

# **CS528**

# **Caching And APP Classification**

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# Outline

- Data Access Optimization
- Roofline Model
- Caching optimization
- App classification based DA:  $N/N$ ,  $N^2/N^2$ ,  $N^3/N^2$

***[Ref: Hager Book, PDF uploaded to Website]***

# Hashing Vs Caching

- Simple Hashing: Direct Map Cache

- Example: Array
- `int A[10]`, each can store one element
- Data stored in  $\text{Addr}\%10$  location

Direct/Random  
Access to  
Element

- Array of List

- `Int LA[10]`, each can store a list of element
- Data stored in List of  $(\text{Addr}\%10)^{\text{th}}$  location
- List size is limited in Set Associative Cache

MIXED

- List of Element

- Full Associative Cache
- All data stored in one list

Serial/Associative Access  
to Element

U  
S  
A  
B  
I  
L  
I  
T  
Y

T  
I  
M  
E

# **Program Cache Behavior: Hit/Miss**

# Cache Model

- Direct mapped 8 word per line



# Program

```
int A[128];  
for (i=0; i<128; i++) {  
    A[i]=i;  
}
```

- Assume &A=000000, **Behavior of only Data**
- Scalar variable {i} mapped to register
- Data have to moved from cache/memory

# Cache perf. : Data Size <= Cache Size

```
int A[128];  
for (i=0; i<128; i++) {  
    A[i]=i;  
}
```

Scalar mapped to register  
Vector mapped to memory

1:7= 1miss:7hit

1:7	0	A[0]	A[1]	A[2]					A[7]
2:14	1	A[8]	A[9]						A[15]
3:21	2	A[16]	A[17]						A[23]
	14								
16:112	15								A[127]

# Strided access: Reduce locality

```
for (i=0; i<N; i++) {  
    for (j=0; j<N; j++) {  
        a[i][j]=i*j  
    }  
    /* (a+i*N+j), j++
```

Row major  
access: Stride 1,  
improve locality,  
cache hit

```
for (i=0; i<N; i++) {  
    for (j=0; j<N; j++) {  
        a[j][i]=i*j  
    }  
    /* (a+j*N+i), j++
```

Column major  
access: Stride N,  
No locality, cache  
miss dominates



## Matrix mult.c

```
int A[8][8], B[8][8], C[8][8];  
for (i=0; i<8; i++) {  
    for (j=0; j<8; j++) {  
        S=0;  
        for (k=0; k<8; k++)  
            S=S+B[i][k]*C[k][j];  
        A[i][j]=S;  
    }  
}
```

## Data Size > Cache Size

- $(64+64+64) > 128$  words
- When we get into cache it can take benefit  
for  $(k=0; k<8; k++)$

$S = S + B[i][k] * C[k][j];$

- Inner loop execute for **64 times**
  - We have to get  $B[j]$  once will have 1miss/7 hit
  - $C[k]$  have to bring every time 8miss
  - Total = **7h+9m**
- 2<sup>nd</sup> loop A have one miss in 8 access (**1miss/7hit**)
  - Total for A = **8m+56h**
- Total program :  $64 * (7h+9m) + 8m+56h = 504h+584m$
- ***Miss Probability =  $584/(504+584)=0.5367$***

# Improving Locality

Matrix Multiplication example

$$[C] = [A] \times [B]$$

$$L \times M$$

$$L \times N$$

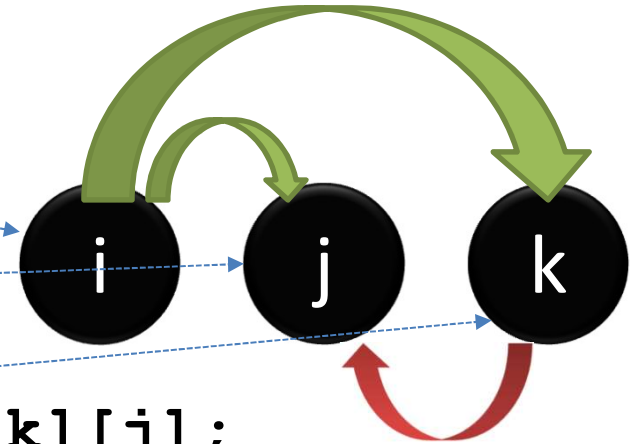
$$N \times M$$

# Cache Organization for the example

- Cache line (or block) = 8 matrix elements.
- Matrices are stored row wise.
- **Cache can't accommodate a full row/column.**
  - **L, M and N are so large w.r.t. the cache size**
  - After an iteration along any of the three indices, when an element is accessed again, it results in a miss.
- Ignore misses due to conflict between matrices.
  - As if there was a **separate cache for each matrix.**

# Matrix Multiplication : Code I

```
for (i = 0; i < L; i++)  
  for (j = 0; j < M; j++)  
    for (k = 0; k < N; k++)  
      C[i][j] += A[i][k] * B[k][j];
```



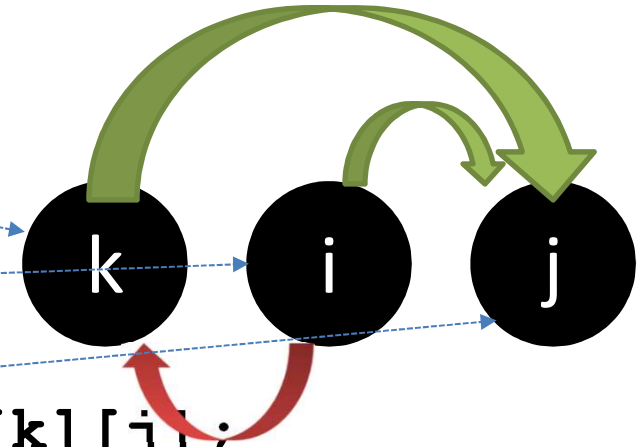
	C	A	B
accesses	LM	LMN	LMN
misses	LM/8	LMN/8	LMN

Total misses =  $LM(9N+1)/8$

$L=M=N=100$ ; miss =  $100*100*901/8=1,126,250$

# Matrix Multiplication : Code II

```
for (k = 0; k < N; k++)  
  for (i = 0; i < L; i++)  
    for (j = 0; j < M; j++)  
      C[i][j] += A[i][k] * B[k][j];
```



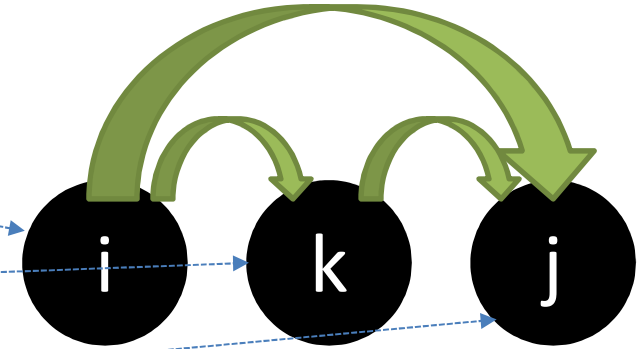
	C	A	B
accesses	LMN	LN	LMN
misses	LMN/8	LN	LMN/8

Total misses =  $LN(2M+8)/8$

$L=M=N=100$ ; miss =  $100*100*208/8=260,000$

# Matrix Multiplication : Code III

```
for (i = 0; i < L; i++)  
  for (k = 0; k < N; k++)  
    for (j = 0; j < M; j++)  
      C[i][j] += A[i][k] * B[k][j];
```



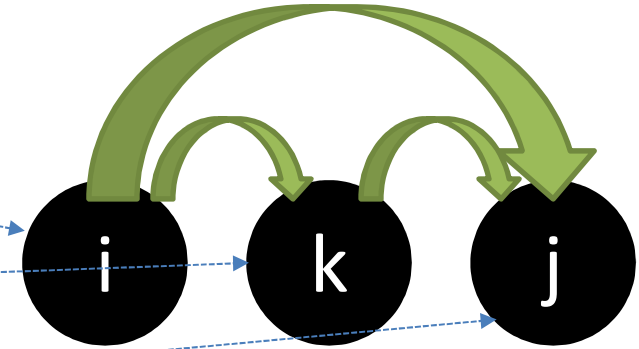
	C	A	B
accesses	LMN	LN	LMN
misses	LMN/8	LN/8	LMN/8

Total misses =  $LN(2M+1)/8$

$L=M=N=100$ ; miss =  $100*100*201/8=251,250$

# Matrix Multiplication : Code III

```
for (i = 0; i < L; i++)  
  for (k = 0; k < N; k++)  
    for (j = 0; j < M; j++)  
      C[i][j] += A[i][k] * B[k][j];
```



All most all modern processor uses

- Cache block pre-fetch
- When ith block is getting used i+1 block prefetched
- **Perfect overlap : only three cache miss**
  - **Each for A, B, C**





# Algorithm Classification and Access Optimization

- $O(N)/O(N)$  : If the # of arithmetic Ops and data transfer (LD/ST) are proportional to Loop Length  $N$ 
  - Optimization potential is limited
  - Example Scalar Product, vector add, sparse MVM
- Memory bound for large  $N$
- Compiler generated code achieve good perf.
  - Using software pipelining and loop nests

# Loop fusion for $O(N)/O(N)$

```
for (i=0; i<N; i++)  
    A[i]=B[i]+C[i];    //Bc=3W/1F  
for (i=0; i<N; i++)  
    Z[i]=B[i]+E[i];    //Bc=3W/1F
```



```
for (i=0; i<N; i++) {  
    A[i]=B[i]+C[i];  
    Z[i]=B[i]+E[i];  
}
```

$B_c = 5W/2F$   
No need get to  
 $B[i]$  again

# $O(N^2)/O(N^2)$ : OPS/DataTransfer

- Typical two loop nests with loop strip count N
  - $O(N^2)$  operation for  $O(N^2)$  loads and stores
- Example: dense MVM, Mat add, MatTrans
- MVM : -> Covert both access to row access

```
for (i=0; i<N; i++) {  
    tmp=C[i]  
    for (j=0; j<N; j++)    tmp=A[i][j]*B[j]  
    C[i]=tmp  
}
```

- Row I of A and vector B
- Original  $Bc=2W/2F$  but  $\rightarrow 2W*m/2F$
- m is miss rate of cache for Row access

# $O(N^3)/O(N^2)$ : OPS/DataTransfer

- Typical three loop nests
  - $O(N^3)$  operation for  $O(N^2)$  loads and stores
- Example: dense Matrix Multiplication
- Implementation of cache Bound
  - Already studied : loop interchange
  - **Blocking : Strassen multiplication, will be discussed later**

# **Multiprocessor Programming using Threading**

# Threading Language and Support

- Initially threading used for
  - Multiprocessing on single core : earlier days
  - Feeling/simulation of doing multiple work simultaneously even if on one processor
  - Used TDM, time slicing, Interleaving
- Now a day threading mostly used for
  - To take benefit of multicore
  - Performance and energy efficiency
  - We can use both TDM and SDM

# Threading Language and Support

- Pthread: POSIX thread
  - Popular, Initial and Basic one
- Improved Constructs for threading
  - c++ thread : available in c++11, c++14
  - **Java thread : very good memory model**
    - **Atomic function, Mutex**
- Thread Pooling and higher level management
  - OpenMP (loop based)
  - Cilk (dynamic DAG based)

# Pthread, C++ Thread, Cilk and OpenMP

```
pthread_t tid1, tid2;  
pthread_create(&tid1, NULL, Fun1, NULL);  
pthread_create(&tid2, NULL, Fun2, NULL);  
pthread_join(tid1, NULL);  
pthread_join(tid2, NULL);
```

Pthread

```
thread t1(Fun1);  
thread t1(Fun2, 0, 1, 2); // 0, 1,2 param to Fun2  
t1.join();  
t2.join();
```

C++  
thread

**#pragma omp parallel for**

```
for(i=0;i<N;i++)
```

```
    A[i]=B[i]*C[i];
```

**//Auto convert serial code to threaded code**

**// \$gcc -fopenmp test.c; export OMP\_NUM\_THREADS=10; ./a.out**

```
cilk fib (int n){//Cilk dynamic parallism, DAG recursive code  
    if (n<2) return n;  
    int x=spawn fib(n-1); //spawn new thread  
    tnt y=spawn fib(n-2); //spawn new thread  
    sync;  
    return x+y;  
}
```