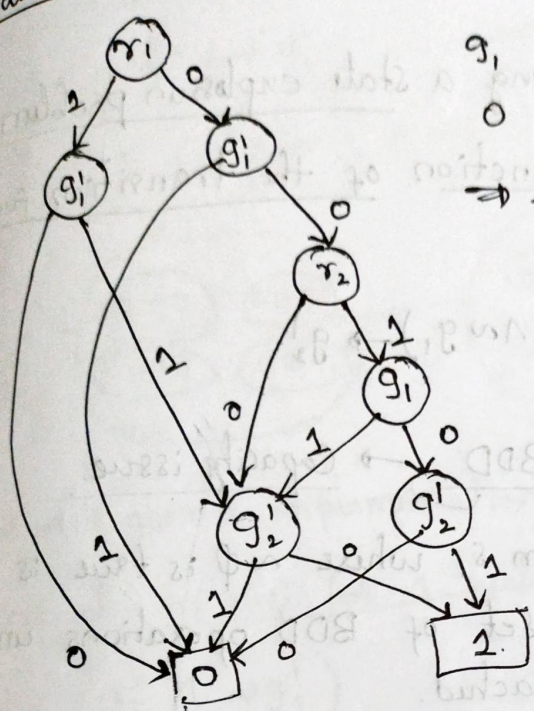


Final BDD

16  $\rightarrow$  valid paths

$g_1$	$g_2$	$r_1$	$r_2$	$g_1'$	$g_2'$
0	X	0	1	0	1

$\Rightarrow$  All paths from root to the node 1 are the valid function transition.



$B_7$

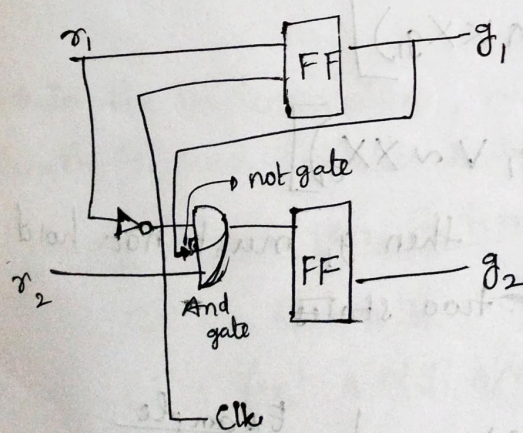
17/10/2023

18/10/2023

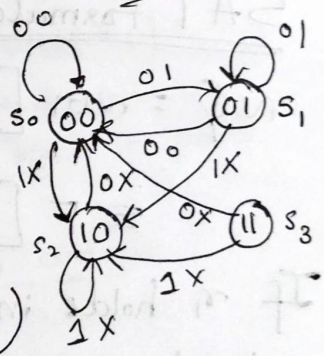
Absent

31/10/2023

SAT based forward property Verification



Kripke structure  
 $\rightarrow$  transition system



states  $\langle g_1, g_2 \rangle$   
transition  $\langle r_1, r_2 \rangle$

$$\phi := G [r_1 \rightarrow X g_1 \wedge X X g_1]$$

$$M, s_1 \models \phi ?$$

open system

$\downarrow$  convert to  
closed system

⇒ Take  $\neg \phi$ , and check for a path from  $S_i$  where  $\neg \phi$  is true

⇒ This may end up having a state explosion problem

⇒ Take characteristic function of the transition function

$$\begin{cases} C_1: r_1 \rightarrow g_1' \\ C_2: (r_1 \wedge \neg r_1 \wedge \neg g_1) \rightarrow g_1' \end{cases}$$

Construct RoBDD → Capacity issue.

⇒ Search for path from  $S_i$  where  $\neg \phi$  is true is done effectively by set of BDD operations until a fixed point is reached.

⇒  $S_1, S_2, S_0$  path doesn't satisfy  $\phi \Rightarrow M, S_1 \not\models \phi$

⇒ Due to Capacity issue, this method may not work as expected.

### SAT Formulation

$$\neg \phi = \neg G_1 [r_1 \rightarrow (Xg_1 \wedge \neg Xg_1)]$$

$$\equiv F [r_1 \wedge (\neg Xg_1 \vee \neg \neg Xg_1)]$$

If  $r_1$  holds in a state, then  $g_1$  must not hold at least one of the next two states.

Set of variables:

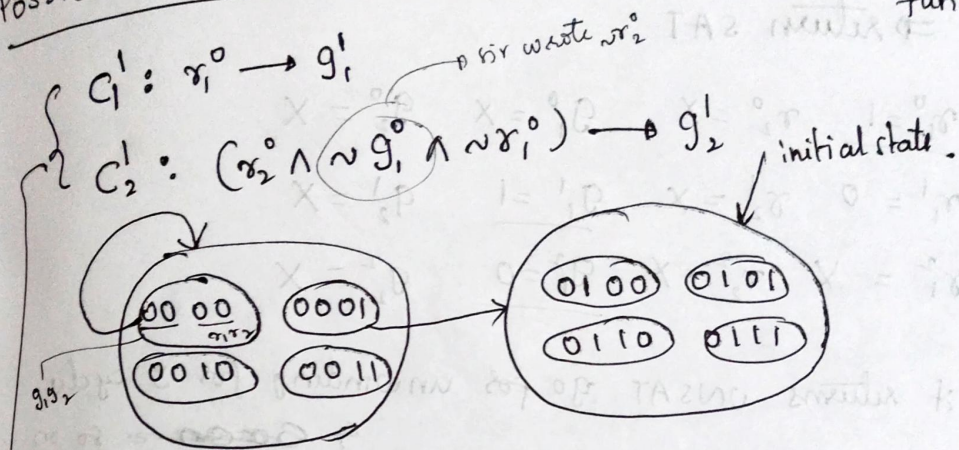
$$\bigcup_i (S_0^i, S_1^i, \dots, S_k^i)$$

Example  
 $cg_1, g_2, r_1, r_2$

Initial state  $I': (\neg g_1^0 \wedge g_2^0)$



Possible transition from initial state (use characteristic function)



Unwind implementation for 1 cycle.

Unwind of  $\neg\phi$  for 1 cycle

$$Z^1: (x_1^0 \wedge \neg g_1^1)$$

check SAT of  $(I \wedge Z^1 \wedge C_1^1 \wedge C_2^1)$

means check if  $\neg\phi$  holds in the implementation upto 1 cycle from the initial state

$\Rightarrow$  returns UNSAT (because  $Z^1$ , and  $C_1^1$  contradicts with each other)

$\Rightarrow$  In the implementation,  $\neg\phi$  does not hold in path length '1' from initial state.

Unwind transition system for two cycles

$$C_1^2: x_1^1 \rightarrow g_1^2$$

$$C_2^2: (x_2^1 \wedge \neg g_1^1 \wedge \neg x_1^1) \rightarrow g_2^2$$

(Note:  $\neg x_1^1$  is circled in the original image, with a note "sir wrote  $\neg g_2^1$ " pointing to it.)

Unwind  $\neg\phi$  for two cycles

$$Z^2: (x_1^0 \wedge (\neg g_1^1 \vee \neg g_1^2)) \vee (x_1^1 \wedge \neg g_1^2)$$



→ check SAT  $(I \wedge G_1' \wedge G_2' \wedge G_1'' \wedge G_2'' \wedge \neg Z^2)$

→ return SAT

$$\begin{cases} x_1^0 = 1 & x_2^0 = X & g_1^0 = X & g_2^0 = X \\ x_1^1 = 0 & x_2^1 = X & g_1^1 = 1 & g_2^1 = X \\ x_1^2 = X & x_2^2 = X & g_1^2 = 0 & g_2^2 = X \end{cases}$$

→ If it returns UNSAT go for unwinding for 3 cycles  
& ~~so on~~ so on...

Bounded Model checking using SAT 01/11/2023

Implementation (Characteristic func<sup>n</sup>)  $R$

Property  $\phi$

$\sim \phi$

$$\begin{array}{ccc} \begin{array}{c} x_1^0 \\ x_2^0 \\ g_1^0 \\ g_2^0 \end{array} \begin{array}{c} S_0^0 \\ S_1^0 \\ \vdots \\ S_k^0 \end{array} & \begin{array}{c} x_1^1 \\ x_2^1 \\ g_1^1 \\ g_2^1 \end{array} \begin{array}{c} S_0^1 \\ S_1^1 \\ \vdots \\ S_k^1 \end{array} & \begin{array}{c} x_1^m \\ x_2^m \\ g_1^m \\ g_2^m \end{array} \begin{array}{c} S_0^m \\ S_1^m \\ \vdots \\ S_k^m \end{array} \\ \text{clk0} & \text{clk1} & \text{clk}m \end{array}$$

~~clk~~  $k'$  state variables

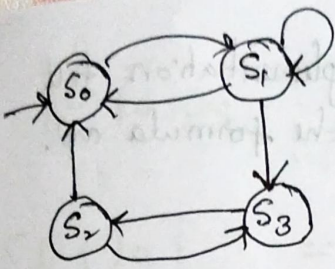
$m$  is bound

→  $k \times m$  variables in the SAT formula.

Clauses (formulas)

- 1)  $I$  :- that captures the initial state
- 2)  $C^j$  :- unfolding the state machine over time and generating new set of clauses

$$C^j = \bigwedge_{i=0, j=1} R(s^i, s^{i+1})$$



$$C^0 = \{S_0\}$$

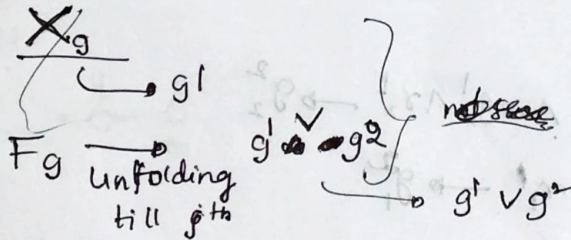
$$C^1 = \{S_0, S_1\}$$

$$C^2 = \{S_0, S_1, S_3\}$$

$$C^3 = \{S_0, S_1, S_2, S_3\}$$

$\Rightarrow R(s^i, s^{i+1})$  is state transition from set of states  $s^i$  to set of states  $s^{i+1}$

3)  $Z^i$  :- Unfolding  $\sim \Phi$  till  $i$ th clk to generate clauses.



Bounded Model checking  $(R, \Phi, m)$  bound

(check transition 'R' holds  $\Phi$  in  $m$  clock cycles)

{  
   $j = 1$   
  while  $(j \leq m)$  {

    1. Construct  $I, C^j, Z^j$ ;

    2. check  $SAT(I \wedge C^j \wedge Z^j)$ ;

    if  $(SAT \text{ returns satisfiable})$  {

$R$  doesnot satisfy  $\Phi$ , SAT instance  
      is the counter example.

    }

$j++$ ;

}

3  $\Phi$  satisfies in  $R$  within bound of  $m$ ;



$\Rightarrow I \wedge C \wedge \neg \epsilon : \text{true} \Rightarrow$  Run on implementation till  $j$ th clk satisfy the formula w.p.

## Unfolding properties

### Unfolding state machines:-

till first clk

$$C^0 = \begin{cases} \neg r_1^0 \wedge \neg g_1^0 \wedge r_2^0 \rightarrow g_2^0 \\ r_1^0 \rightarrow g_1^0 \end{cases}$$

till 2nd clk

$$C^0 \wedge C^1$$

$$C^1 = \begin{cases} \neg r_1^1 \wedge \neg g_1^1 \wedge r_2^1 \rightarrow g_2^1 \\ r_1^1 \rightarrow g_1^1 \end{cases}$$

### Unfolding properties:-

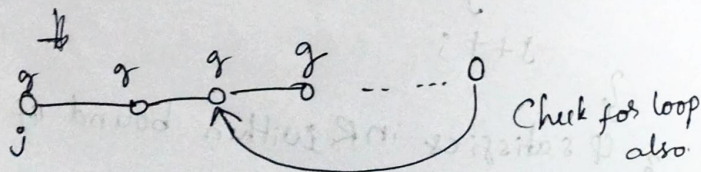
$[F]_{j,m}$  = set of clauses to be considered in order to determine whether a property  $F$  is true at  $j$ th clk, where  $j < m$

$$[X_F]_{j,m} = (j < m) \wedge [F]_{j+1,m}$$

$$[F_g]_{j,m} = \bigvee_{i=j+1}^m [g]_{i,m}$$

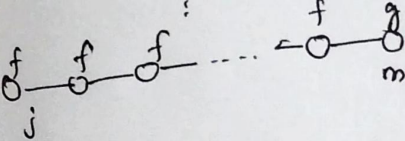
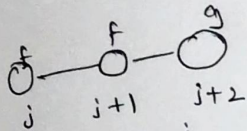
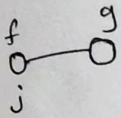
$g$  holds in future

$$[G_f]_{j,m} = \bigwedge_{i=j+1}^m [f]_{i,m} \wedge \text{loop}_m$$

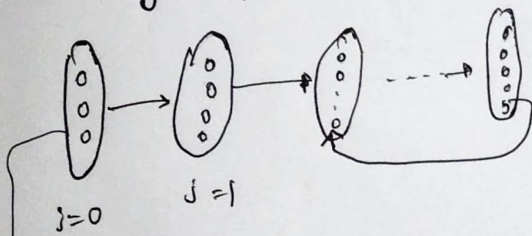


$$[G_f]_{j,m} = \bigwedge_{i=j, \dots, m} [f]_{i,m} \wedge \text{loop}_m$$

$$[f \cup g]_{j,m} = \bigvee_{i=j, \dots, m} \left( [g]_{i,m} \wedge \bigwedge_{n=j, \dots, i-1} [f]_{n,m} \right)$$



### Detecting loop



$s_0$   
 $s_1$   
 $s_2$   
 $\vdots$   
 $s_k$

$$\text{loop}_i = \bigvee_{i=0, i=1} \left( s_0^i = s_0^j \wedge s_1^i = s_1^j \wedge \dots \wedge s_k^i = s_k^j \right)$$

set of states  
 in clk  $j$   
 that are reachable  
 from  $i$   
 if  $i=0 \Rightarrow$  initial state