CS5120 VLSI System Design, Spring 2025

DNN Mapping Part 2: Tiling for Hardware Structure

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Lecture 07

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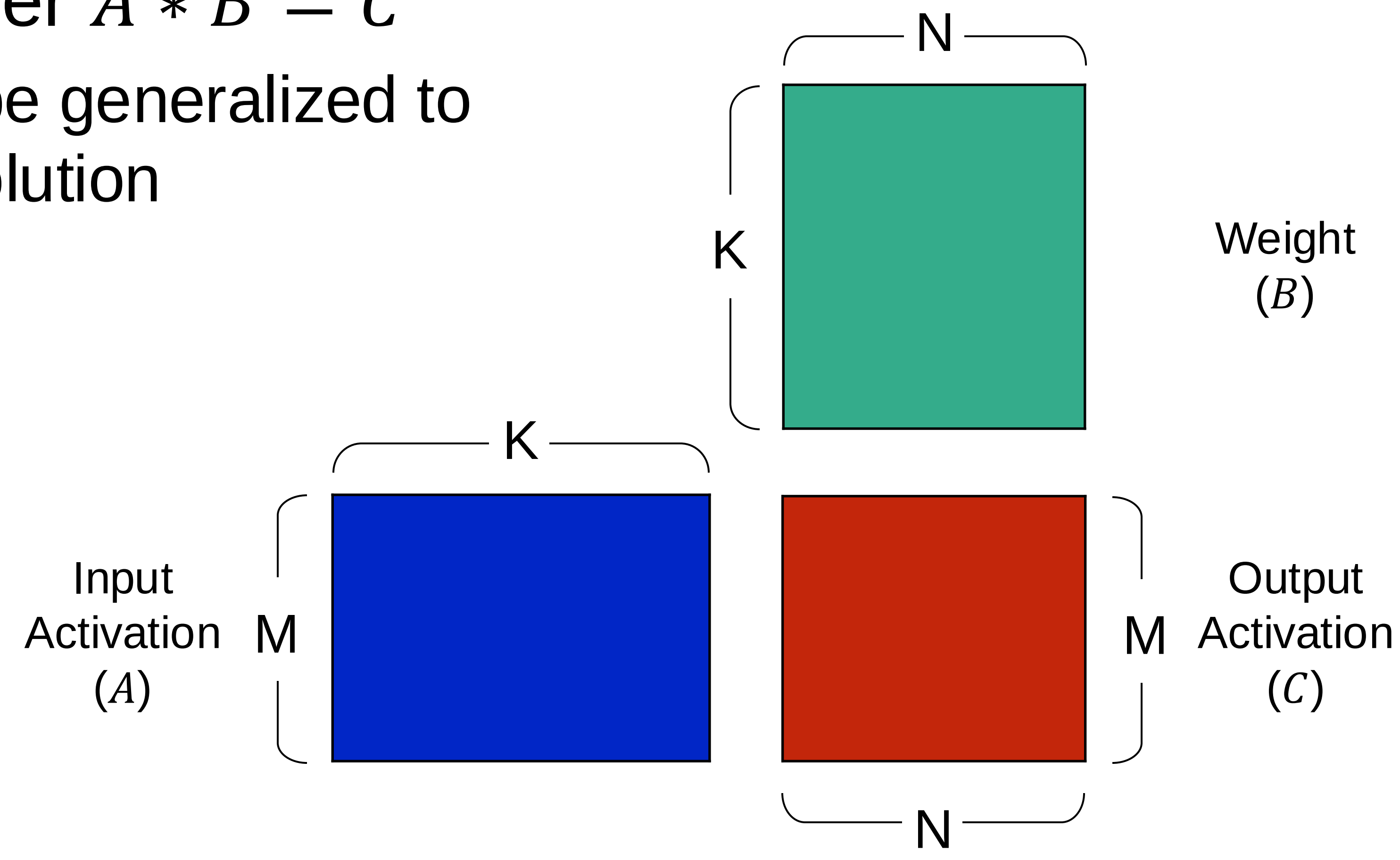
Outline

- Hardware Resource Constraints
- Mapping Space
- Tuning of DNN Mapping

Recap: Matrix Multiplication for Fully-Connected Layer

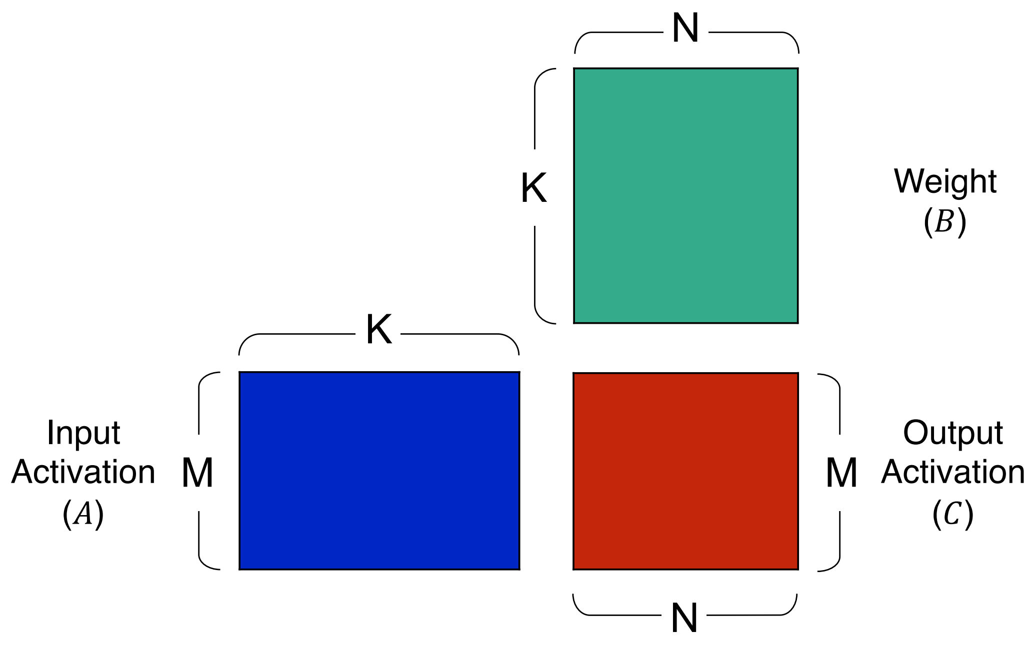
- Consider $A * B = C$
  - Can be generalized to convolution



[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Recap: Loop Nest for Matrix Multiplication

- Consider $A * B = C$



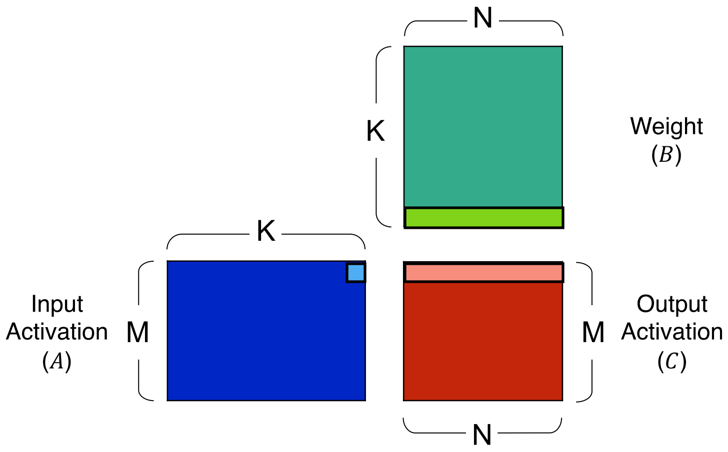

```
for (m=0; m<M; m++) {
  for (n=0; n<N; n++) {
    OA[n,m] = 0;
    for (k=0; k<K; k++) {
      OA[n,m] +=  IA[m, k] * W[k, n];
    }
    OA[n,m] = Activation(OA[n,m]);
  }
}
```

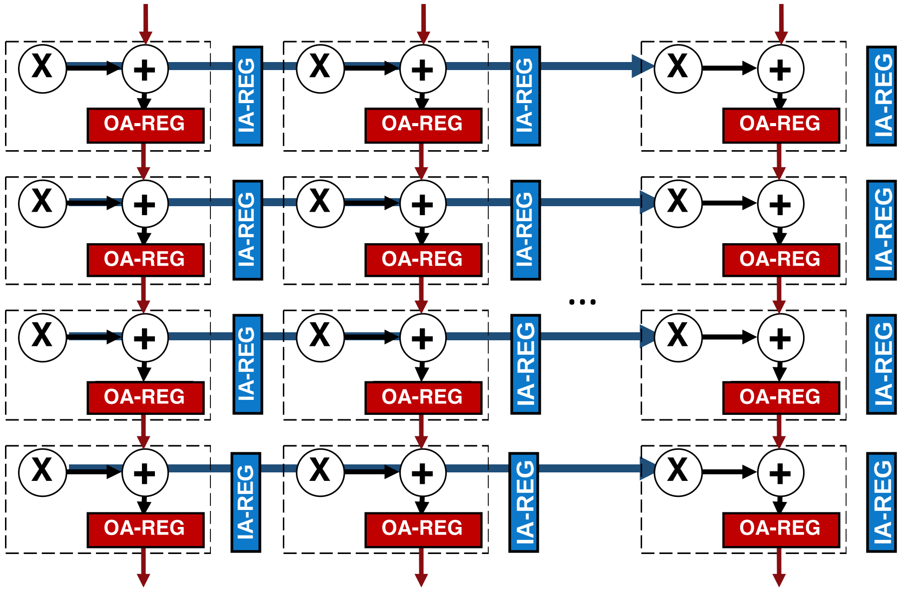
For each output activation

Reduction

[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Recap: Mixed Datapath Optimization: TPU






```
for (m=0; m<M; m++) {
  parallel_for (n=0; n<N; n++) {
    OA[n,m] = 0;
    parallel_for (k=0; k<K; k++) {
      OA[n,m] +=  IA[m,k] * W[k,n];
    }
    OA[n,m] = Activation(OA[n,m]);
  }
}
```

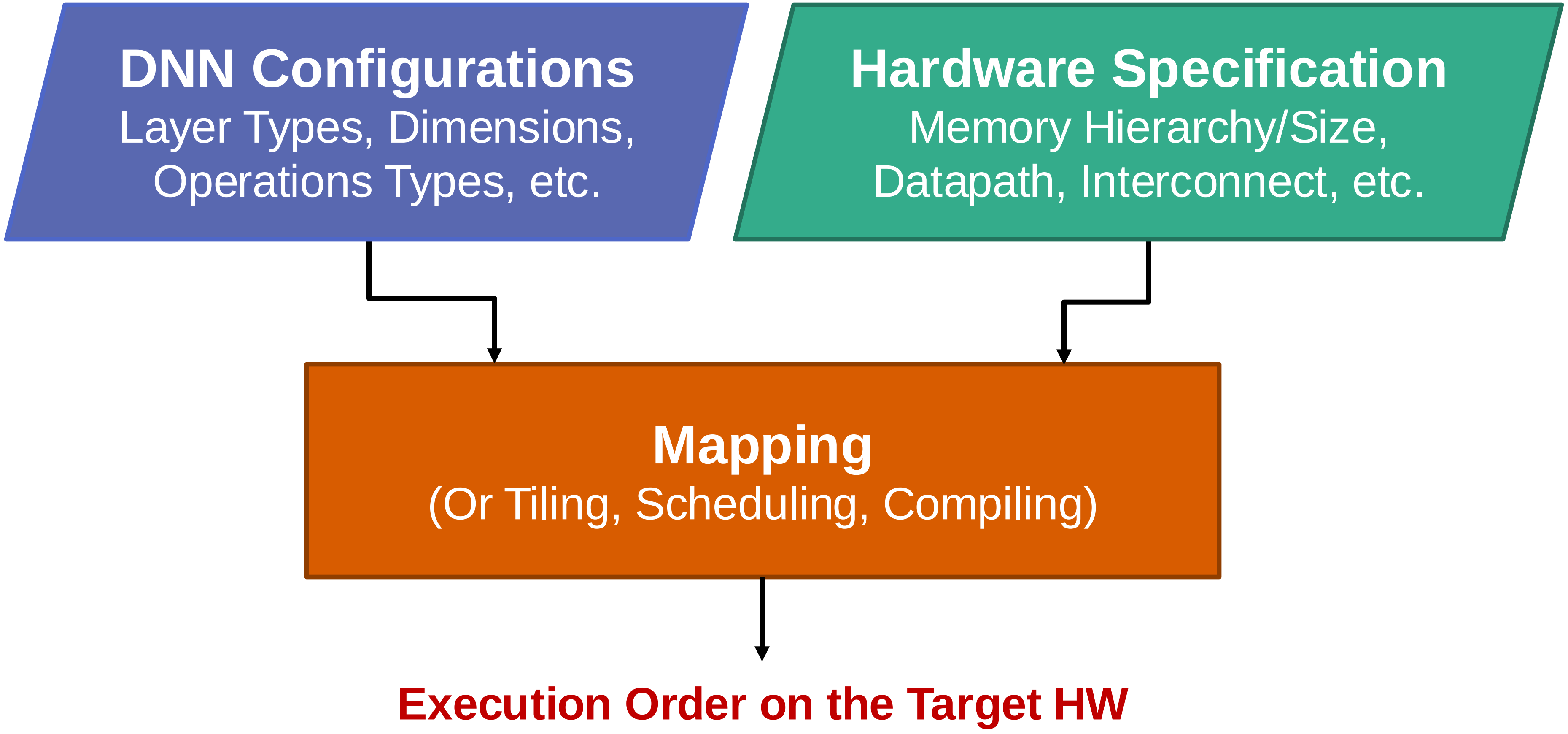
- Systolic accumulation
- Systolic multicast

→ Area vs. scalability

→ Latency vs. pipeline throughput

[Source: Prof. Sophia Shao, EE290-2, Berkeley]

DNN Mapping Problem

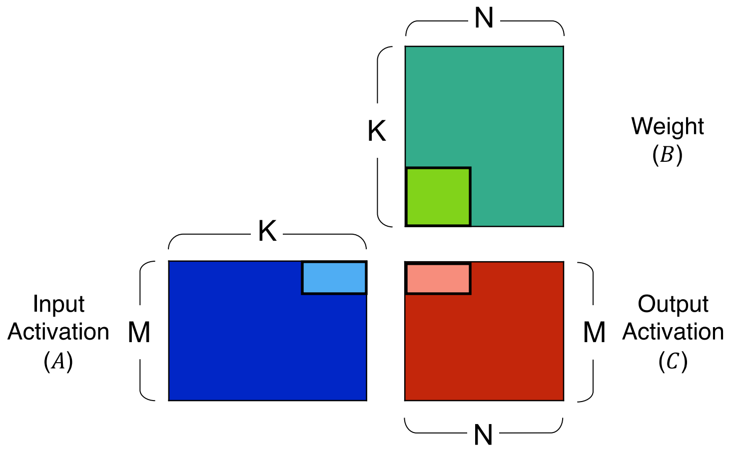


- Mapping objective: efficient for the performance (latency) and/or energy, etc.

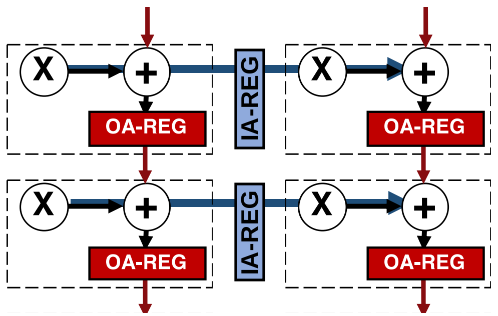
Hardware Resource Constraints

Constraint 1: Systolic Array Size < K and/or N

- DNN Configuration:
  Matrix Multiplication



- Hardware Specification:
  Systolic array size: 2x2



- Notation: K?/N? ➔ loop bounds

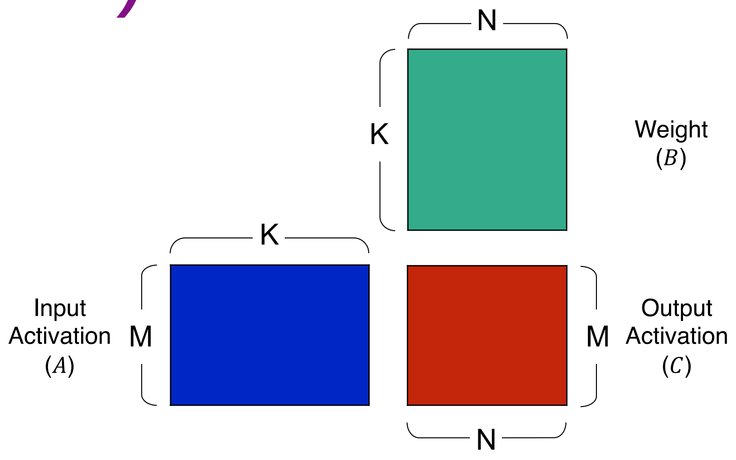
```
for (m=0; m<M; m++) {
  for (n1=0; n1<N1; n1++) {
    OA[n1*N0:(n1+1)*N0,m] = 0;
    for (k1=0; k1<K1; k1++) {
      parallel_for (n0=0; n0<N0; n0++) {
        parallel_for (k0=0; k0<K0; k0++) {
          OA[n1*N0+n0,m] += IA[m,k1*K0+k0]
                     * W[k1*K0+k0,n1*N0+n0];
}}}}}
```

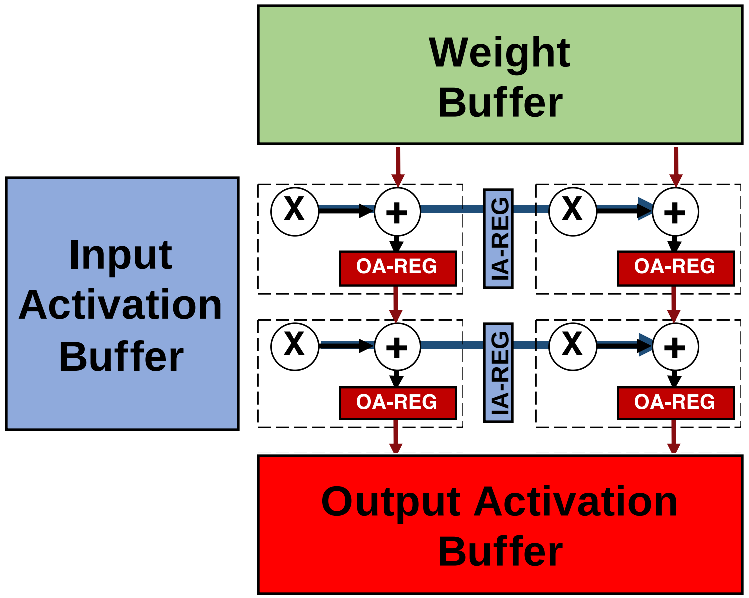
Temporal Tiling

Spatial Tiling

[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Constraint 2: Weight Buffer Size < K * N (1/3)



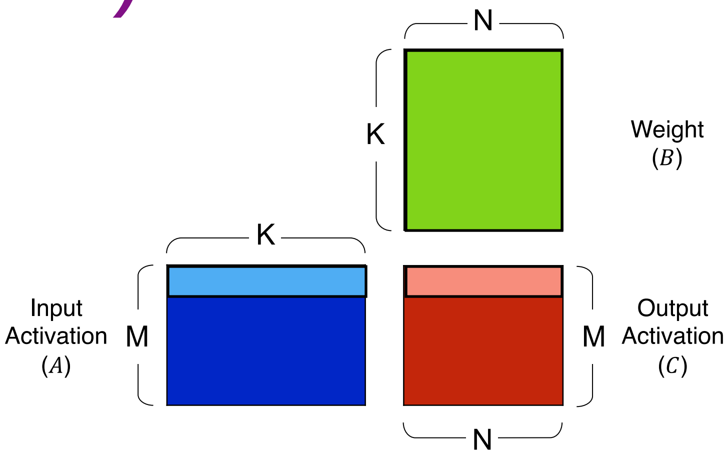


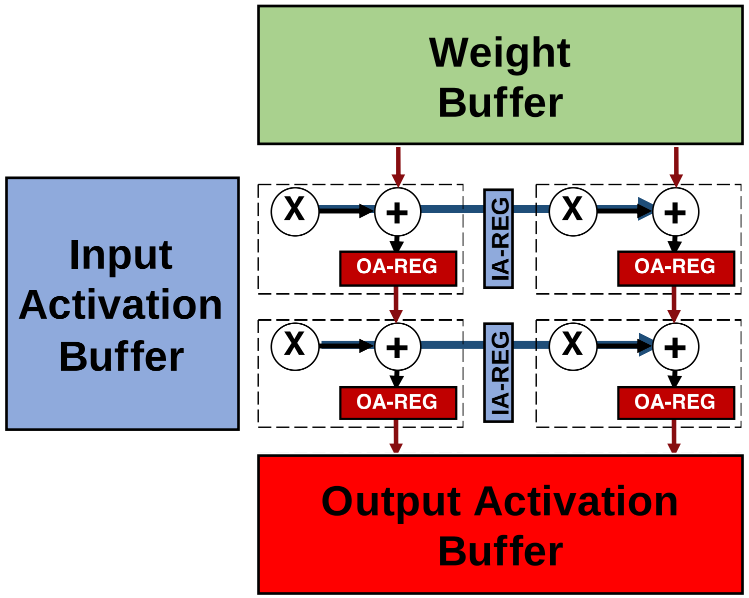
- Hardware Specification:
  - Systolic array size: 2x2
  - Explicit data movement (or data orchestration)

```
for (m=0; m<M; m++) {

  for (n1=0; n1<N1; n1++) {
    OA[n1*N0:(n1+1)*N0,m] = 0;
    for (k1=0; k1<K1; k1++) {
      parallel_for (n0=0; n0<N0; n0++) {
        parallel_for (k0=0; k0<K0; k0++) {
          OA[n1*N0+n0,m] += IA[m,k1*K0+k0]
                       * W[k1*K0+k0,n1*N0+n0];
  }}}}

}
```

[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Constraint 2: Weight Buffer Size < K * N (2/3)



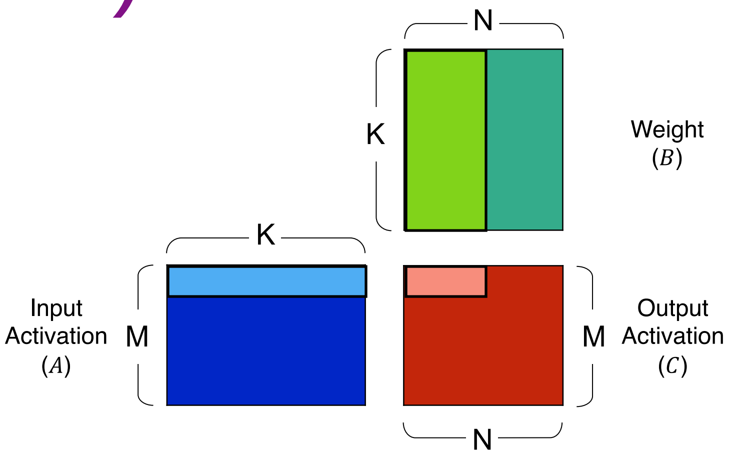


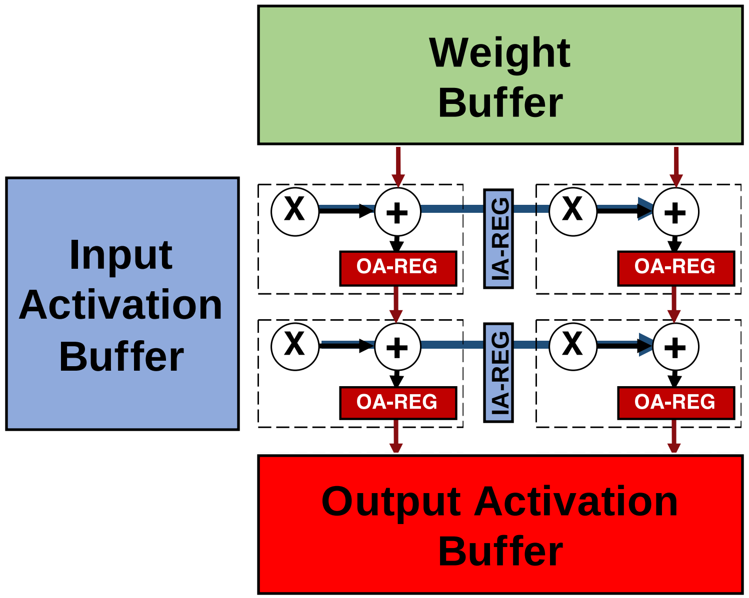
[Source: Prof. Sophia Shao, EE290-2, Berkeley]

- Hardware Specification:
  - Systolic array size: 2x2
  - Explicit data movement (or data orchestration)
  - Weight, input activation, output activation buffer sizes

```
for (m=0; m<M; m++) {
  mvin(W[0:K,0:N]);        // W buffer >= K*N
  mvin(IA[m:m+1,0:K]);    // IA buffer >= 1*K
  for (n1=0; n1<N1; n1++) {
    OA[n1*N0:(n1+1)*N0,m] = 0;
    for (k1=0; k1<K1; k1++) {
      parallel_for (n0=0; n0<N0; n0++) {
        parallel_for (k0=0; k0<K0; k0++) {
          OA[n1*N0+n0,m] += IA[m,k1*K0+k0]
                  * W[k1*K0+k0,n1*N0+n0];
  }}}}
  mvout(OA[0:N,m:m+1]);  // OA buffer >= 1*N
}
```

Constraint 2: Weight Buffer Size < K * N (3/3)

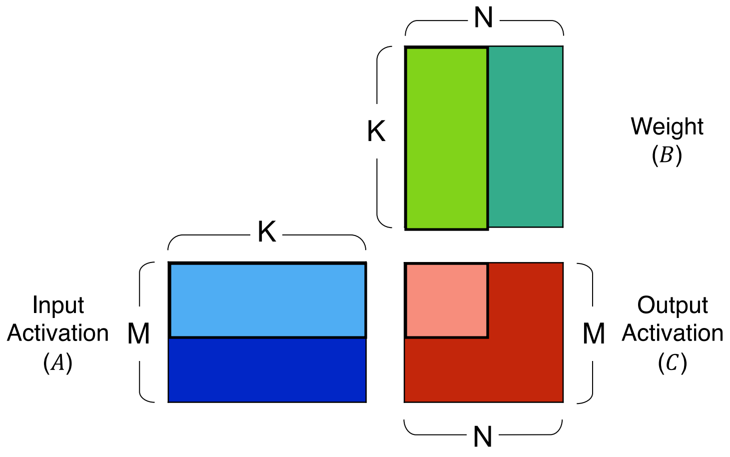


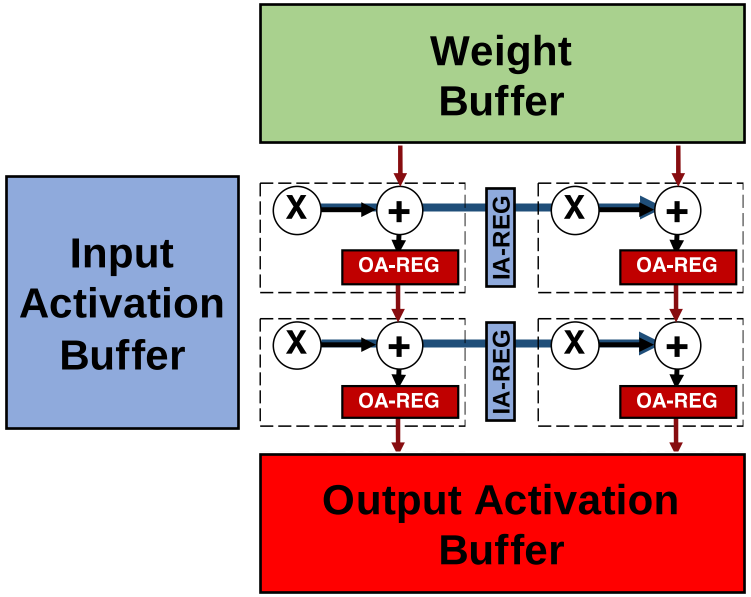



```
for (m=0; m<M; m++) {
  // IA buffer: 1*K
  mvin(IA[m:m+1,0:K]);
  for (n2=0; n2<N2; n2++) {
    // W buffer: N1*N0*K < K*N
    mvin(W[0:K,n2*N1*N0:(n2+1)*N1*N0]);
    OA[n2*N1*N0:(n2+1)*N1*N0,m:m+1]=0;
    for (n1=0; n1<N1; n1++) {
      for (k1=0; k1<K1; k1++) {
        parallel_for (n0=0; n0<N0; n0++) {
          parallel_for (k0=0; k0<K0; k0++) {
            OA[n2*N1*N0+n1*N0+n0,m]
              += IA[m,k1*K1+k0]
              * W[k1*K0+k0,n2*N1*N0+n1*N0+n0];
    }}}}
    mvout(OA[n2*N1*N0:(n2+1)*N1*N0,m:m+1]);
}}
```

[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Constraint 3: Input Buffer Size > 1 * K

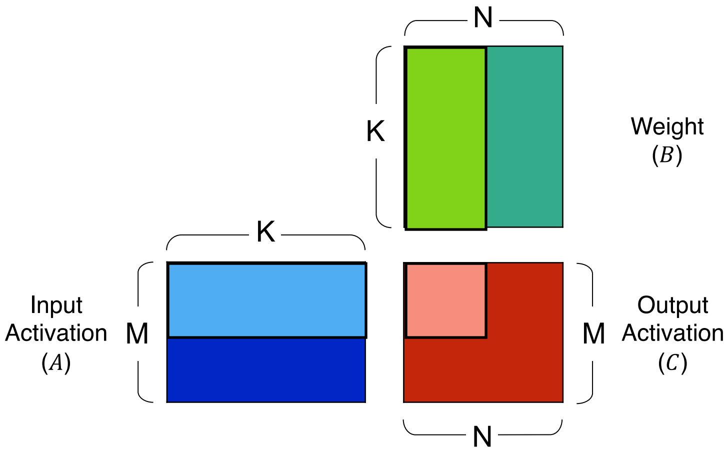


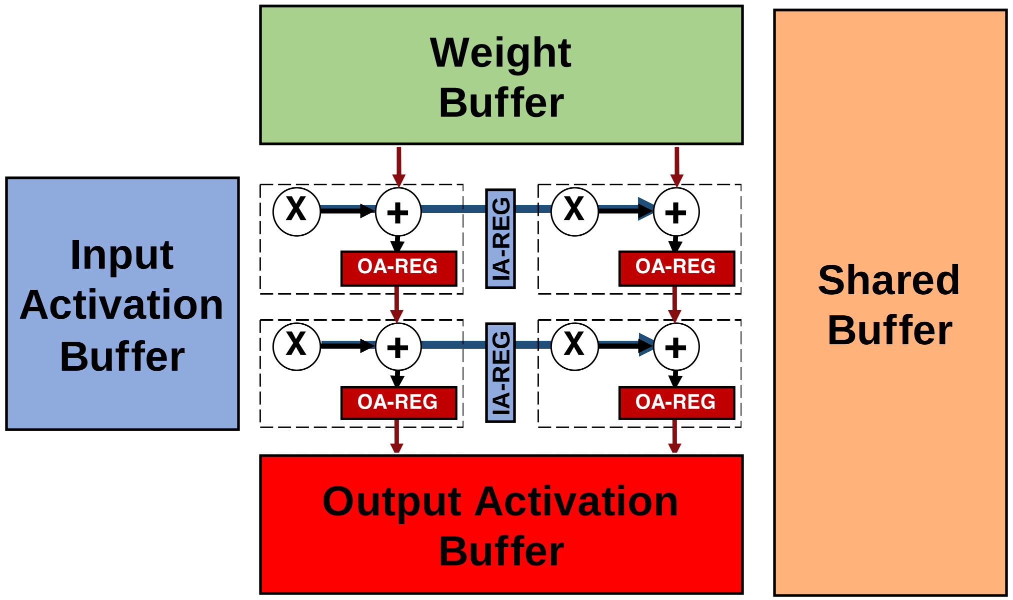



```
for (m2=0; m2<M2; m2++) {
  // IA buffer: M1*K > 1*K
  mvin(IA[m2*M1:(m2+1)*M1,0:K]);
  for (n2=0; n2<N2; n2++) {
    // W buffer: N1*N0*K < K*N
    mvin(W[0:K,n2*N1*N0:(n2+1)*N1*N0]);
    OA[n2*N1*N0:(n2+1)*N1*N0,m2*M1:(m2+1)*M1]=0;
    for (m1=0; m1<M1; m1++) {
      for (n1=0; n1<N1; n1++) {
        for (k1=0; k1<K1; k1++) {
          parallel_for (n0=0; n0<N0; n0++) {
            parallel_for (k0=0; k0<K0; k0++) {
              OA[n2*N1*N0+n1*N0+n0,m2*M1+m1]
                += IA[m2*M1+m1,k1*K1+k0]
                * W [k1*K0+k0,n2*N1*N0+n1*N0+n0];
    }}}}}
    mvout(OA[n2*N1*N0:(n2+1)*N1*N0, m2*M1:(m2+1)*M1]);
}}
```

[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Constraint 4: Adding Another Shared Buffer






```
for (m3=0; m3<M3; m3++) {
 for (n3=0; n3<N3; n3++) {
  // Shared buffer blocking

for (m2=0; m2<M2; m2++) {
  // IA buffer stores: M1*K
  mvin(IA[…:…]);
  for (n2=0; n2<N2; n2++) {
    // W buffer stores: N1*N0*K
    mvin(W[…:…,…:…]);
    OA[…:…,…:…]=0;
    for (m1=0; m1<M1; m1++) {
      for (n1=0; n1<N1; n1++) {
        for (k1=0; k1<K1; k1++) {
          …
  }}}}}
  mvout(OA[…:…,…:…]);
}}
}}
```


[Source: Prof. Sophia Shao, EE290-2, Berkeley]

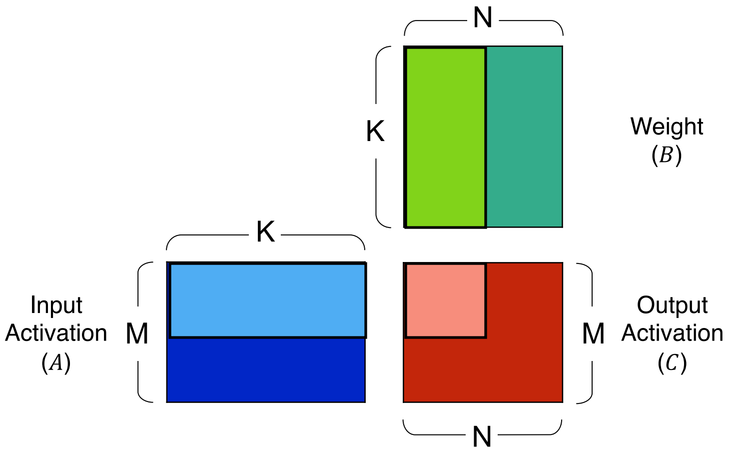
Mapping Space

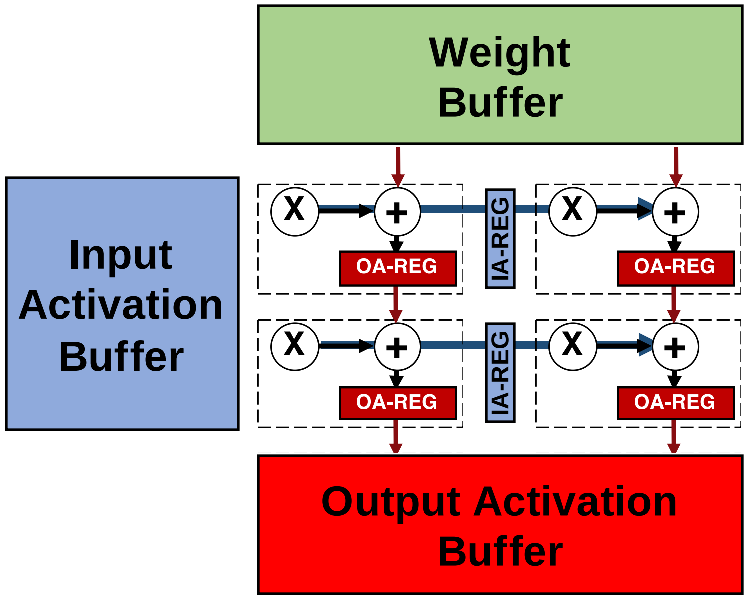
- Loop ordering
- Loop bound
- Spatial choice

Mapping Dimensions

- Loop ordering:
  - Which index goes to the inner/outer loop?
- Loop bounds:
  - What are the loop bounds (i.e., N?/K?/M?) for each loop?
- Spatial choice:
  - Which loop should be spatial/temporal?
    - Data/Model Parallelism

Mapping Problem Example

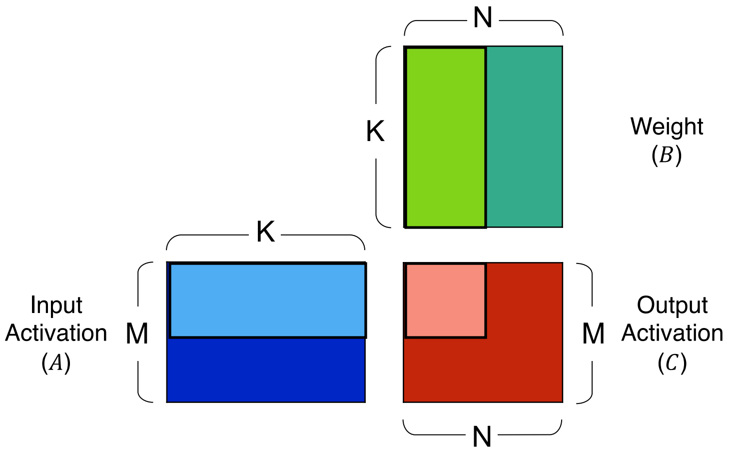


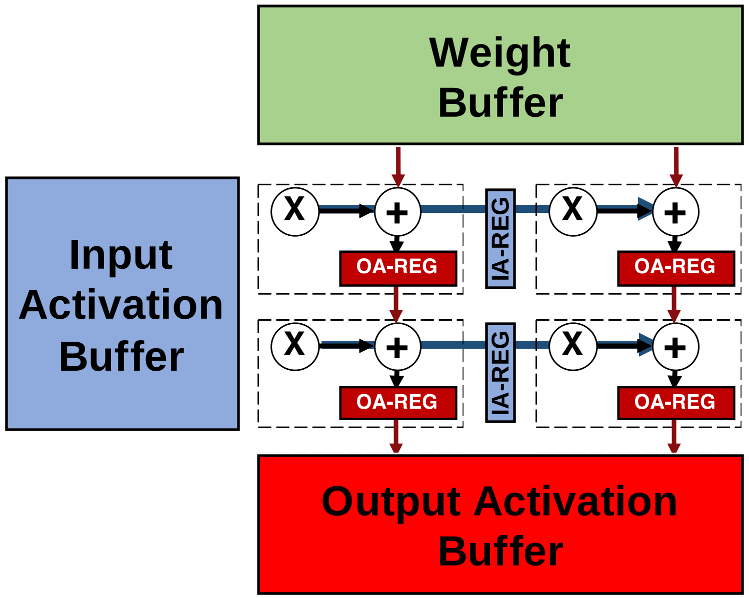



```
for (m2=0; m2<M2; m2++) {
  // IA buffer: M1*K > 1*K
  mvin(IA[m2*M1:(m2+1)*M1,0:K]);
  for (n2=0; n2<N2; n2++) {
    // W buffer: N1*N0*K < K*N
    mvin(W[0:K,n2*N1*N0:(n2+1)*N1*N0]);
    OA[n2*N1*N0:(n2+1)*N1*N0,m2*M1:(m2+1)*M1]=0;
    for (m1=0; m1<M1; m1++) {
      for (n1=0; n1<N1; n1++) {
        for (k1=0; k1<K1; k1++) {
          parallel_for (n0=0; n0<N0; n0++) {
            parallel_for (k0=0; k0<K0; k0++) {
              OA[n2*N1*N0+n1*N0+n0,m2*M1+m1]
                += IA[m2*M1+m1,k1*K1+k0]
                * W [k1*K0+k0,n2*N1*N0+n1*N0+n0];
    }}}}}
    mvout(OA[n2*N1*N0:(n2+1)*N1*N0,m2*M1:(m2+1)*M1]);
}}
```

[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Loop Bound

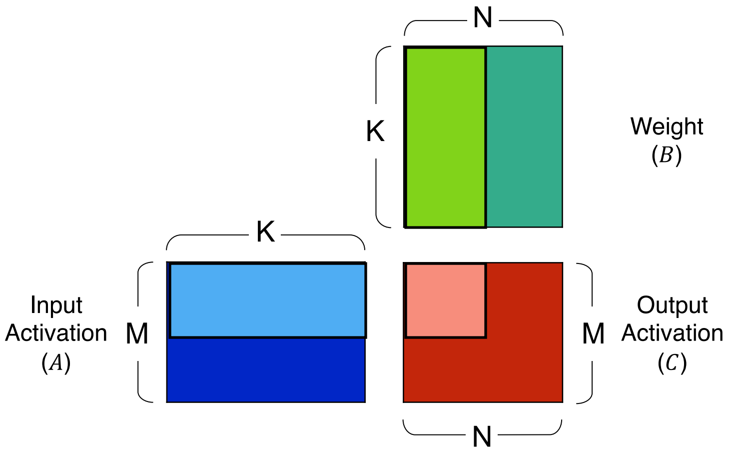


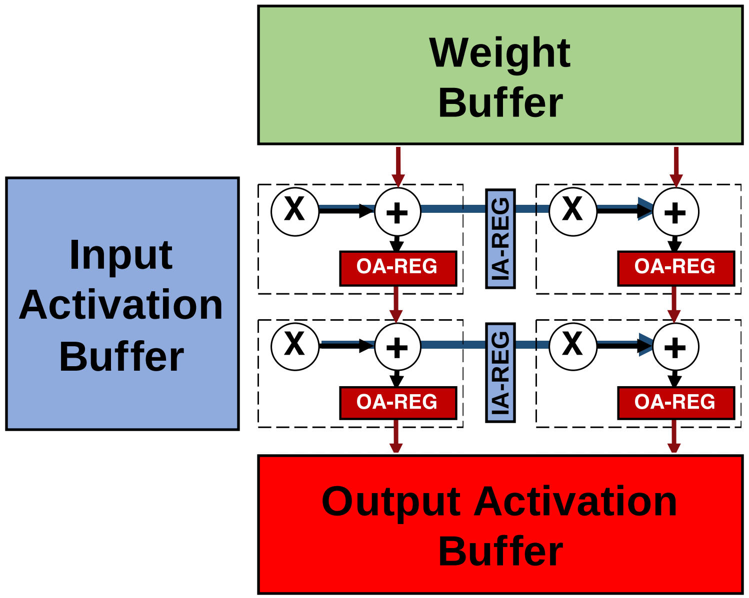



```
for (m2=0; m2<M2; m2++) {
  // IA buffer: M1*K > 1*K
  mvin(IA[m2*M1:(m2+1)*M1,0:K]);
  for (n2=0; n2<N2; n2++) {
    // W buffer: N1*N0*K < K*N
    mvin(W[0:K,n2*N1*N0:(n2+1)*N1*N0]);
    OA[n2*N1*N0:(n2+1)*N1*N0,m2*M1:(m2+1)*M1]=0;
    for (m1=0; m1<M1; m1++) {
      for (n1=0; n1<N1; n1++) {
        for (k1=0; k1<K1; k1++) {
          parallel_for (n0=0; n0<N0; n0++) {
            parallel_for (k0=0; k0<K0; k0++) {
              OA[n2*N1*N0+n1*N0+n0,m2*M1+m1]
                += IA[m2*M1+m1,k1*K1+k0]
                * W [k1*K0+k0,n2*N1*N0+n1*N0+n0];
    }}}}}                                compute_matmul();
    mvout(OA[n2*N1*N0:(n2+1)*N1*N0,m2*M1:(m2+1)*M1]);
}}
```

[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Loop Ordering (1/2)






```
for (m2=0; m2<M2; m2++) {
  // IA buffer: M1*K > 1*K
  mvin(IA[m2*M1:(m2+1)*M1,0:K]);
  for (n2=0; n2<N2; n2++) {
    // W buffer: N1*N0*K < K*N
    mvin(W[0:K,n2*N1*N0:(n2+1)*N1*N0]);
    compute_matmul(*W, *IA, *OA,
                   N2, N1, N0,
                   M2, M1,
                   K1, K0,
                   m2, n2);
    mvout(OA[n2*N1*N0:(n2+1)*N1*N0,
           m2*M1:(m2+1)*M1]);
}}
```

[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Loop Ordering (2/2)

- Option 1: Loops m2 ➔ n2

```
for (m2=0; m2<M2; m2++) {
  // IA buffer: M1*K
  mvin(IA[m2*M1:(m2+1)*M1,0:K]);
  for (n2=0; n2<N2; n2++) {
    // W buffer: N1*N0*K
    mvin(W[0:K,n2*N1*N0:(n2+1)*N1*N0]);
    compute_matmul(*W, *IA, *OA,
                   ...
                   m2, n2);
    mvout(OA[n2*N1*N0:(n2+1)*N1*N0,
             m2*M1:(m2+1)*M1]);
}}
```

IA Movement: M * K
W Movement: M2 * N * K

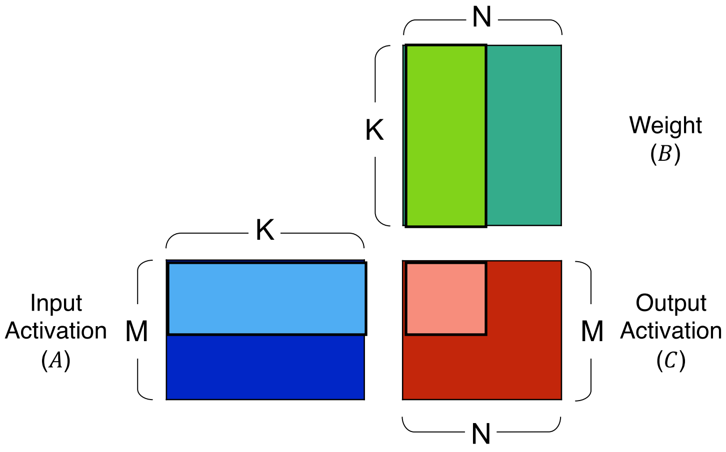
- Option 2: Loops n2 ➔ m2

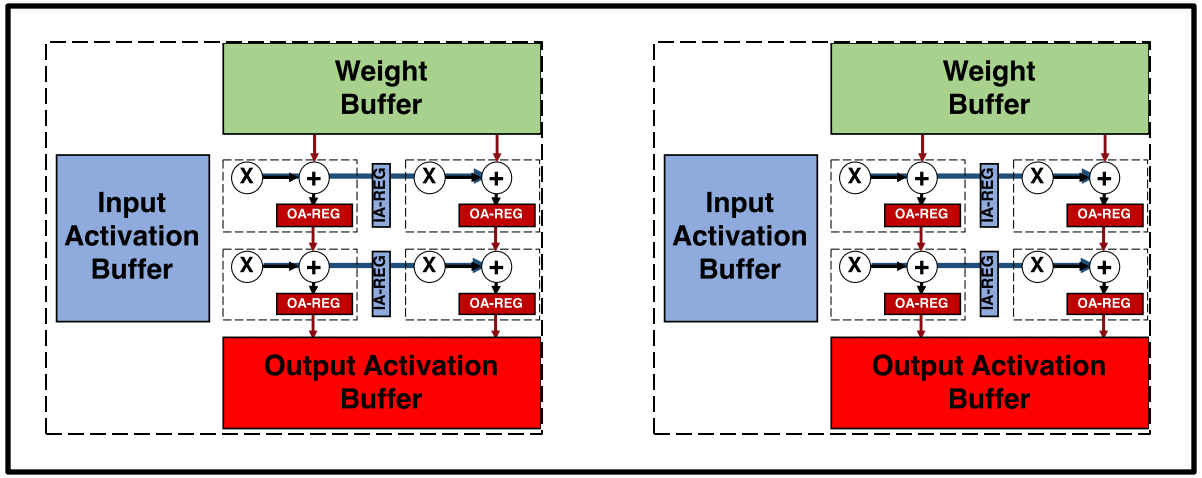
```
for (n2=0; n2<N2; n2++) {
  // W buffer: N1*N0*K
  mvin(W[0:K,n2*N1*N0:(n2+1)*N1*N0]);
  for (m2=0; m2<M2; m2++) {
    // IA buffer: M1*K
    mvin(IA[m2*M1:(m2+1)*M1,0:K]);
    compute_matmul(*W, *IA, *OA,
                   ...
                   m2, n2);
    mvout(OA[n2*N1*N0:(n2+1)*N1*N0,
             m2*M1:(m2+1)*M1]);
}}
```

IA Movement: N2 * M * K
W Movement: N * K

[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Spatial Choice: Model Parallelism

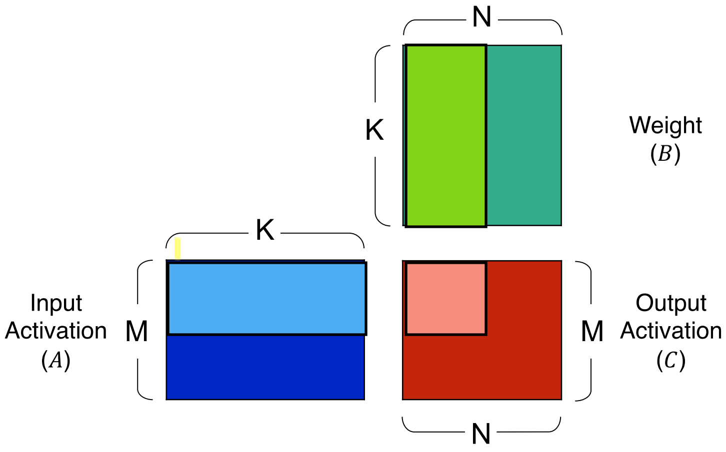


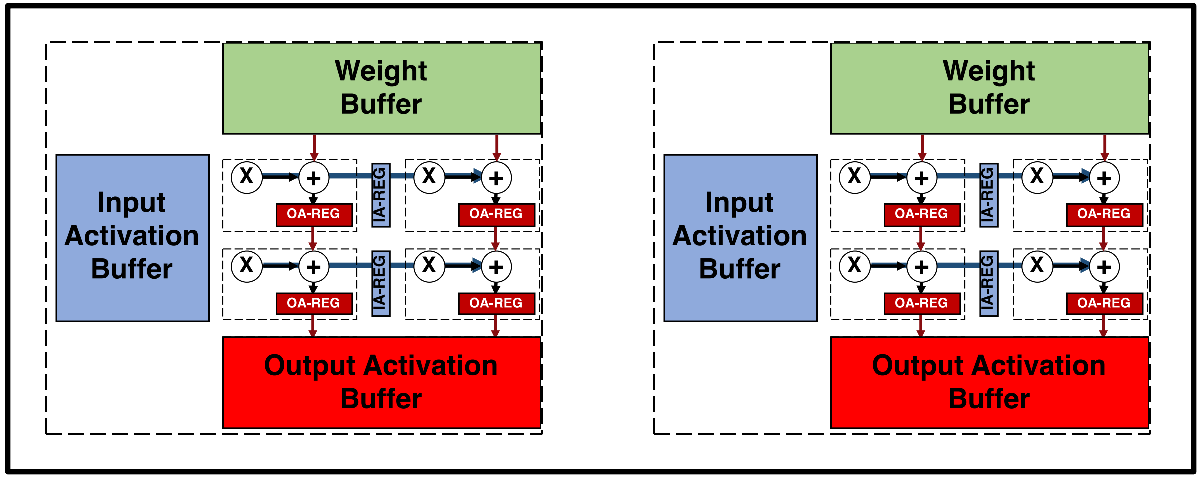



```
parallel_for (n2=0; n2<N2; n2++) {
  // Model Parallelism
  mvin(W[0:K,n2*N1*N0:(n2+1)*N1*N0]);
  for (m2=0; m2<M2; m2++) {
    mvin(IA[m2*M1:(m2+1)*M1,0:K]);
    compute_matmul(*W, *IA, *OA,
                          ...
                          m2, n2);
    mvout(OA[n2*N1*N0:(n2+1)*N1*N0,
              m2*M1:(m2+1)*M1]);
}}
```


[Source: Prof. Sophia Shao, EE290-2, Berkeley]

Spatial Choice: Data Parallelism






```
parallel_for (m2=0; m2<M2; m2++) {
  // Data Parallelism
  mvin(IA[m2*M1:(m2+1)*M1,0:K]);
  for (n2=0; n2<N2; n2++) {
    mvin(W[0:K,n2*N1*N0:(n2+1)*N1*N0]);
    compute_matmul(*W, *IA, *OA,
                        ...
                        m2, n2);
    mvout(OA[n2*N1*N0:(n2+1)*N1*N0,
             m2*M1:(m2+1)*M1]);
}}
```


[Source: Prof. Sophia Shao, EE290-2, Berkeley]

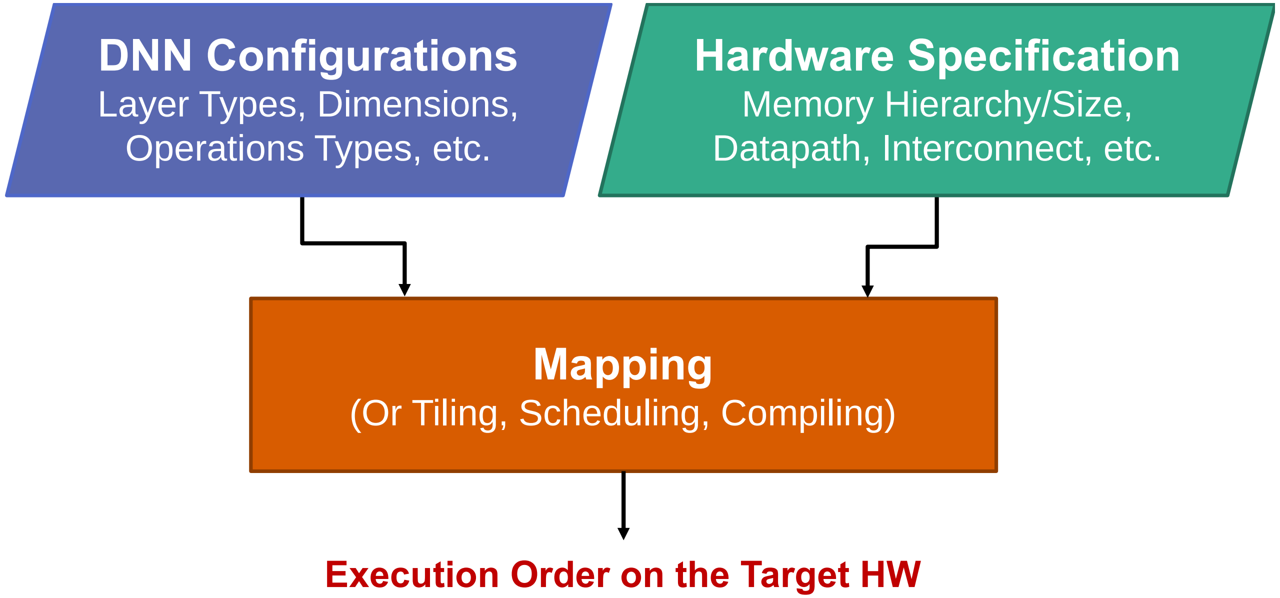
Model Parallelism vs. Data Parallelism

- Data parallelism
  - Each engine processes a subset of the data using the same model
  - Each engine runs the entire model on its assigned data
- Model parallelism
  - The model itself is split across multiple HW engines
- Variants of model parallelism
  - Tensor parallelism
    - Individual tensors are split across multiple engines
    - E.g., splitting weight matrices
    - Aggregation is required during computation
  - Pipeline parallelism
    - The model is divided into stages and executed sequentially in a pipeline
    - Intermediate activations are passed along the pipeline

Tuning of DNN Mapping

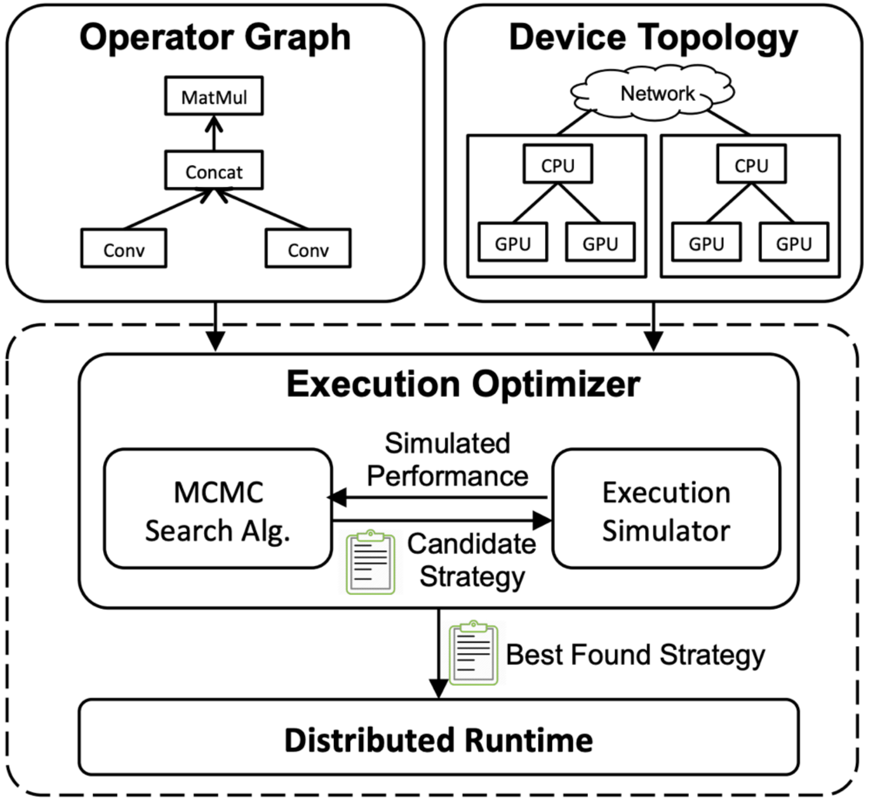
Mapping Dimensions

- DNN mapping problem ➔ an optimization problem
- Given:
  - DNN dimensions (N, H, W, C, R, S, K, stride, padding)
  - Hardware specifications (dataflow, memory hierarchy)
- Objective:
  - An optimal loop nest that minimizes latency and/or energy
    - Both temporal and spatial execution order
- Approaches:
  - Exhaustive search
  - Random search
  - Learning-based algorithms



Example: FlexFlow, SysML'2018

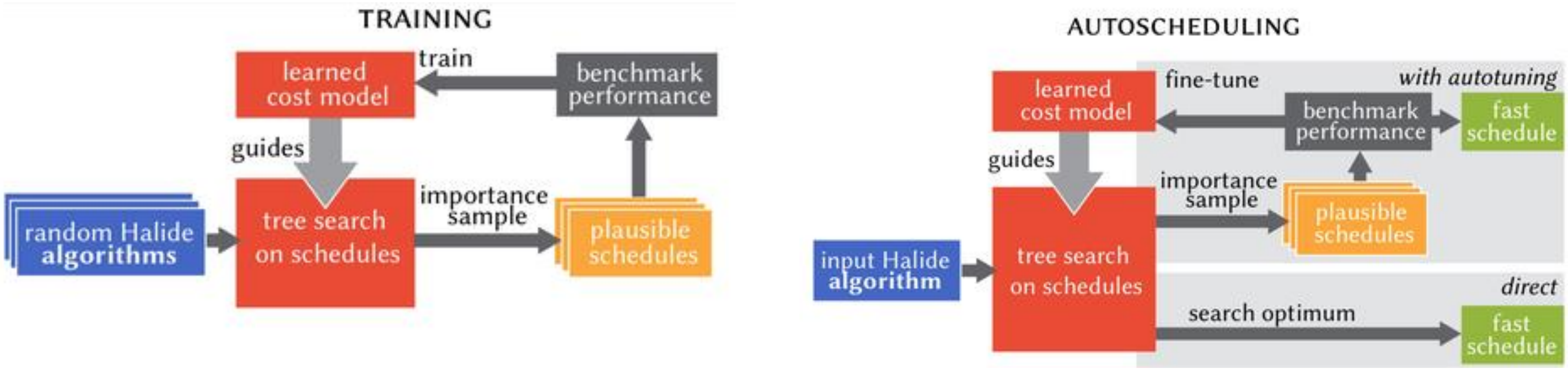
- "The optimizer uses a MCMC (Markov Chain Monte Carlo) search algorithm to explore the space of possible parallelization strategies and iteratively proposes candidate strategies that are evaluated by an execution simulator. "



Beyond Data and Model Parallelism for Deep Neural Networks, SysML 2018

Example: Halide, SIGGRAPH'2019

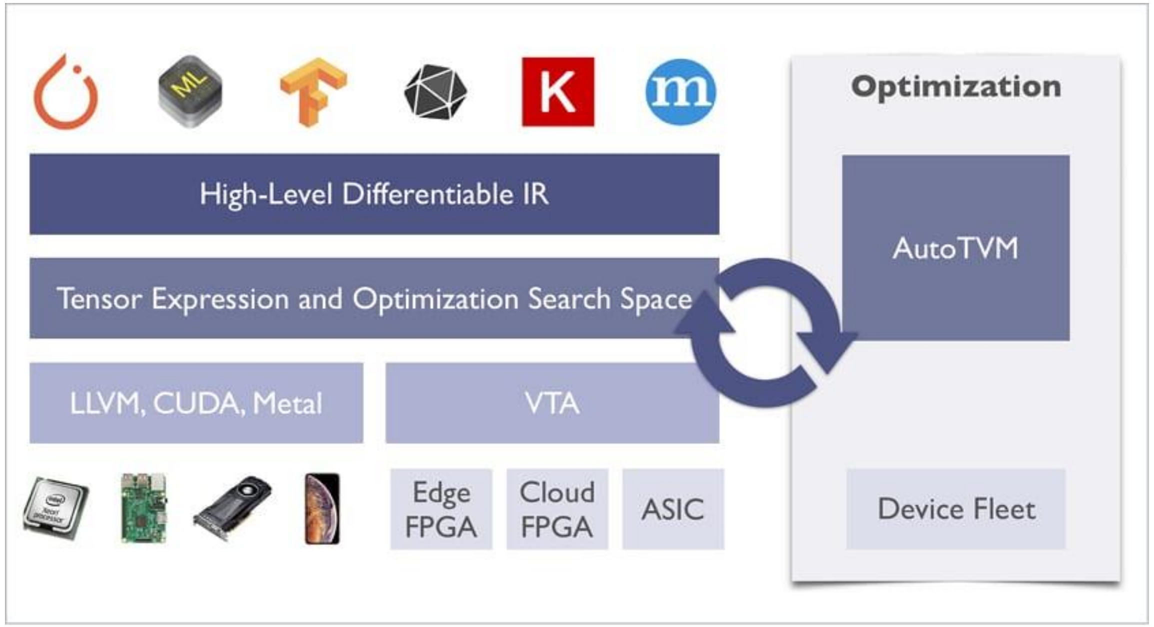
- "We generate schedules for Halide programs using tree search over the space of schedules guided by a learned cost model and optional autotuning. The cost model is trained by benchmarking thousands of randomly-generated Halide programs and schedules. The resulting code significantly outperforms prior work and human experts."

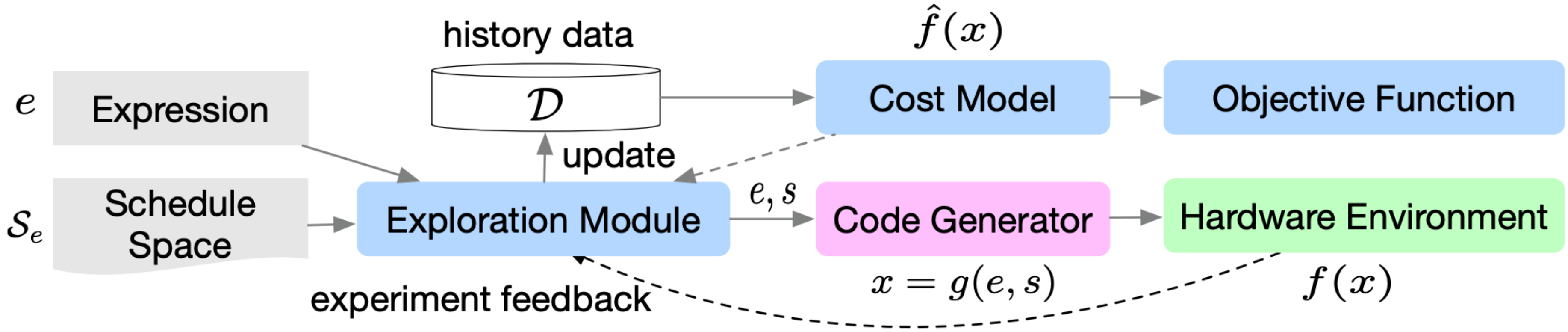


Learning to Optimize Halide with Tree Search and Random Programs, *SIGGRAPH 2019*

Example: TVM, NeurIPS'2018

- Learn domain-specific statistical cost models to guide the search of tensor operator implementations over billions of possible program variants
- Further accelerate the search using effective model transfer across workloads





Learning to Optimize Tensor Programs, *NeurIPS'2018*

Summary

- Temporal and Spatial Mapping based on hardware constraints
  - Memory hierarchy
  - Parallelism
- Tile the loops to improve reuse and parallelism
  - Loop ordering
  - Loop bounds
  - Spatial choices
- Navigate the large mapping space
  - Finding an optimized solution
  - Getting to the solution fast enough