Moderna-Complementaria Tarea 3

10/02/2023

1.

2. Para comenzar hagamos recuento del teorema de Ehrenfest el cual dice

$$\frac{d < A >}{dt} = \frac{i}{h} \left\langle [\hat{H}, \hat{A}] \right\rangle + \left\langle \frac{\partial \hat{A}}{\partial t} \right\rangle. \tag{1}$$

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Y dado que partimos desde

$$\frac{d < xp >}{dt}$$

Ahora bien, tomemos también en cuenta que

$$[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + [\hat{A}, \hat{C}]\hat{B}$$
 (2)

$$[f(\hat{A}), \hat{B}] = \frac{\partial f}{\partial A}[\hat{A}, \hat{B}] \tag{3}$$

$$\hat{K} = \frac{\hat{P}}{2m} \tag{4}$$

$$\hat{p} = \frac{-\hbar}{i} \frac{\partial}{\partial x} \tag{5}$$

Ahora bien, con todo esto partamos el desarrollo

$$\begin{split} \frac{d < xp>}{dt} &= \frac{i}{\hbar} \left\langle [\hat{H}, xp] \right\rangle + \left\langle \frac{\partial xp}{\partial t} \right\rangle \\ &= \frac{i}{\hbar} \left\langle [\hat{H}, x]p + [\hat{H}, p]x \right\rangle + \left\langle p \frac{\partial x}{\partial t} + x \frac{\partial p}{\partial t} \right\rangle \\ \frac{\partial p}{\partial t} &= 0 \\ \frac{\partial x}{\partial t} &= V \\ &= \frac{i}{\hbar} \left\langle [\hat{H}, x]p + [\hat{H}, p]x \right\rangle + \left\langle pV \right\rangle \\ &= \frac{i}{\hbar} \left\langle [\hat{H}, x]p + [\hat{H}, p]x \right\rangle - \frac{i}{\hbar} \left\langle \frac{\partial v}{\partial x} \right\rangle \\ [\hat{H}, x]f &= \hat{H}\hat{x} - \hat{x}\hat{H} \\ &= \left(\frac{\hat{p}^2}{2m} + \hat{V} \right) \hat{x}f - \hat{x} \left(\frac{\hat{p}^2}{2m} + \hat{v} \right) f \\ &= \left(\frac{1}{2m} \left(-i\hbar \frac{\partial}{\partial x} \right)^2 \hat{x}f + \hat{V}\hat{x}f \right) - \hat{x} \left(\frac{1}{2m} \left(-i\hbar \frac{\partial}{\partial x} \right) f + \hat{V}f \right) \\ &= \left(\frac{1}{2m} \left(\hbar^2 \frac{\partial^2}{\partial x^2} \right) \hat{x}f + \hat{V}\hat{x}f \right) - \hat{x} \left(\frac{1}{2m} \left(\hbar^2 \frac{\partial^2}{\partial x^2} \right) f + \hat{V}f \right) \end{split}$$

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