + \alpha) \\ &= \sin\varphi\cos\alpha \\ &\quad + \cos\varphi\sin\alpha \simeq \sin\varphi + \sin\alpha = h/R + h/s.\end{aligned}$$

(4.4) $n_1 \sin\theta_1 = n_2 \sin\theta_2$; $n_1(h/R + h/s_o) = n_2(h/R - h/s_i)$, $n_i/s_o + n_2/s_i = (n_2 - n_1)/R$.

5.6 From Snell's Law $n_1\theta_i = n_2\theta_t$; $\tan\theta_i = y_o/s_o$ and $\tan\theta_t = -y_i/s_i$ since y_i is negative; thus $\theta_i = y_o/s_o$ and $\theta_t = -y_i/s_i$, therefore $M_T = y_i/y_o = -(n_1 s_i)/(n_2 s_o)$.

5.7 From Eq. (5.8), $1/30.0 + 1.33/s_i = (1.333 - 1.000)/5.0$; $s_i = 40.7$ cm and $M_T = -1.02$, thus the image is 3.05 cm tall.

5.8 First surface: $n_1/s_o + n_2/s_i = (n_2 - n_1)/R$, $1/1.2 + 1.5/s_i = 0.5/0.1$, $s_i = 0.36$ m (real image 0.36 m to the right of first vertex). Second surface $s_o = 0.20 - 0.36 = -0.16$ m (virtual object distance). $1.5/(-0.16) + 1/s_i = -0.5/(-0.1)$, $s_i = 0.069$ m. The final image is real ($s_i > 0$), inverted ($M_T < 0$), and 6.9 cm to the right of the second vertex.

5.9 At the first surface from Eq. (5.8), $1/30.0 + 1.33/s_i = (1.333 - 1.000)/5.0$ and $s_i = 40.7$; a real image right of the vertex. For the second surface $s_o = -30.7$ cm and the image will be right of the second vertex, so $1.33/(-30.7) + 1/s_i = (1.000 - 1.333)/(-5.000)$; and $s_i = 9.09$ cm to the right of the second surface. The first surface produces a magnification of $M_T = -1.02$, thus the intermediate image is 3.05 cm tall. The second surface produces a magnification of

$$M_T = -(n_1 s_i)/(n_2 s_o) = -(1.333)(9.09)/(1.000)(-30.7) = 0.395$$

and the total magnification is the product of the two, viz., -0.403. The image is real, inverted and minified.

5.10 (5.16) $1/f = (n-1)(1/R_1 - 1/R_2)$ where $R_2 = -R_1$, so

$$1/f = (n-1)(2/R_1)$$
$$R_1 = (n-1)(2)(f) = (1.5-1)(2)(+10.0 \text{ cm})$$
$$= 10.0 \text{ cm}$$

(5.17) $1/s_o + 1/s_i = 1/f$; $s_o = 1.0$ cm;
$1/s_i = 1/f - 1/s_o = 1/10.0 \text{ cm} - 1/1.0 \text{ cm} = -9.0/10.0$; $s_i = -1.1$ cm.
(5.25) $M_T = -s_i/s_o = -(-1.1 \text{ cm})/(1.0 \text{ cm}) = +1.1$. Image is virtual, erect, and larger than the object.

5.11 (5.14) In the thin lens limit ($d \to 0$) becomes
$n_m/s_o + n_m/s_i = (n_\ell - n_m)(1/R_1 - 1/R_2)$ so,
$1/s_o + 1/s_i = 1/f = (n_\ell - n_m/n_m)(1/R_1 - 1/R_2)$. For a double concave lens $R_1 < 0$, $R_2 > 0$, so that $(1/R_1 - 1/R_2) < 0$. For air lens in water, $n_\ell < n_m$, so that $n_\ell - n_m < 0$; $1/f > 0$, lens is converging.

5.12 (5.15) $1/s_o + 1/s_i = (n_\ell - 1)(1/R_1 - 1/R_2)$ so,

$$1/s_i = (n_\ell - 1)(1/R_1 - 1/R_2) - 1/s_o; \quad 1/s_i = -13.3 \text{ cm}.$$

(5.25) $M_T = -s_i/s_o = -13.3/20.0 = +0.67$. Image is virtual, erect, and smaller than the object.

5.13 $1/8 + 1.5/s_i = 0.5/(-20)$. At the first surface, $s_i = -10$ cm. Virtual image 10 cm to the left of first vertex. At second surface, object is *real* 15 cm from second vertex. $1.5/15 + 1/s_i = -0.5/10$, $s_i = -20/3 = -6.66$ cm. Virtual, to left of second vertex.

5.14 (a) (5.17) $1/s_o + 1/s_i = 1/f$ so,
$1/s_i = 1/f - 1/s_o = 1/(5.00 \text{ cm}) - 1/(1000 \text{ cm})$; $s_i = 5.03 \text{ cm} = 50.3$ mm.
(b) (5.25) $M_T = -s_i/s_o = -5.03 \text{ cm}/1000 \text{ cm} = -.00503$.
Image size $= |M_T|$(object size) $= (.00503)(1700 \text{ mm}) = 8.55$ mm.

5.15 $s_o + s_i = s_o s_i/f$ to minimize $s_o + s_i$, $(d/ds_o)(s_o + s_i) = 0 = 1 + ds_i/ds_o$ or

$$\frac{d}{ds_o}\left(\frac{s_o s_i}{f}\right) = \frac{s_i}{f} + \frac{s_o}{f}\frac{ds_i}{ds_o} = 0.$$

Thus $ds_i/ds_o = -1$ and $ds_i/ds_o = -s_i/s_o$, therefore $s_i = s_o$.

5.16 $1/5 + 1/s_i = 1/10$, $s_i = -10$ cm virtual, $M_T = -s_i/s_o = 10/5 = 2$ erect. Image is 4 cm high. Or $-5(x_i) = 100$, $x_i = -20$, $M_T = -x_i/f = 20/10 = 2$.

5.17 $1/s_o + 1/s_i = 1/f$. For $s_o = 0, f, \infty, 2f, 3f, -f, -2f, f/2$, $s_i = 0, \infty, f, 2f, f3/2, f/2, f2/3, -f$, respectively.

5.18 Draw a ray at 6.0° to the axis passing through the center of the lens. The image is virtual and on the image plane 50.0 cm in front of the lens. The image height y_i is gotten from the fact that $\tan 6.0° = y_i/f$ and so $y_i = 5.3$ cm.

5.19 $s_i < 0$ because image is virtual. $1/100 + 1/(-50) = 1/f$, $f = -100$ cm. Image is 50 cm to the right as well. $M_T = -s_i/s_o = 50/100 = 0.5$. Ant's image is half-sized and erect $(M_T > 0)$.

5.20 (5.16) $1/f = (n_\ell - 1)(1/R_1 - 1/R_2) = (1.5 - 1)(1/20 - 1/(-40)) = 3/80$; $f = 27$ cm. (5.17) $1/s_o + 1/s_i = 1/f$ so $1/s_i = 1/f - 1/s_o = 1/27 - 1/40$; $s_i = +80$ cm. (5.25) $M_T = -s_i/s_o = -80/40 = -2$. Image is real, inverted, at +80 cm and twice the size of the object.

5.21 $1/f = (n_l - 1)[(1/R_1) - (1/R_2)] = 0.5(1/\infty - 1/10) = 0.5/10$, $f = -20$ cm, $\mathcal{D} = 1/f = -1/0.2 = -5$ D.

5.22 (5.16) $1/f = (n_\ell - 1)(1/R_1 - 1/R_2) = (1.5 - 1)(1/(5.00\text{ cm}) - 1/\infty)$
$= 1/(10.0\text{ cm})$; $f = +10.0$ cm.
In a medium where $n_m \neq 0$, (5.16) becomes

$$1/f = ((n_\ell - n_m)/n_m)(1/R_1 - 1/R_2).$$

So, for water

$$(n_m = 1.33), \quad 1/f = ((1.5 - 1.33)/1.33)(1/(5.00\text{ cm}) - 1/\infty).$$

$f = 39.1$ cm (so f increases).

5.23 (5.17) $1/f = 1/s_o + 1/s_i = 1/45 + 1/90 = 3/90$; $f = +30$ cm.

5.24 (a) From the Gaussian lens equation $1/15.0 + 1/s_i = 1/3.00$, $s_i = +3.75$ m. (b) Computing the magnification, we obtain $M_T = -s_i/s_o = -3.75/15.0 = -0.25$. Because the image distance is positive, the image is *real*. Because the magnification is negative, the image is *inverted*, and because the absolute value of the magnification is less than one, the image is *minified*. (c) From the definition of magnification, it follows that $y_i = M_T y_o = (-0.25)(2.25\text{ m}) = -0.563$ m, where the minus sign reflects the fact that the image is inverted. (d) Again from the Gaussian equation $1/17.5 + 1/s_i = 1/3.00$ and $s_i = +3.62$ m. The entire equine image is only 0.13 m long.

5.25 (5.17) $1/f = 1/s_o + 1/s_i$ so,

$$1/s_i = 1/f - 1/s_o = 1/(-30) - 1/(+10) = -4/3.$$

$s_i = -7.5$ cm. (5.25) $M_T = -s_i/s_o = -(-7.5)/30 = 1/4 = 0.25$.
(Image size) $= M_T$(object size) $= (0.25)(6.00\text{ cm}) = 1.50$ cm.

The Image is virtual, 7.5 cm in front of the lens, erect, and 1.50 cm tall.

5.26 $|R_1| = |R_2|$, so (5.16) becomes

$$1/f = (n_\ell - 1)(1/R_1 - 1/(-R_1)) = (n_\ell - 1)(2/R_1) = 1/s_i + 1/s_o;$$

$s_o + s_i = 60$ cm (Image real). $|M_T| = (25\text{ cm})/(5.0\text{ cm}) = 5 = s_i/s_o$ so,

$$s_i = 5(s_o); \quad s_o + 5(s_o) = 60\text{ cm}.$$

$$s_o = 10\text{ cm}; \quad s_i = 50\text{ cm}.$$

$$1/f = 1/s_o + 1/s_i = 1/10 + 1/50 = 6/50; \quad f = 8.3\text{ cm}.$$

$$R_1 = (n_\ell - 1)(2)(f) = (1.5 - 1)(2)(8.3\text{ cm}) = 8.3\text{ cm}.$$

5.27 $1/s_o + 1/s_i = 1/f$ and $M_T = -s_i/s_o = -1/2$ hence $1/s_o + 2/s_o = 1/f$ but $s_o = 60.0$ cm, hence $f = 20.0$ cm; draw a ray cone from an axial image point, it enters the edges of the lens and focuses at 30.0 cm and then spreads out beyond to create a blur on the screen; from the geometry

(0.40 mm)/(10.0 mm) = (R/300 mm), $R = 1.2$ cm so the diameter is 2.4 cm.

5.28 The first thing to find is the focal length in water, using the lensmaker's formula. Taking the ratio
$f_w/f_a = f_w/(10\text{ cm}) = (n_g - 1)/[(n_g/n_w) - 1] = 0.56/0.17 = 3.24$;
$f_w = 32$ cm. The Gaussian lens formula gives the image distance:
$1/s_i + 1/100 = 1/32.4$; $s_i = 48$ cm.

5.29 The image will be inverted if it's to be real, so the set must be upside down or else something more will be needed to flip the image;
$M_T = -3 = -s_i/s_o$; $1/s_o + 1/3s_o = 1/0.60$; $s_o = 0.80$ m, hence
$0.80\text{ m} + 3(0.80\text{ m}) = 3.2$ m.

5.30 $1/f = (n_{lm} - 1)(1/R_1 - 1/R_2)$, $1/f_w = (n_{lm} - 1)/(n_l - 1)f_a = 0.125/0.5f_a$,
$f_w = 4f_a$.

5.31 $1/s_o + 1/s_i = 1/f$ hence for $\vec{A}$ and $\vec{B}$, $1/(1.1f) + 1/s_i = 1/f$ and so $s_i = 11f$, hence $M_T = -s_i/s_o = -(11f)/(1.10f) = -10$; both vectors are imaged inverted and 10 times larger than life, viz, $1f$ long. $\vec{A}$ is in the $-x$-direction and $\vec{B}$ is in the $-z$-direction. As for $\vec{C}$, it stretches from its tail at $11f$ to its tip at infinity.

5.32 Image-to-object distance = $L = s_{o1} + s_{i1} = s_{o2} + s_{i2}$. Also,

$$s_{o1} - s_{o2} = d = s_{i2} - s_{i1} \quad 1/f = 1/s_{i1} + 1/s_{o1};$$

$$1/f = 1/s_{i2} + 1/s_{o2} = 1/(s_{i1} - d) + 1/(s_{o1} + d).$$

Approach: With three is independent equations (two for $1/f$ and $L = s_{o1} + s_{i1}$) eliminate s_{o1} and s_{i1}, leaving $f(L, d)$.

5.33 Find s_{i1} first, and use this position for s_{o2}. (5.17) $1/f = 1/s_o + 1/s_i$, so
$1/s_{i1} = 1/f_1 - 1/s_{o1}$; $1/s_{i1} = 1/(+30) - 1/(+50) = (5 - 3)/150$;
$s_{i1} = 18.75$ cm, which puts s_{o2} at $(20 - 18.75)$ cm $= +1.25$ cm.
$1/s_{i2} = 1/(+50) - 1/(+1.25) = -0.78$; $s_{i2} = -1.3$ cm, (Virtual image).

5.34 For the first lens $s_{i1} = (s_{o1}f_1)/(s_{o1} - f_1) = +37.5$ cm and $M_{T1} = -1.50$; for the second lens $s_{o2} = 60.0 - 37.5 = +22.5$ cm, and $s_{i2} = (s_{o2}f_2)/(s_{o2} - f_2) = +9.00$ cm and $M_{T2} = 9.00/22.5 = +0.40$; the net magnification is $M_T = M_{T1}M_{T2} = -0.60$; the image is real, minified, and inverted.

5.35 $M_{T_1} = -s_{i1}/s_{o1} = -f_1/(s_{o1} - f_1)$, $M_{T_2} = -s_{i2}/s_{o2} = -s_{i2}/(d - s_{i1})$, $M_T = f_1 s_{i2}/(s_{o1} - f_1)(d - s_{i1})$. From (5.30), on substituting for s_{i1}, we have $M_T = f_1 s_{i2}/[(s_{o1} - f_1)d - s_{o1}f_1]$.

5.36 (a) (5.17) $1/f = 1/s_o + 1/s_2$, so

$$1/s_{i1} = 1/f_1 - 1/s_{o1} = 1/(+10.0 \text{ cm}) - 1/(+15.0 \text{ cm}) = (3-2)/(30.0);$$

$$s_{i1} = 30.0 \text{ cm} = 300 \text{ mm}.$$

(b) Image is real, inverted, and larger than the object.
(c) (5.25) $M_{T1} = -s_{i1}/s_{o1} = -30.0/15.0 = -2.00$. (d) s_{i1} sets s_{o2} at -5.00 cm (beyond the second lens, virtual object).

$$1/s_{i2} = 1/f - 1/s_{o2} = 1/(-7.50 \text{ cm}) - 1/(-5.00 \text{ cm}) = (-2+3)/(15.00);$$

$$s_{i2} = 15.0 \text{ cm} = 150 \text{ mm}.$$

(e) $M_{T2} = -s_{i2}/s_{o2} = -(15.0)/(-5.00) = 3.00. M_T = (M_{T1})(M_{T2}) = (-2.00)(3.00) = -6.00$. (Image is real, inverted).

5.37 First lens $1/s_{i1} = 1/30 - 1/30 = 0$, $s_{i1} = \infty$. Second lens $1/s_{i2} = 1/(-20) - 1/(-\infty)$, the object for the second lens is to the right at ∞, that is $s_{o2} = -\infty$. $s_{i2} = -20$ cm, virtual, 10 cm to the left of first lens. $M_T = (-\infty/30)(+20/-\infty) = 2/3$ or from (5.34) $M_T = 30(-20)/[10(30-30) - 30(30)] = 2/3$.

5.38 (5.17) $1/f = 1/s_o + 1/s_i$ so

$$1/s_{i1} = 1/f_1 - 1/s_{o1} = 1/(+15.0) - 1/(+25.0) = (5-3)/75.0;$$

$s_{i1} = +37.5$ cm, which makes $s_{o2} = -12.5$ cm.

$$M_{T1} = -s_{i1}/s_{o1} = -25.0/37.5 = -0.67.$$

$$1/s_{i2} = 1/f_2 - 1/s_{o2} = 1/(-15.0) - 1/(-12.5); \quad s_{i2} = +75.0 \text{ cm}.$$

$$M_{T2} = -s_{i2}/s_{o2} = -(+75.0)/(-12.5) = +6.00.$$

$$M_T = (M_{T1})(M_{T2}) = (-0.67)(+6.00) = -4.00.$$

Image is real, inverted, 75.0 cm beyond the second lens and 4 times the size of the object.

5.39 For the two positive lenses, note that incoming parallel rays result in outgoing parallel rays.

5.40 (5.34) $M_T = \dfrac{f_1 s_{i2}}{d(s_{o1} - f_1) - s_{o1} f_1}$

But $s_{i2} = s_{o1} = s \Rightarrow \infty$. $M_T = \dfrac{f_1 s}{d(s - f_1) - s f_1} = \dfrac{f - 1}{d - (df_1/s)} - f_1$.

$$\lim_{s\to\infty} M_T = \frac{f_1}{d - f_1},$$

where $d = f_1 + f_2 = \dfrac{f_1}{(f_1 + f_2) - f_1} = f_1/f_2;$

$$f_2 = f_1 M_T = (+5.00 \text{ cm})(0.80 \text{ cm}/0.10 \text{ cm}) = +40.0 \text{ cm};$$

$$d = f_1 + f_2 = +45.0 \text{ cm}.$$

5.41 Figure 5.99

5.43 In 5.43a, the rays through h_2 should bend away from the axis (diverging lens). In 5.43b, ray 4 should be directed at F_1'; also $|F_2| \neq |F_2'|$.

5.44 The angle subtended by L_1 at S is $\tan^{-1} 3/12 = 14°$. To find the image of the diaphragm in L_1 we use Eq. (5.23): $x_o x_i = f^2$, $(-6)(x_i) = 81$, $x_i = -13.5$ cm, so that the image is 4.5 cm behind L_1. The magnification is $-x_i/f = 13.5/9 = 1.5$, and thus the image (of the edge) of the hole is $(0.5)(1.5) = 0.75$ cm in radius. Hence the angle subtended at S is $\tan^{-1} 0.75/16.5 = 2.6°$. The image of L_2 in L_1 is obtained from $(-4)(x_i) = 81$, $x_i = -20.2$ cm, in other words, the image is 11.2 cm to the right of L_1. $M_T = 20.2/9 = 2.2$; hence the edge of L_2 is imaged 4.4 cm above the axis. Thus its subtended angle at S is $\tan^{-1} 4.4/(12 + 11.2)$ or $9.8°$. Accordingly, the diaphragm is the A.S., and the entrance pupil (its image in L_1) has a diameter of 1.5 cm at 4.5 cm behind L_1. The image of the diaphragm in L_2 is the exit pupil. Consequently, $1/2 + 1/s_i = 1/3$ and $s_i = -6$, that is, 6 cm in front of L_2. $M_T = 6/2 = 3$, so that the exit pupil diameter is 3 cm.

5.45 Either the margin of L_1 or L_2 will be the A.S.; thus, since no lenses are to the left of L_1, either its periphery or P_1 corresponds to the entrance pupil. Beyond (to the left of) point A, L_1 subtends the smallest angle and is the entrance pupil; nearer in (to the right of A), P_1 marks the edge of the entrance pupil. In the former case P_2 is the exit pupil; in the latter (since there are no lenses to the right of L_2) the exit pupil is the edge of L_2 itself.

5.46 The A.S. is either the edge of L_1 or L_2. Thus the entrance pupil is either marked by P_1 or P_2. Beyond F_{o1}, P_1 subtends the smaller angle; thus Σ_1 locates the A.S. The image of the A.S. in the lens to its right, L_2, locates P_3 as the exit pupil.

5.47 Draw the chief ray from the tip to L_1 such that when extended it passes through the center of the entrance pupil. From there it goes through the center of the A.S., and then it bends at L_2 so as to extend through the center of the exit pupil. A marginal ray from S extends to the edge of the entrance pupil, bends at L_1 so it just misses the edge of the A.S., and then bends at L_2 so as to pass by the edge of the exit pupil.

5.48 Figures P.5.48a and P.5.48b.

5.49 No—although she might be looking at you.

5.50 The mirror is parallel to the plane of the painting, and so the girl's image should be directly behind her and not off to the side.

5.51 $1/s_o + 1/s_i = -2/R$. Let $R \to \infty$: $1/s_o + 1/s_i = 0$, $s_o = -s_i$, and $M_T = +1$. Image is virtual, same size, and erect.

5.52 From Eq. (5.50), $1/100 + 1/s_i = -2/80$, and so $s_i = -28.5$ cm. Virtual ($s_i < 0$), erect ($M_T > 0$), and minified. (Check with Table 5.5.)

5.53 (5.48) $1/s_o + 1/s_i = -2/R$, $R = 0.5$ ft.

$1/s_i = -2/R - 1/s_o = -2/(0.5 \text{ ft}) - 1/(-5 \text{ ft})$, $\quad s_i = -5/21 = -0.24$ ft.

$M_T = -s_i/s_o = (-0.24)/(5) = 0.048$.

Image is virtual (seen in the mirror), erect, and 0.048 times the object size.

5.54 Ant has 3 images: from lens, from mirror, back out from lens.
(i) (5.17) $1/f = 1/s_o + 1/s_i$ so,
$1/s_i = 1/f - 1/s_o = 1/50.0 - 1/250 = 4/250$, $s_i = 62.6$ cm (between lens and mirror). (ii) (5.48) $1/s_o + 1/s_i = -2/R$ so,
$1/s_i = -2/R - 1/s_o = 1/s_o$, $(R = \infty)$, $s_i = -187.5$ cm (virtual image).
(iii) $1/s_i = 1/f - 1/50 = 1/50.0 - 1/(250 + 187.5)$, $s_i = +56.5$ cm. Real image, (left of lens).

5.55 Image on screen must be real, therefore s_i is positive.
$1/25 + 1/100 = -2/R$, $5/100 = -2/R$, $R = -40$ cm.

5.56 The image is erect and minified. That implies (Table 5.5) a convex spherical mirror.

5.57 From Eq. (5.8), $1/\infty + n/s_i = (n-1)/R$; $s_i = 2R$; $n/2R = (n-1)/R$; $n = 2$.

5.58 Want $|M_T| = |s_i/s_o| = y_i/y_0 = 1.0\text{ cm}/100\text{ cm} = .01$. $s_o = 1000$ cm.
$|s_i| = s_o M_T = (1000)(.01) = 10$ cm. Want real image, so $s_i > 0$, and image will be inverted. Detector is 10 cm in front of the mirror.
(5.50) $1/s_o + 1/s_i = 1/f$; $1/f = 1/1000 + 1/10 = 101/1000$; $f = 9.9$ cm.

5.59 To be magnified and erect the mirror must be concave, and the image virtual; $M_T = 2.0 = s_i/(0.015\text{ m})$, $s_i = -0.03$ m, and hence
$1/f = 1/0.015\text{ m} + 1/(-0.03\text{ m})$; $f = 0.03$ m and $f = -R/2$;
$R = -0.06$ m.

5.60 $M_T = y_i/y_o = -s_i/s_o$, using Eq. (5.51), $s_i = fs_o/(s_o - f)$, and since
$f = -R/2$, $M_T = -f/(s_o - f) = -(-R/2)/(s_o + R/2) = R/(2s_o + R)$.

5.61 (5.49) $f = -R/2$ so $R = -2f$. (5.50) $1/s_o + 1/s_i = 1/f$. $M_T = -s_i/s_o$, so
$s_i = -s_o M_T = -(10.0\text{ cm})(0.037) = -0.37$ cm (image is virtual).
$1/f = 1/s_o + 1/s_i = 1/(10.0) + 1/(-0.37)$, $f = -0.38$ cm.
$R = -2(-0.38) = 0.76$ cm.

5.62 (5.50) $1/f = 1/s_o + 1/s_i$; $s_o/f = 1 + s_o/s_i = (s_i + s_o)/s_i$

$$= \frac{(s_i/s_o + 1)}{(s_i/s_o)} = (-M_T + 1)/(-M_T). \quad s_o = f(M_T - 1)/M_T.$$

(5.50) $1/f = 1/s_o + 1/s_i$; $s_i/f = s_i/s_o + 1 = (-M_T) + 1$; $s_i = -f(M_T - 1)$.

5.63 $M_T = -s_i/25\text{ cm} = -0.064$; $s_i = 1.6$ cm. $1/25 + 1/1.6 = -2/R$,
$R = -3.0$ cm.

5.64 Image size in plane mirror equals object size, so $|M_T$ (convex mirror) $| = 0.5$; $|M_T| = |s_i/s_o|$, so $|s_i| = |s_o||M_T| = (5.0)(0.5) = 2.5$ m;
$s_i = -2.5$ m (image is virtual).
(5.50) $1/f = 1/s_o + 1/s_i = 1/(5.0\text{ m}) + 1/(-2.5\text{m})$, $f = -2.5$ m.

5.65 (5.49) $f = -R/2$. Primary $f_p = -(-(200\text{ cm}))/2 = +100$ cm.
(5.50) $1/s_o + 1/s_i = 1/f_p$, $s_o = \infty$, so $1/s_i = 1/f_p$; $s = +100$ cm. Object for secondary is at $s_o = -25$ cm. $1/f_s = 1/s_o + 1/s_i$ so $1/s_i = 1/f_s - 1/s_o$.
Secondary $f_s = -R/2 = -(+60\text{ cm})/2 = -30$ cm.
$1/s_i = 1/(-30) - 1/(-25)$; $s_i = +150$ cm, or 75 cm behind the primary.
The effective focal length of the "lens" is +75 cm.

5.66 See Table 5.3. For $f < s_o < 2f$, a real inverted image is made with $\infty > s_i > 2f$. If this image is directed back at the same angles, the final image will occur at the original object. So, for either type of mirror, it should be placed at the image of the lens (at s_i).

5.67 $M_T = -s_i/s_o$, so, $s_i = -M_T s_o = -1.5s_o$.
(5.50) $1/f = 1/s_o + 1/s_i = 1/s_o + 1/(-1.5s_o)$; $1/10 = 1/3s_o$;
$s_o = 10/3 = 3.3$ cm.
Note in Table 5.5, $s_o < f$ for an erect, magnified image.

5.68 $f = -R/2 = 30$ cm, $1/20 + 1/s_i = 1/30$, $1/s_i = 1/30 - 1/20$. $s_i = -60$ cm, $M_T = -s_i/s_o = 60/20 = 3$. Image is virtual ($s_i < 0$), erect ($M_T > 0$), located 60 cm behind mirror, and 9 inches tall.

5.69 Treat the first surface as a mirror with radius of curvature R.
(5.49) $f_m = -R/2$, which is where the parallel reflected rays converge.
Lens: (5.16) $1/f_\ell = (n_\ell - 1)(1/R_1 - 1/R_2)$; $R_1 = -R$, $R_2 = +R$ so
$1/f_\ell = (2-1)(1/(-R) - 1/R) = -2/R$; $f_\ell = -R/2 = f_m$.

5.70 Image is rotated through 180°.

5.71 From Eq. (5.65),

$$NA = (2.624 - 2.310)^{1/2} = 0.550, \quad \theta_{\max} = \sin^{-1} 0.550 = 33°22'.$$

Maximum acceptance angle is $2\theta_{\max} = 66°44'$. A ray at 45° would quickly leak out of the fiber; in other words, very little energy fails to escape, even at the first reflection.

5.72 Considering Eq. (5.66), $\log 0.5 = -0.30 = -\alpha L/10$, and so $L = 15$ km.

5.73 From Eq. (5.65), $NA = 0.232$ and $N_m = 9.2 \times 10^2$.

5.74 (5.68) $\Delta t = (Ln_f/c)(n_f/n_c - 1)$, so $\Delta t/L = (n_f/c)(n_f/n_c - 1)$.
$\Delta t/L = 1.500/(3 \times 10^{-4}\ \text{km/ns})(1.500/1.485 - 1) = 50.51$ km/ns.

5.75 $M_T = -f/x_o = -1/x_o\mathcal{D}$. For the human eye $\mathcal{D} \approx 58.6$ diopters.
$x_o = 230{,}000 \times 1.61 = 371 \times 10^3$ km,

$M_T = -1/3.71 \times 10^6(58.6) = 4.6 \times 10^{-11}$,
$y_i = 2160 \times 1.61 \times 10^3 \times 4.6 \times 10^{-11} = 0.16$ mm.

5.76 Recall that the angles of a triangle sum to 180°. Recall that at both mirrors $\theta_r = \theta_i$. For the triangle made by the three rays, $(2\theta_{i1}) + (2\theta_{i2}) + (180° - \alpha) = 180°$ so $\alpha = 2(\theta_{i1} + \theta_{i2})$. For the triangle containing "β," $\beta + (90° - \theta_{i1}) + (90° - \theta_{i2}) = 180°$. $\beta = (\theta_{i1} + \theta_{i2})$, so, $\alpha = 2\beta$.

5.77 $1/20 + 1/s_{io} = 1/4$, $s_{io} = 5$ m. $1/0.3 + 1/s_{ie} = 1/0.6$, $s_{ie} = -0.6$ m. $M_{To} = -5/10 = -0.5$, $M_{Te} = -(-0.6)/0.5 = +1.2$, $M_{To}M_{Te} = -0.6$.

5.78 From Table 5.3, the types, positions, and sizes of the images are OK, but the rays from one portion of an object do not consistently trace to the same portion of the image.

5.79 The pinhole allows the eye to get much closer to the object and still see it clearly and that creates a larger retinal image. The pinhole works like a magnifier.

5.80 Want same amount of light to reach the film. $f/\#$ varies as the square root of the time, so we want $f/5.5$.

5.81 See figure. Ray 1 in the figure misses the eye-lens, and there is, therefore, a decrease in the energy arriving at the corresponding image point. This is vignetting.

5.82 Rays that would have missed the eye-lens in the previous problem are made to pass through it by the field-lens. Note how the field-lens bends the chief rays a bit so that they cross the optical axis slightly closer to the eye-lens, thereby moving the exit pupil and shortening the eye relief. (For more on the subject, see *Modern Optical Engineering*, by Smith.)

5.83 From Table 5.3, image is virtual, erect, and magnified. As thickness of lens approaches 0, $|s_i|$ approaches s_o, i.e., $|M_T|$ approaches 1. However, the entire bug is imaged, so that this can be used as a field-lens.

5.84 (a) $\mathcal{D} = 1/f$. If $s_0 = \infty$, $1/f = 1/s_i$. $D = 1/(0.02\text{ m}) = 50\text{ m}^{-1}$. If $s_o = 0.50\text{ m}$, $1/f = 1/(0.50\text{ m}) + 1/(0.02\text{ m}$, $\mathcal{D} = 52\text{ m}^{-1}$.
(b) Accommodation of 2 m^{-1}.
(c) $\mathcal{D} = 1/f = 1/(0.25\text{ m}) + 1/(0.02\text{ m}) = 54\text{ m}^{-1}$. (d) Need to add 2 m^{-1}.

5.85 Unaided eye,

$$\mathcal{D} = 1/f = 1/s_o + 1/s_i, \quad (s_i \simeq 2\text{ cm}), \mathcal{D} = 1/(1.25\text{ m}) + 1/(0.02\text{ m}) = 50.8\text{ m}^{-1}.$$

Want $\mathcal{D} = 1/f = 1/(0.25\text{ m}) + 1/(0.02\text{ cm}) = 54\text{ m}^{-1}$. Lens must have a power of $(54 - 50.8) = 3.2\text{ m}^{-1}$.

5.86 $\mathcal{D}_1 = \mathcal{D}_c/(1 + \mathcal{D}_c d) = 3.2D/[1 + (3.2D)(0.017\text{ m})] = +3.03D$ or to two figures +3.0 D. $f_1 = 0.330$ m, and so the far point is $0.330 \; - 0.017\text{ m} = 0.313\text{ m}$ behind the eye lens. For the contact lens $f_c = 1/3.2 = 0.313$ m. Hence the far point at 0.31 m is the same for both, as it indeed must be.

5.87 (a) (5.77) $MP = d_o \cdot \mathcal{D} + 1 = (0.25\text{ m})(1/.0254\text{ m}) + 1 = 10.8$.
(b) Size $= (MP)$(object size) $= 10.8(5.0\text{ mm}) = 54$ mm diameter.
(c) $\tan\alpha_u = y_o/d_o = (0.0254\text{ m})/(0.25\text{ m})$, $\alpha_u - 5.80° = 0.101$ rad.
(d) $\tan\alpha_a = y_i/L \simeq y_i/d_0 = (0.054\text{ m})/(0.25\text{ m})$, $\alpha_a = 12.19° = 0.213$ rad.

5.88 (a) The intermediate image-distance is obtained from the lens formula applied to the objective; $1/27 + 1/s_i = 1/25$ and $s_i = 3.38 \times 10^2$ mm. This is the distance from the objective to the intermediate image, to which must be added the focal length of the eyepiece to get the lens separation; $3.38 \times 10^2 + 25 = 3.6 \times 10^2$ mm.
(b) $M_{To} = -s_i/s_o = -3.38 \times 10^2/27 = -12.5\times$, while the eyepiece has a magnification of $d_o\mathcal{D} = 254/25 = 10.2\times$. Thus the total magnification is $MP = (-12.5)(10.2) = -1.3 \times 10^2$; the minus sign just means the image is inverted.

5.89 The x-ray "lens" is a mirrored surface that forms a portion of a non-spherical mirror. The reflected rays converge to the focus (F_1).

5.90 (a) These are a parabola and a hyperbola of two sheets. The parabola and left-hand hyperbola share a common focus, F_1. Rays reflected from the parabola head for that focus. Rays directed at the first focus of a hyperbola reflect toward the second focus. (b) Parallel rays coming off the parabola seem to be leaving its first focus. Because this is also the focus of the ellipse the rays reflect toward its second focus.

5.91 The limit of resolution is $1.22\lambda/D$; at 0.50μm, $1.22(0.50 \times 10^{-6})/2.4 = 2.54 \times 10^{-7}$ radians; $1.0 \times 10^{-2} = R2.54 \times 10^{-7}$ and $R = 39$ km.

Chapter 6 Solutions

6.1 (6.7) $M_T = (-s_{i1}/s_{o1})(-s_{i2}/s_{o2}) = -s_i/s_o$. Let $s_o = \infty$ so that $s_o = s_{o1}$, $s_{i1} = f_1$, $s_{i1} = f_1$, $s_{o2} = -(s_{i1} - d)$. Substituting into (6.7),

$$(-f_1/s_o)(-s_{i2}/-(s_{i1}-d)) = -f/s_o.$$
$$f = f_1(s_{i2}/(-s_{o2})) = f_1(s_{i2}/(s_{i1}-d)).$$

From

$$1/s_{o2} + 1/s_{i2} = 1/f_2; \quad 1/s_{i2} = 1/f_2 - 1/s_{o2}, \quad s_{i2} = s_{o2}f_2/(s_{o2}-f_2).$$
$$f = (-f_1/s_{o2})(s_{o2}f_2/(s_{o2}-f_2))$$
$$= -f_1f_2/(s_{o2}-f_2) = f_1f_2/(s_{i1}-d+f_2).$$
$$1/f = (s_{i1}-d+f_2)/f_1f_2 = 1/f_1 + (s_{i1}-d)/f_1f_2.$$

But $s_{i1} = f_1$, so, $1/f = 1/f_1 + 1/f_2 - d/f_1f_2$.

6.2 From Eq. (6.8), $1/f = 1/f' + 1/f' - d/f'f' = 2/f' - 2/3f'$, $f' = 3f'/4$. From Eq. (6.9), $\overline{H_{11}H_1} = (3f'/4)(2f'/3)/f' = f'/2$. From Eq. (6.10), $\overline{H_{22}H_2} = -(3f'/4)(2f'/3)/f' = -f'/2$.

6.3 From Eq. (6.2), $1/f = 0$ when $-(1/R_1 - 1/R_2) = (n_l - 1)d/n_lR_1R_2$. Thus $d = n_l(R_1 - R_2)/(n_l - 1)$.

6.4 $1/f = 0.5[1/6 - 1/10 + 0.5(3)/1.5(6)10]$, $f = +24$;
$h_1 = -24(0.5)(3)/10(1.5) = -2.4$, $h_2 = -24(0.5)(3)/6(1.5) = -4$.

6.5 Since $|h_1| = |h_2|$ it follows from Eqs. (6.3) and (6.4) that $-f(n_l - 1)d_l/|R_2|n_l = -f(n_l - 1)d_l/|R_1|n_l$ and $|R_2| = |R_1|$ which means the lens is a sphere.

6.6 $f = (1/2)nR/(n-1)$; $h_1 = +R$, $h_2 = -R$.

6.7 This is a thick lens with $-R_2 = R_1 = R = 10$ cm and $d = 2R = 20$ cm.

$$(6.2) \qquad 1/f = (n_\ell - 1)[1/R_1 - 1/R_2 + (n_\ell + 1)d/n_\ell R_1 R_2]$$

$$= (1.33-1)[1/10 - 1/(-10) + (1.33-1)(20)/1.33(10)(-10)];$$

$$f = 20.2 \text{ cm}.$$

6.8 From Problem (6.6) or (6.7), (6.2) becomes $1/f = ((n_\ell - 1)/n_\ell)(2/R)$, with $R = +10$ cm. $1/f = ((1.4-1)/1.4)(2/10) = 0.057$ cm^{-1}, $f = 17.5$ cm. (6.1) $1/f = 1/s_o + 1/s_i$, where s_o and s_i are measured from the principal planes. $1/s_i = 1/f - 1/s_o = 1/(17.5)$ cm $- 1/(400-10)$ cm; $s_i = 18.3$ cm. (6.7) $M_T = -s_i/s_o = -18.3/390 = -.047$. Image is real, inverted, and 0.047 times the size of the object.

6.9 (6.2) $1/f = (n_\ell - 1)[1/R_1 - 1/R_2 + (n_\ell - 1)d/n_\ell R_1 R_2]$;

$$(1.5-1)[1/23 - 1/20 + (1.5-1)(9.0)/1.5(23)(20)] = 0,$$

$$f = \infty.$$

Generally, if

$$1/f = 0, \quad [1/R_1 - 1/R_2 + (n_\ell - 1)d/n_\ell R_1 R_2] = 0;$$

$$(n_\ell R_2 - n_\ell R_1 + (n_\ell - 1)d)/n_\ell R_1 R_2 = 0;$$

$$(n_\ell - 1)d = n_\ell(R_1 - R_2); \quad (R_1 - R_2) = (n_\ell - 1)/n_\ell d$$

for $n_\ell = 1.5$, $(R_1 - R_2) = ((1.5-1)/1.5)d = d/3$.

6.10 $f = 29.6 + 0.4 = 30$ cm; $s_o = 49.8 + 0.2 = 50$ cm; $1/50 + 1/s_i = 1/30$ cm. $s_i = 75$ cm from H_2 and 74.6 cm from the back face.

6.12 From Eq. (6.2),

$$1/f = (1/2)[1/4.0 - 1/(-15) + (1/2)4.0/(3/2)(4.0)(-15)] = 0.147$$

and $f = 6.8$ cm. $h_1 = -(6.8)(1/2)(4.0)/(-15)(3/2) = +0.60$ cm, while $h_2 = -2.3$. To find the image $1/(100.6) + 1/s_i = 1/(6.8)$; $s_i = 7.3$ cm or 5 cm from the back face of the lens.

6.13 For both, $-R_2 = R = R$, so (6.2) becomes

$$1/f = (n-1)[2/R + (n-1)d/nR^2];$$

$$1/f = (1.5-1)[2/50 + (1.5-1)(5.0)/1.5(50)^2]; \quad f = 49.2 \text{ cm.}$$

(6.1) $1/f_1 = 1/s_{o1} + 1/s_{i1}$, so $1/s_{i1} = 1/f_1 - 1/s_{o1} = 1/(49.2)$.

6.14 (6.8) $1/f = 1/f_1 + 1/f_2 - d/f_1 f_2 = 1/(+20) + 1/(-20) - 10/(20)(-20)$;
$f = +40$ cm. The principal planes are found from (6.9) and (6.10).
(6.9) $\overline{H_{11}H_1} = fd/f_2 = (+40)(10)/(-20) = -20$ cm.
(6.10) $\overline{H_{22}H_2} = fd/f_1 = (+40)(10)/(20) = +20$ cm.

6.15 $\mathcal{R}_1 = \begin{bmatrix} 1 & -\mathcal{D}_1 \\ 0 & 1 \end{bmatrix}$ from (6.16) where
$\mathcal{D}_1 = (n-1)/R_1 = (1.5-1)/2.5 \text{ cm} = 0.2 \text{ cm}^{-1}$.

$$\mathcal{T}_{21} = \begin{bmatrix} 1 & 0 \\ d_{21}/n & 1 \end{bmatrix} \quad (6.24)$$

$$= \begin{bmatrix} 1 & 0 \\ 1.2/1.5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.8 & 1 \end{bmatrix}$$

$$\mathcal{R}_2 = \begin{bmatrix} 1 & -\mathcal{D}_2 \\ 0 & 1 \end{bmatrix}$$

but $R_2 = \infty$, so $\mathcal{D}_2 = 0$.

$$(6.29) \quad A = \mathcal{R}_2 \mathcal{T}_{21} \mathcal{R}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.2 \\ 0.8 & 1.16 \end{bmatrix}$$

Check: $|A| = 1(1.16) - 0.2(0.8) = 1$.

6.16 Working in centimeters,

$$\mathcal{D}_1 = (2.4-1.9)/R_1 = 0.1 \text{ cm}^{-1}, \quad \mathcal{D}_2 = (1.9-2.4)/R_2 = -0.05 \text{ cm}^{-1}$$

therefore

$$\mathcal{A} = \begin{bmatrix} 1 & -\mathcal{D}_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d/n_l & 1 \end{bmatrix} \begin{bmatrix} 1 & -\mathcal{D}_1 \\ 0 & 1 \end{bmatrix}$$

$$\mathcal{A} = \begin{bmatrix} 1 & -0.05 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.4 & 1 \end{bmatrix} \begin{bmatrix} 1 & +0.1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.02 & -0.052 \\ 0.4 & 0.96 \end{bmatrix}$$

$$(1.02)(0.96) - (0.4)(-0.052) = 0.979 + 0.0208 = 1.$$

6.17 We have

$$\det \mathcal{A} = a_{11}a_{22} - a_{12}a_{21} = 1 - (D_1 + D_2)d_{21}/n_{t1} + D_1 D_2 d_{21}^2/n_{t1}^2$$
$$+ (D_1 + D_2)d_{21}/n_{t1} + D_1 D_2 d_{21}/n_{t1} = 1.$$

6.18 $h_1 = n_{i1}(1 - a_{11})/(-a_{12}) = (D_2 d_{21}/n_{t1})f = -(n_{t1} - 1)d_{21}f/R_2 n_{t1}$, from Eq. (5.64) where $n_{t1} = n_l$; $h_2 = n_{t2}(a_{22} - 1)/(-a_{12}) = -(D_1 d_{21}/n_{t1})f$ from Eq. (5.70), $h_2 = -(n_{i1} - 1)d_{21}f/R_1 n_{t1}$.

6.19 $\mathcal{A} = \mathcal{R}_2 \mathcal{F}_{21} \mathcal{R}_1$, but for the planar surface

$$\mathcal{R}_2 = \begin{bmatrix} 1 & \mathcal{D}_2 \\ 0 & 1 \end{bmatrix} \text{ and } \mathcal{D}_2 = (n_{t1} - 1)/(-R_2) \text{ but } R_2 = \infty \mathcal{R}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which is the unit matrix, hence $\mathcal{A} = \mathcal{F}_{21}\mathcal{R}_1$.

6.20 $\mathcal{D}_1 = (1.5 - 1)/0.5 = 1$ and $\mathcal{D}_2 = (1.5 - 1)/-(-0.25) = 2 \mathcal{A} = \begin{bmatrix} 0.6 & -2.6 \\ 0.2 & 0.8 \end{bmatrix}$ and $|\mathcal{A}| = 0.48 + 0.52 = 1$.

6.21 From the equation above (6.34),

$$-0.2 = -a_{21} = (n_{t1} - 1)\left\{\frac{1}{R_1} + \frac{1}{R_2}\left[\frac{(n_{t1} - d_{21})}{R_1 n_{t1}} - 1\right]\right\}.$$

Solving for the reciprocal of the second radius gives

$$\frac{1}{R_2} = -\left[a_{21} + \frac{(n_{t1} - 1)}{R_1}\right]\frac{R_1 n_{t1}}{(n_{t1} - 1)}\frac{1}{(n_{t1} - d_{21} - R_1 n_{t1})} = 4\ \text{cm}^{-1}.$$

Then $R_2 = 0.25$ cm.

6.22 $\mathcal{R}_1 = \begin{bmatrix} 1 & -\mathcal{D}_1 \\ 0 & 1 \end{bmatrix}$ from (6.16) where

$$\mathcal{D}_1 = (n-1)/R_1 = (3/2-1)/-10.0 \text{ cm} = -0.050 \text{ cm}^{-1}.$$

$$\mathcal{T}_{21} = \begin{bmatrix} 1 & 0 \\ d_{21}/n & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1.00/1.50 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.67 & 1 \end{bmatrix}$$

$$\mathcal{R}_2 = \begin{bmatrix} 1 & -\mathcal{D}_2 \\ 0 & 1 \end{bmatrix}$$

but $R_2 = \infty$, so $\mathcal{D}_2 = 0$.

$$(6.29)\ A = \mathcal{R}_2\mathcal{T}_{21}\mathcal{R}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0.67 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0.05 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.05 \\ 0.67 & 1.03 \end{bmatrix}$$

Check: $|A| = 1(1.03) - (0.05)(0.67) = 1.$

$$\begin{bmatrix} n_t\alpha_t \\ y_t \end{bmatrix} = A \begin{bmatrix} n_i\alpha_i \\ y_i \end{bmatrix}$$

$$\alpha_t = 0, \quad n_i = 1, \quad y_t = y_i.$$

$$\begin{bmatrix} 0 \\ y_i \end{bmatrix} = A \begin{bmatrix} \alpha_i \\ y_i \end{bmatrix} = \begin{bmatrix} 1 & 0.05 \\ 0.67 & 1.03 \end{bmatrix}\begin{bmatrix} \alpha_i \\ y_i \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ y_i \end{bmatrix} = \begin{bmatrix} \alpha_i + (0.05)y_i \\ (0.67)\alpha_i + (1.03)y_i \end{bmatrix}$$

$$0 = \alpha_i + (0.05)y_i, \quad y_i = (0.67)\alpha_i + (1.03)y_i,$$

both yield $\alpha_i = (-0.05)(2.0) = -0.10$ or 0.10 radians above the axis.

6.23 (6.34) $1/f = -a_{12} = -(\mathcal{D}_1 + \mathcal{D}_2 - \mathcal{D}_1\mathcal{D}_2 d/n_\ell)$;

$\mathcal{D}_1 = (n_\ell - 1)/R_1 = (1.5-1)/0.5 = 1.0$;

$\mathcal{D}_2 = (n_\ell - 1)/R_2 = (1.5-1)/(-0.25) = -2.0.$

$1/f = -(1.0 - 2.0 - (10)(2.0)(0.3)/1.5, \quad f = 0.71.$

$\overline{V_1H_1} = (1)(1-a_{11})/-a_{12}$,

(6.36) $a_{11} = 1 - \mathcal{D}_2 d/n_\ell = 1 - (-2.0)(0.3)/1.5 = 1.4.$

$\overline{V_1}H_1 = (1-1.4)/1.4 = -0.29.$

(6.37) $\overline{V_2H_2} = (1)(a_{22} - 1)/ - a_{12};$

$$a_{22} = 1 - (\mathcal{D}_1 d)/n_\ell = 1 - (1.0)(0.3)/1.5 = 0.8;$$
$$\overline{V_2H_2} = (0.8 - 1)/1.4 = -0.14.$$

6.24 For two reflections

$$\begin{bmatrix} -1 & -2/(+r) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -d & 1 \end{bmatrix} \begin{bmatrix} -1 & -2/(-r) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d & 1 \end{bmatrix}$$

and this yields the desired matrix. When $d = r$ the matrix for two traversals becomes

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

and for four it is

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and since this is a unit matrix the light ray is back to where it started.

6.25 See E. Slayter, *Optical Methods in Biology.*

$$\overline{PC}/\overline{CA} = (n_1/n_2)R/R = n_1/n_2,$$

while $\overline{CA}/\overline{P'C'} = n_1/n_2$. Therefore triangles ACP and ACP' are similar; using the sine law

$$\sin \angle PAC/\overline{PC} = \sin \angle APC/\overline{CA}$$

or $n_2 \sin \angle PAC = n_1 \sin \angle APC$, but $\theta_i = \angle PAC$, thus

$$\theta_t = \angle APC = \angle P'AC,$$

and the refracted ray appears to come from P'.

6.26 From Eq. (5.6), let $\cos\varphi = 1 - \varphi^2/2$; then

$$\ell_o = [R^2 + (s_o + R)^2 - 2R(s_o + R) + R(s_o + R)\varphi^2]^{1/2},$$
$$\ell_o^{-1} = [s_o^2 + R(s_o + R)\varphi^2]^{-1/2}, \quad \ell_i^{-1} = [s_i^2 - R(s_i - R)\varphi^2]^{-1/2},$$

where the first two terms of the binomial series are used,

$$\ell_0^{-1} \approx s_o^{-1} - (s_o + R)h^2/2s_o^3 R$$

where

$$\varphi \approx h/R, \quad \ell_i^{-1} \approx s_i^{-1} + (s_i - R)h^2/2s_i^3 R.$$

Substituting into Eq. (5.5) leads to Eq. (6.40).

6.28 Because (a) is symmetrical and looks like a somewhat altered Airy pattern; this is spherical aberration. (b) This pattern is asymmetrical as if the Airy system were pulled off to the side, so it corresponds to a little coma. (c) This pattern is asymmetrical along two axes and must be due to astigmatism.

6.29 Fig. P.6.29a is bi-axially asymmetric and therefore corresponds to astigmatism. (b) is elongated along one axis and is due to coma, and because the pattern isn't very complicated there isn't much of it.

Chapter 7 Solutions

7.1 $E_0^2 = 36 + 64 + 2(6)(8)\cos\pi/2 = 100, E_0 = 10; \tan\alpha = 8/6, \alpha = 53.1° =$ 0.93 rad. $E = 10\sin(120\pi t + 0.93)$.

7.2 $E_1 = E_{01}\cos(\omega t); \quad E_2 = E_{01}\cos(\omega t + \alpha_2)$.

$$\begin{aligned} E = E_1 + E_2 &= E_{01}\cos\omega t + E_{01}\cos(\omega t + \alpha_2) \\ &= E_{01}(2\cos\tfrac{1}{2}(\omega t + \omega t + \alpha_2)\cos\tfrac{1}{2}(\omega t - \omega t - \alpha_2)) \\ &= 2E_{01}\cos(\omega t + \alpha_2/2)\cos(-\alpha_2/2). \end{aligned}$$

Recall $\cos(-\theta) = \cos\theta$, so,

$$E = (2E_{01}\cos(\alpha_2/2))(\cos(\omega t + \alpha_2/2)) = E_0\cos(\omega t + \alpha).$$

To show that this follows from (7.9) and (7.10), recall that $\cos\theta = \sin(\theta + \pi/2)$ so that

$$\alpha_1 \to \alpha_1 + \pi/2 = \pi/2, \quad \alpha_2 \to \alpha_2 + \pi/2.$$

7.3 In phase: $\alpha_1 = \alpha_2 \cos(\alpha_2 - \alpha_1) = \cos(0) = 1$.

$$\begin{aligned} (7.9) \quad E_0^2 &= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1) \\ &= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} = (E_{01} + E_{02})^2. \end{aligned}$$

Out of phase, $\alpha_2 - \alpha_1 = \pi, \cos(\alpha_2 - \alpha_1) = \cos\pi = -1$.

$$(7.9) \quad E_0^2 = E_{01}^2 + E_{02}^2 - 2E_{01}E_{02} = (E_{01} - E_{02})^2.$$

7.4 $OPL = \sum_i n_i x_i = \sum(c/v_i)x_i = \sum_i ct_i$, where t_i is the time spent in medium i. But ct_i is also the distance the light would travel, in vacuum.

7.5 1 m/500 nm $= 0.2 \times 10^7 = 2{,}000{,}000$ waves. In the glass

$$0.05/\lambda_0/n = 0.05(1.5)/500 \text{ nm} = 1.5 \times 10^5;$$

in air

$$0.95/\lambda_0 = 0.19 \times 10^7;$$

total 2,050,000 waves.

$$OPD = [(1.5)(0.05) + (1)(0.95)] - (1)(1),$$

$$OPD = 1.025 - 1.000 = 0.025 \text{ m},$$

$$\Lambda/\lambda_0 = 0.025/500 \text{ nm} = 5 \times 10^4 \text{ waves}.$$

7.6 $OPL_B = nx = (1.00)(100 \text{ cm}) = 100 \text{ cm} = 1.00 \text{ m}.$
$OPL_A = \sum_i n_i x_i = (1.00)(89 \text{ cm}) + 2(1.52)(0.5 \text{ cm})$
$+ (1.33)(10 \text{ cm}) = 103.82 \text{ cm} = 1.0382 \text{ m}.$
$\Lambda = OPL_A - OPL_B = 1.0382 - 1.00 = .00382 \text{ m}.$

(7.16) $\delta = k_0\Lambda = (2\pi/\lambda_0)\Lambda = 2\pi(3.82 \times 10^{-3} \text{ m})/5.00 \times 10^{-9} \text{ m}$
$= 7.64 \times 10^6\pi.$
An integer multiple of 2π, so waves are in phase.

7.7 $E_1 = E_{01}\sin[\omega t - k(x + \Delta x)]$, so $\alpha_1 = -k(x + \Delta x)$. $E_2 = E_{01}\sin[\omega t - kx]$, so $\alpha_2 = -kx$.

(7.9)
$$\begin{aligned} E_0^2 &= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1) \\ &= E_{01}^2 + E_{01}^2 + 2E_{01}^2\cos(-kx - (-k(x + \Delta x))) = 2E_{01}^2(1 + \cos k\Delta x) \\ &= 2E_{01}^2(\cos(0) + \cos(k\Delta x)) = 4E_{01}^2\cos^2(k\Delta x/2), \end{aligned}$$

(see Problem 7.2),

$$E_0 = 2E_{01}\cos(k\Delta x/2).$$

(7.10)
$$\begin{aligned} \tan\alpha &= \frac{E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2}{E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2} \\ &= \frac{E_{01}\sin(-k(x + \Delta x) + E_{01}\sin(-kx))}{E_{01}\cos(-k(x + \Delta x)) + E_{01}\cos(-kx)} \end{aligned}$$

$$= \frac{2\sin\frac{1}{2}(-k(x+\Delta x)-kx)\cos\frac{1}{2}(-k(x+\Delta x)+kx)}{2\cos\frac{1}{2}(-k(x+\Delta x)-kx)\cos\frac{1}{2}(-k(x+\Delta x)+kx)}$$
$$= \tan(-kx-(k\Delta x/2)), \quad \alpha = -k(x+(\Delta x/2)).$$

7.8 $E = E_1 + E_2 = E_{01}\{\sin[\omega t - k(x+\Delta x)] + \sin(\omega t - kx)\}$. Since

$$\sin\beta + \sin\gamma = 2\sin(1/2)(\beta+\gamma)\cos(1/2)(\beta-\gamma),$$
$$E = 2E_{01}\cos(k\Delta x/2)\sin[\omega t - k(x+\Delta x/2)].$$

7.9 $E = E_0\,\mathrm{Re}[e^{i(kx+\omega t)} - e^{i(kx-\omega t)}] = E_0\,\mathrm{Re}[e^{ikx}2i\sin\omega t]$

$$= E_0\,\mathrm{Re}[2i\cos kx\sin\omega t - 2\sin kx\sin\omega t] = -2E_0\sin kx\sin\omega t.$$

Standing wave with node at $x = 0$.

7.10 $E_i = 3\cos wt = 3\angle 0$, $(\alpha_1 = 0)$. $E_2 = 4\sin wt$, but $\sin\theta = \cos(\theta - \pi/2)$, so

$$E_2 = 4\cos(wt - \pi/2) = 4\angle - \pi/2. \qquad E_3 = E_1 + E_1.$$
$$E_{3o}^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$$
$$= 9 + 16 + 2(3)(4)\cos(-\pi/2), E_{3o} = 5.$$

(7.10) $\tan\alpha = (E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2)/(E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2)$

$$= (3(0) + 4(-1))/(3(1) + 4(0)) = -4/3; \qquad \alpha = -53°,$$

so $\varphi = 53° = 0.93$ rad. Note that $\alpha_1 < \varphi$, so E_1 leads E_3.

7.11 By Faraday's law, $\partial E/\partial x = -\partial B/\partial x$. Integrate to get

$$B(x,t) = -\int(\partial E/\partial x)\,dt = -2E_0 k\cos kx\int\cos\omega t\,dt$$
$$= -2E_0(k/\omega)\cos kx\sin\omega t.$$

But $E_0 k/\omega = E_0/c = B_0$; thus $B(x,t) = -2B_0\cos kx\sin\omega t$.

7.12 Fringes are spaces $\lambda/2$ vertically.

$$\sin\theta = \text{(fringes/cm) vertical/(fringes/cm) on film;}$$

$$(\text{fringes/cm}) \text{ on film} = (1/(\lambda/2))/\sin\theta$$
$$= (1/5.50 \times 10^{-7}\text{ cm})/\sin(1^\circ) = 1.04 \times 10^{8}\text{ cm}^{-1}.$$

7.13 Nodes are spaced at $\lambda/2$ apart.

$$c = \nu\lambda, \quad \lambda = c/\nu = 3 \times 10^{8}\text{ m/s}/10^{10}\text{ Hz} = 0.03\text{ m}.$$

Node spacing is .015 m.

7.14 (7.30) E (standing wave) $= 2E_{0\ell}\sin kx \cos\omega t$ from two wave,

$$E_\ell = E_{0\ell}\sin(kx + \omega t); \quad E_R = E_{0\ell}\sin(kx - \omega t),$$

so,

$$E_\ell = 50\sin\left(\tfrac{2}{3}\pi x + 5\pi t\right); \qquad E_R = 50\sin\left(\tfrac{2}{3}\pi x - 5\pi t\right).$$

7.15 Heart beat frequency $= \nu_2 - \nu_1 = 2$ Hz.

7.16 One can see that the relative phase of the two waves varies, and that a maximum occurs (positive or negative), and that a zero occurs when the relative phase is $\pm n\pi$ (n odd). Also at the maxima, the relative phase between one wave and the net wave is zero. At those zeroes where the relative phase between one wave and the net wave is $\pi/2$, the "faster" wave "laps" the slower one, and the relative phase changes abruptly.

7.17 $E_1 = E_{01}\cos[(k_c + \Delta k)x - (\omega_c + \Delta\omega)t]$;

$E_2 = E_{01}\cos[(k_c - \Delta k)x - (\omega_c - \Delta\omega)t]$;

$$E = E_1 + E_2 = 2E_{01}\cos\tfrac{1}{2}[(k_c + \Delta k)x - (\omega_c + \Delta\omega)t$$
$$+ (k_c - \Delta k)x - (w_c - \Delta w)t] \times \cos\tfrac{1}{2}[(k_c + \Delta k)x - (\omega_c + \Delta\omega)t$$
$$- (k_c - \Delta k)x + (\omega_c - \Delta\omega)t] = 2E_{01}[\cos(k_c x - \omega_c t)\cos(\Delta k x - \Delta\omega t)]$$

so that $k_c = \vec{k}$, $\omega_c = \vec{\omega}$, $\Delta k = k_m$, $\Delta\omega = \omega_m$. Wavelength of envelope $\lambda_m = 2\pi/k_m = 2\pi/\Delta k$. Period of envelope $T_m = 2\pi/\omega_m = 2\pi/\Delta\omega$. Speed of envelope $\lambda_m/T_m = (2\pi/\Delta k)/(2\pi/\Delta\omega)$.

7.18
$$E = E_0 \cos \omega_c t + E_0 \alpha \cos \omega_m t \cos \omega_c t$$
$$= E_0 \cos \omega_c t + (E_0 \alpha/2)[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t].$$

Audible range $\nu_m = 20$ Hz to 20×10^3 Hz. Maximum modulation frequency $\nu_m(\max) = 20 \times 10^3$ Hz. $\nu_c - \nu_m(\max) \leq \nu \leq \nu_c + \nu_m(\max)$, $\Delta\nu = 2\nu_m(\max) = 40 \times 10^3$ Hz.

7.19 $v = \omega/k = ak$, $v_g = d\omega/dk = 2ak = 2v$.

7.20 $1/v_g = d(\nu/v)/d\nu$ and the rest follows.

7.21 From the previous problem $1/v_g = (n/c) - (\nu n^2/c^2)[d(c/n)]/d\nu$ and the rest follows.

7.22 $v = \sqrt{g\lambda/2\pi} = \sqrt{g/k}$, $v_g = v + k dv/dk$, where
$$dv/dk = -(1/2k)\sqrt{g/k} = -v/2k, \quad \text{so} \quad v_g = v/2.$$

7.23 We have $\lambda = 2\pi/k$, $d\lambda/dk = -2\pi/k^2 = -\lambda/k$ so that the term $kdv/dk = k(d\lambda/dk)(dv/d\lambda) = k(-\lambda/k)dv/d\lambda = -\lambda dv/d\lambda$ and the expression for v_g follows.

7.24 $v_g = v + kdv/dk$ and $dv/dk = (dv/d\omega)(d\omega/dk) = v_g dv/d\omega$. Since $v = c/n$,
$$dv/d\omega = (dv/dn)(dn/d\omega) = -(c/n^2)dn/d\omega,$$
$$v_g = v - (v_g ck/n^2)dn/d\omega = v/[1 + (ck/n^2)(dn/d\omega)]$$
$$= c/[n + \omega(dn/d\omega)].$$

7.25 (7.40) $n_g \equiv c/\nu_g$. From Problem 7.24 $\nu_g = c/(n + 2(dn/d\omega))$, so
$$n_g = n + \omega(dn/d\omega) = n(\nu) + 2\pi\nu(dn(\nu)/2\pi d\nu) = n(\nu) + \nu(dn(\nu))/d\nu.$$

7.26 For $v = a/\lambda$, $v_g = v - \lambda dv/d\lambda = a/\lambda + \lambda a/\lambda^2 = 2a/\lambda = 2v$.

7.27 (7.38) $\nu_g = \nu + k(d\nu/dk) = c/n - (kc/n^2)(dn/dk)$
$$= c/n - (kc/n^2)(dn/d\lambda)(d\lambda/dk)$$
$$= c/n - (kc/n^2)(dn/d\lambda)(-2\pi/k^2)$$
$$= c/n + (2\pi/k)(c/n^2)(dn/d\lambda) = c/n + (\lambda c/n^2)(dn/d\lambda)$$

7.28 $v = \omega/k = \omega_0/\sin(kl/2)/(kl/2) = \omega_0/\operatorname{sinc}(kl/2)$;
$v_g = d\omega/dk = \omega_0/\cos(kl/2)$.

7.29 $v = \omega/k$ therefore $\omega^2 = \omega_p^2 + c^2(\omega/v)^2$ and

$$v = c/[1-(\omega_p/\omega)^2]^{1/2}; \quad v_g = d\omega/dk = c^2k/\omega = c[1-(\omega_p/\omega)^2]^{1/2}.$$

7.30 For $\omega^2 \gg \omega_i^2$, $n^2 = 1 - (Nq_e^2/\omega^2\epsilon_0 m_e)\sum_i f_i = 1 - Nq_e^2/\omega^2\epsilon_0 m_e$. Using the binomial expansion, we have $(1-x)^{1/2} \approx 1 - x/2$ for $x \ll 1$, so that

$$n = 1 - Nq_e^2/2\omega^2\epsilon_0 m_e, \quad dn/d\omega = Nq_e^2/\epsilon_0 m_e \omega^3.$$
$$v_g = c/[n + \omega(dn/d\omega)] = c/[1 + Nq_e^2/2\epsilon_0 m_e \omega^2]$$

and $v_g < c$, $v = c/n = c/[1 - Nq_e^2/2\epsilon_0 m_e\omega^2]$. By binomial expansion,

$$(1-x)^{-1} \approx 1 + x \quad \text{for} \quad x \ll 1, v = c[1 + Nq_e^2/2\epsilon_0 m_e\omega^2]; \quad vv_g = c^2.$$

7.31 $E_1 = 2E_0\cos\omega t; \quad E_2 = \frac{1}{2}E_0\sin 2\omega t. \quad E = E_1 + E_2$
$$= 2E_0\cos\omega t + \tfrac{1}{2}E_0\sin 2\omega t = E_0(2+\sin\omega t)\cos\omega t.$$

Resultant is anharmonic, but periodic with period ω.

7.32 $$\int_0^\lambda akx\sin bkx\,dx = (1/2k)\left[\int_0^\lambda \cos[(a-b)kx]k\,dx - \int_0^\lambda \cos[(a+b)kx]kdx\right] = 0$$

if $a \neq b$. Whereas if $a = b$,

$$\int_0^\lambda \sin^2 akx\,dx = (1/2k)\int_0^\lambda (1+\cos 2akx)k\,dx = \lambda/2.$$

The other integrals are similar.

7.33 Even function, therefore $B_m = 0$.

$$A_0 = (2/\lambda)\int_{-\lambda/a}^{\lambda/a} dx = (2/\lambda)(2\lambda/a) = 4/a,$$

$$A_m = (2/\lambda)\int_{-\lambda/a}^{\lambda/a} (1)\cos mkx\,dx = (4/mk\lambda)\sin mkx|_0^{\lambda/a}$$
$$= (2/m\pi)\sin 2m\pi/a.$$

7.34 $A_0 = 0$, $A_1 = A$, and all other $A_m = 0$ moreover $B_m = 0$ so $f(x) = A\cos(\pi x/L)$.

7.35 $A_m = 4/m^2$, $m \neq 0$; $A_0 = 8\pi^2/3$; $B_m = -4\pi/m$.

7.36 $A_m = -2(1+\cos m\pi)/\pi(m^2-1)$ where $m \neq 1$ and $A_1 = 0$.

7.37 $$f(x) = \frac{1}{\pi}\int_0^a E_0 L\frac{\sin kL/2}{kL/2}\cos kx\,dk$$
$$= \frac{E_0L}{2\pi}\int_0^a \frac{\sin(kL/2+kx)}{kL/2}dk + \frac{E_0L}{2\pi}\int_0^a \frac{\sin(kL/2-kx)}{kL/2}dk.$$

Let $kL/2 = w$, $(L/2)dk = dw$, $kx = wx'$,

$$f(x) = \frac{E_0}{\pi}\int_0^b \frac{\sin(w+wx')}{w}dw + \frac{E_0}{\pi}\int_0^b \frac{\sin(w-wx')}{w}dw,$$

where $b = aL/2$. Let $w + wx' = t$, $dw/w = dt/t$, $0 \leq w \leq b$ and $0 \leq t \leq (x'+1)b$. Let $w - wx' = -t$ in the other integral, $0 \leq w \leq b$ and $0 \leq t \leq (x'-1)b$.

$$f(x) = \frac{E_0}{\pi}\int_0^{(x'+1)b} \frac{\sin t}{t}dt - \frac{E_0}{\pi}\int_0^{(x'-1)b} \frac{\sin t}{t}dt,$$

$$f(x) = \frac{E_0}{\pi}\mathrm{Si}[b(x'+1)] - \frac{E_0}{\pi}\mathrm{Si}[b(x'-1)],$$

with $x' = 2x/L$.

7.38 By analogy with Eq. (7.61), $A(\omega) = (\Delta t/2)E_0 \operatorname{sinc}(\omega_p - \omega)\Delta t/2$. From Table 1, $\operatorname{sinc}(\pi/2) = 63.7\%$. Not quite 50% actually, $\operatorname{sinc}(\pi/1.65) = 49.8\%$. $|(\omega_p - \omega)\Delta t/2| < \pi/2$ or $-\pi/\Delta t < \omega_p - \omega < \pi/\Delta t$; thus appreciable values of $A(\omega)$ lie in a range $\Delta\omega \sim 2\pi/\Delta t$ and $\Delta\nu\Delta t \sim 1$. Irradiance is proportional to $A^2(\omega)$, and $[\operatorname{sinc}(\pi/2)]^2 = 40.6\%$.

7.39 $\Delta x_c = c\Delta t_c$, $\Delta x_c \sim c/\Delta\nu$. But $\Delta\omega/\Delta k_0 = \bar{\omega}/\bar{k}_0 = c$; thus $|\Delta\nu/\Delta\lambda_0| = \bar{\nu}/\bar{\lambda}_0$, $\Delta x_c \sim c\bar{\lambda}_0/\Delta\lambda_0\bar{\nu}$, $\Delta x_c \sim \bar{\lambda}_0^2/\Delta\lambda_0$. Or try using the uncertainty principle: $\Delta x \sim h/\Delta p$ where $p = h/\lambda$ and $\Delta\lambda_0 \ll \bar{\lambda}_0$.

7.40
$$\Delta x_c = c\Delta t_c = 3 \times 10^8 \text{ m/s} 10^{-8}\text{s} = 3 \text{ m}.$$
$$\Delta\lambda_0 \sim \lambda_0^2/\Delta x_c = (500 \times 10^{-9} \text{ m})^2/3 \text{ m},$$
$$\Delta\lambda_0 \sim 8.3 \times 10^{-14} \text{ m} = 8.3 \times 10^{-5} \text{ nm},$$
$$\Delta\lambda_0/\bar{\lambda}_0 = \Delta\nu/\bar{\nu} = 8.3 \times 10^{-5}/500 = 1.6 \times 10^{-7} \sim 1 \text{ part in } 10^7.$$

7.41 $\Delta\nu = 54 \times 10^3$ Hz; $\Delta\nu/\bar{\nu} = (54 \times 10^3)(10{,}600 \times 10^{-9} \text{ m})/(3 \times 10^8 \text{m/s})$ $= 1.91 \times 10^{-9}$. $\Delta x_c = c\Delta t_c \sim c/\Delta\nu, \Delta x_c \sim (3 \times 10^8)/(54 \times 10^3)$ $= 5.55 \times 10^3$ m.

7.42 $\Delta\nu/\nu = 2/10^{10}$; $c = \nu\lambda$, so

$$\nu = c/\lambda = 3 \times 10^8 \text{ m/s}/632.8 \times 10^{-9} \text{ m} = 4.74 \times 10^{14} \text{ Hz}.$$

$$(7.64) \quad \Delta\ell_c = c\Delta t_c.$$

Frequency range is $\pm 2(4.74 \times 10^4$ Hz) or 9.48×10^4 Hz, so

$$\Delta t \simeq 1.05 \times 10^{-5} \text{ s}. \quad \Delta\ell_c = (3 \times 10^8 \text{ m/s})(1.05 \times 10^{-5} \text{ s}) = 3.15 \times 10^3 \text{ m}.$$

7.43 $\Delta x_c = c\Delta t_c = 3 \times 10^8 \times 10^{-10} = 3 \times 10^{-2}$ m, $\Delta\nu \sim 1/\Delta t_c = 10^{10}$ Hz, $\Delta\lambda_0 \sim \bar{\lambda}_0^2/\Delta x_c$ (see Problem 7.35),

$\Delta\lambda_0 \sim (632.8 \text{ nm})^2/(3 \times 10^{-2} \text{ m}) = 0.013$ nm. $\Delta\nu = 10^{15}$ Hz,
$\Delta x_c = c \times 10^{-15} = 300$ nm, $\Delta\lambda_0 \sim \bar{\lambda}_0^2/\Delta x_c = 133478$ nm.

7.44 $\Delta\nu/\nu = \Delta\lambda/\lambda$, (see Table 7.1)

$$= (1 \times 10^{-10} \text{ m})/(600 \times 10^{-9} \text{ m}) = 1.67 \times 10^{-4}.$$

$$c = \nu\lambda, \quad \text{so} \quad \nu = c/\lambda = (3 \times 10^8 \text{ m/s})/(600 \times 10^{-9} \text{ m})$$

$$= 5.00 \times 10^{14} \text{ Hz.} \qquad \Delta v = (1.67 \times 10^{-4})(5 \times 10^{14} \text{ Hz})$$

$$= 8.35 \times 10^{14} \text{ Hz}, \quad \text{so} \quad \Delta t \simeq 1.20 \times 10^{-11} \text{ s.}$$

(7.64) $\Delta\ell_c = c\Delta t_c = (3 \times 10^8 \text{ m/s})(1.20 \times 10^{-11} \text{ s}) = 3.60 \times 10^{-3}$ m.

7.45 $\Delta\ell_c = 20\lambda_0$. (7.64) $\Delta\ell_c = c\Delta t_c$, so
$\Delta t_c = \Delta\ell_c/c = 20\lambda_0/c = 20(500 \times 10^{-9}\text{m})/(3 \times 10^8\text{m/s}) = 3.33 \times 10^{-15}$ sec.
$\Delta\nu \simeq 1/\Delta t_c = 3 \times 10^{14}$ Hz.

7.46 $\Delta\nu/\nu = \Delta\lambda/\lambda$, (see Table 7.1) $= (1.2 \times 10^{-9} \text{ m})/(500 \times 10^{-9} \text{ m}) = 0.0024$.
$c = \nu\lambda$, so $\nu = c/\lambda = (3 \times 10^8 \text{ m/s})/(500 \times 10^{-9} \text{ m}) = 6.00 \times 10^{14}$ Hz.
$\Delta\nu =$ Frequency Bandwidth $= (0.0024)(6.00 \times 10^{14} \text{ Hz}) = 1.44 \times 10^{12}$ Hz.
$\Delta t_c \simeq 1/\Delta\nu = 6.94 \times 10^{-13}$ s.
(7.64) $\Delta\ell_c = c\Delta t_c = (3 \times 10^8 \text{ m/s})(6.94 \times 10^{-13} \text{ s}) = 2.08 \times 10^{-4}$ m.

Chapter 8 Solutions

8.1 In each part the x and y components have the same amplitude E_0.
(a) $\vec{E} = (\hat{\imath} - \hat{\jmath})E_0 \cos(kz - \omega t)$ is a $\mathcal{P}$ state at 135° or −45°.
(b) $\vec{E} = (\hat{\imath} - \hat{\jmath})E_0 \sin(kz - \omega t)$ is also a $\mathcal{P}$ state at 135° or −45°. (c) E_x leads E_y by $\pi/4$. Therefore it is an $\mathcal{E}$ state and left-handed. (d) E_y leads E_x by $\pi/2$. Therefore it is an $\mathcal{R}$ state.

8.2 E_x leads E_y by $\pi/2$. Therefore it is a left-handed circularly-polarized standing wave.

8.3 $\vec{E}_{\mathcal{R}} = \hat{\imath}E_0 \cos(kz - \omega t) + \hat{\jmath}E_0 \sin(kz - \omega t)$.
$\vec{E}_{\mathcal{L}} = \hat{\imath}E_0' \cos(kz - \omega t) - \hat{\jmath}E_0' \sin(kz - \omega t)$.
$\vec{E} = \vec{E}_{\mathcal{R}} + \vec{E}_{\mathcal{L}} = \hat{\imath}(E_0 + E_0') \cos(kz - \omega t) + \hat{\jmath}(E_0 - E_0') \sin(kz - \omega t)$.

Let

$$E_0 + E_0' = E_{0x}'' \quad \text{and} \quad E_0 - E_0' = E_{0y}'';$$

then

$$\vec{E} = \hat{\imath}E_{0x}'' \cos(kz - \omega t) + \hat{\jmath}E_{0y}'' \sin(kz - \omega t).$$

From Eqs. (8.11) and (8.12) it is clear that we have an ellipse where $\epsilon = -\pi/2$ and $\alpha = 0$.

8.4 $E_{0y} = E_0 \cos 25°$; $E_{0z} = E_0 \sin 25°$;
$\vec{E}(x,t) = (0.91\hat{\jmath} + 0.42\hat{k})E_0 \cos(kx - \omega t + \pi/2)$.

8.5 $\vec{k} = k(\hat{\imath} + \hat{\jmath})/\sqrt{2}$.

8.6 $\vec{E} = E_0[\hat{\jmath} \sin(kx - \omega t) - \hat{k} \cos(kx - \omega t)]$.

8.7 $\theta = 0°$, $I = I_1 \cos^2 0° = 200\ \text{W/m}^2$.

8.8 Half the energy is removed and half transmitted, hence $I = 150\ \text{W/m}^2$.

8.9 (8.24) $I(\theta) = I(0)\cos^2\theta$ so $I(\theta)/I(0) = \cos^2(60°) = (1/2)^2 = 0.25$.

8.10 $HN - 32$, so 32% of incident light is (ideally) transmitted.
$I_t = (0.32)(0.32)I_i = 0.10 I_i$.

8.11 In natural light each filter passes 32% of the incident beam. Half of the incoming flux density is in the form of a $\mathcal{P}$-state parallel to the extinction axis, and effectively none of this emerges. Thus, 64% of the light parallel to the transmission axis is transmitted. In the present problem $32\% I_i$ enters the second filter, and $64\%(32\% I_i) = 21\% I_i$ leaves it.

8.12 $I = I_1 \cos^2\theta = (200\ \text{W/m}^2)\cos^2 40° = 153\ \text{W/m}^2$.

8.13 50% means all the light from the first polarizer, viz. $(1/2)I_i$, is passed by the second. Hence the angle is 90°.

8.14 $I = I_1 \cos^2\theta = (200\ \text{W/m}^2)\cos^2 60° = (1/4)200\ \text{W/m}^2 = 50\ \text{W/m}^2$.

8.15 $I = I_1 \cos^2 45° = (100\ \text{W/m}^2)0.50 = 50\ \text{W/m}^2$.

8.16 The light from the first polarizer is at 10° and has an irradiance of $I_i \cos^2 30° = 0.75 I_i$; this makes an angle of 60° with the next filter, hence $I_2 = (0.75 I_i)\cos^2 50° = 0.31 I_i$.

8.17 $I' =$ flux density through middle polarizer $= I_1 \cos^1\theta$ (8.24).

$$\begin{aligned} I' &= I_1 \cos^2(45°) = I_1/2 \\ I_2 &= I' \cos^2(45°) \\ &= (I_1/2)/2 = I_1/4 \end{aligned}$$

8.18 Without middle polarizer, $I_t = (I_i/2)\cos^2(50°) = 207\ \text{W/m}^2$. With middle polarizer, $I_t = (I_i/2)\cos^2(25°)\cos^2(25°) = 337\ \text{W/m}^2$.

8.19 $I_1 = (1/2)I_i$; $I_2 = I_1 \cos^2 30°$; $I_3 = I_2 \cos^2 30°$; $I_4 = I_3 \cos^2 30°$ hence $I_4 = (1/2)I_i \cos^2 30° \cos^2 30° \cos^2 30° = 0.21 I_i = 0.21(200\ \mathrm{W/m^2}) = 42\ \mathrm{W/m^2}$.

8.20 $I_2 = I_1 \cos^2\theta = 30\% I_i$ and $I_i = 2I_1$; $30\% I_i = (1/2)I_i \cos^2\theta$; $0.60 = \cos^2\theta$, $\theta = 39°$.

8.21 With $\theta = \omega t$, the emergent flux density is

$$I = \tfrac{1}{2}E_{01}^2 \sin^2\theta \cos^2\theta = (E_{01}^2/8)(1 - \cos 2\theta)(1 + \cos 2\theta)$$
$$= (E_{01}^2/8)(1 - \cos^2 2\theta) = (E_{01}^2/16)(1 - \cos 4\theta) = (I_1/8)(1 - \cos 4\theta).$$

8.22 No. The crystal performs as if it were two oppositely oriented specimens in series. Two similarly oriented crystals in series would behave like one thick specimen and thus separate the *o*-and *e*-rays even more.

8.23 The polarization of the light is lost in the specular reflection from the pencil dot.

8.24 Light scattered from the paper passes through the polaroids and becomes linearly polarized. Light from the upper left filter has its $\vec{E}$-field parallel to the principal section (which is diagonal across the second and fourth quadrants) and is therefore an *e*-ray. Notice how the letters P and T are shifted downward in an extraordinary fashion. The lower right filter passes an *o*-ray so that the letter C is undeviated. Note that the ordinary image is closer to the blunt corner.

8.25 (a) and (c) are two aspects of the previous problem. (b) shows double refraction because the polaroid's axis is at roughly 45° to the principal section of the crystal. Thus both an *o*- and an *e*-ray will exist.

8.26 When $\vec{E}$ is perpendicular to the CO_3 plane the polarization will be less than when it is parallel. In the former case, the field of each polarized oxygen atom tends to reduce the polarization of its neighbors. In other words, the induced field is down while $\vec{E}$ is up. When $\vec{E}$ is in the carbonate plane two dipoles reinforce the third and vice versa. A reduced polarizability leads to a lower dielectric constant, a lower refractive index, and a higher speed. Thus $v_{\parallel} > v_{\perp}$.

8.27 (8.25) $\tan\theta_p = n_t/n_i = 9.0/1.0$, $\theta_p = 83.7°$. The dipole is perpendicular to the plane of incidence.

8.28 $\tan\theta_p = n_t/n_i = 1.33/1.00$, $\theta_p = 53°$.

8.29 $\tan\theta_p = n_t/n_i = 1.65/1.33$, $\theta_p = 51.1°$.

8.30 $\tan\theta_p = n_t/n_i$; $n_t = \tan 54.30° = 1.39$.

8.31 $\tan\theta_p = n_g/n_e = 1.65/1.36 = 1.21$ and $\theta_p = 50.5°$; $n_e \sin\theta_p = n_g \sin\theta_t$; $\sin\theta_t = (1.36/1.65)\sin 50.50° = 0.636$ and $\theta_t = 39.5°$.

8.32 (4.5) $\sin\theta_i/\sin\theta_t = n_{ti}$; $\sin\theta_t = \sin\theta_i/n_{ti} = \sin(40°)/1.5$; $\theta_t = 25.4°$.
(8.26) $R_{\parallel} = \tan^2(\theta_i - \theta_t)/\tan^2(\theta_i + \theta_t) = \tan^2(-14.6°)/\tan^2(65.4°) = 0.014$.
(8.27) $R_{\perp} = \sin^2(\theta_i - \theta_t)/\sin^2(\theta_i + \theta_t) = \sin^2(-14.6°)/\sin^2(65.4°) = 0.077$.
(8.28) $R = \frac{1}{2}(R_{\parallel} + R_{\perp}) = 0.0455$.
(9.29) $V = I_p/(I_p + I_n) = (R_{\perp} + R_{\parallel})/(R_{\perp} + R_{\parallel} + R) = 67\%$.

8.33 (4.5) $\sin\theta_i/\sin\theta_t = n_{ti}$; $\sin\theta_t = \sin\theta_i/n_{ti} = \sin(70°)/1.5$; $\theta_t = 38.8°$.
(8.26) $R_{\parallel} = \tan^2(\theta_i - \theta_t)/\tan^2(\theta_i + \theta_t) = \tan^2(-31.2°)/\tan^2(108.8°) = 0.045$.
(8.27) $R_{\perp} = \sin^2(\theta_i - \theta_t)/\sin^2(\theta_i + \theta_t) = \sin^2(-31.2°)/\sin^2(108.8°) = 0.299$.
(8.28) $R = \frac{1}{2}(R_{\parallel} + R_{\perp}) = 0.172$.

8.34 $n_o = 1.6584$, $n_e = 1.4864$. Using Snell's law, $\sin\theta_i = n_o \sin\theta_{to} = 0.766$, $\sin\theta_i = n_e \sin\theta_{te} = 0.766$, $\sin\theta_{to} \approx 0.463$, $\theta_{to} \approx 27°35'$; $\sin\theta_{te} \approx 0.516$, $\theta_{te} \approx 31°4'$; $\Delta\theta \approx 3°29'$.

8.35 (3.59) $n \equiv c/\nu = \lambda_o/\lambda_n$, so $\lambda_n = \lambda_o/n$.
Ordinary $\lambda_n = \lambda_o/n_o = 589.3\text{ nm}/1.5443 = 381.6\text{ nm}$.
Extraordinary $\lambda_n = \lambda_o/n_o = 589.3\text{ nm}/1.5533 = 379.4\text{ nm}$. Same frequency $\nu = c/\lambda_o = (3 \times 10^8\text{ m/s})/5.893 \times 10^{-7}\text{ m} = 5.091 \times 10^{14}\text{ Hz}$.

8.36 For calcite, $n_o > n_e$. Two spectra will be visible when (b) or (c) is used in a spectrometer. The indices are computed in the usual way, using $n = \sin[(\alpha + \delta_m)/2]/\sin(\alpha/2)$, where δ_m is the angle of minimum deviation of either beam.

8.37 E_x leads E_y by $\pi/2$. They were initially in phase and $E_x > E_y$. Therefore the wave is left-handed, elliptical, and horizontal.

8.38 $\sin\theta_c = n_{\text{balsam}}/n_o = 1.55/1.658 = 0.935$; $\theta_c \sim 69°$.

抛弃

8.40 (c) Undesired energy in the form of one of the $\mathcal{P}$-states can be disposed of without local heating problems. (d) The Rochon transmits an undeviated beam (the *o*-ray), which is therefore achromatic as well.

8.41 Each half wave plate rotates $\bar{E}$ by $2\theta = 2(\pi/40 \text{ rad}) = \pi/20$ rad. Stack of plates rotates $\bar{E}$ by $10(\pi/20) = \pi/2$ rad. Ignoring surface reflections, I is reduced by 1/2 at the first polarizer, but since the beam's polarization is rotated by $\pi/2$ rad, I is not further reduced by the second polarizer.

效应

8.42 Placing the quarter wave plate first will have no effect on the irradiance. The irradiance will be affected with the quarter wave plate following the
影响
polarizer.

8.43 Emerging wave is elliptically polarized with $\mathcal{E} = (\pi/2 - \pi/4) = \pi/4$.

8.44 The polarizers are aligned. The cellophane is a half wave plate, so is seen as "dark" (no beam passing through in this region).

外部

8.45 $\Delta\varphi = 2\pi d\Delta n/\lambda_0$ but $\Delta\varphi = (1/4)(2\pi)$ because of the fringe shift. Therefore $\Delta\varphi = \pi/2$ and $d = 589.3 \times 10^{-9}/2(10^{-2}) = 2.94 \times 10^{-5}$ m.

8.46 The $\mathcal{R}$-state incident on the glass screen drives the electrons in circular orbits, and they reradiate reflected circular light whose $\vec{E}$-field rotates in the same direction as that of the incoming beam. But the propagation direction has been reversed on reflection, so that although the incident light is in an $\mathcal{R}$-state, the reflected light is left-handed. It will therefore be completely absorbed by the right-circular polarizer.

8.47 Yes. If the amplitudes of the $\mathcal{P}$-states differ. The transmitted beam, in a pile-of-plates polarizer, especially for a small pile.

8.48 Concentration is 10 g/1000 cm^3 = .01 g/cm^3, so rotatory power = .01(+66.45°)/10 cm = 0.06645°/cm. Light travels through 1 m = 100 cm, so emerging light is at 6.645° from vertical (clockwise).

8.49 Place the photoelastic material between circular polarizers with both retarders facing it. Under circular illumination no orientation of the stress axes is preferred over any other, and they will thus all be indistinguishable. Only the birefringence will have an effect, and so the isochromatics will be visible. If the two polarizers are different, that is, one an $\mathcal{R}$, the other an $\mathcal{L}$, regions where Δn leads to $\Delta\varphi = \pi$ will appear bright. If they are the same, such regions appear dark.

8.50 From (8.32), $\Delta\varphi = (2\pi/\lambda_o)\ell(|n_o - n_e|)$ so $|n_o - n_e| = \lambda_o\Delta\varphi/2\pi\ell$.
(8.40) $\Delta n = \lambda_o KE^2 = \lambda_o K(V/d)^2$ so $\lambda_o\Delta\varphi/2\pi\ell = \lambda_o K(V/d)^2$;
$\Delta\varphi = 2\pi\ell K(V/d)^2$.

8.51 $V_{\lambda/2} = \lambda_0/2n_0^3 r_{63} = 550\times10^{-9}/2(1.58)^3 5.5\times10^{-12} = 10^5/2(3.94) = 12.7\,\text{kV}$.

8.52 $J = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos 45° \\ \sin 45° \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

8.53 $\vec{E}_1 \cdot \vec{E}_2^* = (1)(e_{21}^*) + (-2i)(e_{22})^* = 0$, $\vec{E}_2 = (2, i)^T$.

8.54 (a) $E_1 = (1, 1, 0, 0)$ has relative irradiance of 1, and is horizontally polarized. $E_2 = (3, 0, 0, 3)$ has relative irradiance of 3, is right circularly polarized. For both, $V = 1$. (b) $E = E_1 + E_2 = (4, 1, 0, 3)$, and has both a horizontal $\mathcal{P}$ component and an $\mathcal{R}$ component.
(c) (8.48) $V = (S_1^2 + S_2^2 + S_3^2)^{1/2}/S_o = (1^2 + 0^2 + 3^2)^{1/2}/4 = 0.79$.
(d) $E = (1, 1, 0, 0) + (1, -1, 0, 0) = (2, 0, 0, 0)$ and is "natural" light (unpolarized).

8.55 (See Tables 8.5 and 8.6.)

$$S_t = \tfrac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \tfrac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \text{Vertical } \mathcal{P}$$

Relative irradiance $= 1/2$. (8.48) $V = (S_1^2 + S_2^2 + S_3^2)^{1/2}/S_o = 1$.

8.56 (See Tables 8.5 and 8.6).

$$S_t = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = (45^\circ\ \mathcal{P})$$

Relative irradiance $= 1/2$. (8.48) $V = (S_1^2 + S_2^2 + S_3^2)^{1/2}/S_o = 1$.

8.57

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

8.58

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Therefore light polarized at 45° is unchanged, as expected.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

A horizontal $\mathcal{P}$-state is changed to an $\mathcal{R}$ state.

8.59 Fast axis $\alpha = +45°$, $\cos 90° = 0$, $\sin 90° = 1$. Quarter wave plate, $\Delta\varphi = \pi/2$; $\cos(\pi/2) = 0$, $\sin(\pi/2) = 1$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c^2 + s^2 \cos\Delta\varphi & cs(1 - \cos\Delta\varphi) & -s\sin\Delta\varphi \\ 0 & cs(1-\cos\Delta\varphi) & s^2 + c^2\cos\Delta\varphi & c\sin\Delta\varphi \\ 0 & s\sin\Delta\varphi & -c\sin\Delta\varphi & \cos\Delta\varphi \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0+1(0) & 0(1)(1-0) & -1(1) \\ 0 & 0(1)(1-0) & 1+0(0) & 0(1) \\ 0 & 1(1) & -(0)(1) & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

8.60 Quarter wave plate, $\Delta\varphi = \pi/2$; $\cos(\pi/2) = 0$, $\sin(\pi/2) = 1$. Vertical fast axis $\alpha = 0$, $\cos(0) = 1$, $\sin(0) = 0$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c^2 + s^2 \cos\Delta\varphi & cs(1 - \cos\Delta\varphi) & -s\sin\Delta\varphi \\ 0 & cs(1-\cos\Delta\varphi) & s^2 + c^2\cos\Delta\varphi & c\sin\Delta\varphi \\ 0 & s\sin\Delta\varphi & -c\sin\Delta\varphi & \cos\Delta\varphi \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1+0(0) & 1(0)(1-0) & -0(1) \\ 0 & 1(0)(1-0) & 0+1(0) & 1(1) \\ 0 & 0(1) & -1(1) & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

8.61 $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8.62 (From Problem 8.60).

$$S_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \mathcal{R}\text{-state.}$$

E_y leads E_x by $\pi/2$ (see third part from right of Figure 8.7a).

$$S_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \mathcal{L}\text{-state.}$$

E_x leads E_y by $\pi/2$ (see third part from left of Figure 8.7a).

8.63
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

8.64
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

8.65 (8.42) $\Delta\varphi = 2\pi n_o^3 r_{63} V/\lambda_o$. At maximum transmission, $\Delta\varphi = \pi$. For $I_t = 0$, $\sin^2(\Delta\varphi/2) = 0$, so $(\Delta\varphi/2) = \pi$. $2\pi = 2\pi n_o^3 r_{63} V/\lambda_o$.

$V = \lambda_o/n_o^3 r_{63} = (5.461 \times 10^{-7}\text{ m})/((1.52)^3(8.5 \times 10^{-12}\text{ m/V}) = 18.29\text{ kV}$.

If polarizers are parallel, I_t is a maximum at $V = 0$. Equivalent to $I_t = I_i \cos^2(\Delta\varphi/2)$, so

$$I_t/I_i = \cos^2(\pi V/2V_{\lambda/2}) \quad \text{(from 8.43)} = \cos^2(\pi/2) = 0.$$

8.66 $\begin{bmatrix} te^{i\varphi} & 0 \\ 0 & te^{i\varphi} \end{bmatrix}$, where a phase increment of φ is introduced into both components as a result of traversing the plate. vacuum: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, perfect absorber: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

8.67
$$\begin{bmatrix} t^2 & 0 & 0 & 0 \\ 0 & t^2 & 0 & 0 \\ 0 & 0 & t^2 & 0 \\ 0 & 0 & 0 & t^2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

8.68 $V = I_p/(I_p + I_u) = (S_1^2 + S_2^2 + S_3^2)^{1/2}/S_0$,

$I_p = (S_1^2 + S_2^2 + S_3^2)^{1/2}; \quad I - I_p = I_u.$

$$S_0 - (S_1^2 + S_2^2 + S_3^2)^{1/2} = I_u$$

$$\begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$5 - (0 + 0 + 1)^{1/2} = I_u.$$

Chapter 9 Solutions

9.1 $\vec{E}_1 \cdot \vec{E}_2 = (1/2)(\vec{E}_1 e^{-i\omega t} + \vec{E}_1^* e^{i\omega t}) \cdot (1/2)(\vec{E}_2 e^{-i\omega t} + \vec{E}_2^* e^{i\omega t})$, where

$$\mathrm{Re}(z) = (1/2)(z + z^*).$$

$$\vec{E}_1 \cdot \vec{E}_2 = (1/4)[\vec{E}_1 \cdot \vec{E}_2 e^{-2i\omega t} + \vec{E}_1^* \cdot \vec{E}_2^* e^{2i\omega t} + \vec{E}_1 \cdot \vec{E}_2^* + \vec{E}_1^* \cdot \vec{E}_2].$$

The last two terms are time independent, while $\langle \vec{E}_1 \cdot \vec{E}_2 e^{-2i\omega t} \rangle \to 0$ and $\langle \vec{E}_1^* \cdot \vec{E}_2^* e^{2i\omega t} \rangle \to 0$ because of the $1/T\omega$ coefficient. Thus

$$I_{12} = 2\langle \vec{E}_1 \cdot \vec{E}_2 \rangle = (1/2)(\vec{E}_1 \cdot \vec{E}_2^* + \vec{E}_1^* \cdot \vec{E}_2).$$

9.2 The largest value of $r_1 - r_2$ is equal to a. Thus if $\epsilon_1 = \epsilon_2$, $\delta = k(r_1 - r_2)$ varies from 0 to ka. If $a \gg \lambda$, $\cos\delta$ and therefore I_{12} will have a great many maxima and minima and therefore average to zero over a large region of space. In contrast, if $a \ll \lambda$, δ varies only slightly from 0 to $ka \ll 2\pi$. Hence I_{12} does not average to zero, and from Eq. (9.17), I deviates little from $4I_0$. The two sources effectively behave as a single source of double the original strength.

9.3 Dropping the common time factor $E_1 = E_0 \exp(2\pi i z/\lambda)$ and $E_2 = E_0 \exp[(2\pi i/\lambda)(z\cos\theta + y\sin\theta)]$, adding these at the $z = 0$ plane yields $E = E_0\{1 + \exp[(2\pi i/\lambda)(y\sin\theta)]\}$. The absolute square of this is the irradiance viz.

$$I(y) = 2E_0^2\left[1 + \cos\left(\frac{2\pi}{\lambda} y \sin\theta\right)\right]$$

and the rest follows from the identity $\cos 2\theta = 2\cos^2\theta - 1$. The cosine squared has zeros at $y = m\lambda/(2\sin\theta)$ where m is an odd integer. The fringe separation is $\lambda/\sin\theta$. As θ increases, the separation decreases.

9.4 A bulb at S would produce fringes. We can imagine it as made up of a very large number of incoherent point sources. Each of these would generate an independent pattern, all of which would then overlap. Bulbs at S_1 and S_2 would be incoherent and could not generate detectable fringes.

9.5 $y_m = sm\lambda/a \approx 14.5 \times 10^{-2}$ m and $\lambda = 0.0145$ m: $\nu = v/\lambda = 23.7$ kHz. This is Young's Experiment with the sources out-of-phase.

9.6 This is comparable to the "two-slit" configuration, (Figure 9.8), so we can use (9.29) $a \sin\theta_m = m\lambda$ (θ_m may not be "small"). Let $m = 1$, $\sin\theta = y/(s^2 + y^2)^{1/2}$, so,

$$ay = \lambda(s^2 + y^2)^{1/2}; \quad (a^2 - \lambda^2)y^2 = \lambda^2 s^2;$$

$$y = \lambda s/(a^2 - y^2)^{1/2}. \quad c = v\lambda,$$

so $\lambda = c/v = (3 \times 10^8 \text{ m/s})/(1.0 \times 10^6 \text{ Hz}) = 300$ m.
$y = (300 \text{ m})(2000 \text{ m})/((600 \text{ m})^2 - (300 \text{ m})^2)^{1/2} = 1.15 \times 10^3$ m

9.7 (a) $r_1 - r_2 = \pm\lambda/2$, hence $a \sin\theta_1 = \pm\lambda/2$ and

$$\theta_1 \approx \pm\lambda/2a = \pm(1/2)(632.8 \times 10^{-9} \text{ m})/(0.220 \times 10^{-3} \text{ m})$$
$$= \pm 1.58 \times 10^{-3} \text{ rad},$$

or since

$$y_1 = s\theta_1 = (1.00 \text{ m})(\pm 1.58 \times 10^{-3} \text{ rad}) = \pm 1.58 \text{ mm}.$$

(b) $y_5 = s5\lambda/a = (1.00 \text{ m})5(632.8 \times 10^{-9})/(0.200 \times 10^{-3} \text{ m}) = 1.582 \times 10^{-2}$ m. (c) Since the fringes vary as cosine-squared and the answer to (a) is half a fringe width, the answer to (b) is 10 times larger.

9.8 θ_m is "small," so we can use (9.28) $\theta_m = m\lambda/a$, θ_m is radian,

$$a = m\lambda/\theta_m = [4(6.943 \times 10^{-7} \text{ m})]/[1°(2\pi \text{ rad}/360°)] = 1.59 \times 10^{-4} \text{ m}.$$

9.9 $\Delta y \simeq (s/a)\lambda$, so,

$$s = a\Delta y/\lambda = [(1.0 \times 10^{-4} \text{ m})(10 \times 10^{-3} \text{ m})]/(4.8799 \times 10^{-7} \text{ m}) = 2.05 \text{ m}.$$

9.10 (9.28) $\theta_m = m\lambda/a$. Want $\theta_{1,\text{red}} = \theta_{2,\text{violet}}$; $(1)\lambda_{\text{red}}/a = (2)\lambda_{\text{violet}}/a$; $\lambda_{\text{violet}} = 390$ nm.

9.11 Follow section (9.3.1), except that (9.26) becomes $r_1 - r_2 = (2m' - 1)(\lambda/2)$ for destructive interference, where $m' = \pm 1, \pm 2, \ldots$, so that $(2m' - 1)$ is an odd integer. This leads to an expression equivalent to (9.28), $\theta_{m'} = (2m - 1)\lambda/2a$.

9.12 Follow section (9.3.1), except that (9.26) becomes $r_1 - r_2 + \Lambda = m\lambda$, where Λ = Optical path differences in beam. Following r_1, $\Lambda = nd$ (for θ_m "small").

$$(r_1 - r_2) = m\lambda - \Lambda; \quad a\theta_m = m\lambda - nd; \quad \theta_m = (m\lambda - nd)/a.$$

9.13 As in section (9.3.1), we have constructive interference when $OPD = m\lambda$. There is an added OPD due to the angle, θ, of the plane wave equal to $a \sin\theta$, so (9.26) becomes $r_1 - r_2 + a\sin\theta = m\lambda$. (9.24) $\theta_m \simeq y/s$ and (9.25) $r_1 - r_2 \simeq ay/s$ are unchanged, for small θ_m so $r_1 - r_2 = m\lambda - a\sin\theta = a(y/s) = a\theta_m$; $\theta_m = (m\lambda/a) - \sin\theta$.

9.14 (9.27) $y_m = (s/a)m\lambda$; $y_{1,\text{red}} = [(2.0 \text{ m})/(2.0 \times 10^{-4} \text{ m})](1)(4 \times 10^{-7} \text{ m})$
$= 4.0 \times 10^{-3}$ m.
$y_{1,\text{violet}} = [(2.0 \text{ m})/(2.0 \times 10^{-4} \text{ m})](2)(6 \times 10^{-7} \text{ m}) = 12.0 \times 10^{-3}$ m.
Distance $= 8.0 \times 10^{-3}$ m.

9.15 $r_2^2 = a^2 + r_1^2 - 2ar_1 \cos(90° - \theta)$. The contribution to $\cos\delta/2$ from the third term in the Maclaurin expansion will be negligible if

$$(k/2)(a^2 \cos^2\theta/2r_1) \ll \pi/2; \qquad \text{therefore} \qquad r_1 \ll a^2/\lambda.$$

9.16 $E = mv^2/2$; $v = 0.42 \times 10^6$ m/s; $\lambda = h/mv = 1.73 \times 10^{-9}$ m; $\Delta y = s\lambda/a = 3.46$ mm.

9.17 $\Delta\nu/\Delta\lambda = \nu/\lambda$; $\quad \delta\nu = \nu\Delta\lambda/\lambda = 1/\Delta t_c$;

$$c = \nu\lambda, \quad \text{so} \quad \nu = c/\lambda.$$

$$\Delta = (c/\lambda)\Delta\lambda/\lambda = c\Delta\lambda/\lambda^2;$$

$$\Delta t_c = \lambda^2/c\Delta\lambda; \quad \Delta\ell_c = c\Delta t_c = c(\lambda^2/\Delta\lambda)$$

$$= \lambda^2/\Delta\lambda = (500 \text{ nm})^2/(2.5 \times 10^{-3} \text{ nm})$$

$$= 1 \times 10^8 \text{ nm} = 0.1 \text{ m} \simeq \Lambda.$$

9.18 $\bar{E} = E_o e^{i\omega t} + E_o e^{\omega t+\delta} + E_o e^{i(\omega t+5\delta/2)}$. $I = \langle \bar{E}^2 \rangle_T = \langle \bar{E} \cdot \bar{E} \rangle_T$, so, as in section 9.1, $I = (3/2)E_o^2 + 2E_o^2\{\frac{1}{2}(\cos\delta + \cos(3\delta/2) + \cos(5\delta/2))\}$ (three terms of $\bar{E}_i \cdot \bar{E}_i$, 3 cross terms of $\bar{E}_i \cdot \bar{E}_j$). For each beam,

$$I_i = \langle \bar{E}_i^2 \rangle_T = \frac{1}{2}E_o^2,$$

at $\theta = 0$, so that for all three together $I(\theta = 0) = \frac{3}{2}E_j^2$. Note that $(r_2 - r_1) = a\sin\theta$ so that

$$\delta_2 = k(r_2 - r_1) = k(a\sin\theta); \quad (r_3 - r_1) = (5a/2)\sin\theta$$

so that $\delta_3 = k(r_3 - r_1) = k(\frac{5}{2}a\sin\theta)$ where $\delta = ka\sin\theta$. So,

$$I(\theta) = I(0)/3 + (2I(0)/9)(\cos\delta + \cos(3\delta/2) + \cos(5\delta/2))$$

when $\theta = 0$, the second term is zero.

9.19 A ray form S hits the biprism at an angle θ_i (w.r.t normal), is refracted at angle θ_t, and hits the second face at angle $(\theta_t + \alpha)$.
(4.4) $(1)\sin\theta_i = (n)\sin\theta_t$. $(n)\sin(\theta_t + \alpha) = (1)\sin(\theta/2 + \alpha)$, where angle θ is defined in Figure 9.13. As $\theta_i \to 0$, $\theta_t \to 0$; α, θ are both "small."
$n\sin\alpha = \sin(\theta/2 + \alpha)$, so $n\alpha \simeq (\theta/2) + \alpha$, $\theta = 2(n-1)\alpha$. From the figure $\tan(\theta/2) = (a/2)/d$, so

$$\theta/2 \simeq (a/2)/d, \quad \theta = a/d. \qquad a/d = 2(n-1)\alpha, \quad a = 2d(n-1)\alpha.$$

9.21 From Problem 9.19, $a = 2d(n-1)\alpha$; $s = 2d$, so $d = 1m$.

$$\Delta y = (s/a)\lambda = s\lambda/2d(n-1)\alpha; \quad \alpha = s\lambda/2d(n-1)\Delta y$$
$$= [(2m)(5.00\times 10^{-7}\text{ m})]/[2(1\text{ m})(1.5-1)(5\times 10^{-4}\text{ m})] = 0.002\text{ rad}.$$

9.22 $\Delta y = s\lambda_0/2d\alpha(n-n')$.

9.23 $\Delta y = (s/a)\lambda$, $a = 10^{-2}$ cm, $a/2 = 5\times 10^{-3}$ cm.

9.24 $\delta = k(r_1 - r_2) + \pi$ Lloyd's mirror,

$$\delta = k(a/2\sin\alpha - [\sin(90^\circ - 2\alpha)]a/2\sin\alpha) + \pi,$$
$$\delta = ka(1-\cos 2\alpha)/2\sin\alpha + \pi,$$

maximum occurs for $\delta = 2\pi$ when $\sin\alpha(\lambda/a) = (1-\cos 2\alpha) = 2\sin^2\alpha$. First maximum $\alpha = \sin^{-1}(\lambda/2a)$.

9.25 E_{1r} is reflected once. $E_{1r} = E_{oi}\, r_{\theta=0}$ (see 4.47) $= E_{oi}(n-1)/(n+1) = E_{oi}(1.52-1)/(1.52+1) = 0.206E_{oi}$.
E_{2r} is transmitted once, reflected once, then transmitted.
$E_{2r} = E_{oi}(t_{\theta=0})(r'_{\text{glass-air}})(t'_{\text{glass-air}}) = E_{oi}[2/(1+n)][(1-n)/(1+n)][2n/(n+1)] = 4n(1-n)/(n+1)^3 = E_{oi}[4(1.52)(1-1.52)]/(1+1.52)^3 = -0.198E_{oi}$, (see 4.48) ($-$ indicates π phase changed).
E_{3r} is transmitted, reflected 3 times (internally), and then transmitted.

$$E_{3r} = E_{oi}t(r')^3t' = E_{oi}[2/(1+n)][(1-n)/(1+n)]^3[(2n)/(n+1)]$$
$$= [4n(1-n)^3]/(n+1)^5 = E_{oi}[4(1.52)(1-1.52)^3]/(1.52+1)^5$$
$$= -0.008E_{oi}$$

for water in air.

$$E_{1r} = E_{oi}(1.333-1)/(1.333+1) = 0.143E_{oi}.$$
$$E_{2r} = E_{oi}[4(1.333)(1-1.333)]/(1+1.333)^3 = -0.140E_{oi}.$$
$$E_{3r} = E_{oi}[4(1.333)(1-1.333)^3]/(1.333+1)^5 = -0.003E_{oi}.$$

9.26 Here $1.00 < 1.34 < 2.00$, hence from Eq. (9.36) with $m = 0$,
$d = (0 + 1/2)(633 \text{ nm})/2(1.34) = 118$ nm.

9.27 (9.36) $d \cos\theta_t = (2m+1)(\lambda_f)/4$ for a maximum at (near) normal incidence, and taking $m = D$ (lowest value)

$$d = \lambda f/4 = \lambda_o/4n = (5.00 \times 10^{-7} \text{ m})/4(1.36) = 9.2 \times 10^{-6} \text{ m}.$$

9.28 (9.37) $d \cos\theta_t = 2m(\lambda_f/4)$ for minimum reflection $= 2m(\lambda_o/n)$
at $\theta \simeq 0$, $\lambda_o = nd/2m = [(1.34)(550.0 \text{ nm})]/2 \text{ m} = 368.5(1/m)$ nm,
$m = 1, 2, 3, \ldots$ or $\lambda_o = 368.5$ nm, 184.25 nm, 122.83 nm,. . . .

9.29 Eq. (9.37) $m = 2n_f d/\lambda_0 = 10{,}000$. A minimum, therefore central dark region.

9.30 The fringes are generally a series of fine jagged bands, which are fixed with respect to the glass.

9.31 $x^2 = d_1[(R_1 - d_1) + R_1] = 2R_1 d_1 - d_1^2$. Similarly $x^2 = 2R_2 d_2 - d_2^2$.
$d = d_1 - d_2 = (x^2/2)(1/R_1 - 1/R_2)$, $d = m\lambda_f/2$. As $R_2 \to \infty$, x_m approaches Eq. (9.43).

9.32 (9.42) $x_m = [(m + 1/2)\lambda_f R]^{1/2}$, air film, $n_f = 1$, so $\lambda_f = \lambda_o$.
$R = x_m^2/(m + 1/2)\lambda_o = (0.01 \text{ m})^2/(20.5)(5 \times 10^{-7} \text{ m}) = 9.76$ m.

9.33 $\Delta x = \lambda_f/2\alpha$, $\alpha = \lambda_0/2n_f\Delta x$, $\alpha = 5 \times 10^{-5}$ rad= 10.2 seconds.

9.34 (9.40) $\Delta x = \lambda_f/2\alpha$ for fringe separation where $\alpha = d/x$.
$\Delta x = \lambda_f/2(d/x) = x\lambda_f/2d$. Number of fringes = (length)/(separation) $= x/\Delta x$ so,

$$x/\Delta x = 2d/\lambda_f = [2(7.618 \times 10^{-5} \text{ m})]/(5.00 \times 10^{-7} \text{ m}).$$

9.35 A motion of $\lambda/2$ causes a single fringe pair to shift past, hence
$92\lambda/2 = 2.53 \times 10^{-5}$ m and $\lambda = 550$ nm.

9.36 $\Delta d = N(\lambda_o/2) = (1000)(5.00 \times 10^{-7} \text{ m})/2 = 2.50 \times 10^{-4}$ m.

9.37 $\Lambda = \Delta d = N(\lambda_o/2)$; $\Lambda = (n_{\text{air}}x - n_{\text{vacuum}}x)$;
$N = 2\Lambda/\lambda_o = [2(1.00029 - 1.00000)(0.10\text{ m})]/(6.00 \times 10^{-7}\text{ m}) = 97$.

9.38 Fringe pattern comes from the interference of two beams, one that passes through the lower medium (n_1), and is reflected off its mirror, one that passes through the top medium (n_2) and is reflected off its mirror. The two beams reflect off the front surface of the other medium.
It might be used to compare n_1 and n_2 (especially if one changes, such as due to pressure or temperature), or compare the flatness of one surface, to a known optically flat surface.

9.39 $E_t^2 = E_t E_t^* = E_0^2(tt')^2/(1 - r^2e^{-i\delta})(1 - r^2e^{i\delta})$,
$I_t = I_i(tt')^2/(1 - r^2e^{-i\delta} - r^2e^{i\delta} + r^4)$.

9.40 (a) $R = 0.80$, therefore $F = 4R/(1-R)^2 = 80$.
(b) $\gamma = 4\sin^{-1} 1/\sqrt{F} = 0.448$. (c) $\mathcal{F} = 2\pi/0.448$. (d) $C = 1 + F$.

9.41
$$2/[1 + F(\Delta\delta/4)^2] = 0.81[1 + 1/(1 + F(\Delta\delta/2)^2)],$$
$F^2(\Delta\delta)^4 - 15.5F(\Delta\delta)^2 - 30 = 0$.

9.42 $I = I_{\max}\cos^2\delta/2$, $I = I_{\max}/2$ when $\delta = \pi/2$, therefore $\gamma = \pi$. Separation between maxima is 2π. $\mathcal{F} = 2\pi/\gamma = 2$.

9.43 (4.47) $r_{\theta_i=0} = (n_t - n_i)/(n_t + n_i)$. Bare substrate: $r = (n_s - 1)/(n_s + 1)$. Substrate with film: $r' = t_{o-f}r_{f-s}t_{f-o}$. (4.48) $t_{\theta_i=0} = 2n_i/(n_i + n_t)$, so, $r' = [2/(1 + n_f)][(n_s - n_f)/(n_s + n_f)][2n_f/(n_f + 1)]$, where $n_f = n$. Note that for $n_s > n_f > 1$, both r and r' are positive. But, with thickness $\lambda_f/4$, a π phase shift occurs due to the OPD in the r' beam, so $r_{\text{net}} = r - r'$. Thus, the r' beam (partially) cancels the r beam.

9.44 At near normal incidence ($\theta_i \approx 0$) the relative phase shift between an internally and externally reflected beam is π rad. That means a total relative phase difference of $(2\pi/\lambda_f)[2(\lambda_f/4)] + \pi$ or 2π. The waves are in phase and interfere constructively.

9.45 $n_0 = 1, \quad n_s = n_g, \quad n_1 = \sqrt{n_g}.$

$\sqrt{1.54} = 1.24, \quad d = \lambda_f/4 = \lambda_0/4n_1 = 540/4(1.24)$ nm.

No relative phase shift between two waves.

9.46 The refracted wave will traverse the film twice, and there will be no relative phase shift on reflection. Hence $d = \lambda_0/4n_f = (550 \text{ nm})/4(1.38)$ $= 99.6$ nm.

9.47 (9.36) $d\cos\theta_t = (2m+1)(\lambda_f/4)$. Let $\theta_t = 0$, $m = 0$, (minimum thickness). $d = \lambda_o/4n = (5.50 \times 10^{-7} \text{ m})/4(1.55) = 8.87 \times 10^{-6}$ m.

9.48 Note that in the triangle including θ and r_1, the length of the side from P_1 to a plane, parallel to the surface, and containing point $z(x)$ is $r_1 \cos\theta$. So, from zero elevation, $h = r_1\cos\theta + z(x)$ or $z(x) = h - r_1\cos\theta$.
(9.108) can be demonstrated on the triangle (a, r_1, r_2), where a is the length of the boom:

$$r_2^2 = r_1^2 + a^2 - 2r_1 a\cos(\alpha + 90° - \theta) = \sin(\gamma) = -\cos(90° + \gamma)$$

and $\delta = k(r_2 - r_1) = (2\pi/\lambda)(r_2 - r_1)$.

Chapter 10 Solutions

10.1 $(R+\ell)^2 = R^2 + a^2$; therefore $R = (a^2 - \ell^2)/2\ell \approx a^2/2\ell$, $\ell R = a^2/2$, so for $\lambda \ll \ell$, $\lambda R \gg a^2/2$. Therefore $R = (1 \times 10^{-3})^2 10/2\lambda = 10$ m.

10.2 $E_0/2 = R\sin(\delta/2)$, $E = 2R\sin(N\delta/2)$ chord length;

$$E = [E_0 \sin(N\delta/2)]/\sin(\delta/2), \quad I = E^2.$$

10.3 A "constant" phase shift is added due to the angle of the incident wave reaching the ends of the slit at different phase, so that (10.11) becomes $r = R - y(\sin\theta - \sin\theta_i) + \ldots$. This constant carries through the integration, so that the definition of β in 10.18 (or 10.14) becomes $\beta = (kb/2)(\sin\theta - \sin\theta_i)$.

10.4 $d\sin\theta_m = m\lambda$, $\theta = N\delta/2 = \pi$, $7\sin\theta = (1)(0.21)$, $\delta = 2\pi/N = kd\sin\theta$, $\sin\theta = 0.03$ so $\theta = 1.7°$. For $\sin\theta = 0.0009$, $\theta = 3$ min.

10.5 Converging spherical wave in image space is diffracted by the exit pupil.

10.6 $\beta = \pm\pi$, $\sin\theta = \pm\lambda/b \approx \theta$, $L\theta \approx \pm L\lambda/b$, $L\theta \approx \pm f_2\lambda/b$.

10.7 Far field if $R > b^2/\lambda$, $b^2/\lambda = (1 \times 10^{-4}\text{ m})^2/(4.619 \times 10^{-7}\text{ m})^2 = 0.02$. *Yes*, far field. $\sin\theta_1 = \lambda/b$.

$$\theta_1 = \sin^{-1}(\lambda/b) = \sin^{-1}(4.619 \times 10^{-7}\text{ m}/1 \times 10^{-4}\text{ m}) = 0.26°.$$

Angular width $= 2\theta_1 = 0.52°$.

10.8 $b\sin\theta_m = m\lambda$, so,

$$b = m\lambda/\sin\theta_m = 10(1.1522 \times 10^{-6}\text{ m})/\sin(6.2°) = 1.07 \times 10^{-4}\text{ m}.$$

In water, $b \sin\theta_m = m\lambda$, where $\lambda = n_{\text{air}}\lambda_o/n_{\text{water}}$.

$$\sin\theta_m = mn_{\text{air}}\lambda_o/n_{\text{water}}/b;$$

$$\theta_m = \sin^{-1}[10(1.00029)(1.1522 \times 10^{-6}\text{ m})/(1.33)(1.07 \times 10^{-4}\text{ m})]$$

$$= 4.7^\circ.$$

10.9 $\lambda = (20\text{ cm}) \sin 36.87^\circ = 12\text{ cm}$.

10.10 $\alpha = (ka/2)\sin\theta$, $\beta = (kb/2)\sin\theta$. $a = mb$, $\alpha = m\beta$, $\alpha = m2\pi$,
N = number of fringes $= \alpha/\pi = m2\pi/\pi = 2m$.

10.11 Is $R > b^2/\lambda$?, b = slit width.

$$b^2\lambda = (1 \times 10^{-4}\text{ m})^2/(5 \times 10^{-7}\text{ m}) = .02\text{ m} \ll 2.5\text{ m}.$$

Fraunhofer.
(Half) angular width of central maximum from

$$\beta = \pi = (kb/2)\sin\theta_1.$$

$$\sin\theta_1 = 2\pi/kb = \lambda/b = (5 \times 10^{-7}\text{ m})/(1 \times 10^{-4}\text{ m}); \quad \theta_1 = 0.29^\circ.$$

To what order Young's fringe does θ_1 correspond?

$$\alpha = m'\pi = (ka/2)\sin\theta_1. \quad m' = (ka/2\pi)\sin\theta_1 = (a/\lambda)\sin\theta_1$$

$$= (2 \times 10^{-4}\text{ m})/(5 \times 10^{-7}\text{ m})\sin(0.29^\circ) = 2.$$

So there are 4 "Young's Fringes" in the central maximum.

10.12 $\alpha = 3\pi/2N = \pi/2$, $I(\theta) = I(0)[(\sin\beta)/\beta]^2/N^2$ and $I/I(0) \approx 1/9$.

10.13 (10.17) $I(\theta) = I(0)(\sin\beta/\beta)^2$, where $\beta \equiv (kD/2)\sin\theta$. "Miniscule Area" corresponds to the limit $D \to 0$. As $D \to 0$, $\beta \to 0$, so

$$\lim_{D\to 0} I(\theta) = \lim_{\beta\to 0}(I(0)(\sin\beta/\beta)^2);$$

As $\beta \to 0$, $\text{sinc}(\beta) \to 1$, so $\lim_{D\to 0} I(\theta) = I(0)$, i.e., same in all directions.

10.14 (from 10.41) $\tilde{E} \propto \int\int e^{ik(Yy+Zz)/R} dS$. and $I(Y,Z) \propto \langle \tilde{E}^2 \rangle$. If $\tilde{E}$ is an even function of (Y,Z), $\tilde{E}(-Y,-Z) = \tilde{E}(Y,Z)$. If $\tilde{E}$ is an odd function of (Y,Z), $\tilde{E}(-Y,-Z) = -\tilde{E}(Y,Z)$, but $I(-Y,-Z) = I(Y,Z)$.

10.15 If the aperture is symmetrical about a line, the pattern will be symmetrical about a line parallel to it. Moreover, the pattern will be symmetrical about yet another line perpendicular to the aperture's symmetry axis. This follows from the fact that Fraunhofer patterns have a center of symmetry.

10.16 For the solution to this problem, please refer to the textbook.

10.17 Three parallel short slits.

10.18 Two parallel short slits.

10.19 An equilateral triangular hole.

10.20 A cross-shaped hole.

10.21 The E-field of a rectangular hole.

10.22 From section 10.2.5, first "ring" (maximum) occurs for $u = kaq/R = 5.14$. Interpolating from Table 10.1, $J_1(5.14) \simeq -0.33954$

From (10.55) $I/I(0) = \left[\dfrac{2J_1(u)}{u}\right]^2 = \left[\dfrac{2(-0.33954)}{5.14}\right]^2 = 0.0175$

10.23 From Eq. (10.58), $q_1 \approx 1.22(f/D)\lambda \approx \lambda$.

10.24 For the solution to this problem, please refer to the textbook.

10.25 (10.57) $q_1 = 1.22(R\lambda/2a)$

$$= 1.22[(3.76 \times 10^8 \text{ m})(6.328 \times 10^{-7} \text{ m})]/2[1 \times 10^{-3} \text{ m}]$$

$$= 1.45 \times 10^5 \text{ m}.$$

10.26 (10.59) $(\Delta\varphi)_{\min} = 1.22\lambda/D = [(1.22)(5.50 \times 10^{-7}\text{ m})]/(7.5 \times 10^{-4}\text{ m})$

$$= 8.9 \times 10^{-5}\text{ rad,}$$

which is about half the angular resolution of the pupil.

10.27 1 part in 1000. 3 yd $\approx$ 100 inches.

10.28 (10.59) $\Delta\varphi_{\min} = 1.22\lambda/D = [1.22(5.50 \times 10^{-7}\text{ m})]/5.08\text{ m}$

$$= 1.32 \times 10^{-7}\text{ rad;}$$

or $\Delta\varphi = 1.32 \times 10^{-7}\text{ rad}(360^\circ/2\pi\text{ rad}) = 7.55 \times 10^{-6\,\circ}$; or
$\Delta\varphi = (7.55 \times 10^{-6\,\circ})(3600\text{ sec/degree}) = 2.72 \times 10^{-2}$ arc sec.
To be resolved, $s = r\Delta\varphi$ ($\Delta\varphi$ in radians).

$$s = (3.844 \times 10^{8}\text{ m})(1.32 \times 10^{-7}) = 50.7\text{ m.}$$

To be resolved by eyes,

$$s = r\Delta\varphi = r(1.22\lambda/D)$$
$$= (3.844 \times 10^{8}\text{ m})[1.22(5.50 \times 10^{-7}\text{ m})/(4 \times 10^{-3}\text{ m})] = 6.44 \times 10^{4}\text{ m.}$$

10.29 (10.32) $a\sin\theta_m = m\lambda; \sin\theta_1 \simeq y/R$, so

$$a(Y/R) = \lambda;\; Y = R\lambda/a = (2.0\text{ m})(6.943{\times}10^{-7}\text{ m})/(3.0{\times}10^{-6}\text{ m}) = 0.46\text{ m.}$$

10.30 (10.32) $a\sin\theta_m = m\lambda$; $\sin\theta_3 = 3\lambda/a = 3(5.00 \times 10^{-7}\text{ m})/(6.0 \times 10^{-6}\text{ m})$;
$\theta_3 = 14^\circ$.

10.31 (10.32) $a\sin\theta_m = m\lambda$.

$$a = 2\lambda/\sin\theta_2 = 2(5.50 \times 10^{-7}\text{ m})/\sin(25^\circ) = 2.6 \times 10^{-6}\text{ m.}$$

10.32 From Eq. (10.32), where $a = 1/(1000$ lines per cm$) = 0.001$ cm per line (center to center), $\sin\theta_m = 1(650 \times 10^{-9} m)/(0.001 \times 10^{-2}\text{ m}) = 6.5 \times 10^{-2}$ and $\theta_1 = 3.73^\circ$.

10.33 (10.32) $a\sin\theta_m = m\lambda$. $\sin\theta_m \simeq Y_m/R; Y_m = (m\lambda/a)R = 10{,}000$ lines/cm $= 10^{6}$ lines/m so $a = 10^{-6}$ m.

$$Y_1(589.5923\text{ nm}) = [1(5.895923 \times 10^{-7}\text{ m})/10^{-6}\text{ m}](1.00\text{ m})$$

$$= 0.5895923 \text{ m}.$$

$$Y_1'(588.9953 \text{ nm}) = [1(5.889553 \times 10^{-7} \text{ m})/10^{-6} \text{ m}](1.00 \text{ m})$$

$$= 0.5889953 \text{ m}.$$

$$\text{Separation} = Y_1 - Y_1' = 5.97 \times 10^{-4} \text{ m}.$$

10.34 (10.32) $a \sin\theta_m = m\lambda$, so $\sin\theta_m = m\lambda/a$; is $\theta_2(\text{red}) > \theta_3(\text{violet})$?

$$5000 \text{ lines/cm} = 5 \times 10^5 \text{ lines/m}; \quad a = 2 \times 10^{-6} \text{ cm}.$$

$$\sin\theta_2(\text{red}) = 2(7.8 \times 10^{-7} \text{ m})/(2 \times 10^{-6} \text{ m}); \quad \theta_2(\text{red}) = 51.3°.$$

$$\sin\theta_3(\text{violet}) = 3(3.90 \times 10^{-7} \text{ m})/(2 \times 10^{-6} \text{ m}) = 35.8°.$$

Spectra do not overlap. Note: Can see "by inspection" by comparing factor of 2 in wavelength to factor of 3/2 in m's.

10.35 The largest value of m in Eq. (10.32) occurs when the sine function is equal to one, making the left side of the equation as large as possible, then $m = a/\lambda = (1/10 \times 10^5)/(3.0 \times 10^8 \text{ m/s} \div 4.0 \times 10^{14} \text{ Hz}) = 1.3$, and only the first-order spectrum is visible.

10.36 (10.32) $a \sin\theta_m = m\lambda$, where $\lambda = \lambda_o/n$.

$$\sin\theta_m = m\lambda/a; \quad \sin\theta_1(\text{vacuum})/\sin\theta_1(\text{Mongo})$$

$$= [(1)\lambda_o/a]/[(1)\lambda_o/na]; \quad n = \sin(20.0°)/\sin(18.0°) = 1.11.$$

10.37 $\sin\theta_i = n \sin\theta_n$ Optical path length difference is $m\lambda, a\sin\theta_m - na\sin\theta_n = m\lambda$. $A(\sin\theta_m - \sin\theta_n) = m\lambda$.

10.38 $\mathcal{R} = mN = 10^6$, $N = 78 \times 10^3$. Therefore $m = 10^6/78 \times 10^3$,
$\Delta\lambda_{fsr} = \lambda/m = 500 \text{ nm}/(10^6/78 \times 10^3) = 39 \text{ nm}$.
$\mathcal{R} = \mathcal{F}m = \mathcal{F}2b_fd/\lambda = 10^6\Delta\lambda_{fsr} = \lambda^2/wn_fd = 0.0125 \text{ nm}$.

10.39 $\mathcal{R} = \lambda/\Delta\lambda = 5892.9/5.9 = 999$, $N = \mathcal{R}/m = 333$.

10.40 Except on the central axis, there will be no regular pattern. A circular Fraunhofer pattern (as in Figure 10.33d) could occur, with the intensity dependent on the degree of coherence. If the sources are completely incoherent, the intensity goes to zero.

10.41 $y = L\lambda/d$, $d = 12 \times 10^{-6}/12 \times 10^{-2} = 10^{-4}$ m.

10.42 (From 10.75) $E_\ell = [-K_\ell \mathcal{E}_A \rho\lambda/(\rho + r)][\sin(wt - k\rho - kr)]_{r=r_{\ell-1}}^{r=r_\ell}$;

$$\begin{aligned}&\sin(wt - k\rho - r_\ell) - \sin(wt - k\rho - kr_{\ell-1})\\ &= \sin(wt - k\rho - k(r_o + \ell\lambda/2))\\ &\quad - \sin(wt - k\rho - k(r_o + (\ell - 1)(\lambda/2)))\\ &= \sin(wt - k(\rho + r_o) - 2\pi\ell\lambda/2\lambda) - \sin(wt - k(\rho + r_o)\\ &\quad - (2\pi(\ell - 1)\lambda/2\lambda)).\end{aligned}$$

Recall $\sin(\alpha - \beta) = \sin\alpha\cos\beta^{+}\cos\alpha\sin\beta$ and $\sin\pi\ell = \sin(\ell - 1)\pi = 0$. $\cos(\ell - 1)\pi = -\cos\ell\pi$; $\cos\ell\pi = (-1)^\ell\pi$, so

$$E_\ell = (-1)^{\ell+1}(2K_\ell\mathcal{E}_A\rho\lambda)/(\rho + r_o)\sin(wt - k(\rho + r_o)).$$

10.43 $A = 2\pi\rho^2 \int_0^\varphi \sin\varphi d\varphi = 2\pi\rho^2(1 - \cos\varphi)$,
$\cos\varphi = [\rho^2 + (\rho + r_0)^2 - r_l^2]/2\rho(\rho + r_0)$, $r_l = r_0 + l\lambda/2$.
Area of first l zones $A = 2\pi\rho^2 - \pi\rho(2\rho^2 + 2\rho r_0 - l\lambda r_0 - l^2\lambda^2/r)/(\rho + r_0)$,
$A_l = A - A_{l-1} = \lambda\pi\rho(r_0 + (32l - 1)\lambda/4)(\rho + r_0)$.

10.44 (10.78) becomes

$$E = |E_1|/2 + (|E_1|/2 - |E_2| + |E_3|/2) + \cdots + (|E_{m-1}|/2 - |E_m|),$$

so that (10.80) becomes $E < |E_2|/2 - |E_m|/2$ and (10.82) becomes $E > |E_1|/2 - |E_m|/2$ so that (10.84) $E \approx E < |E_1|/2 - |E_m|/2$.

10.45 For the solution to this problem, please refer to the textbook.

10.46 $I = (I_0/2)([1/2 - \mathcal{C}(v_1)]^2 + [1/2 - \mathcal{S}(v_1)]^2)$,
$I = (I_0/2)(1/\pi v_1)^2[\sin^2(\pi v_1^2/2) + \cos^2(\pi v_1^2/2)] = I_0/2(\pi v_1)^2$.

10.47 Fringes in both the clear and shadow region [see M. P. Givens and W. L. Goffe, *Am. J. Phys.*, **34**, 248 (1966)].

10.48 $u = y[2/\lambda r_0]^{1/2}$; $\Delta u = \Delta y \times 10^3 = 2.5$.

10.49 For the solution to this problem, please refer to the textbook.

10.50 We should see symmetry through the x-y plane in both patterns. The keyhole should bear some resemblance to the combined patterns of a circle and a rectanglar aperture. The image of the triangle should have nearly 3-fold symmetry.

10.51 As the slit widens, the pattern becomes more like that of a rectangular aperture (see Figure 10.49).

10.52 (10.91) $R_m^2 = mr_o\lambda$ so $R_m = (mr_o\lambda)^{1/2}$;

$$R_1 = (1(1.00\text{ m})5.6819 \times 10^{-7}\text{ m})^{1/2} = 7.54 \times 10^{-4}\text{ m}.$$

10.53 The full first zone has a radius $q_1 = 1.22R\lambda/2a$. Since area $= \pi q^2$, half the first zone corresponds to $q = q_1/\sqrt{2} = 1.22R\lambda/2\sqrt{2}a$; $I_o = \mathcal{E}_A^2 A^2$, for a plane wave, so (10.55) becomes

$$I = \frac{I_o}{2R^2}\left[\frac{J_1(kaq/R)}{kaq/R}\right]^2 = \frac{I_o}{2R^2}\left[\frac{J_1\left(ka(1.22R\lambda/2\sqrt{2}a)/R\right)}{ka(1.22R\lambda/2\sqrt{2}a)/R}\right]^2$$

$$= \frac{I_o}{2R^2}\left[\frac{J_1(1.22\pi/\sqrt{2})}{1.22\pi/\sqrt{2}}\right]^2 \simeq \frac{I_o}{2R^2}(0.026) = \frac{I_o}{R^2}(0.013)\text{ (using Table 10.1)}$$

10.54 (From 10.42 and 10.43), $I(0) \propto \frac{1}{2}(A\mathcal{E}_A/R)^2$, recall (3.46) $I = \epsilon_o c\langle E^2\rangle_T$ so, $I(0) = \frac{1}{2}\epsilon_o c(A\mathcal{E}_A/R)^2$; $I(\text{incident}) = \frac{1}{2}\epsilon_o c(A\mathcal{E}_A)^2 = (\text{flux})(\text{area}) =$ $10\text{ W/m}^2(5.0 \times 10^{-3}\text{ m})^2 = 2.5 \times 10^{-4}\text{ W}$; $I(0) = I(\text{incident})A/R^2 =$ $(2.5 \times 10^{-4}\text{ W})(5.0 \times 10^{-3}\text{ m})^2/(2.50\text{ m})^2 = 1.0 \times 10^{-9}\text{ W}$.

Chapter 11 Solutions

11.1 $E_0 \sin k_p x = E_0(e^{ik_px} - e^{-ik_px})/2i;$

$$F(k) = \frac{E_0}{2i}\left[\int_{-L}^{L} e^{i(k+k_p)x}\,dx - \int_{-L}^{L} e^{i(k-k_p)x}dx\right]$$
$$= -\frac{iE_0 \sin(k+k_p)L}{(k+k_p)} + \frac{iE_0 \sin(k-k_p)L}{(k-k_p)}$$

$F(k) = iE_0L[\mathrm{sinc}(k-k_p)L - \mathrm{sinc}(k+k_p)L].$

11.2 (11.5) $F(K) = \int_{-\infty}^{\infty} f(x)e^{ikxdx} = \int_{-L}^{L} \sin^2 k_p e^{ik_px}\,dx$

$$= \int_{-L}^{L} \sin^2 k_p x \cos k_p x\,dx + i\int_{-L}^{L} \sin^3 k_p x\,dx$$
$$= \frac{1}{3k_p}\sin^3 k_p x\Big|_{-L}^{L} + 0 = (2/3k_p)(\sin^3 k_p L).$$

11.3 $\cos^2\omega_p t = 1/2 + (1/2)\cos 2\omega_p t = 1/2 + (1/4)(e^{2i\omega_p t} + e^{-2i\omega_p t}).$

$$F(\omega) = \frac{1}{2}\int_{-T}^{T} e^{i\omega t}\,dt + \frac{1}{4}\int_{-T}^{T} e^{i(\omega+2\omega_p)t}\,dt + \frac{1}{4}\int_{-T}^{T} e^{i(\omega-2\omega_p)t}\,dt$$
$$= \frac{1}{\omega}\sin\omega T + \frac{1}{2(\omega+2\omega_p)}\sin(\omega+2\omega_p)T + \frac{1}{2(\omega-2\omega_p)}\sin(\omega-2\omega_p)T$$

$F(\omega) = T\,\mathrm{sinc}\,\omega T + (T/2)\,\mathrm{sinc}(\omega+2\omega_p)T + (T/2)\,\mathrm{sinc}(\omega-2\omega_p)T.$

11.4 Show that $\mathcal{F}^{-1}\{F(K)\} = f(x)$, where

$$f(x) = 1, \quad F(K) = 2\pi\delta(K).$$

$$(11.4) \qquad f(x) = (1/2\pi)\int_{-\infty}^{\infty} F(K)e^{-iKx}\,dK$$
$$= (1/2\pi)\int_{-\infty}^{\infty} 2\pi\delta(K)e^{-iKx}\,dK = e^0 = 1.$$

11.5 $f(x) = A\cos K_o x = (A/2)(e^{iK_o x} + e^{-ik_o x})$;

$$F(K) = (A/2)\int_{-\infty}^{\infty}(e^{iK_o x} + e^{-iK_o x})e^{iKx}\,dx$$

$$= (A/2)[2\pi/(K+K_o) + 2\pi/(K-K_o)] = \pi A[2k/(K^2-K_o^2)].$$

11.6 $\mathcal{F}[af(x) + bh(x)] = aF(k) + bH(k)$.

11.8 $F(k) = L\,\mathrm{sinc}^2\,kL/2$ at $k=0$, $F(0) = L$, and $F(\pm 2\pi/L) = 0$.

11.9 $F(K) = \int_{-\infty}^{\infty} f(x)e^{ikx}\,dx$, (11.5).Let $x \to x/a$;

$$F(K') = \int_{-\infty}^{\infty} f(x/a)e^{ik'(x/a)}\,d(x/a).$$

So, $K' \to Ka$, and $\mathcal{F}\{f(x/a)\} = F(Ka)$. If, $a=-1$, $\mathcal{F}\{f(-x)\} = F(-K)$.

11.10 $F(K) = \mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x)e^{ikx}\,dx$, (11.5) (a function of K).

$$\mathcal{F}\{\mathcal{F}\{f(x)\}\} = \mathcal{F}\{F(K)\} = \int_{-\infty}^{\infty} F(K)e^{ikx}\,dK.$$

(11.4) $f(x) = (1/2\pi)\int_{-\infty}^{\infty} F(K)e^{-iKx}\,dK$,

so, $2\pi f(-x) = \int_{-\infty}^{\infty} F(K)e^{iKx}\,dK = \mathcal{F}\{\mathcal{F}\{f(x)\}\} \neq f(x)$.

11.11 $\mathcal{F}\{\mathrm{rect}\,|(x-x_o)/a|\} = \int_{-\infty}^{\infty}\mathrm{rect}\,|(x-x_o)/a|\}e^{iKx}\,dx = \int_{-1/2}^{1/2} e^{iKx}\,dx =$
$1/(iK)e^{iKx}|_{-1/2}^{1/2} = 1/iK(e^{iK/2} - e^{-iK/2}) = (2/K)\sin(K/2) = \mathrm{sinc}(K/2)$.

11.12 $\mathcal{F}\{\mathrm{rect}\,|x|\} = \mathrm{sinc}(K/2)$, from Problem 11.10, $\mathcal{F}\{\mathcal{F}\{f(x)\}\} = 2\pi f(-x)$.
So, $\mathcal{F}\{(1/2\pi)\mathcal{F}\{f(-x)\} = f(x)$, let

$$f(x) = \mathrm{rect}\,|x|, \quad \mathcal{F}\{\mathrm{rect}\,|x|\} = \mathrm{sinc}(K/2);$$

$$\mathcal{F}\{(1/2\pi)\mathcal{F}\{\mathrm{rect}\,|x|\}\} = \mathcal{F}\{(1/2\pi)\,\mathrm{sinc}(K/2)\} = f(x) = \mathrm{rect}\,|x|,$$

since $\mathrm{sinc}(-x) = \mathrm{sinc}(x)$.

11.13
$$\mathcal{F}^{-1}\{\mathcal{F}\{f(x)\}\} = (1/2\pi)\int_{-\infty}^{\infty} e^{-iKx}\,dK \int_{-\infty}^{\infty} f(x')e^{iKx'}\,dx'$$
$$= \int_{-\infty}^{\infty} dx' \left(\int_{-\infty}^{\infty} e^{iK(x'-x)}\,dK\right) f(x')$$
$$= \int_{-\infty}^{\infty} \delta(x-x')f(x')\,dx' = f(x),$$
since the integral is zero except at $x = x'$.

11.14 $\mathcal{F}\{f(x-x_o)\} = \int_{-\infty}^{\infty} f(x-x_o)e^{iKx}\,dx$. Change variables, $x' \equiv x - x_o$, $dx' = dx$. $\mathcal{F}\{f(x')\} = \int_{-\infty}^{\infty} f(x')e^{iK(x'+x_o)}\,dx' = e^{iKx_o}\int_{-\infty}^{\infty} f(x')e^{iKx'}\,dx'$. so that $\mathcal{F}\{f(x-x_o)\}$ differs from $\mathcal{F}\{f(x)\}$ by only the phase factor e^{iKx_o}.

11.15 $\int_{-\infty}^{\infty} f(x)h(X-x)\,dx = -\int_{\infty}^{-\infty} f(X-x')h(x')\,dx' = \int_{-\infty}^{\infty} h(x')f(X-x')\,dx'$ where $x' = X - x$, $dx = -dx'$. $f * h = h * f$ or

$$\mathcal{F}(f*h) = \mathcal{F}(f)\cdot\mathcal{F}(h) = \mathcal{F}(h)\cdot\mathcal{F}(f) = \mathcal{F}(h*f).$$

11.17 A point on the edge of $f(x,y)$, for example, at $(x = d, y = 0)$, is spread out into a square 2ℓ on a side centered on $X = d$. Thus it extends no farther than $X = d + \ell$, and so the convolution must be zero at $X = d + \ell$ and beyond.

11.19 $f(x-x_0) * h(x) = \int_{-\infty}^{\infty} f(x-x_0)h(X-x)\,dx$, and setting $x - x_0 = \alpha$, this becomes $\int_{-\infty}^{\infty} f(\alpha)h(X-\alpha-x_0)d\alpha = g(X-x_0)$.

11.20
$$g(X) = \int_{-\infty}^{\infty} f(x)h(X-x)\,dx, \quad (11.52)$$
$$= \int_{-\infty}^{\infty} \delta(x)h(X-x)\,dx = h(X-0)\int_{-\infty}^{\infty}\delta(x)\,dx, \text{ (see Section 11.2.3)},$$
$$= h(X), \quad \text{since } \int_{-\infty}^{\infty}\delta(x)\,dx = 1.$$

11.21 For the solution to this problem, please refer to the textbook.

11.22
$$\begin{aligned}\mathcal{F}\{f(x)\cos K_o x\} &= \mathcal{F}\{f(x)(1/2)(e^{iK_o x}+e^{-iK_o X})\}\\ &= (1/2)\left[\int_{-\infty}^{\infty} f(x)e^{i(K+K_o)x}\,dx + \int_{-\infty}^{\infty} f(x)e^{i(K-K_o)x}\,dx\right]\\ &= (1/2)[F(K+K_o)+F(K-K_o)].\\ \mathcal{F}\{f(x)\sin K_o x\} &= \mathcal{F}\{f(x)(1/2i)(e^{iK_o x}-e^{-iK_o x})\}\\ &= (1/2i)\left[\int_{-\infty}^{\infty} f(x)e^{i(K+K_o)x}\,dx - \int_{-\infty}^{\infty} f(x)e^{i(K-K_o)x}\,dx\right]\\ &= (1/2i)[F(K+K_o)-F(K-K_o)].\end{aligned}$$

11.24 We see that $f(x)$ is the convolution of a rect-function with two δ-functions, and from the convolution theorem,

$$\begin{aligned}F(k) &= \mathcal{F}\{(\text{rect}(x) * [\delta(x-a)+\delta(x+a)]\}\\ &= \mathcal{F}[\text{rect}(x)]\cdot\mathcal{F}\{[\delta(x-a)+\delta(x+a)]\}\\ &= a\,\text{sinc}(ka/2)\cdot(e^{ikx}+e^{-ika}) = a\,\text{sinc}(ka/2)\cdot 2\cos ka.\end{aligned}$$

11.25 $f(x)*h(x) = [\delta(x+3)+\delta(x-2)+\delta(x-5)]*h(x) = h(x+3)+h(x-2)+h(x-5)$.

11.27
$$\mathcal{F}\{\text{rect}\,|x/(d/2)|\} = \text{sinc}(K/d) = G(K);$$
$$\mathcal{F}\left\{\sum_{n=-\infty}^{\infty}\delta(x-nd)\right\} = \sum_{n=-\infty}^{\infty} e^{iKnd} = H(K); \quad F(K) = G(K)H(K);$$
$$F(K) = \mathcal{F}\{f(x)\} \text{ is zero at } Knd = n\pi \text{ or } Kd = \pi.$$

11.28 For the solution to this problem, please refer to the textbook.

11.29 $\mathcal{A}(y,z) = \mathcal{A}(-y,-z)$.

$$E(Y,Z,t) \propto \int\!\!\int \mathcal{A}(y,z)e^{i(k_Y y+k_Z z)}\,dy\,dz.$$

Change Y to $-Y$, Z to $-Z$, y to $-y$, z to $-z$, then k_Y goes to $-k_Y$ and k_Z to $-k_Z$.

$$E(Y,Z,t) \propto \int\int \mathcal{A}(-y,-z)e^{i(k_Y y+k_Z z)}\,dy\,dz.$$

Therefore $E(-Y,-Z) = E(Y,Z)$.

11.30 From Eq. (11.63),

$$E(Y,Z) = \int\int \mathcal{A}(y,z)e^{ik(Yy+Zz)/R}dydz.$$

$$E'(Y,Z) = \int\int \mathcal{A}(\alpha y,\beta z)e^{ik(Yy+Zz)/R}dydz;$$

now let $y' = \alpha y$, $z' = \beta z$:

$$E'(Y,Z) = \frac{1}{\alpha\beta}\int\int \mathcal{A}(y',z')e^{ik[(Y/\alpha)y'+(Z/\beta)z']/R}\,dy'\,dz'$$

or $E'(Y,Z) = (1/\alpha\beta)E(Y/\alpha, Z/\beta)$.

11.31 $C_{ff} = \lim_{T\to\infty}\dfrac{1}{2T}\displaystyle\int_{-T}^{T} A\sin(\omega t+\epsilon)A\sin(\omega t-\omega\tau+\epsilon)\,dt$

$$= \lim_{T\to\infty}\frac{A^2}{2T}\int_{-T}^{T}\left[\frac{1}{2}\cos(\omega\tau) - \frac{1}{2}\cos(2\omega t-\omega\tau+2\epsilon)\right]dt,$$

since $\cos\alpha - \cos\beta = -2\sin(1/2)(\alpha+\beta)\sin(1/2)(\alpha-\beta)$. Thus $C_{ff} = (A^2/2)\cos(\omega\tau)$.

11.32 $E(k_Z) = \displaystyle\int_{-b/2}^{b/2} \mathcal{A}_0\cos(\pi z/b)e^{ik_Z z}\,dz$

$$= \mathcal{A}_0\int\cos(\pi z/b)\cos(k_Z z)\,dz + i\mathcal{A}_0\int\cos(\pi z/b)\sin(k_Z z)\,dz$$

$$E(k_Z) = \mathcal{A}\cos\frac{bk_Z}{2}\left[\frac{1}{\pi/b-k_Z}+\frac{1}{\pi/b+k_Z}\right].$$

11.33 (From 11.52). $h(X) = f(x) \circledast g(x) = \int_{-\infty}^{\infty} f(x)g(X-x)\,dx$, so, $f(x) * g(-x) = h'(x) = \int_{-\infty}^{\infty} f(x)g(X+x)\,dx$, which is the form of (11.86), so $f(x) * g(-x) = f(x) \odot g(x)$.

11.35 $f(t) = g(t)\odot h(t) = A\cos(\omega_o t)\odot e^{-i\omega_o t}$.

$F(\omega) = G(\omega)H(\omega)$; $G(\omega) = (A/2)(2\pi/(\omega+\omega_o) + 2\pi/(\omega-\omega_o))$;

$H(\omega) = 2\pi/\omega - \omega_o$,

so, $F(\omega) = 2\pi^2 A/(\omega - \omega_o)(2\omega/\omega^2 - \omega_o^2)$.

Chapter 12 Solutions

12.1 At low pressures, the intensity emitted from the lamp is low, the bandwidth is narrow, and the coherence length is large. The fringes will initially display a high contrast, although they'll be fairly faint. As the pressure builds, the coherence length will decrease, the contrast will drop off, and the fringes might even vanish entirely.

12.2 Over a long time interval, $E_1 \times E_2$ averages to zero. So, $\langle (E_1 + E_2)^2 \rangle_T \approx \langle E_1^2 \rangle_T + \langle E_2^2 \rangle_T$.

12.3 The net irradiance becomes more uniform as more waves are added. There will be a less distinct pattern, which corresponds to a smaller coherence length. The irradiance will become constant as the bandwidth goes to infinity.

12.4 Each sine function in the signal produces a cosinusoidal autocorrelation function with its own wavelength and amplitude. All of these are in phase at the zero delay point corresponding to $\tau = 0$. Beyond that origin the cosines soon fall out of phase, producing a jumble where destructive interference is more likely. (The same sort of thing happens when, say, a square pulse is synthesized out of sinusoids—everywhere beyond the pulse all the contributions cancel.) As the number of components increases and the signal becomes more complex—resembling random noise—the autocorrelation narrows, ultimately becoming a δ-spike at $\tau = 0$.

12.5 (12.1) $\mathcal{V} = (I_{\max} - I_{\min})/(I_{\max} + I_{\min}) = 2|\operatorname{sinc}(a\pi w/s\lambda)|/2$ (from 12.8, 12.9), $= |\operatorname{sinc}(5 \times 10^{-4}\pi/1 \times 10^{-3})| = \operatorname{sinc}(\pi/2) = 0.64$.

12.6 The irradiance at Σ_0 arising from a point source is $4I_0\cos^2(\delta/2) = 2I_0(1+\cos\delta)$. For a differential source element of width dy at point S', y from the axis, the OPD to P at Y via the two slits is $\Lambda = (\overline{S'S_1}+\overline{S_1P})-(\overline{S'S_2}+\overline{S_2P} = (\overline{S'S_1}-\overline{S'S_2})+(\overline{S_1P}-\overline{S_2P} = ay/l+aY/s$ from Section 9.3. The contribution to the irradiance from dy is then $dI \propto (1+\cos k\Lambda)\,dy$, $I \propto \int_{-b/2}^{b/2}(1+\cos k\Lambda)\,dy$,

$$I \propto b + \frac{d}{ka}\left[\sin\left(\frac{aY}{s}+\frac{ab}{2l}\right) - \sin\left(\frac{aY}{s}-\frac{ab}{2l}\right)\right] I \propto b$$

$$+ (d/ka)[\sin(kaY/s)\cos(kab/2l) + \cos(kaY/s)\sin(kab/2l)$$

$$- \sin(kaY/s)\cos(kab/2l) + \cos(kaY/s)\sin(kab/2l)],$$

$$I \propto b + (2l/ka)\sin(kab/2l)\cos(kaY/s).$$

12.7 $\mathcal{V} = (I_{\max} - I_{\min})/(I_{\max}+I_{\min})$, $\quad I_{\max} = I_1 + I_2 + 2\sqrt{I_1I_2}|\tilde{\gamma}_{12}|$, $I_{\min} = I_1 + I_2 - 2\sqrt{I_1I_2}|\tilde{\gamma}_{12}|$, $\quad \mathcal{V} = 4\sqrt{I_1I_2}|\tilde{\gamma}_{12}|/2(I_1+I_2)$.

12.8 When $S''S_1O' - S'S_1O' = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots$, the irradiance due to S' is given by $I' = 4I_0\cos^2(\delta'/2) = 2I_0(1+\cos\delta')$, while the irradiance due to S'' is $I'' = 4I_0\cos^2(\delta''/2) = 4I_0\cos^2(\delta'+\pi)/2 = 2I_0(1-\cos\delta')$. Hence $I' + I'' = 4I_0$.

12.9 Fringes disappear when $w = s\lambda/a$ so, $a = \lambda(s/w)$, from Figure 12.3,

$$\ell/b = s/w, \quad a = \lambda(\ell/b) = (5.893\times10^{-7}\text{ m})(1\text{ m})/(1\times10^{-4}\text{ m})$$

$$= 5.893\times10^{-3}\text{ m}.$$

12.10 $\theta = \frac{1}{2}^\circ = 0.0087$ rad; $h = 0.32\bar{\lambda}_0/\theta$ using $\bar{\lambda}_0 = 550$ nm, $h = 0.32(550\text{ nm})/0.0087 = 2\times10^{-2}$ mm.

12.11 $I_1(t) = \Delta I_1(t) + \langle I_1\rangle$; hence

$$\langle I_1(t+\tau)I_2(t)\rangle = \langle[\langle I_1\rangle + \Delta I_1(t+\tau)][\langle I_2\rangle + \Delta I_2(t)]\rangle,$$

since $\langle I_1\rangle$ is independent of time.

$$\langle I_1(t+\tau)I_2(t)\rangle = \langle I_1\rangle\langle I_2\rangle + \langle \Delta I_1(t+\tau)\Delta I_2(t)\rangle,$$

if we recall that $\langle \Delta I_1(t)\rangle = 0$. Eq. (12.34) follows by comparison with Eq. (12.32).

12.13 From Eq. (12.22), $\mathcal{V} = 2\sqrt{(10I)I}/(10I + I) = 2\sqrt{10}/11 = 0.57$.

12.14 Fringes disappear when $w = s\lambda/a$, so, $a = \lambda(s/w)$, from Figure 12.3, $\ell/b = s/w$ where ℓ = (mean) distance to sun; b = diameter of sun. $a = \lambda(\ell/b) = [(5.50 \times 10^{-7}\text{ m})(1.50 \times 10^{11}\text{ m})]/2(6.96 \times 10^{8}\text{ m}) = 5.93 \times 10^{-5}\text{ m}$.

12.15 Using the van Cittert-Zernike theorem, we can find $\tilde{\gamma}_{12}(0)$ from the diffraction pattern over the apertures, and that will yield the visibility on the observation plane: $\mathcal{V} = |\tilde{\gamma}_{12}(0)| = |\operatorname{sinc}\beta|$. From Table 1, $\sin u/u = 0.85$ when $u = 0.97$, hence $\pi b y/l\lambda = 0.97$, and if $y = \overline{P_1P_2} = 0.50$ mm, then

$$b = 0.97(l\lambda/\pi y) = 0.97(1.5\text{ m})(500 \times 10^{-9}\text{ m})/\pi(0.50 \times 10^{-3}\text{ m})$$
$$= 0.46\text{ mm}.$$

12.16 (12.23) $\mathcal{V} = |\tilde{\gamma}_{12}(\tau)|$.
(12.1) $\mathcal{V} = (I_{\max} - I_{\min})/(I_{\max} + I_{\min}) = 2|\operatorname{sinc}(a\pi w/s\lambda)|/2$ (from 12.8, 12.9), $\mathcal{V} = 0.90 = |\operatorname{sinc}(a\pi w/s\lambda)| = |\operatorname{sinc}(a\pi(1.0 \times 10^{-3}\text{ m})/(10.0\text{ m})$ $(5.00 \times 10^{-7}\text{ m}))| = |\operatorname{sinc}(200\pi a)|$; $\sin x \simeq x - x^2/3!$, so $\operatorname{sinc}(x) \simeq 1 - x^2/3!$; $0.90 = 1 - [(200\pi a)^2/6]$; $a = 1.23 \times 10^{-3}$ m.

12.17 $\mathcal{V} = |\operatorname{sinc}(a\pi b/\ell\lambda)|$; as shown in Figure 12.6, $\mathcal{V}$ is a minimum when $(a\pi b/\ell\lambda) = m\pi$, $(m \neq 0)$. $b/\ell \simeq \sin(\alpha_2 - \alpha_1) \simeq (\alpha_2 - \alpha_1)$ for small angle, so minimum $\mathcal{V}$ when $[a(\alpha_2 - \alpha_1)\pi/\lambda] = m\pi$; $a(\alpha_2 - \alpha_1) = m\lambda$.

12.18 From the van Cittert-Zernike theorem, the degree of coherence can be obtained from the Fourier transform of the source function, which itself is a series of δ-functions corresponding to a diffraction grating with spacing a, where $a\sin\theta_m = m\lambda$. The coherence function is therefore also a series of

δ-functions. Hence the $\overline{P_1P_2}$, the slit separation d, must correspond to the location of the first-order diffraction fringe of the source if $\mathcal{V}$ is to be maximum. $a\theta_1 \approx \lambda$, and so

$$d \approx l\theta_1 \approx \lambda l/a = (500 \times 10^{-9}\ \mathrm{m})(2.0\ \mathrm{m})/(500 \times 10^{-6}\ \mathrm{m}) = 2.0\ \mathrm{mm}.$$

Chapter 13 Solutions

13.1 $T = 673$ K, area of each face is $A = 10^{-2}$ m^2, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}, then $0.97AI_e = 0.97A\sigma T^4 = 110$ W.

13.2 $0.97I_e = 0.97\sigma(T^4 - T_E^4) = 76.9$ W/m^2 with $T = 306$ K and $T_e = 293$ K is the temperature of the environment. Then $0.97AI_e = 108$ W for the radiated power.

13.3 $I_e = 22.8 \times 10^4$ W/m^2, $T = (I_e/\sigma)^{1/4} = 1420$ K.

13.4 $E \sim T^4$, so the energy radiated increases by a factor of 10^4.

13.5 $T = 306$ K, $\lambda_{\text{max}} = 2.8978 \times 10^{-3}$ m$K/T = 9.45 \times 10^{-6}$ m $= 9.5\mu$ m (in the infrared).

13.6 If the blackbody is at $T = 293$ K, then
$\lambda_{\text{max}} = 2.8978 \times 10^{-3}$ m K/T $= 9.9\mu$ m (in the IR).

13.7 $T = 4.0 \times 10^4$ K, $\nu_{\text{max}} = c/\lambda_{\text{max}} = cT/2.8978 \times 10^{-3}$ m K $= 4.1 \times 10^{15}$ Hz (in the UV).

13.8 $T = 2.8978 \times 10^{-3}$ m K$/\lambda_{\text{max}} = 2.8978 \times 10^{-3}$ mK$/4.65 \times 10^{-7}$ m $= 6230$ K.

13.9 $T = 2.8978 \times 10^{-3}$ m K$/\lambda_{\text{max}} = 4300$ K.

13.10 We have for the total radiated power per unit area of the blackbody

$$\begin{aligned} P(T) &= \int_0^\infty I_\lambda d\lambda = 2\pi hc^2 \int_0^\infty \frac{d\lambda}{\lambda^5(e^{hc/\lambda k_B T} - 1)} \\ &= 2\pi hc^2 \left(\frac{k_B T}{hc}\right)^4 \int_0^\infty \frac{x^3 dx}{(e^x - 1)}, \end{aligned}$$

by putting $x = hc/\lambda k_B T$, $d\lambda = -hcdx/k_B T x^2$. The value of the integral over x is $\Gamma(4)\zeta(4) = 3!\pi^4/90 = \pi^4/15$. Therefore the Stefan-Boltzmann law follows,

$$P(T) = \frac{2\pi^5}{15}\frac{(k_B T)^4}{h^3 c^2}.$$

13.11 $E = hc/\lambda = 1.99 \times 10^{-25}$ J m/λ. Since 1 eV = 1.602×10^{-19} J and 1 nm = 10^{-9} m, this gives $E = 1240$ eV nm/λ. Therefore the energy of a 600 nm photon is 2.1 eV.

13.12 $\lambda(\text{min}) = 300$ nm, $E = h\nu = hc/\lambda = 1240$ eV nm/300 nm = 4.14 eV = 6.63×10^{-19} J.

13.13 If the $P = 100$ W light bulb has an efficiency of $\epsilon = 2.5\%$, then the radiated power is $\epsilon P = Nh\nu/t$, where N is the number of photons, ν is their frequency, and $t = 1$ ms. In terms of the wavelength $\lambda = 550$ nm, this gives $N = \epsilon Pt/h\nu = \epsilon Pt\lambda/hc$. The solid angle subtended by the $d = 3$ cm diameter aperture at distance $r = 100$ ms is: aperature area/$r^2 = \pi d^2/4r^2$. Making the assumption that the light bulb emits isotropically, this is the fraction of photon that passes through the aperature,
$N\pi d^2/4r^2 = \epsilon Pt\lambda\pi d^2/4r^2 hc = 4.9 \times 10^8$ photons.

13.14 $Nh\nu = Nhc/\lambda = (1.4 \times 10^3 W/m^2)(1m^2)(1s)$ gives $N = 49 \times 10^{20}$.

13.15 The number of Ar atoms present in the chamber is $N = pV/k_B T = 2.69 \times 10^{17}$. Taking 1% of this number and using the given excited-state lifetime yields $0.01N/1.4 \times 10^{-8}$ s $= 1.9 \times 10^{23}$ transitions per second.

13.16 With energy density

$$\rho(\nu) = \frac{A_{21}/B_{21}}{(B_{12}/B_{21})e^{h\nu/k_B T} - 1}$$

and $B_{12} = B_{21}$, the ratio of interest is $B_{21}\rho(\nu)/A_{21} = 1/(e^{h\nu/k_B T} - 1)$.

13.17 $k_BT = 4.14 \times 10^{-21}$ J, $E = 3.2 \times 10^{-19}$ J, $E/k_BT = 77.3$. Since $\exp(E/k_BT) \gg 1$, the ratio

$$1/(\exp(E/k_BT) - 1) \simeq \exp(-E/k_BT) \approx 2.7 \times 10^{-34},$$

extremely low.

13.18 $k_BT = 4.14 \times 10^{-19}$ J, $E = 3.2 \times 10^{-19}$ J, $E/k_BT = 0.773$. Then $1/(\exp(E/k_BT) - 1) = 0.86$, much higher than in the previous problem. Stimulated emission is quite likely at this high temperature (as at the surface of a hotter star).

13.19 $N_j/N_i = e^{-(E_j-E_i)/k_BT} \approx 1 - (E_j - E_i)/k_BT$ for $|E_j - E_i| \ll k_BT$. Therefore as $T \to \infty$, N_j/N_i tends to 1.

13.20 $T = 3.0$ K, $\quad k_BT = 4.14 \times 10^{-23}$ J,
$E = hc/\lambda = 1.99 \times 10^{-25}$ J m$/\lambda = 9.48 \times 10^{-25}$ J,
$E/k_BT = 2.29 \times 10^{-2}$
and the ratio $1/(\exp(E/k_BT) - 1) \simeq 43$, so stimulated emission is very likely, if not dominant. (The number of significant figures is important in such a case.)

13.21 $\Phi \approx 2.44\lambda/D = 5.15 \times 10^{-4}$ rad, $s = r\Phi = 5.15 \times 10^{-2}$ m or the diameter of the spot on the wall is 5.1 cm.

13.22 The volume of the crystal is $V = (\pi/4)D^2L = 9.8 \times 10^{-7}$ m^3. Therefore the mass of Cr_2O_3 present is

$$0.05 \times 10^{-2}(3.7 \times 10^3)9.8 \times 10^{-7} = 1.81 \times 10^{-6} \text{ Kg}.$$

The mass of one Cr_2O_3 molecule is 152 amu or 2.52×10^{-25} Kg. Therefore approximately 7.17×10^{18} Cr_2O_3 molecules. are present. Assuming that each contributes two Cr^{3+} ions to lasing, $N_{\text{ions}} = 1.4 \times 10^{19}$ ions participate in the lasing action, at $\Delta E = 2.87 \times 10^{-19}$ J. Then $E_{\text{tot}} = N_{\text{ions}}\Delta E = 4.0$ J; the corresponding power is $E_{\text{tot}}/t = 4.0$ J$/5.0 \times 10^{-6}$ s $= 0.8$ mW.

13.23 $N/t = P/\Delta E = 1.0 \times 10^{-3}$ J s$^{-1}/(1.96)(1.602 \times 10^{-19}$ J $= 3.2 \times 10^{15}$ transitions per second.

13.24 $\Delta\lambda_0 = \bar{\lambda}_0^2 \Delta\nu/c = 8.0 \times 10^{-5}$ nm.

13.25 $\Delta\nu = c/2L = 6 \times 10^8$ Hz, using $v = c$ when $n = 1$.

13.26 The condition $\Delta\nu = 1.4 \times 10^9 = c/2L$ for $n = 1$ gives $L = c/2\Delta\nu = 11$ cm.

13.27 $I = (v/2)\epsilon E_0^2 = (n/2)(\epsilon_0/\mu_0)^{1/2} E_0^2$, where $\mu \approx \mu_0$, $E_0^2 = 2(\mu_0/\epsilon_0)^{1/2} I/n$, $(\mu_0/\epsilon_0)^{1/2} = 376.730\Omega$, so $E_0 = 27.4(I/n)^{1/2}$.

13.28 $\Phi \approx 2.44\lambda/D = 2.6 \times 10^{-3}$ rad.

13.29 The three crossed gratings form a type of triangular lattice. The diffraction spots will appear along the directions of the dual lattice, which are directions connecting the centroids of the original lattice. As usual, there will be a central spot of highest irradiance (intensity) and the irradiance decreases with distance from this central spot. Reciprocals of multiples of lattice constants of the original lattice are proportional to the spatial frequencies present in the diffraction pattern. (Strictly speaking, the lattice should have infinite extent in order to consider it a mathematically periodic structure.)

13.30 In this case the four crossed gratings form a sort of rectangular lattice, whose dual lattice is again rectangular. Therefore the diffraction spots will be located along horizontal and vertical lines. The central spot has the highest irradiance and the irradiance of the others decreases with distance from the center. A horizontal slit filter will enhance the vertical lined grating in the altered image, and vice versa.

13.31 The horizontal grating gives a row of diffraction spots, with the central spot of highest irradiance. The details in the picture image are contained in many high spatial frequency components. The picture can be enhanced by using a filter which blocks out the diffraction pattern of the horizontal grating.

13.32 The circular grating present will generate a central spot of highest irradiance, together with successive rings. In order to enhance the picture, a spatial filter which blocks out these contributions should be used.

13.33 The filter is a long slit, perpendicular to the observed image.

13.34 From the geometry, $f_t\theta = f_i\Phi$: $k_O = k\sin\theta$ and $k_I = k\sin\Phi$, hence $\sin\theta \approx \theta \approx k_O\lambda/2\pi$ and $\sin\Phi \approx \Phi \approx k_I\lambda/2\pi$, therefore $\theta/\Phi = k_O/k_I$ and $k_I = k_O(\Phi/\theta) = k_O(f_t/f_i)$. When $f_i > f_t$ the image will be larger than the object, the spatial periods in the image will also be larger, and the spatial frequencies in the image will be smaller than in the object.

13.35 $a = (1/50)$ cm: $a\sin\theta = m\lambda$, $\sin\theta \approx \theta$, hence $\theta = (5000\text{ m})\lambda$, and the distance between orders on the transform plane is $f\theta = 5000\lambda f = 2.7$ mm.

13.36 (a) As in Figure 11.10, the transform of the cosine function will be a pair of δ-functions, at $x = \pm d$, where d is the spatial period of the cosine. To pass only the first order terms, we need a filter with holes at these positions, for the specific wavelength, as given by $x/f \simeq \sin\theta_1 = \lambda(1/d)$; $x = f\lambda/d = [(2.0\text{ m})(5.00\times10^{-7}\text{ m})/(1\times10^{-5}\text{ m}) = 0.1$ m, above and below center. (b) Any "DC" components, and all high order components, are removed. A smoothly varying cosine amplitude should be seen in the image. (c) A filter with a hole in the center would pass only the DC term, resulting in a lower intensity, uniform image.

13.37 Each point on the diffraction pattern corresponds to a single spatial frequency, and if we consider the diffracted wave to be made up of plane waves, it also corresponds to a single-plane wave direction. Such waves, by themselves, carry no information about the periodicity of the object and produce a more or less uniform image. The periodicity of the source arises in the image when the component plane waves interfere.

13.39 The relative field amplitudes are 1.00, 0.60, and 0.60; hence $E \propto 1 + 0.60\cos(+ky') + 0.60\cos(-ky') = 1 + 1.2\cos ky'$. This is a cosine oscillating about a line equal to 1.0. It varies from +2.2 to −0.2. The square of this will correspond to the irradiance, and it will be a series of tall peaks with a relative height of $(2.2)^2$, between each pair of which there will be a short peak proportional to $(0.2)^2$; notice the similarity with Fig. 11.32.

13.40 $a\sin\theta = \lambda$, here $f\theta = 50\lambda f = 0.20$ cm; hence $\lambda = 0.20/50(100) = 400$ nm. The magnification is 1.0 when the focal lengths are equal, hence the spacing is again 50 wires/cm.

13.41 The random dots will add considerable "noise" to the pattern. The spatial frequency is $1/(0.1\text{ mm}) = 10\text{ mm}^{-1}$. A filter that is the transform of the regular pattern will remove the random dots.

13.42 The array of top hats corresponds to the pixels, so that each "selects" the amplitude (density) of the picture within its radius. The transform will look like a regular arrray of dots of varying amplitude. As in Figure 13.39, filtering out the higher frequency components will yield a continuous image.

13.43 The pinhole blocks the high-frequency components, which correspond to the rapid spatial variations in the beam.

13.44 The randomly, but more or less uniformly, distributed particles in the milk will tend to block the "regular" part of the beam, and thus enhance the relative intensity of the speckle.

13.45 The inherent motion of the medium would cause the speckle pattern to vanish.