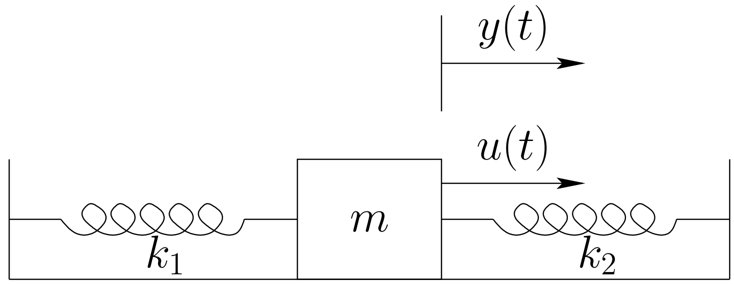
# CS 522, Embedded Systems
# Midsem Exam, Monsoon 2023-24
# Department of Computer Science and Engineering
# IIT Guwahati
# Time: Two hours

**Important**

1. Write your answers in the space provided for each question in the answer sheet.
2. Mention your roll number at the top of every page in the answer sheet.
3. A supplementary sheet is being provided for rough work. **Do not attach your rough work to the answer sheet.**
4. This exam has 5 questions over 6 pages, with a total of 60 marks.

1. Consider the spring-mass system shown below, where the mass $m$ is connected by two springs with spring constants $k_1$ and $k_2$ on a frictionless floor. Let $u(t)$, the external force applied to the mass (different from the forces due to the springs), be the input to the system. Let $y(t)$, the displacement of the mass from its neutral position (*i.e.*, the position where the total force due to the springs is zero), be the output of the system. For both $u(t)$ and $y(t)$ a positive magnitude indicates direction to the right. Assume the initial values $y_0$ and $v_0$ for the initial displacement and velocity.




(a) Identify the state variables of the system. Then find the state space equations for the system, one for each state variable, *i.e.*, equations of the form $\frac{d\mathbf{X}(t)}{dt} = F(\mathbf{X}(t), u(t))$ where $\mathbf{X}(t)$ is the state vector and $u(t)$ the input at time $t$.

(10)

**Solution:** The state variables are $x_1 = y$ and $x_2 = \dot{y}$. From Newton's second law we have $u(t) - k_1 y(t) - k_2 y(t) = m\ddot{y}$ which gives $\ddot{y} = \frac{1}{m}[u(t) - (k_1 + k_2)y(t)]$. The state space equations are:

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \frac{1}{m}[u(t) - (k_1 + k_2)x_1(t)] \end{aligned}$$

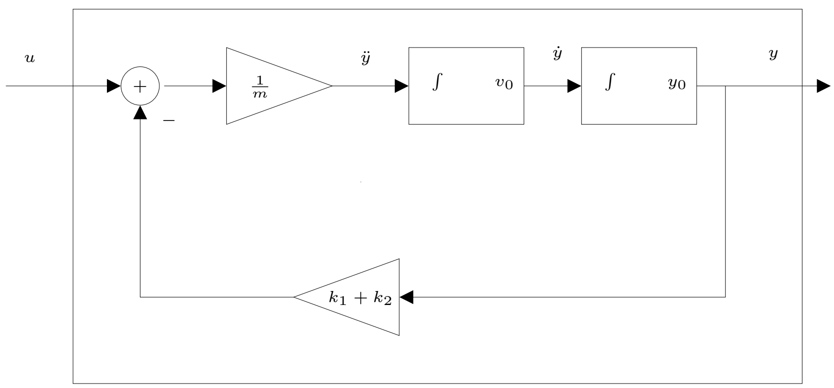
(b) Construct a model of the system using only actors such as Integrators and basic arithmetic actors. Note that you don't have to solve the differential equation, only model the system using actors.

(10)

**Solution:** We have the following equations:

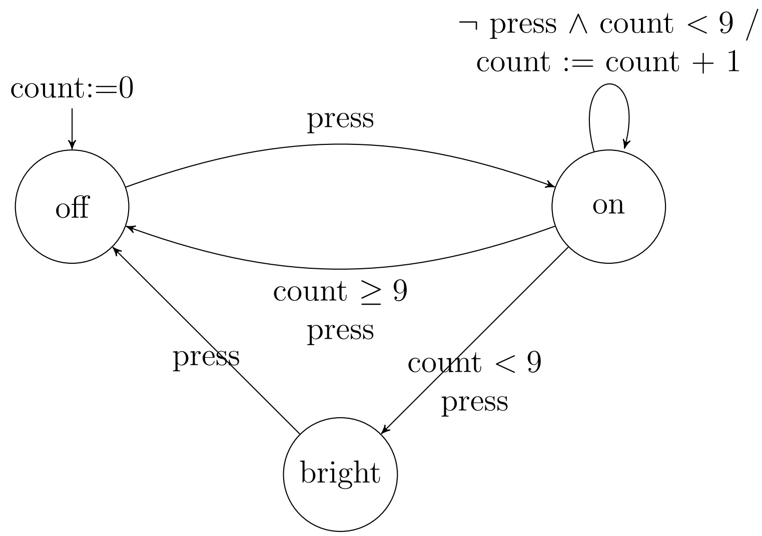
$$
\begin{aligned}
y(t) &= y_0 + \int_0^t \dot{y}(\tau)d\tau \\
\dot{y}(\tau) &= v_0 + \int_0^\tau \ddot{y}(\alpha)d\alpha \\
&= v_0 + \frac{1}{m}\int_0^\tau [u(\alpha) - (k_1 + k_2)y(\alpha)]d\alpha
\end{aligned}
$$

From the above equation we have the following actor diagram:




2. Design an **extended state machine** for a light switch with two levels of brightness. Assume that the machine is time-triggered and reacts once every time unit. It has a single pure input signal called `press` and no output. Initilly the light is off. When the swich is pressed, it is turned on with normal brightness. If another press happens at or after ten time units the light is switched off. But if the swich is pressed strictly before ten time units when it is on, the light gets brighter. In the bright mode, a press always turns off the light. (Note: You have to design an extended state machine and *not* a timed or hybrid automaton for this problem.)

(10)



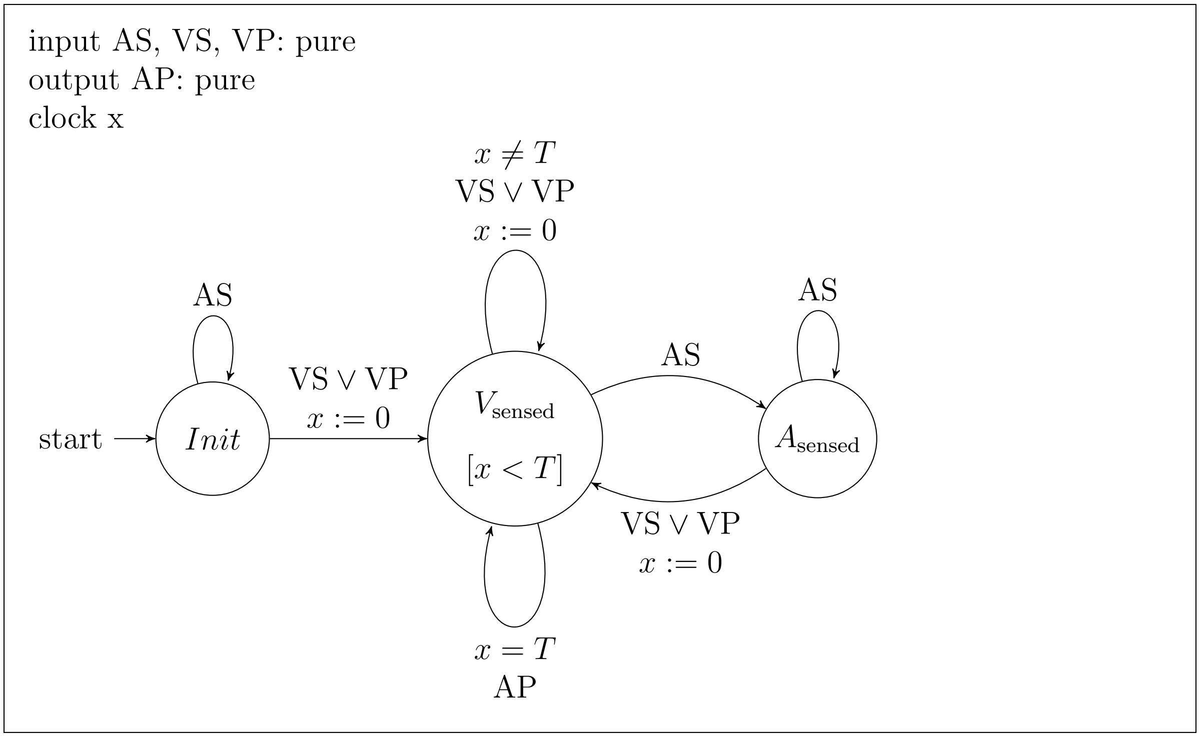

**Solution:**

3. The function of a pacemaker is to keep the heart rate above a minimum value. For this purpose, the pacemaker keeps track of the input events Ventricular Sense (VS), Ventricular Pace (VP) and Atrial Sense (AS), and produces the output event Atrial Pace (AP). At all times the pacemaker has the following behaviour: if no atrial event (*i.e.*, AS) or ventricular event (*i.e.*, VS or VP) has been sensed since the last ventricular event (*i.e.*, VS or VP) and a constant $T$ amount of time has elapsed, then the pacemaker delivers an AP output. It then waits for the next ventricular event before repeating the behaviour. On the other hand, the occurrence of an AS at any time makes the pacemaker wait for the next ventricular event before repeating the behaviour. Model this behaviour of the pacemaker as a timed automaton.

   **Hint:** Here all signals are pure. You can assume that the input event AS will not occur simultaneously with the input events VS or VP. You can use the notation "VS $\vee$ VP" to model "either or both of the events VS and VP are present". For this probem assume a guard only enables a transition but does not force it, as is true in the orginal timed automaton model by Alur and Dill. To force an exit when a condition is true location invariants must be used.

(10)

**Solution:**



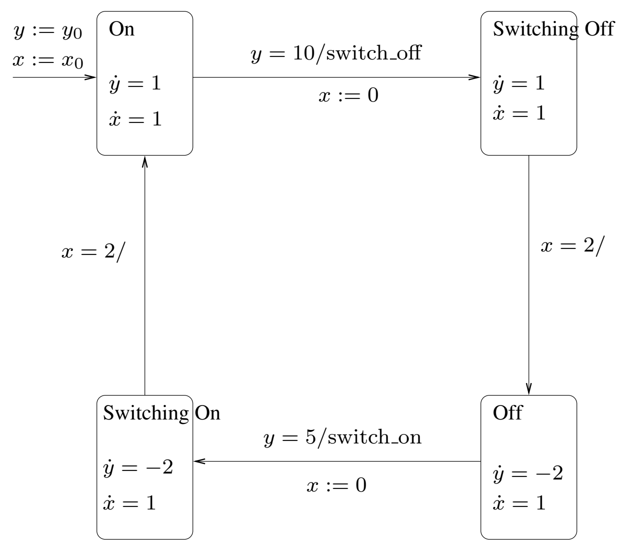

4. A water level control system behaves as follows. The water level rises one unit per second when the pump is on and falls two units per second when the pump is off. When the water level rises to 10 units the controller sends a `switch_off` signal, which after a delay of two seconds results in the pump turning off. When the water level falls to 5 units, the controller sends a `switch_on` signal, which after a delay of two seconds causes the pump to switch on. There are two continuous state variables in this model – the water level $y(t)$ and the time elapsed $x(t)$ since the last signal sent by the controller. Model the control system as a hybrid system with outputs `switch_off` and `switch_on`. Assume the initial values of $y(t)$ and $x(t)$ are $y_0$ and $x_0$, respectively and the pump is initially on. Do not use any variables or signals in the model other than the ones mentioned.

   For this problem you can assume that a transition is taken the moment its guard becomes true *i.e.*, we have an urgent semantics for transitions.

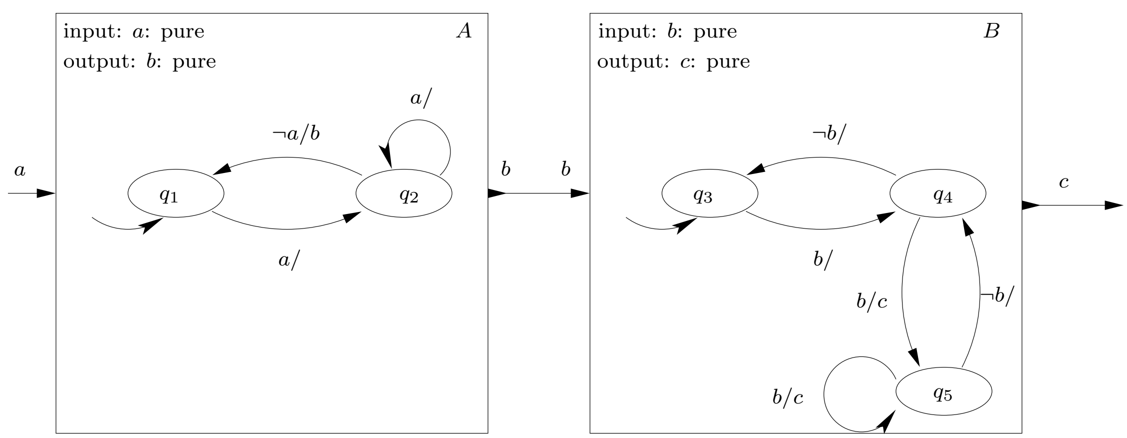
(10)

**Solution:** The hybrid system is shown in the figure below.

continuous variable x,y : $\mathbb{R}$
output switch_off, switch_on: pure



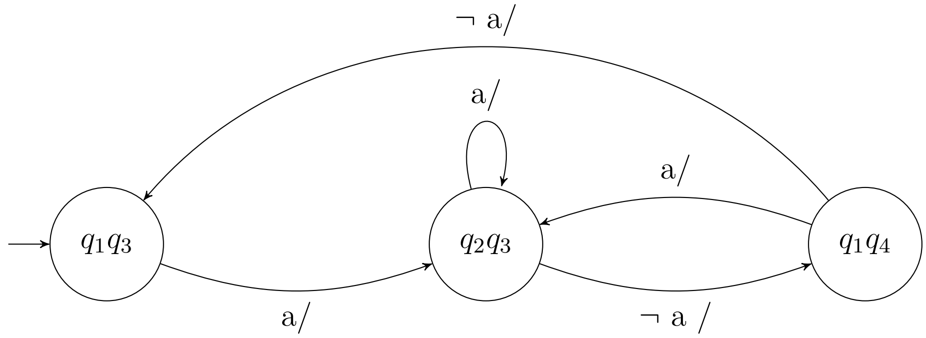

5. Consider the following synchronous cascade composition of two FSMs $A$ and $B$. Note that all transitions not shown in the figure are assumed to be default transitions. Construct a single state machine $C$ representing the composition. You can drop the unreachable states from the composition.




(10)

**Solution:**

input a: pure
output c: pure




Note that the self-loop on the state $q_2q_3$ is redundant, as it is a default transition. Similarly, in the question the loop on $q_2$ is redundant.