

CS 5291: Stochastic Processes for Networking

HW4

1. Suppose that whether or not it rains today depends on previous weather conditions through the last three days. If it has rained for the past three days, then it will rain today with probability 0.8; if it did not rain for any of the past three days, then it will rain today with probability 0.2; and in any other case the weather today will, with probability 0.6, be the same as the weather yesterday. Determine P for this Markov chain.

Hint: Let the state space be $\{RRR,RRD,RDR,RDD,DRR,DRD,DDR,DDD\}$, where R denotes rain day and D denotes dry day. For example, RDD means that it rained in three days ago and it did not rain in past two days. State changes from RDD to DDD means that it does not rain today.

2. A Markov chain $\{X_n, n \geq 0\}$ with states 0, 1, 2, has the transition probability matrix shown below. If $P\{X_0 = 0\} = P\{X_0 = 1\} = \frac{1}{4}$, find $E[X_3]$.

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

3. Given the following transition matrix,

$$P = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

- (a) Draw the state transition diagram for this chain.
 - (b) Determine the classes and specify which are recurrent and which are transient.
4. Leonard moves between the airport A and two hotels B and C according to the

following rules. If he is at the airport, he will be at one of the two hotels next with equal probability. If at a hotel then he returns to the airport with probability $3/4$ and goes to the other hotel with probability $1/4$.

- (a) Find the transition matrix for the chain.
- (b) Suppose Leonard begins at the airport at time 0. Find the probability for each of his three possible locations at time 2 and find the probability he is at hotel B at time 3.