Deep Reinforcement Learning

Learn to make good sequence of decision

Outlines

- What is Reinforcement Learning?
- Markov Decision Processes
- Q-Learning
- Policy Gradients

Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn a *function* to map x -> y

Examples: Classification, regression, object detection, semantic segmentation, image captioning, etc.

$\longrightarrow$ Cat

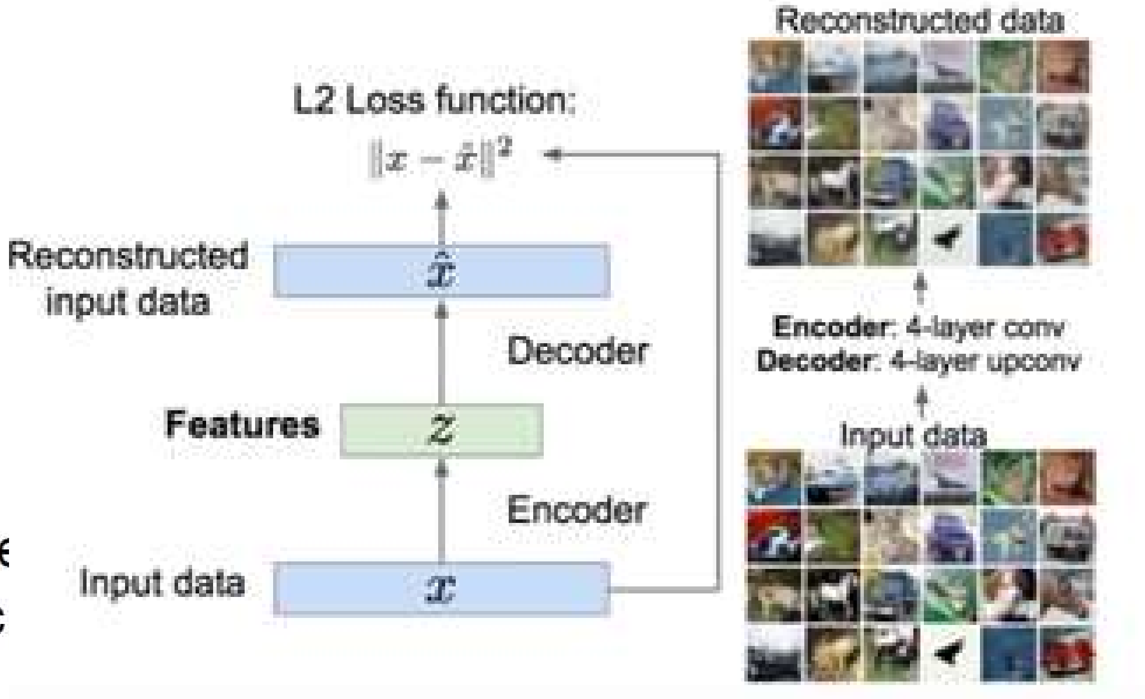
Classification

Un-Supervised Learning

Data: x
Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, featur learning, density estimation, etc



Reinforcement Learning

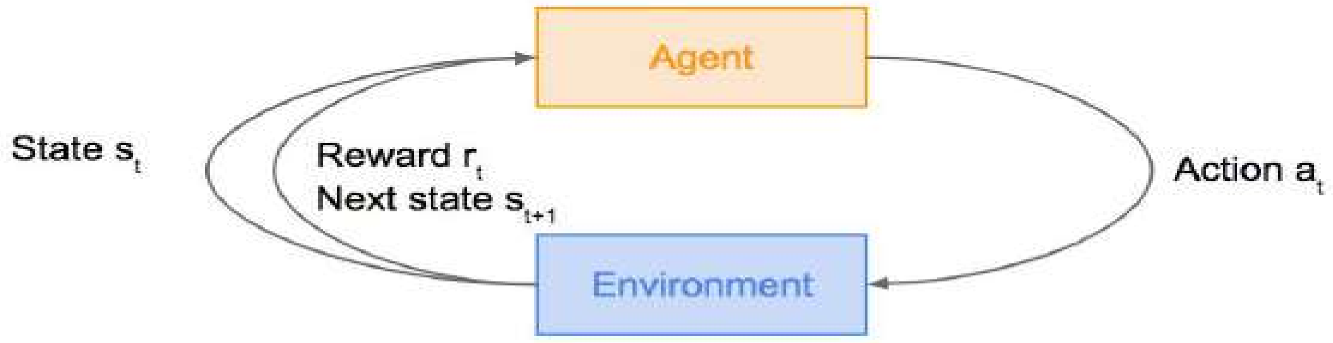
Repeated interaction with world

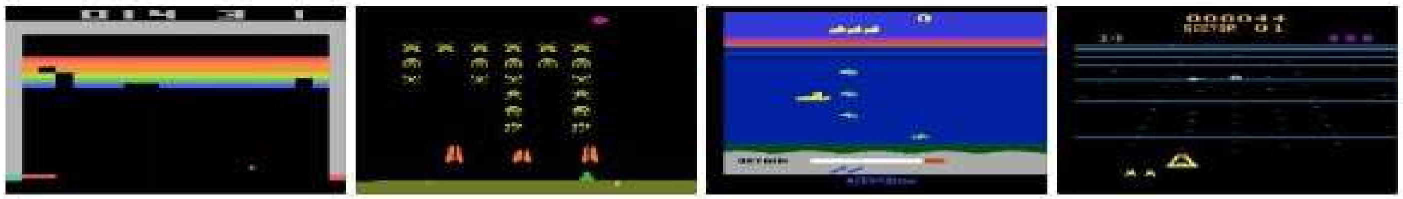
Learn to make good sequence of decisions

Reinforcement Learning

Problems involving an **agent** interacting with an **environment,** which provides numeric **reward** signals

Goal: Learn how to take actions in order to maximize reward

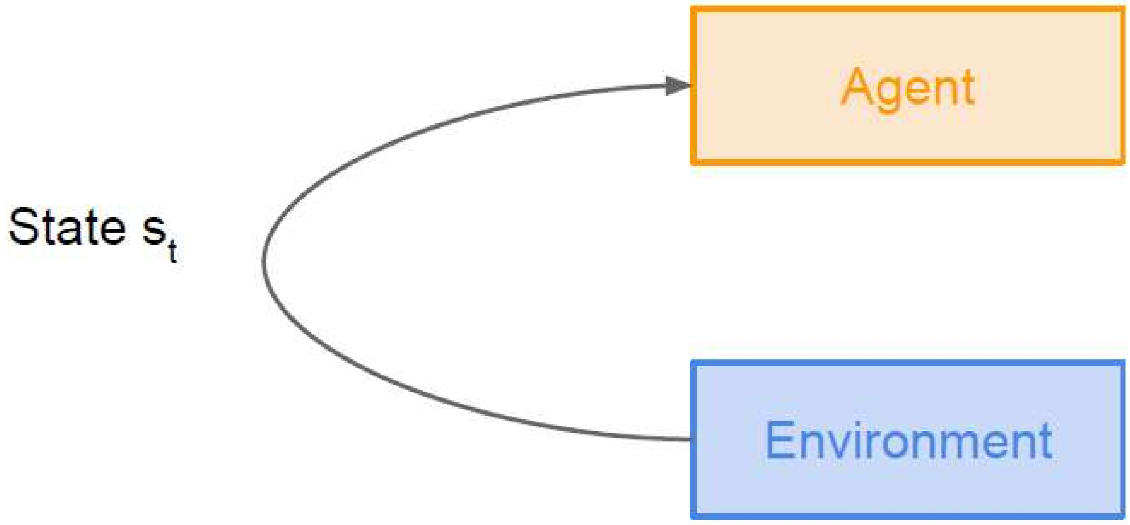




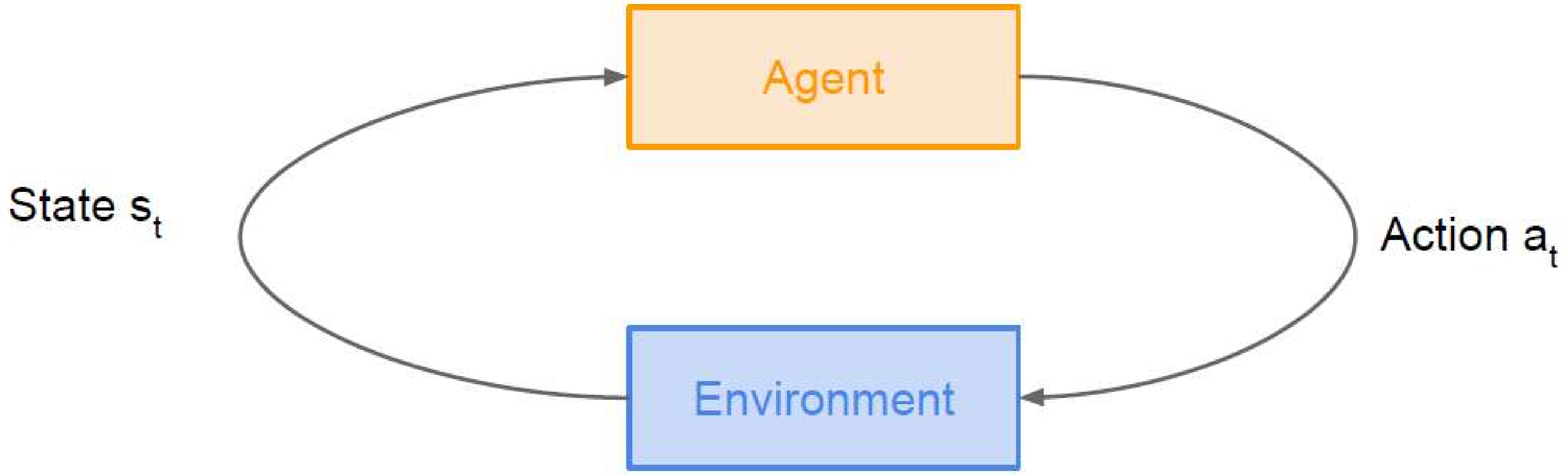
Reinforcement Learning

Agent
Environment

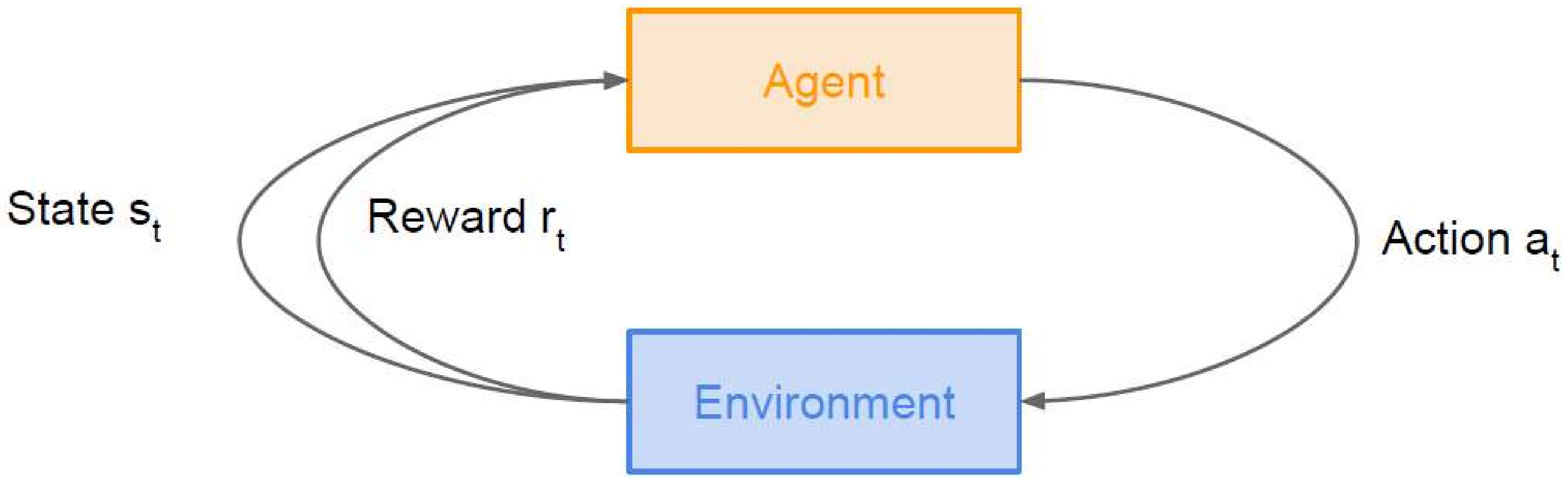
Reinforcement Learning



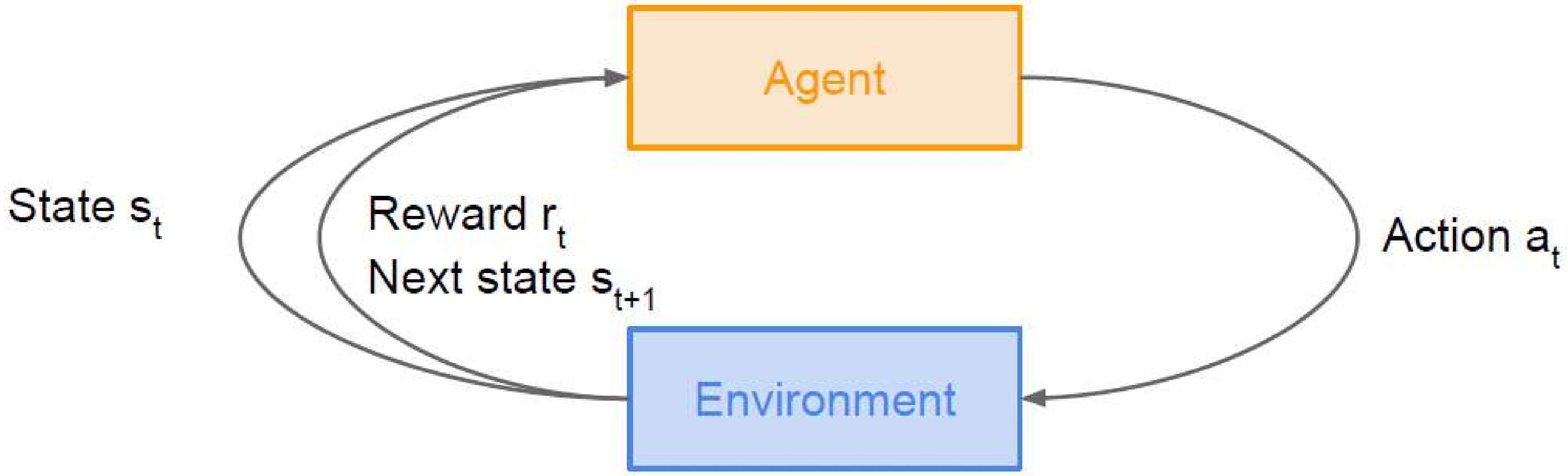
Reinforcement Learning



Reinforcement Learning

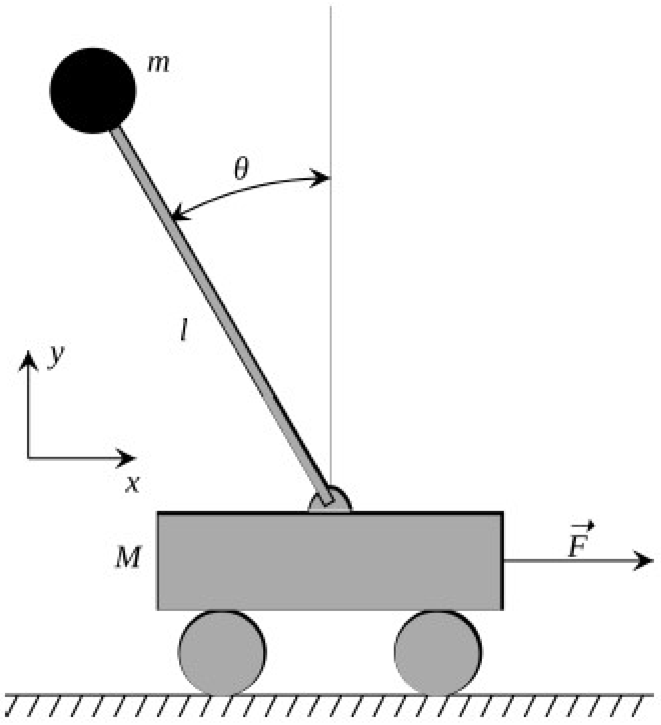


Reinforcement Learning



Cart-Pole Problem

Objective : Balance a pole on top of a movable cart
State : angle, angular speed, position, horizontal velocity
Action : horizontal force applied on the cart
Reward : 1 at each time step if the pole is upright



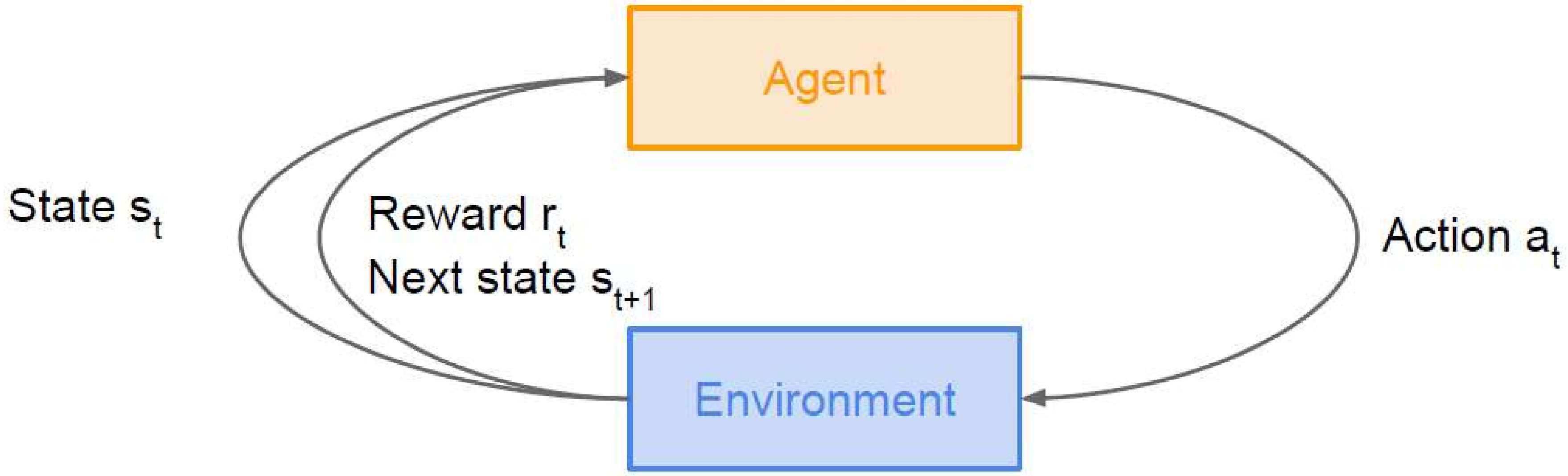
Robot Locomotion

Objective: Make the robot move forward State: Angle and position of the joints
Action: Torques applied on joints
Reward: 1 at each time step upright + forward movement

Atari Games

Objective	: Complete the game with the highest score
State	: Raw pixel inputs of the game state
Action	: Game controls e.g. Left, Right, Up, Down
Reward	: Score increase/decrease at each time step

RL: Mathematical Formulation



Markov Decision Process

- Mathematical formulation of the RL problem
- **Markov property**: Current state completely characterises the state of the world

Defined by: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{P}, \gamma)$

$\mathcal{S}$	: set of possible states
$\mathcal{A}$	: set of possible actions
$\mathcal{R}$	: distribution of reward given (state, action) pair
$\mathbb{P}$	: transition probability i.e. distribution over next state given (state, action) pair
γ	: discount factor

Markov Decision Process

S is a set of a finite state that describes the environment.

A is a set of a finite actions that describes the action that can be taken by the agent

P is a probability matrix that tells the probability of moving from one state to the other.

R is a set of rewards that depend on the state and the action taken. Rewards are not necessarily positive, they should be seen as outcome of an action done by the agent when it is at a certain state. So negative reward indicates bad result, whereas positive reward indicates good result.

γ is a discount factor, that tells how important future rewards are to the current state. Discount factor is a value between 0 and 1. A reward R that occurs N steps in the future from the current state, is multiplied by γ^N to describe its importance to the current state. For example consider $\gamma = 0.9$ and a reward $R = 10$ that is 3 steps ahead of our current state. The importance of this reward to us from where we stand is equal to $(0.9^3)*10 = 7.29$.

Value Functions

Now with the MDP in place, we have a description of the environment but still we don't know how the agent should act in this environment.

The rule we impose on the agent is that it must act in a way to maximize the collected rewards.

But to be able to do that, the agent should have something to estimate the position or state it is currently in.

Consider again the labyrinth, if the agent is few steps from the exit, his position has much more value than a position at the centre of the labyrinth.

We call this is to estimate the Value of the position or the Value of the state.

Once we know the value of each state, we can figure out what is the best way to act (simply by following the state that with the highest value).

State Value Function

Still we need to figure out a way to quantify the value of each state. This is done by a function called Sate-Value Function:

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s', r | s, a) \left[r + \gamma v_\pi(s') \right], \quad \text{for all } s \in \mathcal{S}$$

What is important to learn about this equation is that the value of each state is the average of discounted future rewards.

If we look closely at the equation we see that ***v(s)*** is expressed in terms of immediate rewards ***r*** and the value of neighbouring states ***v(s')***.

State Value Function

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)\Big[r + \gamma v_\pi(s')\Big], \quad \text{for all } s \in \mathcal{S}$$

1. When at a state ***s***, consider all possible actions, then,

2. for each action ***a*** get the probability ***p(s', r | s, a)*** of getting immediate reward ***r***, and moving to neighbouring state ***s'***, knowing that we are at state ***s*** and we performed action ***a***.

3. Add the reward ***r*** to the discounted value of neighbour state ***s'*** given by ***γ*v(s')***. Multiply the result with ***p(s', r | s, a)***, and we get ***p(s', r | s, a)*[r+γ*v(s')]***.

4. Repeat steps 2 and 3 for all states **s'** neighbouring ***s,*** as well as all the possible rewards ***r***, and compute the sum of the results. Now we have ***Sum(p(s', r | s, a)*[r+γ*v(s')])*** over all ***s'*** and ***r.***

5. This will give the average of discounted future rewards when taking only one action ***a***. However there are multiple actions to be taken so we have to average over all actions. To do so we multiply by probability ***π(a|s)*** that action ***a*** be performed at state ***s***, and we sum over all possible actions in that state.

Action Value Function

1. It suffices to think that when we are at a state S, we have a probability to take some actions that might lead us to different states with different rewards.

2. So the value of our state is the average of all discounted rewards that we might get when performing all of the possible actions at the current state.

3. Now we have a function that gives us the value of each state. It tells us how good is to be at each one of them.

4. Of course we still have to make the computation in order to get a number that represents the value of that state.

5. The **v(s)** tells how good to be in state s, but it does not tell how good to perform action a while in state s. This is the purpose of the Action-Value function:

$$q_\pi(s,a) = \sum_{s',r} p(s',r|s,a)\Big[r + \gamma v_\pi(s')\Big]$$

Action Value Function

$$q_\pi(s,a) = \sum_{s',r} p(s',r|s,a)\Big[r + \gamma v_\pi(s')\Big]$$

The explication is simple, we take ***v(s)*** and instead of performing all actions, we decide to perform only one action.

We no more ask ourselves what is the probability of using action ***a*** over all possible actions, but we say we will use action ***a***.

So the sum of ***π(a|s)*** does not apply anymore.

With this logic we reduce ***v(s)*** from averaging over all possible actions to simply using one selected action, we denote this as ***q(s,a)***.

Action Value Function

Notice that ***q(s,a)*** checks the value of the action at state ***s*** while the values ***v(s')*** of neighboring states are kept unchanged. So in this sense it checks how good this action is in the current state while keeping everything else the same.

It is also easy to notice that ***v(s)*** is the weighted average of ***q(s,a)*** over all possible actions at state ***s***

$$v_\pi(s) = \sum_a \pi(a|s) q_\pi(s,a)$$

Policy

The act of selecting an action at each state is called "policy" and is denoted as π.

- some policies are better than others due to the selection of actions over others in one or more states.
- It is important to note that a policy π is better than π' if all ***v(s)*** under π are greater or equal to all ***v(s)*** under π'.
- It follows that in order to **maximize the collected rewards** we have to find the best possible policy, called optimal policy and denoted π^*.

Optimal Value Functions and Policy

An optimal value state function ***v*(s)*** is a function that gives the maximum value at each state among all policies:

$$v_*(s) = \max_\pi v_\pi(s), \quad \text{for all } s \in \mathcal{S}$$ where

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)\Big[r + \gamma v_\pi(s')\Big], \quad \text{for all } s \in \mathcal{S}$$

Similarly an optimal action state function ***q*(s)*** is the function that gives the maximum ***q*** value at each state among all policies:

$$q_*(s,a) = \max_\pi q_\pi(s,a), \quad \text{for all } s \in \mathcal{S} \text{ and } a \in \mathcal{A}(s)$$

it follows that

$$v_*(s) = \max_{a\in\mathcal{A}(s)} q_{\pi_*}(s,a)$$ where $$q_\pi(s,a) = \sum_{s',r} p(s',r|s,a)\Big[r + \gamma v_\pi(s')\Big]$$

$$v_*(s) = \max_a \sum_{s',r} p(s',r|s,a)[r + \gamma v_*(s')]$$

Optimal Policy

Notice that ***v(s)*** is the average of values produced by all actions, while ***v*(s)*** is the maximum value that one action can produce among all other actions.

$$q_*(s,a) = \sum_{s',r} p(s',r|s,a)\left[r + \gamma \max_{a'} q_*(s',a')\right]$$

Since ***q(s,a)*** is the value produced by a specific action ***a*** at state ***s***, ***q*(s,a)*** is the value produced by a specific action ***a*** at state ***s***, while selecting maximum ***q*** values in the other states.

Earlier we said that a policy π is better than another policy ***π'*** if it produces higher or equal ***v(s)*** value at each state:

$$\pi \geq \pi' \text{ if } v_\pi(s) \geq v_{\pi'}(s), \forall s$$

Optimal Policy

It follows that the optimal policy π* produces, at each state, higher or equal ***v(s)*** values than any other policy π .

$$\pi_* \geq \pi \ \text{ if } v_{\pi_*}(s) \geq v_\pi \ (s), \forall s \ \forall \pi$$

This means that any ***v(s)*** and ***q(s,a)*** following the optimal policy π* are equal to ***v*(s)*** and ***q*(s,a)*** respectively.

All optimal policies achieve the optimal value function,
$v_{\pi_*}(s) = v_*(s)$
All optimal policies achieve the optimal action-value function,
$q_{\pi_*}(s, a) = q_*(s, a)$

How to find the optimal policy π^* ?

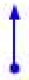
So the question that remains is, how to find the optimal policy π^* ?
The optimal policy should always return the action that produces ***q*(s,a)***

$$\pi_*(a|s) = \begin{cases} 1 & \text{if } a = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\; q_*(s,a) \\ 0 & otherwise \end{cases}$$

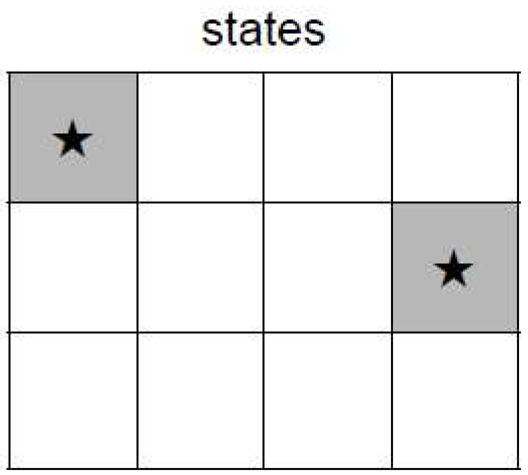
- There is always a deterministic optimal policy for any MDP
- If we know ***q*(s,a)*** it will be very easy to find π^*.

MDP Example: Grid World

actions = {

1. right
2. left
3. up 
4. down

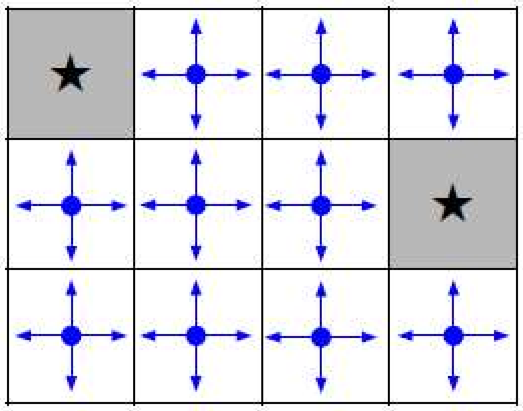
}



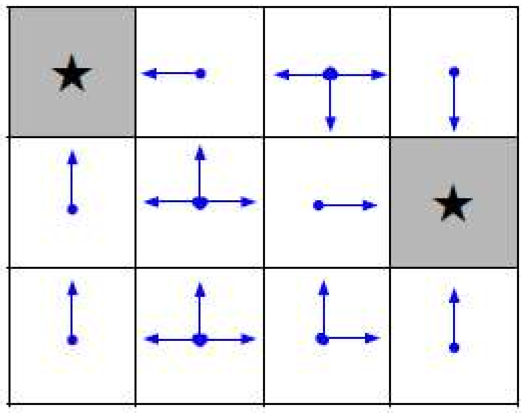
Set a negative "reward" for each transition (e.g. $r = -1$)

Objective: reach one of terminal states (greyed out) in least number of actions

MDP Example: Grid World



Random Policy



Optimal Policy

Optimal Policy

Note that the optimal policy is not unique, and this can be easily verified.

Suppose an agent in the Labyrinth has arrived to a state with 3 possible moves (or actions), go left, go right, or go forward. Suppose going forward leads the agent to a dead end, while going left or right will lead the agent to the exit using the same number of steps.

It is clear that there are two optimal policies (not only one) at this state, one that instructs the agent to go left and the other instructs it to go right.

Markov Decision Process

- At time step t=0, environment samples initial state $s_0 \sim p(s_0)$
- Then, for t=0 until done:
  - Agent selects action a_t
  - Environment samples reward $r_t \sim R(. \mid s_t, a_t)$
  - Environment samples next state $s_{t+1} \sim P(. \mid s_t, a_t)$
  - Agent receives reward r_t and next state s_{t+1}

- A policy **π** is a function from S to A that specifies what action to take in each state
- **Objective**: find policy **π*** that maximizes cumulative discounted reward: $\sum_{t \geq 0} \gamma^t r_t$

The optimal policy π^*

We want to find optimal policy **π*** that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability…)?

Maximize the **expected sum of rewards!**

Formally: $\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi\right]$ with $s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$

As we see earlier

$$\pi_*(a|s) = \begin{cases} 1 & \text{if } a = \underset{a \in \mathcal{A}}{\text{argmax}}\ q_*(s,a) \\ 0 & \textit{otherwise} \end{cases}$$

Value function and Q-value function

Following a policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \ldots$

How good is a state?
The **value function** at state s, is the expected cumulative reward from following the policy from state s:

$$V^\pi(s) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, \pi\right]$$

As we see earlier

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)\Big[r + \gamma v_\pi(s')\Big], \quad \text{for all } s \in \mathcal{S}$$

How good is a state-action pair?
The **Q-value function** at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^\pi(s,a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

As we see earlier

$$q_\pi(s,a) = \sum_{s',r} p(s',r|s,a)\Big[r + \gamma v_\pi(s')\Big]$$

Bellman equation

The optimal Q-value function Q* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Q* satisfies the following **Bellman equation**:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s',a') | s, a\right]$$

Intuition: if the optimal state-action values for the next time-step Q*(s',a') are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s',a')$

As we see earlier

$$q_*(s,a) \;=\; \sum_{s',r} p(s',r|s,a)\Big[r + \gamma \max_{a'} q_*(s',a')\Big]$$

The optimal policy **π*** corresponds to taking the best action in any state as specified by Q*

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s,a) = \mathbb{E}\left[r + \gamma \max_{a'} Q_i(s',a')|s,a\right]$$

Q_i will converge to Q^* as i -> infinity

What's the problem with this?
Not scalable. Must compute Q(s,a) for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

Solution: use a function approximator to estimate Q(s,a). E.g. a neural network!

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$\underset{\text{function parameters (weights)}}{Q(s, a; \theta)} \approx Q^*(s, a)$$

If the function approximator is a deep neural network => **deep q-learning**!

Example: Playing Atari Games

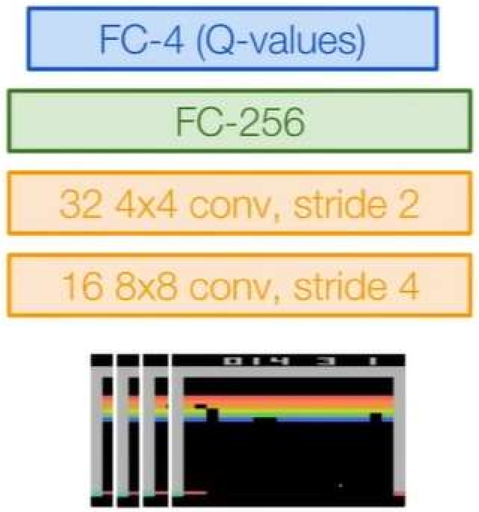
Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

Q Network Architecture: Atari games

$Q(s, a; \theta)$:
neural network
with weights θ

A single feedforward pass to compute Q-values for all actions from the current state => efficient!



Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t, a_4)$

Number of actions between 4-18 depending on Atari game

Current state s_t: 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s',a') | s,a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)} \left[(y_i - Q(s,a;\theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) | s,a \right]$

Iteratively try to make the Q-value close to the target value (y_i) it should have, if Q-function corresponds to optimal Q* (and optimal policy π^*)

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) - Q(s,a;\theta_i)) \nabla_{\theta_i} Q(s,a;\theta_i) \right]$$

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

Address these problems using **experience replay**

- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

Each transition can also contribute to multiple weight updates
=> greater data efficiency

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

```
Initialize replay memory $\mathcal{D}$ to capacity $N$
Initialize action-value function $Q$ with random weights
for episode $= 1, M$ do
    Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
    for $t = 1, T$ do
        With probability $\epsilon$ select a random action $a_t$
        otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
        Execute action $a_t$ in emulator and observe reward $r_t$ and image $x_{t+1}$
        Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
        Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
        Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
        Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    end for
end for
```

$\leftarrow$ Initialize replay memory, Q-network

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

```
Initialize replay memory $\mathcal{D}$ to capacity $N$
Initialize action-value function $Q$ with random weights
**for** episode $= 1, M$ **do**    ← Play M episodes (full games)
    Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
    **for** $t = 1, T$ **do**
        With probability $\epsilon$ select a random action $a_t$
        otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
        Execute action $a_t$ in emulator and observe reward $r_t$ and image $x_{t+1}$
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        Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
        Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    **end for**
**end for**
```

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

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        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    end for
end for
```

$\leftarrow$ Initialize state (starting game screen pixels) at the beginning of each episode

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Deep Q-Learning with Experience Replay

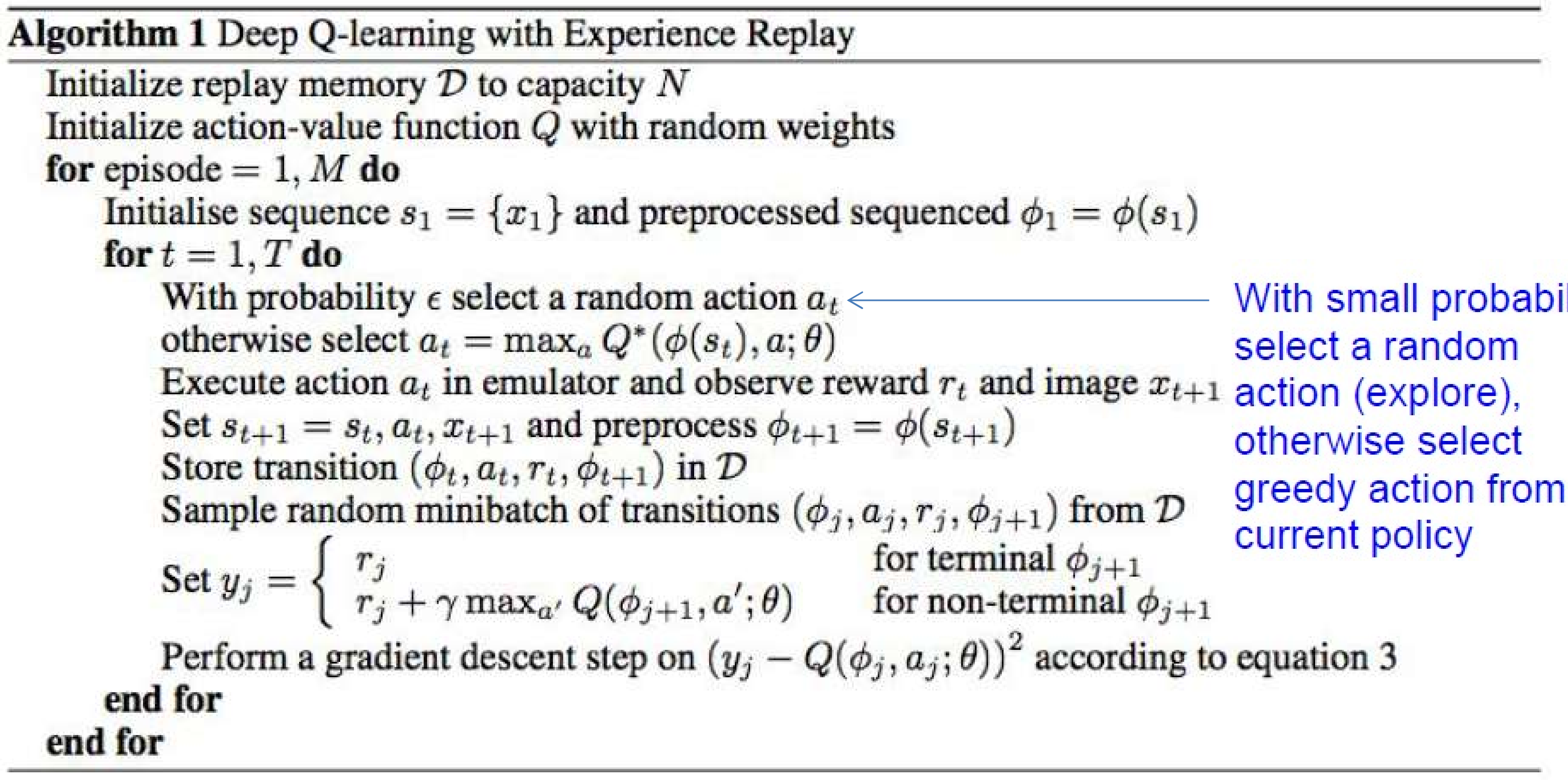
Algorithm 1 Deep Q-learning with Experience Replay

```
Initialize replay memory $\mathcal{D}$ to capacity $N$
Initialize action-value function $Q$ with random weights
for episode $= 1, M$ do
    Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
    for $t = 1, T$ do
        With probability $\epsilon$ select a random action $a_t$
        otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
        Execute action $a_t$ in emulator and observe reward $r_t$ and image $x_{t+1}$
        Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
        Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
        Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
        Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    end for
end for
```

For each timestep t of the game

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Deep Q-Learning with Experience Replay



Algorithm 1 Deep Q-learning with Experience Replay

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    end for
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```

With small probability, select a random action (explore), otherwise select greedy action from current policy

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Deep Q-Learning with Experience Replay

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          Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
      end for
  end for
```

Take the action (a_t), and observe the reward r_t and next state s_{t+1}

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

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        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    end for
end for
```

Store transition in replay memory

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Deep Q-Learning with Experience Replay

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        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    end for
end for
```

Experience Replay: Sample a random minibatch of transitions from replay memory and perform a gradient descent step

[Mnih et al. NIPS Workshop 2013; Nature 2015]

to continue...