Fisica Estadistica
Tarea 2

Sergio Montoya Ramirez

202112171

Contents

Chapter 1

1.1

En este caso simplemente tenemos que despejar:

$$
\begin{aligned}
S\left(N,V,E\right) &= Nk\ln\left[\frac{V}{h^3}\left(\frac{4\pi mE}{3N}\right)^{\frac{3}{2}}\right]+\frac{3}{2}Nk\\
S &= Nk\ln\left[\frac{V}{h^3}\left(\frac{4\pi mE}{3N}\right)^{\frac{3}{2}}\right]+\frac{3}{2}Nk\\
S &= \ln\left[\left(\frac{V}{h^3}\left(\frac{4\pi mE}{3N}\right)^{\frac{3}{2}}\right)^{Nk}\right]+\frac{3}{2}Nk\\
e^S &= e^{\ln\left[\left(\frac{V}{h^3}\left(\frac{4\pi mE}{3N}\right)^{\frac{3}{2}}\right)^{Nk}\right]+\frac{3}{2}Nk}\\
e^S &= \left(\frac{V}{h^3}\right)^{Nk}\left(\frac{4\pi mE}{3N}\right)^{\frac{3}{2}Nk}e^{\frac{3}{2}Nk}\\
e^S\left(\frac{V}{h^3}\right)^{-Nk}e^{-\frac{3}{2}Nk} &= \left(\frac{4\pi mE}{3N}\right)^{\frac{3}{2}Nk}\\
\left(e^S\left(\frac{V}{h^3}\right)^{-Nk}e^{-\frac{3}{2}Nk}\right)^{\frac{2}{3Nk}} &= \left(\frac{4\pi mE}{3N}\right)\\
e^{\frac{2S}{3Nk}}\left(\frac{V}{h^3}\right)^{-\frac{2}{3}}e^{-1} &= \left(\frac{4\pi mE}{3N}\right)\\
E &= e^{\frac{2S}{3Nk}}\frac{h^2}{V^{\frac{2}{3}}}e^{-1}\left(\frac{3N}{4\pi m}\right)\\
E &= e^{\frac{2S}{3Nk}-1}\left(\frac{3Nh^2}{4\pi mV^{\frac{2}{3}}}\right).
\end{aligned}
$$

1.2

Para este caso vamos a usar:

$$
\begin{aligned}
E &= e^{\frac{2S}{3Nk}-1}\left(\frac{3Nh^2}{4\pi mV^{\frac{2}{3}}}\right)\\
T &= \frac{\partial E}{\partial S}\\
T &= \left(\frac{3Nh^2}{4\pi mV^{\frac{2}{3}}}\right)e^{-1}\frac{\partial e^{\frac{2S}{3Nk}}}{\partial S}\\
T &= \left(\frac{3Nh^2}{4\pi mV^{\frac{2}{3}}}\right)e^{-1}\frac{2}{3Nk}e^{\frac{2S}{3Nk}}\\
T &= \left(\frac{h^2}{k2\pi mV^{\frac{2}{3}}}\right)e^{-1}e^{\frac{2S}{3Nk}}\\
T\left(\frac{k2\pi mV^{\frac{2}{3}}}{h^2}\right) &= e^{\frac{2S}{3Nk}-1}\\
E &= e^{\frac{2S}{3Nk}-1}\left(\frac{3Nh^2}{4\pi mV^{\frac{2}{3}}}\right)\\
E &= T\left(\frac{k2\pi mV^{\frac{2}{3}}}{h^2}\right)\left(\frac{3Nh^2}{4\pi mV^{\frac{2}{3}}}\right)\\
E &= T\left(\frac{3Nk}{2}\right)\\
E &= \frac{3}{2}NkT.
\end{aligned}
$$

1.3

Ahora desarrollemos:

$$
\begin{aligned}
E &= e^{\frac{2S}{3Nk}-1}\left(\frac{3Nh^2}{4\pi mV^{\frac{2}{3}}}\right)\\
P &= -\frac{\partial E}{\partial V}\\
P &= -e^{\frac{2S}{3Nk}-1}\left(\frac{3Nh^2}{4\pi m}\right)\frac{\partial V^{-\frac{2}{3}}}{\partial V}\\
P &= \frac{2}{3}e^{\frac{2S}{3Nk}-1}\left(\frac{3Nh^2}{4\pi m}\right)V^{-\frac{5}{3}}\\
T &= \left(\frac{h^2}{k2\pi mV^{\frac{2}{3}}}\right)e^{\frac{2S}{3Nk}-1}\\
\frac{P}{T} &= \frac{\frac{2}{3}e^{\frac{2S}{3Nk}-1}\left(\frac{3Nh^2}{4\pi m}\right)V^{-\frac{5}{3}}}{\left(\frac{h^2}{k2\pi mV^{\frac{2}{3}}}\right)e^{\frac{2S}{3Nk}-1}}\\
\frac{P}{T} &= \frac{2}{3}\frac{\left(\frac{3Nh^2}{4\pi m}\right)}{\left(\frac{h^2}{k2\pi mV^{\frac{2}{3}}}\right)}V^{-\frac{5}{3}}\\
\frac{P}{T} &= \frac{2}{3}\left(\frac{3Nh^2k2\pi mV^{\frac{2}{3}}}{4\pi mh^2}\right)V^{-\frac{5}{3}}\\
\frac{P}{T} &= \frac{4}{3}\left(\frac{3Nh^2k\pi mV^{\frac{2}{3}}}{4\pi mh^2V^{\frac{2}{3}}}\right)V^{-1}\\
\frac{P}{T} &= (Nk)\,V^{-1}\\
PV &= NkT.
\end{aligned}
$$

1.4

$$
\begin{aligned}
C_v &= \frac{\partial E}{\partial T} \\
&= \frac{\partial \left(\frac{3}{2}NkT\right)}{\partial T} \\
&= \frac{3}{2}Nk \\
C_p &= \frac{\partial (E + PV)}{\partial T} \\
&= \frac{\partial \left(\frac{3}{2}NkT + NkT\right)}{\partial T} \\
&= \frac{\partial NkT \left(\frac{3}{2} + 1\right)}{\partial T} \\
&= \frac{\partial \frac{5}{2}NkT}{\partial T} \\
&= \frac{5}{2}Nk \\
\frac{C_p}{C_v} &= \frac{\frac{5}{2}Nk}{\frac{3}{2}Nk} \\
&= \frac{5 \cdot 2}{3 \cdot 2} \\
&= \frac{5}{3}.
\end{aligned}
$$

1.5

Para este caso necesitamos

$$
\begin{aligned}
E &= e^{\frac{2S}{3Nk}-1}\left(\frac{3Nh^2}{4\pi m V^{\frac{2}{3}}}\right) \\
\mu &= \frac{\partial E}{\partial N} \\
&= \frac{\partial e^{\frac{2S}{3Nk}-1}\left(\frac{3Nh^2}{4\pi m V^{\frac{2}{3}}}\right)}{\partial N} \\
&= \frac{\partial e^{\frac{2S}{3Nk}-1}}{\partial N}\left(\frac{3Nh^2}{4\pi m V^{\frac{2}{3}}}\right) + e^{\frac{2S}{3Nk}-1}\frac{\partial\left(\frac{3Nh^2}{4\pi m V^{\frac{2}{3}}}\right)}{\partial N} \\
&= -\frac{2S}{3N^2k}e^{\frac{2S}{3Nk}-1}\left(\frac{3Nh^2}{4\pi m V^{\frac{2}{3}}}\right) + e^{\frac{2S}{3Nk}-1}\left(\frac{3h^2}{4\pi m V^{\frac{2}{3}}}\right) \\
&= -\frac{2S}{3Nk}e^{\frac{2S}{3Nk}-1}\left(\frac{3h^2}{4\pi m V^{\frac{2}{3}}}\right) + e^{\frac{2S}{3Nk}-1}\left(\frac{3h^2}{4\pi m V^{\frac{2}{3}}}\right) \\
&= e^{\frac{2S}{3Nk}-1}\left(\frac{3h^2}{4\pi m V^{\frac{2}{3}}}\right)\left(1 - \frac{2S}{3Nk}\right).
\end{aligned}
$$

Y con esto podemos probar si

$$
\begin{aligned}
\mu(\lambda N, \lambda V, \lambda S) = \lambda\mu(N, V, S) \\
&= e^{\frac{2\lambda S}{3\lambda Nk}-1}\left(\frac{3h^2}{4\pi m \lambda^{\frac{2}{3}} V^{\frac{2}{3}}}\right)\left(1 - \frac{2\lambda S}{3\lambda Nk}\right) \\
&= e^{\frac{2S}{3Nk}-1}\left(\frac{3h^2}{4\pi m \lambda^{\frac{2}{3}} V^{\frac{2}{3}}}\right)\left(1 - \frac{2S}{3Nk}\right)
\end{aligned}
$$

Que como se ve no se coincide con una cantidad intensiva.

1.6

1.7

Chapter 2

Tenemos

$$\begin{aligned}
\varepsilon &= \frac{hc}{2L}\sqrt{n_x^2 + n_y^2 + n_z^2} \\
\sqrt{n_x^2 + n_y^2 + n_z^2} &= \frac{2L}{hc}\varepsilon = \varepsilon^*.
\end{aligned}$$

Ahora, extendiendo esto a N partículas y tres dimensiones tenemos:

$$\begin{aligned}
E^* &= \sum_{n=1}^{N} \varepsilon^* \\
V &= L^3 \\
E &= \sum_{n=1}^{N} \varepsilon.
\end{aligned}$$

Con lo cual podemos desarrollar:

$$\begin{aligned}
\Omega = E^* &= \frac{2V^{\frac{1}{3}}}{hc}\sum_{i=1}^{N} \varepsilon_i \\
&= \frac{2V^{\frac{1}{3}}}{hc} E \\
\Omega &\propto V^{\frac{1}{3}} E \\
S &= k \ln(\Omega) \\
\implies S &\propto V^{\frac{1}{3}} E.
\end{aligned}$$

Para un proceso reversible adiabatico:

$$\begin{aligned}
V^{\frac{1}{3}} E &= CTE \\
E &= \frac{CTE}{V^{\frac{1}{3}}}.
\end{aligned}$$

Con lo cual

$$\begin{aligned}
P &= -\left(\frac{\partial E}{\partial V}\right)_{N,S} = -\frac{1}{3}\frac{1}{V} - E \\
&= \frac{CTE}{3V^{\frac{4}{3}}} \\
&= \frac{CTE}{3V^{\frac{4}{3}}} \\
PV^{\frac{4}{3}} &= \frac{CTE}{3} \\
PV^{\gamma} &= cte \\
\frac{4}{3} &= \gamma.
\end{aligned}$$

Chapter 3

3.1

Tenemos

$$\begin{aligned}
\varepsilon &= nhv \\
\frac{\varepsilon}{hv} &= n = \varepsilon^* \\
E^* &= \sum_{n=1}^{N} \varepsilon_n^* \\
E^* &= \frac{1}{hv} \sum_{n=1}^{N} \varepsilon_n .
\end{aligned}$$

Ahora, para esto necesitamos entonces

$$\begin{aligned}
\Omega &= \frac{(E^* + N - 1)!}{E^*!\,(N-1)!} \\
S &= k \ln \Omega \\
&= k \ln \left(\frac{(E^* + N - 1)!}{E^*!\,(N-1)!} \right) \\
&= k \ln ((E^* + N - 1)!) - k \ln E^*! - k \ln ((N-1)!) \\
\frac{S}{k} &= (E^* + N - 1) \ln ((E^* + N - 1)) - (E^* + N - 1) - E^* \ln E^* + E^* - (N-1) \ln ((N-1)) + (N-1) \\
\frac{S}{k} &= (E^* + N - 1) \ln ((E^* + N - 1)) - E^* \ln E^* - (N-1) \ln ((N-1)) \\
\frac{S}{k} &= (E^* + N - 1) \ln ((E^* + N - 1)) - E^* \ln E^* - (N-1) \ln ((N-1)) \\
\frac{S}{k} &= (E^*) \ln ((E^* + N - 1)) + (N-1) \ln ((E^* + N - 1)) - E^* \ln E^* - (N-1) \ln ((N-1)) \\
\frac{S}{k} &= E^* \left(\ln \left(\frac{E^* + N - 1}{E^*} \right) \right) + (N-1) \ln \frac{E^* + N - 1}{N - 1} \\
N &\gg 1 \\
\frac{S}{k} &= E^* \left(\ln \left(\frac{E^* + N}{E^*} \right) \right) + (N) \ln \frac{E^* + N}{N} \\
\frac{S}{k} &= \frac{E}{hv} \left(\ln \left(\frac{\frac{E}{hv} + N}{\frac{E}{hv}} \right) \right) + (N) \ln \frac{\frac{E}{hv} + N}{N} \\
\frac{S}{k} &= \frac{E}{hv} \left(\ln \left(\frac{E + Nhv}{E} \right) \right) + (N) \ln \frac{E + Nhv}{Nhv} \\
S &= k \left(\frac{E}{hv} \left(\ln \left(\frac{E + Nhv}{E} \right) \right) + (N) \ln \frac{E + Nhv}{Nhv} \right).
\end{aligned}$$

3.2

Ahora para este caso:

$$\begin{aligned}
\frac{1}{T} &= \left(\frac{\partial S}{\partial E}\right)_N \\
\frac{1}{T} &= k\left(\frac{\partial \frac{E}{hv}\left(\ln\left(\frac{E+Nhv}{E}\right)\right) + (N)\ln\frac{E+Nhv}{Nhv}}{\partial E}\right)_N \\
\frac{1}{kT} &= \frac{1}{hv}\ln\left(\frac{E+Nhv}{E}\right) + \frac{E}{hv}\frac{1}{\frac{E+Nhv}{E}} + \frac{1}{hv}\frac{1}{\frac{E+Nhv}{Nhv}} \\
\frac{hv}{kT} &= \ln\left(\frac{E+Nhv}{E}\right) + \frac{1}{\frac{E+Nhv}{E}}\left(1-\frac{1}{E}\right) + \frac{1}{\frac{E+Nhv}{Nhv}} - Nhv \\
\frac{hv}{kT} &= \ln\left(\frac{E+Nhv}{E}\right) + E\frac{E}{E+Nhv} + \frac{Nhv}{E+Nhv} - Nhv \\
\frac{hv}{kT} &= \ln\left(\frac{E+Nhv}{E}\right) + \frac{Nhv+E}{E+Nhv} - Nhv \\
\frac{hv}{kT} &= \ln\left(\frac{E+Nhv}{E}\right) + 1 - Nhv \\
\frac{hv}{k\ln\left(\frac{E+Nhv}{E}\right) + 1 - Nhv} &= T.
\end{aligned}$$

3.3

Para este caso entonces el termino que mas aporta es el primero por lo tanto quedamos con:

$$\begin{aligned}
T &= \frac{hv}{k\ln\left(1+\frac{Nhv}{E}\right)} \\
&= \frac{hv}{k\frac{Nhv}{E}} \\
&= \frac{Ehv}{kNhv} \\
&= \frac{E}{Nk}.
\end{aligned}$$

Chapter 4

4.1

Podemos expresar este sistema simplemente reemplazando:

$$\begin{aligned}
S\left(N,V,E\right) &= Nk\ln\left[\frac{V}{h^3}\left(\frac{4\pi mE}{3N}\right)^{\frac{3}{2}}\right] + \frac{3}{2}Nk \\
E &= \frac{3}{2}NkT \\
S\left(N,V,T\right) &= Nk\ln\left[\frac{V}{h^3}\left(\frac{4\pi m\frac{3}{2}NkT}{3N}\right)^{\frac{3}{2}}\right] + \frac{3}{2}Nk \\
S\left(N,V,T\right) &= Nk\ln V + Nk\ln\left[\frac{1}{h^3}\left(\frac{2\pi mkT}{1}\right)^{\frac{3}{2}}\right] + \frac{3}{2}Nk \\
S\left(N,V,T\right) &= Nk\ln V + Nk\ln\left[\left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}}\right] + \frac{3}{2}Nk \\
S\left(N,V,T\right) &= Nk\ln V + \frac{3}{2}Nk\ln\left[\left(\frac{2\pi mkT}{h^2}\right)\right] + \frac{3}{2}Nk \\
S\left(N,V,T\right) &= Nk\ln V + \frac{3}{2}Nk\left\{1 + \ln\left[\left(\frac{2\pi mkT}{h^2}\right)\right]\right\} \\
&.
\end{aligned}$$

Con esto entonces lo unico que queda es reemplazar:

$$\begin{aligned}
S_1\left(N_1,V_1,T\right) &= N_1k\ln V_1 + \frac{3}{2}N_1k\left\{1 + \ln\left[\left(\frac{2\pi mkT}{h^2}\right)\right]\right\} \\
S_2\left(N_2,V_2,T\right) &= N_2k\ln V_2 + \frac{3}{2}N_2k\left\{1 + \ln\left[\left(\frac{2\pi mkT}{h^2}\right)\right]\right\} \\
S_T &= \sum_{i=1}^{2}\left[N_ik\ln V + \frac{3}{2}N_ik\left\{1 + \ln\left(\frac{2\pi mkT}{h^2}\right)\right\}\right].
\end{aligned}$$

4.2

$$\Delta S = S_T - S_1 - S_2.$$

Notemos que con estas expresiones el segundo termino se cancelan mutuamente. Por lo tanto, solo nos interesa quedarnos con $N_i k \ln V$ lo que implica

$$\begin{aligned}\Delta S &= N_1 k \ln V + N_2 k \ln V - N_1 k \ln V_1 - N_2 k \ln V_2 \\ &= k \left[N_1 \ln V + N_2 \ln V - N_1 \ln V_1 - N_2 \ln V_2\right] \\ &= k \left[N_1 \ln \left(\frac{V}{V_1}\right) + N_2 \ln \left(\frac{V}{V_2}\right)\right] \\ &= k \left[N_1 \ln \left(\frac{V_1 + V_2}{V_1}\right) + N_2 \ln \left(\frac{V_1 + V_2}{V_2}\right)\right].\end{aligned}$$

4.3

Ahora en este caso partimos de que $\frac{N_1}{V_1} = \frac{N_2}{V_2} = \frac{N_1+N_2}{V_1+V_2} = \delta$. Por lo tanto, podemos despejar como:

$$\begin{aligned}\frac{N_1 + N_2}{\delta} &= V_1 + V_2 \\ \frac{N_1}{\delta} &= V_1 \\ \frac{N_2}{\delta} &= V_2.\end{aligned}$$

Ahora si lo ponemos todo dentro nos queda:

$$\begin{aligned}\Delta S &= k \left[N_1 \ln \left(\frac{\frac{N_1+N_2}{\delta}}{\frac{N_1}{\delta}}\right) + N_2 \ln \left(\frac{\frac{N_1+N_2}{\delta}}{\frac{N_2}{\delta}}\right)\right] \\ &= k \left[N_1 \ln \left(\frac{N_1 + N_2}{N_1}\right) + N_2 \ln \left(\frac{N_1 + N_2}{N_2}\right)\right].\end{aligned}$$

4.4

En este caso volvemos a partir de la expresión anterior:

$$\begin{aligned}\Delta S &= k \left[N_1 \ln \left(\frac{N_1 + N_2}{N_1}\right) + N_2 \ln \left(\frac{N_1 + N_2}{N_2}\right)\right] \\ &= k \left[N_1 \ln (N_1 + N_2) - N_1 \ln (N_1) + N_2 \ln (N_1 + N_2) - N_2 \ln (N_2)\right] \\ &= k \left[(N_1 + N_2) \ln (N_1 + N_2) - N_1 \ln (N_1) - N_2 \ln (N_2)\right] \\ &= k \left[(N_1 + N_2) \ln (N_1 + N_2) - N_1 \ln (N_1) - N_2 \ln (N_2) + N_1 - N_1 + N_2 - N_2\right] \\ &= k \left[(N_1 + N_2) \ln (N_1 + N_2) - (N_1 + N_2) - N_1 \ln (N_1) + N_1 - N_2 \ln (N_2) + N_2\right] \\ &= k \left[(N_1 + N_2) \ln (N_1 + N_2) - (N_1 + N_2) - (N_1 \ln (N_1) - N_1) + (N_2 \ln (N_2) - N_2)\right].\end{aligned}$$

Esto dado que N es grande nos permite usar la aproximación de Stirling $\ln (N!) = N \ln (N) - N$. Lo que entonces nos permite poner esto como se nos pide:

$$\begin{aligned}\Delta S &= k \left[(N_1 + N_2) \ln (N_1 + N_2) - (N_1 + N_2) - (N_1 \ln (N_1) - N_1) - (N_2 \ln (N_2) - N_2)\right] \\ \Delta S &= k \left[\ln ((N_1 + N_2)!) - \ln (N_1!) - \ln (N_2!)\right].\end{aligned}$$

4.5

Cuando despejamos con la consideración de Gibbs queda:

$$
\begin{aligned}
S(N,V,E) &= Nk\ln\left[\frac{V}{h^3}\left(\frac{4\pi mE}{3N}\right)^{\frac{3}{2}}\right]+\frac{3}{2}Nk-k\ln(N!)\\
S(N,V,E) &= Nk\ln\left[\frac{V}{h^3}\left(\frac{4\pi mE}{3N}\right)^{\frac{3}{2}}\right]+\frac{3}{2}Nk-kN\ln(N)+Nk\\
S(N,V,E) &= Nk\ln\left[\frac{V}{Nh^3}\left(\frac{4\pi mE}{3N}\right)^{\frac{3}{2}}\right]+\frac{5}{2}Nk\\
&= Nk\ln\left(\frac{V}{N}\right)+Nk\ln\left[\left(\frac{4\pi mE}{3Nh^2}\right)^{\frac{3}{2}}\right]+\frac{5}{2}Nk\\
&= Nk\ln\left(\frac{V}{N}\right)+\frac{3}{2}Nk\ln\left[\left(\frac{4\pi mE}{3Nh^2}\right)\right]+\frac{5}{2}Nk\\
&= Nk\ln\left(\frac{V}{N}\right)+\frac{3}{2}Nk\ln\left[\left(\frac{4\pi m\frac{3}{2}NkT}{3Nh^2}\right)\right]+\frac{5}{2}Nk\\
&= Nk\ln\left(\frac{V}{N}\right)+\frac{3}{2}Nk\left\{\frac{5}{3}+\ln\left(\frac{2\pi mkT}{h^2}\right)\right\}.
\end{aligned}
$$

Ahora volvamos a notar que en el caso de que ΔS el segundo termino se cancelaría mutuamente. Por lo tanto solo nos interesa el primer caso con lo cual tendríamos:

$$
\Delta S = k\left[(N_1+N_2)\ln\left(\frac{V_1+V_2}{N_1+N_2}\right)-N_1\ln\left(\frac{V_1}{N_1}\right)-N_2\ln\left(\frac{V_2}{N_2}\right)\right].
$$

Ahora, en el caso de $\frac{N_1}{V_1}=\frac{N_2}{V_2}=\frac{N_1+N_2}{V_1+V_2}=\delta^{-1}$ nos queda

$$
\begin{aligned}
\Delta S &= k\left[(N_1+N_2)\ln(\delta)-N_1\ln(\delta)-N_2\ln(\delta)\right]\\
\Delta S &= k\left[(N_1+N_2)\ln(\delta)-(N_1+N_2)\ln(\delta)\right]\\
\Delta S &= k\left[0\right]\\
\Delta S &= 0.
\end{aligned}
$$

Chapter 5

5.1

Si partimos de un espacio de fase entonces la manera en la que estos puntos cambian en el espacio es:

$$\frac{\partial}{\partial t}\int_{\omega}\rho d\omega$$
$$\omega = d^{3N}qd^{3N}p.$$

Ahora bien, el ratio neto en que los puntos se mueven fuera de nuestra superficie queda:

$$\int_{\sigma}\rho\left(v\cdot\hat{n}\right)d\sigma = \int_{\omega}div\left(\rho v\right)d\omega$$
$$div(pv) = \sum_{i=1}^{3N}\left\{\frac{\partial}{\partial q_i}\left(\rho\dot{q}_i\right)+\frac{\partial}{\partial p_i}\left(\rho\dot{p}_i\right)\right\}.$$

Dado que no hay fuentes ni sumideros entonces sabemos que:

$$\frac{\partial}{\partial t}\int_{\omega}\rho d\omega = -\int_{\omega}div\left(\rho v\right)d\omega$$
$$\int_{\omega}\left\{\frac{\partial\rho}{\partial t}+div\left(\rho v\right)\right\}d\omega = 0$$
$$\frac{\partial\rho}{\partial t}+div\left(\rho v\right) = 0$$
$$\frac{\partial\rho}{\partial t}+\sum_{i=1}^{3N}\left(\frac{\partial\rho}{\partial q_i}\dot{q}_i+\frac{\partial\rho}{\partial p_i}\dot{p}_i\right)+\rho\sum_{i=1}^{3N}\left(\frac{\partial\dot{q}_i}{\partial q_i}+\frac{\partial\dot{p}_i}{\partial p_i}\right) = 0$$
$$\frac{\partial\dot{q}_i}{\partial q_i} = \frac{\partial^2 H}{\partial q_i\partial p_i} = -\frac{\partial\dot{p}_i}{\partial p_i}$$
$$\frac{\partial\rho}{\partial t}+\sum_{i=1}^{3N}\left(\frac{\partial\rho}{\partial q_i}\dot{q}_i+\frac{\partial\rho}{\partial p_i}\dot{p}_i\right)+\rho\sum_{i=1}^{3N}\left(-\frac{\partial\dot{p}_i}{\partial p_i}+\frac{\partial\dot{p}_i}{\partial p_i}\right) = 0$$
$$\frac{\partial\rho}{\partial t}+\sum_{i=1}^{3N}\left(\frac{\partial\rho}{\partial q_i}\dot{q}_i+\frac{\partial\rho}{\partial p_i}\dot{p}_i\right) = 0$$

.

5.2

Partimos del teorema:

$$\frac{\partial\rho}{\partial t} + \sum_{i=1}^{3N}\left(\frac{\partial\rho}{\partial q_i}\dot{q}_i + \frac{\partial\rho}{\partial p_i}\dot{p}_i\right) = 0$$

$$\rho = cte$$

$$\frac{\partial cte}{\partial t} + \sum_{i=1}^{3N}\left(\frac{\partial cte}{\partial q_i}\dot{q}_i + \frac{\partial cte}{\partial p_i}\dot{p}_i\right) = 0$$

$$0 + \sum_{i=1}^{3N}\left(0\dot{q}_i + 0\dot{p}_i\right) = 0$$

$$0 = 0.$$

5.3

Partimos del teorema:

$$\frac{\partial\rho}{\partial t} + \sum_{i=1}^{3N}\left(\frac{\partial\rho}{\partial q_i}\dot{q}_i + \frac{\partial\rho}{\partial p_i}\dot{p}_i\right) = 0$$

$$\frac{\partial\rho\left[H(q_i,p_i)\right]}{\partial t} + \sum_{i=1}^{3N}\left(\frac{\partial\rho\left[H(q_i,p_i)\right]}{\partial q_i}\dot{q}_i + \frac{\partial\rho\left[H(q_i,p_i)\right]}{\partial p_i}\dot{p}_i\right) = 0$$

$$0 + \sum_{i=1}^{3N}\left(\rho\frac{\partial\left[H(q_i,p_i)\right]}{\partial q_i}\dot{q}_i + \rho\frac{\partial\left[H(q_i,p_i)\right]}{\partial p_i}\dot{p}_i\right) = 0$$

$$\frac{\partial\left[H(q_i,p_i)\right]}{\partial p_i} = \dot{q}_i$$

$$\frac{\partial\left[H(q_i,p_i)\right]}{\partial q_i} = -\dot{p}_i$$

$$0 + \sum_{i=1}^{3N}\left(-\rho\dot{p}_i\dot{q}_i + \rho\dot{q}_i\dot{p}_i\right) = 0$$

$$0 = 0.$$

Chapter 6

6.1

En este caso tenemos solo un grado de libertad por lo tanto tenemos solo una variable a la que llamaremos θ. Por lo tanto, su momento angular seria $p_\theta = mL^2\theta$