

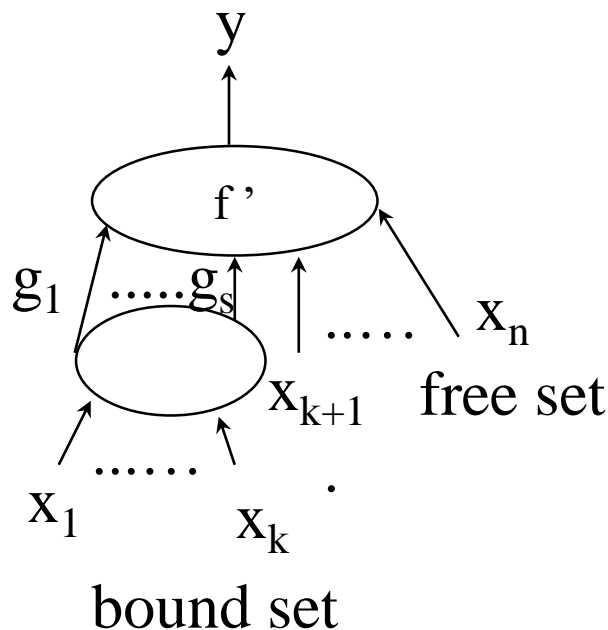
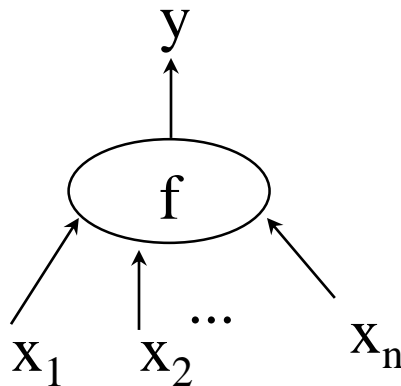
# **Functional Decomposition**

# Functional Decomposition

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- Functional Decomposition  
(Roth-Karp Decomposition)

$$f(x_1, x_2, \dots, x_n) = f'(g_0(x_1, \dots, x_k), g_1(x_1, \dots, x_k), \dots, g_s(x_1, \dots, x_k), x_{k+1}, \dots, x_n))$$



## Three Questions

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- (1) With respect to a partitioning, what is the minimum number of sub-functions  $g$ ?
- (2) How to synthesize the subfunctions  $g$  , and the top function  $f$  ?
- (3) How to select a partition ?

# Minimum Subfunctions

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(1) With respect to a partitioning,

$\Pi = (B, F)$  (B: Bound set; F: Free set)

what is the minimum number of sub-functions  $g$ ?

Ex: The comparison function  $(a_2a_1, b_2b_1)$  and

$\Pi = (\{a_1b_1\}, \{a_2b_2\})$

$a_1b_1$	$a_2b_2$					
	00	01	10	11		
00	1	0	0	1	←	1
01	0	0	0	0	←	0
10	0	0	0	0	←	0
11	1	0	0	1	←	1

the number of row patterns = 2

$\log(\# \text{row patterns}) = 1$

# Equivalent Classes

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1.  $x_1 \approx x_2$  for  $x_1, x_2 \in B$   
if  $f(x_1, y) = f(x_2, y)$  for all  $y \in F$   
 $x_1$  and  $x_2$  are in the same equivalence class
2. The number of row patterns is the number of equivalence class
3. The minimum number of g functions is  
 $\log(\text{the number of equivalent classes})$

# Example

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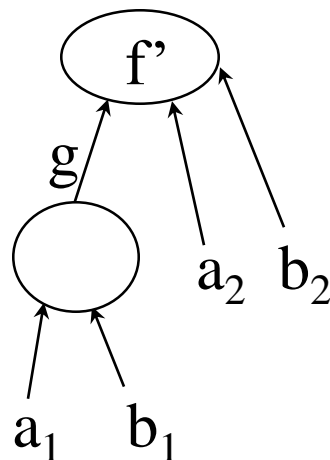
(2) How to synthesize  $g$ ,  $f'$  ?

Ex:

$a_2b_2$	00	01	10	11		$g(a_1 b_1)$
$a_1b_1$						$a_1b_1 \quad g$
00	1	0	0	1	$\rightarrow 1$	00 $\left  \begin{array}{c} 1 \end{array} \right.$
01	0	0	0	0	$\rightarrow 0$	01 $\left  \begin{array}{c} 0 \end{array} \right.$
10	0	0	0	0		10 $\left  \begin{array}{c} 0 \end{array} \right.$
11	1	0	0	1		11 $\left  \begin{array}{c} 1 \end{array} \right.$

$f'(g a_2 b_2)$

$g$	00	01	10	11
1	1	0	0	1
0	0	0	0	0



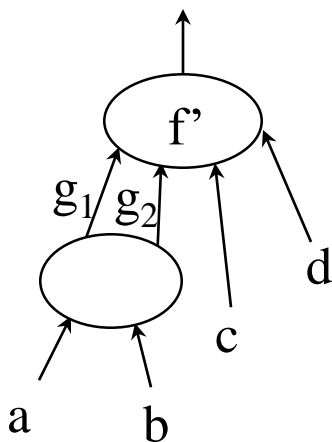
# Example

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$\backslash$ cd	00	01	10	11	$g_1 g_2$
ab					
00	1	1	0	0	$\leftarrow$ 00
01	1	0	0	1	$\leftarrow$ 01
10	1	1	0	0	$\leftarrow$ 00
11	0	0	0	1	$\leftarrow$ 10

$\backslash$ $g_1$		$\backslash$ $g_2$	
ab		ab	
00	0	00	0
01	0	01	1
10	0	10	0
11	1	11	0

$f' \backslash$ cd	00	01	10	11
$g_1 g_2$				
00	1	1	0	0
01	1	0	0	1
10	0	0	0	1
11	d	d	d	d



# How to Select a Partition

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(3) How to select a partition ?

$a_2b_2$	00	01	10	11
$a_1b_1$				
00	1	0	0	1
01	0	0	0	0
10	0	0	0	0
11	1	0	0	1

$b_1b_2$	00	10	11	10
$a_1a_2$				
00	1	0	0	0
01	0	1	0	0
10	0	0	1	0
11	0	0	0	1

row pattern=4  
 $\log(\# \text{row patterns})$   
 $= 2$



## Example

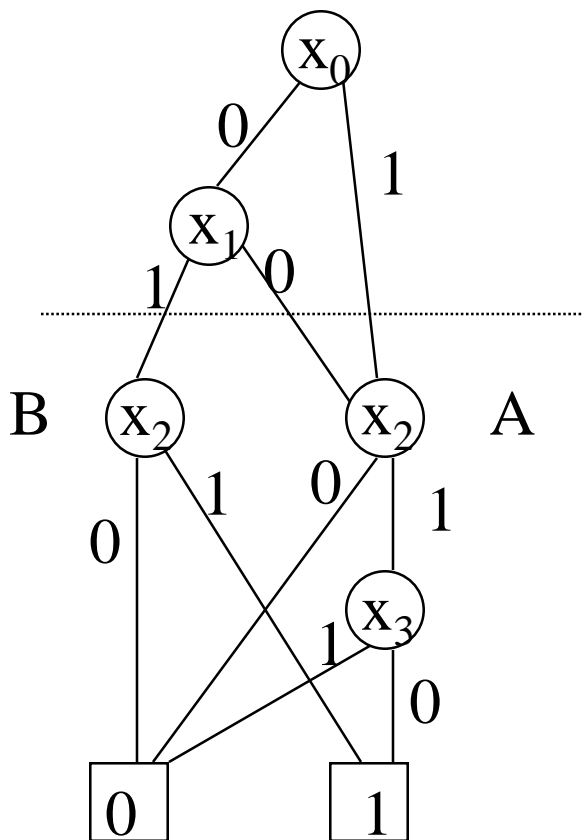
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- Compute the number of equivalence classes using BDD

$x_0x_1$	$x_2x_3$ 00 01 10 11				
00	0	0	1	0	A
01	0	0	1	1	B
10	0	0	1	0	A
11	0	0	1	0	A

## Example (cont.)

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1. Order BDD such that the variables in the bound set is on the top of BDD
2. Draw a line between the bound set and the free set
3. The number of equivalence classes is the number of sub-trees rooted at the line