Reduce

- Reduce:

  Replace each prime by a smaller cube contained in it.
- $|\underline{F}| = |F|$ after reduce
- Since some of cubes of $\underline{F}$ are not prime, Expand can be applied to $\underline{F}$ to yield a different cover that may have fewer cubes.
- $|\underline{F}| \leqslant |F|$ after Expand
- Move from locally optimal solutions to a better one.

Reduce

- Gives two possibilities for decreasing the size of cover

  - The reduced cube can be covered by a neighboring cube after the EXPAND.

  - The reduced cube can expand in different direction to cover some neighboring cube.

Reduce

Definition:

Smallest cube $\underline{C_i}$ is a cube containing all minterms in C_i not covered by D and $F \setminus C_i$

$$F(i) = (F \setminus C_i) \cup D$$

$$\underline{C_i} = \text{smallest cube containing } (C_i \cap F(i)') = SCC(C_i \cap F(i)')$$

$(F \setminus C_i) \cup \underline{C_i}$ is still a cover.

Reduce

$\underline{C}_i = \mathrm{SCC}(C_i \cap F(i)')$

$?= \mathrm{SCC}(C_i) \cap \mathrm{SCC}(F(i)')$

$?= C_i \cap \mathrm{SCC}(F(i)')$

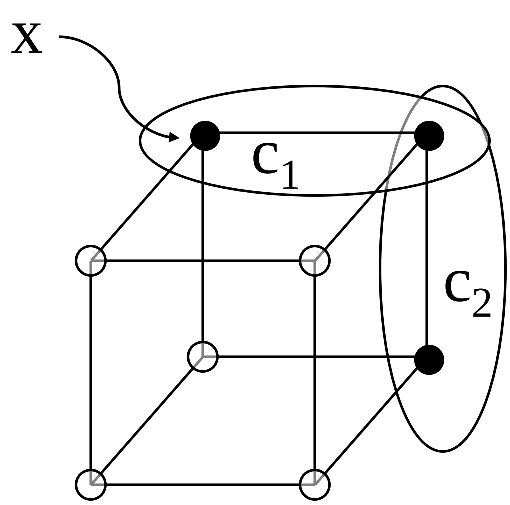
Ex:

$F = C_1 + C_2$

$F(1) = F - C_1 = C2$

$\mathrm{SCC}(F(1)') = 1$

$C_1 \cap 1 = C_1$



But, $\mathrm{SCC}(C_1 \cap F(1)') =$ a single vertex x

so $C_1 \cap \mathrm{SCC}(F(1)') \neq \mathrm{SCC}(C_1 \cap F(1)')$

Reduce

$$\underline{\boldsymbol{c_i}} = \mathrm{SCC}(\mathrm{C_i} \cap \overline{F(i)}) = \mathrm{C_i} \cap \mathrm{SCC}(\overline{F(i)_{C_i}})$$

$\mathrm{SCC}(\overline{\boldsymbol{F(i)_{C_i}}})$

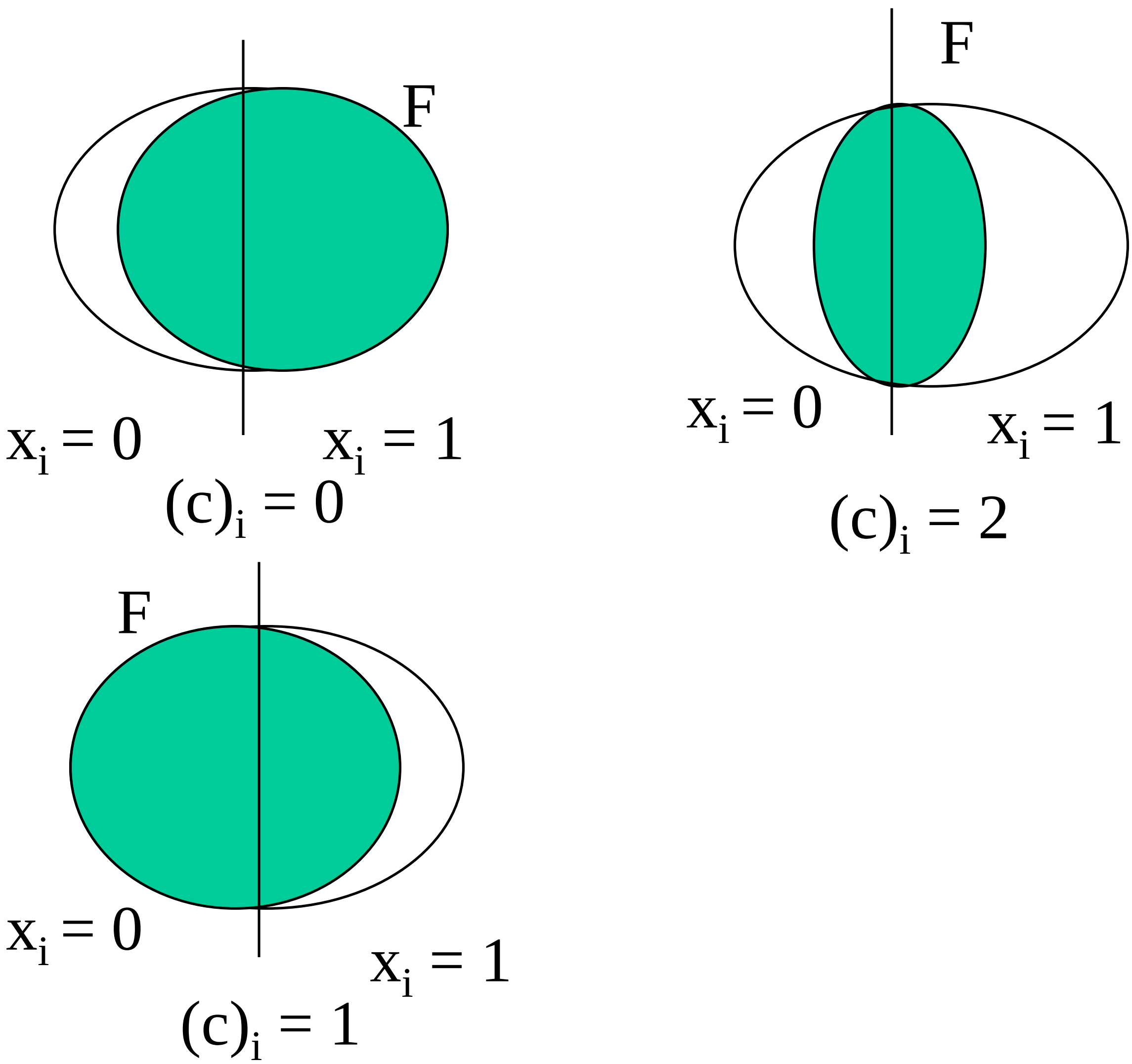
1. Complementation
2. Smallest cube containing problem
   => Find a cube containing the complement of a cover

Find a Cube Containing the Complement of a Cover

- Proposition : The smallest cube c containing the complement of a cover F, F $\neq$ 1, satisfies

$$I(C)_i = \begin{cases} 0 \text{ if } x_i \subseteq F \\ 1 \text{ if } x_i' \subseteq F \\ 2 \text{ otherwise} \end{cases}$$

Reduce



For general function, the above test employ difficult covering test. However, if it is a unate function, all test can be done easily.

Reduce

Proposition : Let F be a unate cover. Then $x_i \subseteq F$ if and only if there exists a cube, $c_i \in F$, such that

$$I(x_i) \subseteq I(c_k)$$

pf : A single output unate cover contains all primes of that function.

Reduce

ex:

$$
\begin{array}{llll}
 & x_1 & x_2 & x_3 \\
F & 2 & 1 & 2 \\
 & 2 & 2 & 0
\end{array}
$$

Find the smallest cube containing F'.

$x_1 \notin F$

$x_1' \notin F$

$\Longrightarrow c_1 = 2$

$x_2 \in F$

$\Longrightarrow c_2 = 0$

$x_3' \in F$

$\Longrightarrow c_3 = 1$

$\underline{C} = (2\ 0\ 1)$

Smallest Cube Containing Problem

- Let $\overline{F(i)} = g$

$$\boldsymbol{SCC(g)}$$

$$= \boldsymbol{SCC(x_j SCC(g_{xj}) + x_j{}' SCC(g_{xj'}))}$$

- R-merge

 Let a and b be two cubes. Then, the smallest cube C containing {a, b} is

 a $\bigcup$ b , where $\bigcup$ denotes the coordinate wise union.

ex: a = 0 1 2 1
 b = 1 1 2 0

a $\bigcup$ b= 2 1 2 2

Reduce

ex:

$$F = \begin{matrix} 2 & 2 & 2 & 0 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 1 & 2 \end{matrix}$$

reduce $C_1 = 2\ 2\ 2\ 0$

$$F(1) = \begin{matrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 1 & 2 \end{matrix}$$

$$F(1)_{C1} = \begin{matrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 1 & 2 \end{matrix}$$

Reduce

{2 2 0 2} = SCC(F(1)$_{C1}$’)

1 2 1 2
1 1 2 2
0 0 2 2
0 2 1 2

{0 1 0 2} x_1’ {1 0 0 2} x_1

2 0 2 2
2 2 1 2

x_1’ {2 1 0 2}

2 2 1 2
2 1 2 2

x_1 {2 0 0 2}

$$\underline{c_1} = \mathrm{SCC}(\ \overline{F(1)_{C_1}}\ ) \cap c_1$$

$$= 2\ 2\ 0\ 2 \cap 2\ 2\ 2\ 0$$

$$= 2\ 2\ 0\ 0$$

Reduce

- order dependent
  - reduce the largest cube
  - reduce those cubes that are nearest to it.
- compute pseudo distance

  (the number of mismatch)

  Ex:

  0 1 0 1

  0 2 1 1 PD = 2