Física Estadistica
Tarea 5

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Índice general

Capítulo 1

1.1.

En las secciones 6.1 y 6.2 del librio Pathria se llego a

$$\frac{PV}{kT} = \sum_{\varepsilon} \ln\left(1 + ze^{-\beta\varepsilon}\right) \tag{1.1}$$

$$N = \sum_{\varepsilon} \frac{1}{z^{-1}e^{\beta\varepsilon} + 1} \tag{1.2}$$

Sin embargo

$$\sum_{\varepsilon} \to \int_0^{\infty} g(\varepsilon)d\varepsilon$$

donde

$$g(\varepsilon)d\varepsilon = \frac{Vg\sqrt{\varepsilon}}{2\pi^2\hbar^3}(2m)^{3/2}d\varepsilon,$$

Ademas usaremos:

$$f_n(z) = \frac{1}{\Gamma(n)} \int_0^{\infty} \frac{x^{n-1}}{z^{-1}e^x + 1}$$

por lo tanto aplicando en 1.1 y 1.2 tenemos

1. Para 1.1

$$\begin{aligned}
\frac{PV}{kT} &= \sum_{\varepsilon} \ln\left(1 + ze^{-\beta\varepsilon}\right) \\
\frac{PV}{kT} &= \int_0^{\infty} \ln\left(1 + ze^{-\beta\varepsilon}\right) \frac{Vg\sqrt{\varepsilon}}{2\pi^2\hbar^3}(2m)^{3/2}d\varepsilon \\
\frac{PV}{kT} &= \frac{Vg}{2\pi^2\hbar^3}(2m)^{3/2} \int_0^{\infty} \ln\left(1 + ze^{-\beta\varepsilon}\right)\sqrt{\varepsilon}d\varepsilon \\
x &= \beta x \\
\varepsilon &= kTx \\
d\varepsilon &= kTdx \\
\frac{PV}{kT} &= \frac{Vg}{2\pi^2\hbar^3}(2m)^{3/2} \int_0^{\infty} \ln\left(1 + ze^{-x}\right)\sqrt{kTx}kTdx \\
\frac{PV}{kT} &= \frac{Vg}{2\pi^2\hbar^3}(2m)^{3/2}(kT)^{\frac{3}{2}} \int_0^{\infty} \ln\left(1 + ze^{-x}\right)\sqrt{x}dx
\end{aligned}$$

Ahora para solucionar la integral podemos hacerla por partes de la siguiente manera

$$
\begin{aligned}
u &= \ln\left(1+ze^{-x}\right) \\
du &= \frac{-ze^{-x}}{1+ze^{-x}}dx \\
dv &= \sqrt{x}dx \\
v &= \frac{2}{3}x^{\frac{3}{2}} \\
\int udv &= uv - \int vdu \\
\int_0^\infty \ln\left(1+ze^{-x}\right)\sqrt{x}dx &= \left[\ln\left(1+ze^{-x}\right)\frac{2}{3}x^{\frac{3}{2}}\right]_0^\infty - \int_0^\infty \frac{2}{3}x^{\frac{3}{2}}\frac{-ze^{-x}}{1+ze^{-x}}dx \\
&= \frac{2}{3}\int_0^\infty \frac{x^{\frac{3}{2}}ze^{-x}}{1+ze^{-x}}dx \\
&= \frac{2}{3}\Gamma\left(\frac{5}{2}\right)f_{\frac{5}{2}}\left(z\right) \\
\Gamma\left(\frac{5}{2}\right) &= \frac{3}{4}\sqrt{\pi} \\
&= \frac{\sqrt{\pi}}{2}f_{\frac{5}{2}}\left(z\right)
\end{aligned}
$$

Con esto entonces

$$
\begin{aligned}
\frac{PV}{kT} &= \frac{Vg}{2\pi^2\hbar^3}(2m)^{3/2}\left(kT\right)^{\frac{3}{2}}\frac{\sqrt{\pi}}{2}f_{\frac{5}{2}}\left(z\right) \\
\frac{PV}{kT} &= \frac{Vg}{2\pi^2\hbar^3}(2mkT)^{3/2}\frac{\sqrt{\pi}}{2}f_{\frac{5}{2}}\left(z\right) \\
\frac{PV}{kT} &= \frac{Vg}{2\pi^2\frac{h^3}{8\pi^3}}(2mkT)^{3/2}\frac{\sqrt{\pi}}{2}f_{\frac{5}{2}}\left(z\right) \\
\frac{PV}{kT} &= \frac{Vg}{\frac{h^3}{2\pi}}(2mkT)^{3/2}\frac{\sqrt{\pi}}{2}f_{\frac{5}{2}}\left(z\right) \\
\frac{PV}{kT} &= 2\pi\frac{Vg}{h^3}(2mkT)^{3/2}\frac{\sqrt{\pi}}{2}f_{\frac{5}{2}}\left(z\right) \\
\frac{PV}{kT} &= \frac{Vg}{h^3}(2\pi mkT)^{3/2}f_{\frac{5}{2}}\left(z\right) \\
\lambda &= \frac{h}{\sqrt{2\pi mkT}} \\
\lambda^3 &= \frac{h^3}{\left(2\pi mkT\right)^{\frac{3}{2}}} \\
\frac{1}{\lambda^3} &= \frac{\left(2\pi mkT\right)^{\frac{3}{2}}}{h^3} \\
\frac{PV}{kT} &= \frac{Vg}{\lambda^3}f_{\frac{5}{2}}\left(z\right) \\
\frac{P}{kT} &= \frac{g}{\lambda^3}f_{\frac{5}{2}}\left(z\right)
\end{aligned}
$$

2. Para 1.2

$$
\begin{aligned}
N &= \sum_{\varepsilon} \frac{1}{z^{-1}e^{\beta\varepsilon}+1} \\
&= \int_0^\infty \frac{1}{z^{-1}e^{\beta\varepsilon}+1} g(\varepsilon) d\varepsilon \\
&= \int_0^\infty \frac{1}{z^{-1}e^{\beta\varepsilon}+1} \frac{Vg\sqrt{\varepsilon}}{2\pi^2\hbar^3}(2m)^{3/2} d\varepsilon \\
&= \frac{Vg}{2\pi^2\hbar^3}(2m)^{3/2} \int_0^\infty \frac{1}{z^{-1}e^{\beta\varepsilon}+1}\sqrt{\varepsilon} d\varepsilon \\
&= \frac{Vg}{2\pi^2\hbar^3}(2m)^{3/2} \int_0^\infty \frac{\varepsilon^{1/2}}{z^{-1}e^{\beta\varepsilon}+1} d\varepsilon \\
&= \frac{Vg}{2\pi^2\hbar^3}(2m)^{3/2} \int_0^\infty \frac{\varepsilon^{1/2}}{z^{-1}e^{\beta\varepsilon}+1} d\varepsilon \\
x &= \beta\varepsilon \\
kTx &= \varepsilon \\
d\varepsilon &= kTdx \\
&= \frac{Vg}{2\pi^2\hbar^3}(2m)^{3/2} \int_0^\infty \frac{(kTx)^{1/2}}{z^{-1}e^{x}+1} kTdx \\
&= \frac{Vg}{2\pi^2\hbar^3}(2mkT)^{3/2} \int_0^\infty \frac{(x)^{1/2}}{z^{-1}e^{x}+1} dx \\
&= \frac{Vg}{2\pi^2\hbar^3}(2mkT)^{3/2}\Gamma\left(\frac{3}{2}\right) f_{\frac{3}{2}}(z) \\
&= \frac{Vg}{2\pi^2\hbar^3}(2mkT)^{3/2}\frac{\sqrt{\pi}}{2} f_{\frac{3}{2}}(z) \\
&= \frac{Vg}{\frac{h^3}{2\pi}}(2mkT)^{3/2}\frac{\sqrt{\pi}}{2} f_{\frac{3}{2}}(z) \\
&= 2\pi\frac{Vg}{h^3}(2mkT)^{3/2}\frac{\sqrt{\pi}}{2} f_{\frac{3}{2}}(z) \\
&= \frac{Vg}{h^3}(2\pi mkT)^{3/2} f_{\frac{3}{2}}(z) \\
&= \frac{Vg}{\lambda^3} f_{\frac{3}{2}}(z) \\
N &= \frac{Vg}{\lambda^3} f_{\frac{3}{2}}(z) \\
\frac{N}{V} &= \frac{g}{\lambda^3} f_{\frac{3}{2}}(z)
\end{aligned}
$$

1.2.

Tenemos

$$U = kT^2 \left(\frac{\partial}{\partial T} \frac{PV}{kT} \right)$$
$$U = kT^2 \left(\frac{\partial}{\partial T} \frac{Vg}{\lambda^3} f_{\frac{5}{2}}(z) \right)$$
$$U = kT^2 Vg \left(\frac{\partial}{\partial T} \frac{1}{\lambda^3} f_{\frac{5}{2}}(z) \right)$$
$$U = kT^2 Vg \left(\frac{\partial}{\partial T} \frac{1}{\lambda^3} f_{\frac{5}{2}}(z) + \frac{1}{\lambda^3} \frac{\partial}{\partial T} f_{\frac{5}{2}}(z) \right)$$
$$U = kT^2 Vg \left(\frac{3}{2\lambda^3 T} f_{\frac{5}{2}}(z) + \frac{1}{\lambda^3} 0 \right)$$
$$U = kT^2 Vg \frac{3}{2\lambda^3 T} f_{\frac{5}{2}}(z)$$
$$U = \frac{3kT^2 Vg}{2\lambda^3 T} f_{\frac{5}{2}}(z)$$
$$\frac{N}{V} = \frac{g}{\lambda^3} f_{\frac{3}{2}}(z)$$
$$\frac{N}{f_{\frac{3}{2}}(z)} = \frac{gV}{\lambda^3}$$
$$U = \frac{3kTN}{2 f_{\frac{3}{2}}(z)} f_{\frac{5}{2}}(z)$$
$$U = \frac{3}{2} kTN \frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)}$$

1.3.

Para esto usaremos

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V$$

Con lo cual:

$$U = \frac{3}{2}kTN\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)}$$

$$C_V = \left(\frac{\partial}{\partial T}\frac{3}{2}kTN\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)}\right)_V$$

$$C_V = \frac{3}{2}Nk\left(\frac{\partial}{\partial T}T\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)}\right)_V$$

$$C_V = \frac{3}{2}Nk\left(\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)} + T\frac{\partial}{\partial T}\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)}\right)_V$$

$$C_V = \frac{3}{2}Nk\left(\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)} + T\frac{f_{\frac{3}{2}}(z)\frac{\partial f_{\frac{5}{2}}(z)}{\partial T} - f_{\frac{5}{2}}(z)\frac{\partial f_{\frac{3}{2}}(z)}{\partial T}}{\left[f_{\frac{3}{2}}(z)\right]^2}\right)_V$$

$$C_V = \frac{3}{2}Nk\left(\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)} + T\frac{f_{\frac{3}{2}}(z)\frac{\partial f_{\frac{5}{2}}(z)}{\partial z}\frac{\partial z}{\partial T} - f_{\frac{5}{2}}(z)\frac{\partial f_{\frac{3}{2}}(z)}{\partial z}\frac{\partial z}{\partial T}}{\left[f_{\frac{3}{2}}(z)\right]^2}\right)_V$$

$$C_V = \frac{3}{2}Nk\left(\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)} + T\frac{f_{\frac{3}{2}}(z)\frac{f_{\frac{3}{2}}(z)}{z}\frac{\partial z}{\partial T} - f_{\frac{5}{2}}(z)\frac{f_{\frac{1}{2}}(z)}{z}\frac{\partial z}{\partial T}}{\left[f_{\frac{3}{2}}(z)\right]^2}\right)_V$$

$$C_V = \frac{3}{2}Nk\left(\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)} + \frac{T}{z}\frac{\partial z}{\partial T}\frac{f_{\frac{3}{2}}(z)^2 - f_{\frac{5}{2}}(z)f_{\frac{1}{2}}(z)}{\left[f_{\frac{3}{2}}(z)\right]^2}\right)_V$$

$$C_V = \frac{3}{2}Nk\left(\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)} + \frac{T}{z}\left(-\frac{3}{2}\frac{z}{T}\frac{f_{\frac{3}{2}}(z)}{f_{\frac{1}{2}}(z)}\right)\frac{f_{\frac{3}{2}}(z)^2 - f_{\frac{5}{2}}(z)f_{\frac{1}{2}}(z)}{\left[f_{\frac{3}{2}}(z)\right]^2}\right)$$

$$C_V = \frac{3}{2}Nk\left(\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)} - \frac{3}{2}\frac{f_{\frac{3}{2}}(z)^2 - f_{\frac{5}{2}}(z)f_{\frac{1}{2}}(z)}{f_{\frac{3}{2}}(z)f_{\frac{1}{2}}(z)}\right)$$

$$C_V = \frac{3}{2}Nk\left(\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)} - \frac{3}{2}\frac{f_{\frac{3}{2}}(z)^2}{f_{\frac{3}{2}}(z)f_{\frac{1}{2}}(z)} + \frac{3}{2}\frac{f_{\frac{5}{2}}(z)f_{\frac{1}{2}}(z)}{f_{\frac{3}{2}}(z)f_{\frac{1}{2}}(z)}\right)$$

$$C_V = \frac{3}{2}Nk\left(\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)} - \frac{3f_{\frac{3}{2}}(z)}{2f_{\frac{1}{2}}(z)} + \frac{3f_{\frac{5}{2}}(z)}{2f_{\frac{3}{2}}(z)}\right)$$

$$C_V = \frac{3}{2}Nk\left(\frac{5f_{\frac{5}{2}}(z)}{2f_{\frac{3}{2}}(z)} - \frac{3f_{\frac{3}{2}}(z)}{2f_{\frac{1}{2}}(z)}\right)$$

$$C_V = Nk\frac{3}{2}\left(\frac{5f_{\frac{5}{2}}(z)}{2f_{\frac{3}{2}}(z)} - \frac{3f_{\frac{3}{2}}(z)}{2f_{\frac{1}{2}}(z)}\right)$$

$$C_V = Nk\left(\frac{15}{4}\frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)} - \frac{9}{4}\frac{f_{\frac{3}{2}}(z)}{f_{\frac{1}{2}}(z)}\right)$$

1.4.

En el Apendice E del libro de Pathria explican que para z pequeños se cumple que:

$$f_v(z) = z - \frac{z^2}{2^v} + \frac{z^3}{3^v} - \ldots$$

Nos piden encontrar esta serie en terminos de $n\lambda^3$ por lo tanto partamos de la expresión para $n = \frac{N}{V}$ con lo cual:

$$\begin{aligned} n &= \frac{g}{\lambda^3} f_{\frac{3}{2}}(z) \\ n &= \frac{g}{\lambda^3}\left(z - \frac{z^2}{2^{\frac{3}{2}}} + \frac{z^3}{3^{\frac{3}{2}}} - \ldots\right) \\ \frac{n\lambda^3}{g} &= \left(z - \frac{z^2}{2^{\frac{3}{2}}} + \frac{z^3}{3^{\frac{3}{2}}} - \ldots\right) \end{aligned}$$

1.5.

1.6.

1.7.

Capítulo 2

2.1.

2.2.

2.3.

2.4.

Capítulo 3

3.1.

Partimos desde

$$\chi = \frac{2n\mu^{*2}}{\left(\frac{\partial \mu_0(xN)}{\partial x}\right)_{x=1/2}}$$

Tenemos que considerar que segun Pathria en la ecuación 8.1.34

3.2.

3.3.

Capítulo 4

4.1.

4.2.

4.3.

4.4.

4.5.

4.6.

4.7.