Irredundant Cover

- After performing Expand, we have a prime cover without single cube containment now.
- We want to find a proper subset which is also a cover (irredundant).

Irredundant Cover

- Proposition:

  A set of cubes C covers a cube p if and only if C_p is a tautology.

  proof:

  => $C \cap p = p$

  => $(C \cap p)_p = p_p$

  => $C_p \cap p_p = 1$

  => $C_p = 1$

  <= $C_p = 1$

  => $C_P \cap P = P$

  => $C \cap P = P$

Example

Ex: $p = 1\ 1\ 2$

$$C = \begin{matrix} 2 & 1 & 0 \\ 1 & 2 & 1 \end{matrix}$$

$$C_p = \begin{matrix} 2 & 2 & 0 \\ 2 & 2 & 1 \end{matrix}$$

Check tautology(C_p)

If (C_p) is a tautology, C covers p.

Property of Unate Function

- Proposition:

  A unate cover is a tautology if and only if it contains a row of 2's.

  pf:

  (<=) trivial

  (=>) Assume the cover represent a monotone increasing function, the function contains 1's and 2's.

  The minterm (0,0,....0) must be covered. Unless the cover contains (2,2,....2) , the minterm (0,0,...0) will not be covered.

Tautology Check

1. speed-up by unate variables

Let $F(x_1,x_2,...x_n)$, x_1 : positive unate

– $F(x_1,x_2,...x_n) = x_1A(x_2,...x_n) + B(x_2,...x_n)$

A : terms with x_1

B : terms without x_1

$F_{x1} = A + B \quad F_{x1'} = B$

– If ($F_{x1'} = B$) is tautology, $F_{x1} = A + B$ is a tautology.

– If ($F_{x1'} = B$) is not a tautology, F is not a tautology.

If x is positive unate, test if F is a tautology.

<=> $F_{x'}$ is a tautology

Tautology Check

2. Other techniques
   - a row of 2's, answer 'Yes'
   - a column of all 1's or all 0's, answer 'No'
   - compute an upper bound on the no. of minterms of on-set

     Ex: number of minterms

0 2 0 1	2
1 1 2 2	4
1 2 1 1	2

     $2^n = 16 \quad > \quad 8$ answer 'No'

   - n $\leq$ 7, test by truth table

Irredundant Cover

Primes in F

(1) relatively essential cube c of F (E)

F - {c} is not a cover

(2) redundant cubes

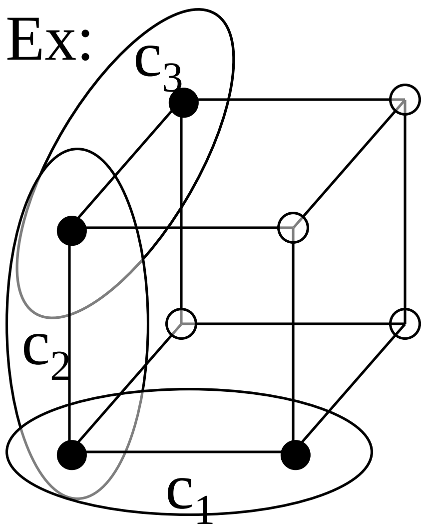
- Totally redundant (R_t)

  a cube covered by relatively essential cubes and Don’t care

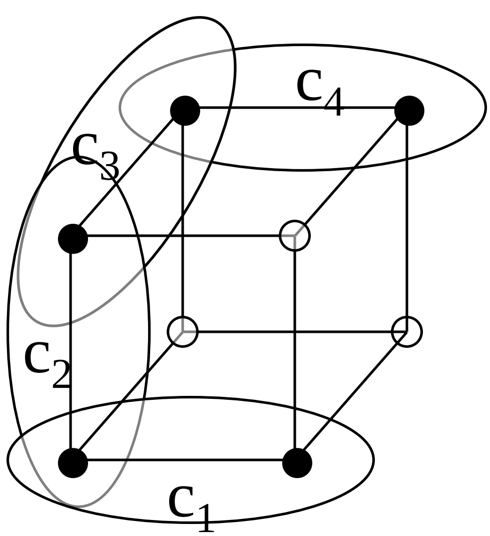
- Partially redundant (R_p)

Example

Ex:



$F = \{c_1, c_2, c_3\}$
$E = \{c_1, c_3\}$
$R_t = \{c_2\}$
$R_p = \phi$



$F = \{c_1, c_2, c_3, c_4\}$
$E = \{c_1, c_4\}$
$R_t = \phi$
$R_p = \{c_2, c_3\}$

Irredundant Cover

(1) Check whether cubes are relatively essential

1. tautology $(F\text{-}c)_C$

2. If not tautology $(F\text{-}c)_c$, c is relatively essential.

(2) Check partially, totally redundant

1. tautology $(E \cup D)_C$ (D : don't care)

2. If yes, c is totally redundant

   If no, c is partially redundant

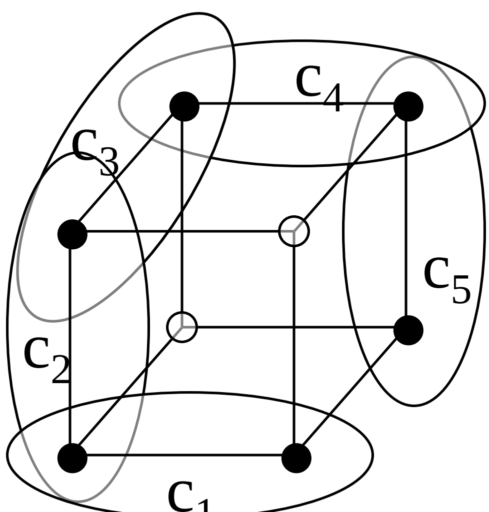
Irredundant Cover

(3) Minimal Irredundant (R_p)

- Find a minimum subset of partially irredundant set such that combining with relatively essential cubes, it forms a cover.

- For each $r \in R_p$, define the minimal set S such that $(R_p - S) \cup E \cup D$ is NOT a cover

ex:



$R_p = \{c_4, c_3, c_2\}$

$S_4 = \{\{c_4, c_3\}\}$

$S_3 = \{\{c_3, c_4\}, \{c_3, c_2\}\}$

$S_2 = \{\{c_2, c_3\}\}$

Irredundant Cover

- Define a matrix B. Each rows corresponds to a S_i

$B_{ij} = $ 1 if R_{pj} in S_i
0 otherwise

Ex: $R_p = \{c_4, c_3, c_2\}$
$S_4 = \{\{c_4, c_3\}\}$
$S_3 = \{\{c_3, c_4\}, \{c_3, c_2\}\}$
$S_2 = \{\{c_2, c_3\}\}$

	c_2	c_3	c_4
S_4		1	1
$S_{3.1}$		1	1
$S_{3.2}$	1	1	
S_2	1	1	

minimum column cover of B
$\equiv$ minimal irredundant cover

Irredundant Cover

- Two things:
  1. find minimal set S
  2. find minimal column covering

Find Minimal Set S

For each r in R_p, find all minimal sets S

$(R_p - S) \cup E \cup D$ does not cover r

$\equiv$ Tautology$((R_p - S) \cup E \cup D)_r$ is not a tautology.

$\equiv$ Let $A = (R_p)_r$ $B = (E \cup D)_r$

$A \cup B \equiv 1$ determine all minimal subset $S \subseteq A$

such that $(A - S) \cup B \not\equiv 1$

Find Minimal Set S

Let $E \cup D = \phi$

$A = (R_p)_r$

taut(A)

If A is a unate cover with ($A \equiv 1$), then

there must be at least one row of 2's in the cover A

take S to be the set of all such cubes, then

$A - S \not\equiv 1$

else

binate select x_j

split A_{xj}, $A_{xj'}$

$S_{xj} \leftarrow \text{taut}(A_{xj})$

$S_{xj'} \leftarrow \text{taut}(A_{xj'})$

merge: $S = S_x \cup S_{x'}$

($\because (A-S) \not\equiv 1 \Leftrightarrow (A_x - S_x) \not\equiv 1$ OR

$(A_{x'} - S_{x'}) \not\equiv 1$)

Irredundant Cover

Let $A = (R_p)_r$ $B = (E \cup D)_r$

$A \cup B \equiv 1$

find subset $S \subset A$ such that $(A - S) \cup B \not\equiv 1$

If $B \neq 0$ then

$$\frac{A}{B}$$

listed in two parts

Split the function until it becomes unate.

If $(2, 2, \ldots 2)$ appears in B part, then there will be no "S" set for this subspace.

Example

ex: $A = (R_p)_r$ $\quad B = \phi$

$x_4 x_5 x_6$

0 2 2(4) {{5,7},{5,6},{4,7},{4,6}}

1 2 2(5)

2 0 2(6)

2 1 2(7)

2 2 1(8)

1 2 1(9)

{{5,7},{5,6}} x_4 $\quad$ $x_{4'}$ {{4,7},{4,6}}

2 2 2(5)

2 0 2(6)

2 1 2(7)

2 2 1(8)

2 2 1(9)

{5,7} x_5 $\quad$ $x_{5'}$ {5,6}

2 2 2(5)

2 2 2(7)

2 2 1(8)

2 2 1(9)

2 2 2(5)

2 2 2(6)

2 2 1(8)

2 2 1(9)

2 2 2(4)

2 0 2(6)

2 1 2(7)

2 2 1(8)

x_5 {4,7} $\quad$ $x_{5'}$ {4,6}

2 2 2(4)

2 2 2(7)

2 2 1(8)

2 2 2(4)

2 2 2(6)

2 2 1(8)