嚴格競爭遊戲
Strictly Competitive Game

- 兩人嚴格競爭遊戲是一種兩人遊戲，對於每兩個策略配置 (strategy profiles)
- $u_1(s) > u_1(s')$ if and only if $u_2(s) < u_2(s')$.

Strictly Competitive Game

- u_1(A,X) =3, u_1(A,Y) =0,
- u_1(B,X) =6, u_1(B,Y) =1
- $\boldsymbol{u_1}$**(B,X) >** $\boldsymbol{u_1}$**(A,X) >** $\boldsymbol{u_1}$**(B,Y) >** $\boldsymbol{u_1}$**(A,Y)**
- u_2(A,X) =2, u_2(A,Y) =4,
- u_2(B,X) =1, u_2(B,Y) =3
- $\boldsymbol{u_2}$**(B,X) <** $\boldsymbol{u_2}$**(A,X) <** $\boldsymbol{u_2}$**(B,Y) <** $\boldsymbol{u_2}$**(A,Y)**
- The rankings are exactly reversed between the two players
- 因此，這是嚴格競爭遊戲

嚴格競爭遊戲
Strictly Competitive Game

- 匹配硬幣(Matching pennies)是嚴格競爭遊戲的特例。 它實際上是一個零和遊戲（並非所有嚴格競爭的遊戲都是零和遊戲）。
- 如果玩家更喜歡獲勝(winning)而不是平局(tying)，更喜歡平局而不是失敗(losing)
- 體育遊戲（網球、足球、棒球等）和休閒遊戲（跳棋、國際象棋等）都是嚴格競技遊戲。

安全策略 Security strategy

- **安全策略為玩家 1 提供了最壞情況中最好的情況：**

$max_{s_1}\ w_1(s_1) = max_{s_1}\ min_{s_2}\ u_1(s_1, s_2)$

- 解決此最大化問題的策略稱為安全策略，或 Max-min 策略。

這個博弈有唯一的unique的納許均衡：(B,Y)

		Player 2	
		X	Y
Player 1	A	(3, 2)	(0,4*)
	B	(6*,1)	(1*,3*)

安全策略 Security strategy

- $max_{s_1}\ w_1(s_1) = max_{s_1}\ min_{s_2}\ u_1(s_1, s_2)$
- $max_{s_1}\ w_1(s_1)$通常被稱為安全收益水平 (Security-payoff level.)。
- 我最大化我的收益，因為我知道我的對手會最小化它（因為他想最大化自己的收益，因為
- $u_2(s_1,\ s_2) = -u_1(s_1,\ s_2)$

Security or Max-min strategy

- 如果一個兩人博弈是嚴格競爭的並且有一個納許均衡 $s^*=(\ s_1^*, s_2^*\ )$
- s_1^*, 是玩家 1 的安全策略，s_2^*是 玩家 2 的安全策略

鞍點 “saddlepoint”

- 兩人恆和博弈中 (in a two-person constant-sum game)的 “鞍點” (“saddlepoint”)是理性玩家選擇的結果
- 鞍點名字的來源於是矩陣中一行的最小值，也是一列的最大值(對應於馬鞍的形狀)

Minimax strategy

- Minimax strategy
- = $\arg(min_{s_2} max_{s_1}\ u_1(s_1, s_2)\ )$

兩人恆和博弈中 (a two-person constant-sum game)

- 兩個政黨，A 和 B，必須各自決定如何處理某次選舉中的一個有爭議的問題。
- 每一方都可以支持這個問題，反對它，或者通過模棱兩可來逃避它 (either support the issue, oppose it, or evade it by being ambiguous.)

Min-Max定理

- 該定理歸因於約翰·馮·諾伊曼（John Von Neumann）和奧斯卡·摩根斯坦（Oscar Morgenstern）。
- 在零和遊戲中，一個玩家應該嘗試最小化對手的最大收益。
- **最大收益的最小值等於最小收益的最大值**。

Nash Equilibrium Payoff Matrix with saddlepoint

A \ B	Support	Oppose	Evade
support	(60%,40%)	(20%,80%*)	(80%*,20%)
oppose	(80%*,20%)	(25%,75%*)	(75%,25%)
Evade	(35%,65%)	(30%*,70%*)	(40%,60%)

Security (Max-min) Strategy - Tennis Game

- 本博弈存在純策略納許均衡 (Evade, Oppose)

Security (Max-min) Strategy by Player B with B's Payoff

A \ B	Support	Oppose	Evade
support	40%	80%	20%
oppose	20%	75%	25%
Evade	65%	70%	60%

The Maxmin solution by player B is oppose

安全策略 Maxmin

B's minimizing A's payoff ={20%,25%,30%}
A's maximizing {20%,25%,30%}=30%
The Maxmin solution by player 1 is Evade

Both players play security strategy

- The Maxmin solution by player A is Evade
- The Maxmin solution by player B is Oppose
- The solution of security strategy by both players is (Evade, Oppose)

Security (Max-min) Strategy - Tennis Game by Player A with A's Payoff

A \ B	Support	Oppose	Evade
support	60%	20%	80%
oppose	80%	25%	75%
Evade	35%	30%	40%

The Maxmin solution by player A is Evade

Use Minimax

Minimax by Player B with A's Payoff

A \ B	Support	Oppose	Evade
support	60%	20%	80%*
oppose	80%	25%	75%
Evade	35%	30%	40%

The Minimax solution by player B is oppose

Minimax by Player A with B's Payoff

- A's maximum payoff ={80%,75%,70%} B's minimum of {80%,75%,70%} = 70%
- The Minimax solution by player A is Evade

Minimax by Player B with A's Payoff

- A's maximum payoff ={80%,30%,80%} B's minimum of {80%,30%,80%}=30%
- The Minimax solution by player B is oppose

Both players play Minimax

- The Maxmin solution by player A is Evade
- The Maxmin solution by player B is Oppose
- The solution of security strategy by both players is (Evade, Oppose)

Minimax by Player A with B's Payoff

A \ B	Support	Oppose	Evade
support	40%	80%	20%
oppose	20%	75%	25%
Evade	65%	70%	60%

The Maxmin solution by player A is Evade

The minimax theorem

- 馮諾依曼在 1928 年證明的極大極小定理指出，每個有限的兩人恆和博弈都有純策略或混合策略的解決方案。

Max-min strategy

- 納許均衡 $s^*=(\ s_1^*, s_2^*\ )$ = Security strategy = Max-min strategy
- Security strategy = Max-min strategy
- $max_{s_1}\ w_1(s_1) = \ max_{s_1}\ \ min_{s_2}\ u_1(s_1, s_2)$

Tennis Game

		Player 2	
		Right	Left
Player1	Right	20,80	70,30
	Left	90,10	30,70

The minimax theorem

- Minimax strategy
- = arg($min_{s_2} max_{s_1}\ u_1(s_1, s_2)$)
- = Max-min strategy
- =arg($max_{s_1}\ \ min_{s_2}\ u_1(s_1, s_2)$)

Security (Max-min) Strategy - Tennis Game

- 這是一個恆和博弈，因為對於所有可能的策略組合(all possible strategy profiles)，由於玩家的收益總和等於常數（100）
- 這不是零和遊戲，因為對於所有策略組合，玩家收益的總和不等於零

恆和博弈 constant-sum game

- 在零和博弈:

$u_1(s_1, s_2) + u_2(s_1, s_2) = 0$ ，

在零和博弈:

$u_1(s_1, s_2) + u_2(s_1, s_2) = \text{Constant}$

- 在零和博弈,常數正好等於零

Security (Max-min) Strategy - Tennis Game

- 本博弈不存在純策略納許均衡
- 存在混合策略均衡，兩個玩家隨機選擇策略

Use Maxmin

The mxmin solution is (left, left)

Security (Max-min) Strategy - Tennis Game by Player 2

		Player 2	
		Right	Left
Player1	Right	80	**30**
	Left	**10**	70

Player 2 choose left

Security (Max-min) Strategy - Tennis Game by Player 1

		Player 2	
		Right	Left
Player1	Right	20	70
	Left	90	30

Player 1 choose left

Security (Max-min) Strategy - Tennis Game by Player 2

- **Player 2** will choose the strategy that **maximizes** their minimum payoff:
- The minimum payoffs are **10** (if Player 2 chooses Right) and **30** (if Player 2 chooses Left).
- So, Player 2 will choose **Left**, since 30 is larger than 10.

Security (Max-min) Strategy by Player 1

- Player 1 will choose the strategy that maximizes their minimum payoff:
- The minimum payoffs are **20** (if Player 1 chooses Right) and **30** (if Player 1 chooses Left).
- So, Player 1 will choose Left, since 30 is larger than 20.

Both players play security strategy

- The Maxmin solution by player 1 is **Left**
- The Maxmin solution by player 2 is **Left**
- The solution of security strategy by both players is (Left, Left)

Use Minimax

The Minimax solution is (left, left)

Tennis Game by Player 2

		Player 2	
		Right	Left
Player1	Right	20	70
	Left	90	30

Player 1 choose left

Minimax
Tennis Game by Player 1

		Player 2	
		Right	Left
Player1	Right	80	**30**
	Left	**10**	70

Player 2 choose left

Minimax by Player 2

- Player 2 will choose the strategy that minimizes Player 1's maximum payoff
- The maximum payoffs for Player 1 are 90 (if Player 2 chooses Right) and 70 (if Player 2 chooses Left).
- Player 2 will choose Left, since it minimizes Player 1's maximum payoff (70 instead of 90).

Minimax by Player 1

- Player 1 will choose the strategy that minimizes Player 2's maximum payoff:
- The maximum payoffs for Player 2 are 80 (if Player 1 chooses Right) and **70** (if Player 1 chooses Left).
- Player 1 will choose Left, since it minimizes Player 2's maximum payoff (70 instead of 80).