Stochastic Processes for Networking
HW4: Solution

Problem 1 (25 %)

Let the state space be $\{\text{RRR}, \text{RRD}, \text{RDR}, \text{RDD}, \text{DRR}, \text{DRD}, \text{DDR}, \text{DDD}\}$

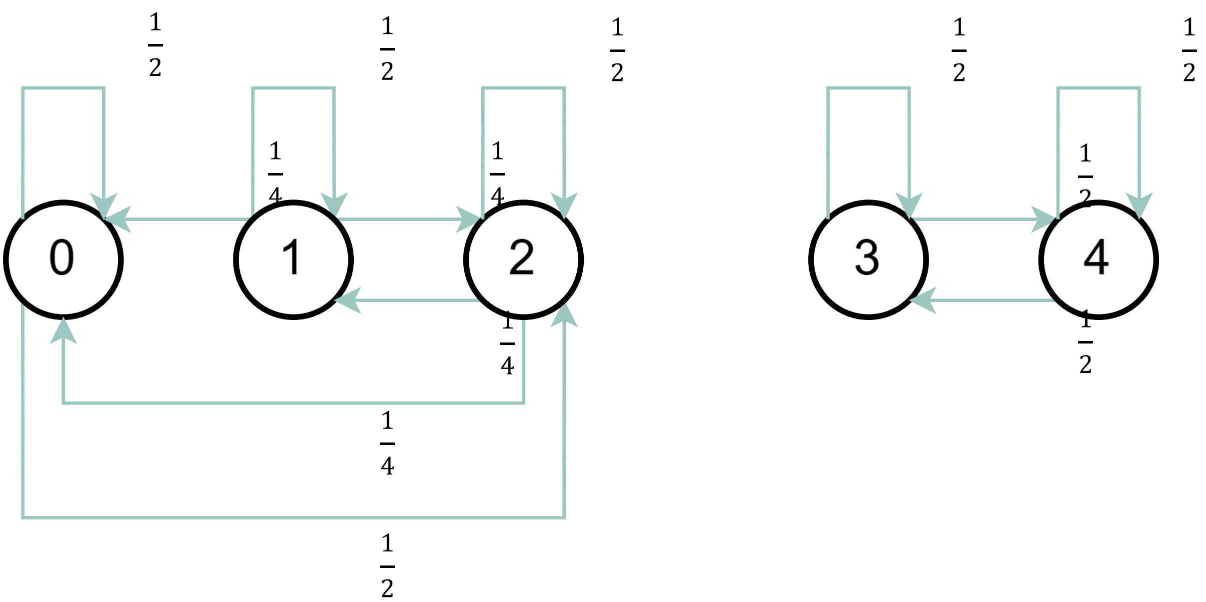
$$P = \begin{bmatrix} 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \\ 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.8 \end{bmatrix}$$

Problem 2 (10 %, 15 %)

Cubing the transition probability matrix, we obtain $P^3 = \begin{bmatrix} \frac{13}{36} & \frac{11}{54} & \frac{47}{108} \\ \frac{4}{9} & \frac{4}{27} & \frac{11}{27} \\ \frac{5}{12} & \frac{2}{9} & \frac{13}{36} \end{bmatrix}$

Thus, $E[X_3] = P(X_3 = 1) + 2P(X_3 = 2) = \frac{1}{4}P_{01}^3 + \frac{1}{4}P_{11}^3 + \frac{1}{2}P_{21}^3 + 2[\frac{1}{4}P_{02}^3 + \frac{1}{4}P_{12}^3 + \frac{1}{2}P_{22}^3]$

Problem 3 (10 %, 15 %)



State 0: recurrent

State 1: recurrent

State 2: recurrent

State 3: recurrent

State 4: recurrent

Problem 4 (25 %)

(a)$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & 0 & \frac{1}{4} \\ \frac{3}{4} & \frac{1}{4} & 0 \end{bmatrix}$

(b)$P^2 = \begin{bmatrix} \frac{3}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{3}{16} & \frac{7}{16} & \frac{3}{8} \\ \frac{3}{16} & \frac{3}{8} & \frac{7}{16} \end{bmatrix}$

At time 2, the probability of Leonard at A is $\frac{3}{4}$, at B is $\frac{1}{8}$, and at C is $\frac{1}{8}$

At time 3, the probability of Leonard at B is

$$\begin{aligned} P_{AB}^{(3)} &= P_{AA}^{(2)}P_{AB} + P_{AB}^{(2)}P_{BB} + P_{AC}^{(2)}P_{CB} \\ &= \frac{3}{4}\cdot\frac{1}{2} + \frac{1}{8}\cdot 0 + \frac{1}{8}\cdot\frac{1}{4} \\ &= \frac{13}{32} \end{aligned}$$