Two Objects Rolling Down an Inclined Plane
IB Physics SL

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Mr. Shaw

1 Background information

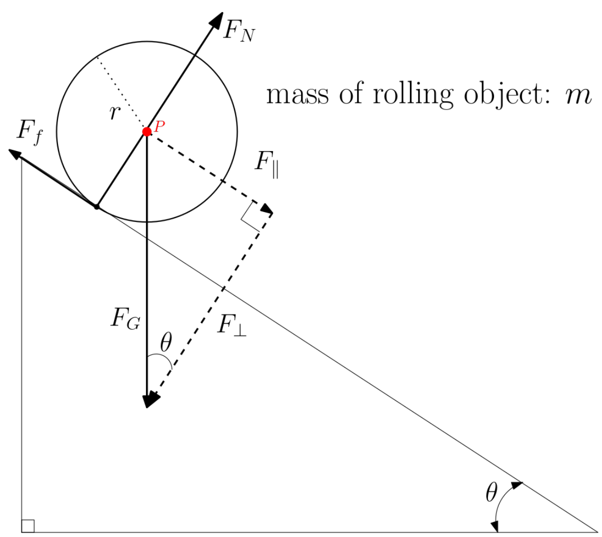

Figure 1: Diagram of rolling object on inclined ramp labelled with forces

Referring to Figure 1, the translational F_{net} is given by

$$F_{net} = F_{\parallel} - F_f$$

as every force in the system other than frictional force (F_f) and the force component of gravitational force parallel to the incline of the ramp ($F_{\parallel}$) is being cancelled out by some other force.

The rolling object is also experiencing a torque relative to the point P in Figure 1. This torque is only arising from frictional force, as while F_N passes through point P and F_G is originating from point P, F_f is the only force that is creating a force that originates from some point other than P and is perpendicular to the line of the force's own origin to P.

Using this information, the formula for the translational acceleration of the two rolling objects (a hollow cylinder and a solid sphere) can be derived.

1.1 Derivation of the formula for translational acceleration of the hollow cylinder

The moment of inertia for a hollow cylinder is $I = mr^2$. The derivation for the translational acceleration of this cylinder is shown below.

$$\Gamma = I\alpha,\ \Gamma = F_f r,\ \alpha = \frac{a}{r}$$
$$F_f r = mr^2\left(\frac{a}{r}\right)$$
$$F_f = ma$$

$$\sin(\theta) = \frac{F_\parallel}{F_G},\ F_G = mg$$
$$F_\parallel = mg\sin(\theta)$$

$$F_{net} = ma$$
$$F_\parallel - F_f = ma$$
$$mg\sin(\theta) - ma = ma$$
$$a = \frac{1}{2}g\sin\theta$$

1.2 Derivation of the formula for translational acceleration of the solid sphere

The moment of inertia for a hollow cylinder is $I = \frac{2}{5}mr^2$. The derivation for the translational acceleration of this cylinder is shown below.

$$\Gamma = I\alpha,\ \Gamma = F_f r,\ \alpha = \frac{a}{r}$$
$$F_f r = \frac{2}{5}mr^2\left(\frac{a}{r}\right)$$
$$F_f = \frac{2}{5}ma$$

$$\sin(\theta) = \frac{F_\parallel}{F_G},\ F_G = mg$$
$$F_\parallel = mg\sin(\theta)$$

$$F_{net} = ma$$
$$F_\parallel - F_f = ma$$
$$mg\sin(\theta) - \frac{2}{5}ma = ma$$
$$a = \frac{5}{7}g\sin\theta$$

1.3 How the predicted time to reach the end of the ramp is calculated

Given that we will be able to calculated the angle of inclination of the ramp using the length and height of the ramp, we will be able to find what the translational acceleration of the rolling object will be. Additionally, we know the incline length of the ramp and that the initial translational velocity of the rolling object will be zero. Since we know the values of a, u, s, then we can calculate the predicted time to reach the end of the ramp.

$$
\begin{aligned}
& s = ut + \frac{1}{2}at^2 \\
& \text{Because } u = 0 \\
& s = \frac{1}{2}at^2 \\
& at^2 = 2s \\
& t^2 = \frac{2s}{a} \\
& t = \sqrt{\frac{2s}{a}}
\end{aligned}
$$

2 Raw data

2.1 Qualitative observations

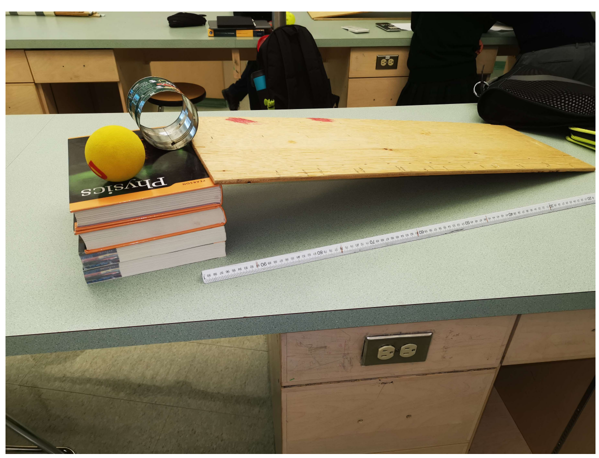

Figure 2: Photo of the sphere, cylinder and ramp

2.1.1 Hollow cylinder

Upon close inspection, the hollow cylinder does not appear to be completely circular and is rather in a slight elliptical shape. This is assumed to be as a result of the cylinder's malleable material.

Additionally, the cylinder has grooves and holes along the exterior of it. This may or may not affect the frictional force of the cylinder moving down the ramp.

2.1.2 Solid sphere

The solid sphere, being made of foam, is soft and squishy. This may affect the normal force of the sphere, as rather than only having on point of contact for the normal force to originate from, the sphere will have a considerable area of contact, in which various parts of the area of contact will have different magnitudes of normal force.

When conducting the trials, the sphere was found to sometimes roll diagonally down the ramp. This may be as a result of the sphere being soft and squishy, causing a larger area of contact and potentially a normal force directed to one side or another.

The material of foam may also have a slight affect on the moment of inertia of the sphere, as the air in the sphere may cause a slightly different distribution of mass in the sphere.

2.2 Quantitative data

Table 1: Raw data of time to roll down ramp for both hollow cylinder and solid sphere

Time for hollow cylinder to roll down ramp /s $\Delta t \pm 0.3$s	Time for solid sphere to roll down ramp /s $\Delta t \pm 0.2$s
1.4	1.2
1.4	1.5
1.5	1.6
1.6	1.5
2.0	1.6
1.6	1.3
1.7	1.4
1.6	1.3
1.7	1.3
1.7	1.5
1.7	1.5
1.6	1.6
1.6	1.5
1.7	1.4
1.8	1.5
1.6	1.5
2.0	1.4
1.7	1.6
1.7	1.4
1.7	1.5

Phyphox has determined that the angle of inclination when the phone is laid down on the table is 0.80°, and the angle of inclination when put on the ramp is 10.10°.

3 Processed data

Note that the values used in the following calculations are unrounded and therefore are not entirely reflected by Table 1.

3.1 Averaged values

The averaged values of the time it takes for either the hollow cylinder or the solid sphere to roll completely down the ramp is calculated using the following formula:

$$\bar{t} = \frac{\sum t}{n}$$

where $\bar{t} =$ averaged value of all the trials /s
$t =$ a single time value of one of the trials /s
$n =$ number of trials

3.1.1 Averaged values from raw data for hollow cylinder

$$\begin{aligned}\bar{t} &= \frac{\sum t}{n}\\ \bar{t} &= \frac{33.12\text{s}}{20}\\ &= 1.66\text{s}\end{aligned}$$

3.1.2 Averaged values from raw data for solid sphere

$$\begin{aligned}\bar{t} &= \frac{\sum t}{n}\\ \bar{t} &= \frac{29.19\text{s}}{20}\\ &= 1.47\text{s}\end{aligned}$$

3.2 Calculation of uncertainty from human error

The uncertainty for both the timing of the hollow cylinder and the solid sphere is calculated using the following formula.

$$err = \frac{MAX - MIN}{2}$$

3.2.1 Uncertainty for hollow cylinder

The hollow cylinder had a maximum time of 1.99 s and a minimum time of 1.36 s. The calculation for the human error uncertainty of the hollow cylinder is shown below.

$$\begin{aligned}err &= \frac{MAX - MIN}{2}\\ err &= \frac{1.99\text{s} - 1.36\text{s}}{2}\\ err &= 0.315\text{s}\\ err &= 0.3\text{s}\end{aligned}$$

3.2.2 Uncertainty for solid sphere

The solid sphere had a maximum time of 1.64 s and a minimum time of 1.24 s. The calculation for the human error uncertainty of the solid sphere is shown below.

$$err = \frac{MAX - MIN}{2}$$
$$err = \frac{1.64\text{s} - 1.24\text{s}}{2}$$
$$err = 0.2\text{s}$$

3.3 Formal experimental results

Using the averaged values and calculated uncertainty for both the hollow cylinder and the solid sphere, the formal experimental results can be presented.

$$\bar{t}_{hollowcylinder} = 1.7\text{s} \pm 0.3\text{s}$$

$$\bar{t}_{solidsphere} = 1.5\text{s} \pm 0.2\text{s}$$

4 Determining predicted times

4.1 Angle of inclination of the ramp

Given that we know what the height and hypotenuse of the ramp is, we can calculate the angle of inclination using trigonometry. Note that eventually in our calculation of the predicted times, we can simply substitute $\sin\theta$ for $\frac{height}{hypotenuse}$. However, we will nevertheless be calculating the angle of inclination to ensure that the calculated value is reasonable by comparing it to the angle of inclination measured by phyphox.

Because there is measurement uncertainty in the height and hypotenuse of the ramp, we will approach calculating the angle of inclination by determining the maximum and minimum value of the angle of inclination.

The height of the ramp is $14.70\text{cm} \pm 0.05\text{cm}$, and the hypotenuse of the ramp is $90.90\text{cm} \pm 0.05\text{cm}$.

$$\begin{aligned}\theta_{max} &= \sin^{-1}\left(\frac{height}{hypotenuse}\right)\\ &= \sin^{-1}\left(\frac{14.75\text{cm}}{90.85\text{cm}}\right)\\ &= 9.3436^\circ\\ &= 9.34^\circ\end{aligned}$$

$$\begin{aligned}\theta_{min} &= \sin^{-1}\left(\frac{height}{hypotenuse}\right)\\ &= \sin^{-1}\left(\frac{14.65\text{cm}}{90.95\text{cm}}\right)\\ &= 9.2694^\circ\\ &= 9.27^\circ\end{aligned}$$

The averaged value and absolute error of the maximum and minimum values will then be calculated.

$$\begin{aligned}\theta &= \frac{\theta_{max} + \theta_{min}}{2}\\ &= \frac{9.3436^\circ + 9.2694^\circ}{2}\\ &= 9.3065^\circ\\ &= 9.31^\circ\end{aligned}$$

$$\begin{aligned}err &= \frac{\theta_{max} - \theta_{min}}{2}\\ &= \frac{9.3436^\circ - 9.2694^\circ}{2}\\ &= 0.0371^\circ\\ &= 0.04^\circ\end{aligned}$$

The angle of inclination can then be presented as:

$$\theta = 9.31^\circ \pm 0.04^\circ$$

Phyphox measured that the angle of inclination of the table was 0.80°and the angle of inclination of the ramp was 10.10°. By subtracting these values, we determine that the angle of inclination between the table and the ramp is 9.30°, indicating that the angle of inclination calculated through trigonometry is reasonable as the value of 9.30°fits within the range of $9.31^\circ \pm 0.04^\circ$.

4.2 Calculating predicted time for hollow cylinder

Recalling the following formulas for the hollow cylinder:

$$a = \frac{1}{2} g \sin\theta \tag{1}$$

$$t = \sqrt{\frac{2s}{a}} \tag{2}$$

By substituting Equation 1 for a in Equation 2, we derive the formula:

$$t = \sqrt{\frac{4s}{g \sin\theta}}$$

We can then calculate the predicted time for the hollow cylinder to roll down the ramp.

Calculation of t

$$t = \sqrt{\frac{4s}{g \sin\theta}}$$
$$= \sqrt{\frac{4(0.9090\text{m} \pm 0.0005\text{m})}{(9.81\text{ms}^{-2})(\frac{0.1470\text{m}\pm0.0005\text{m}}{0.9090\text{m}\pm0.0005\text{m}})}}$$
$$= 1.5139\text{s}$$

Calculation of Δt

$$\frac{\Delta t}{t} = \sum \frac{\Delta x}{x}$$

where $\Delta t =$ uncertainty value of the predicted time /s

$t =$ calculated predicted value of time /s

$\Delta x =$ uncertainty value of a value in the equation with a uncertainty

$x =$ a value in the equation with an uncertainty

$$\frac{\Delta t}{1.5139\text{s}} = \frac{0.0005\text{m}}{0.9090\text{m}} + \frac{0.0005\text{m}}{0.1470\text{m}} + \frac{0.0005\text{m}}{0.9090\text{m}}$$
$$\Delta t = 0.00681\text{s}$$
$$= 0.007\text{s}$$

$$t = 1.514\text{s} \pm 0.007\text{s}$$

4.3 Calculating predicted time for solid sphere

Recalling the following formulas for the solid sphere:

$$a = \frac{5}{7} g \sin\theta \tag{3}$$

$$t = \sqrt{\frac{2s}{a}} \tag{4}$$

By substituting Equation 3 for a in Equation 4, we derive the formula:

$$t = \sqrt{\frac{14s}{5g \sin\theta}}$$

We can then calculate the predicted time for the solid sphere to roll down the ramp.

Calculation of t

$$t = \sqrt{\frac{14s}{5g\sin\theta}}$$

$$t = \sqrt{\frac{14(0.9090\text{m} \pm 0.0005\text{m})}{5(9.81\text{ms}^{-2})(\frac{0.1470\text{m}\pm0.0005\text{m}}{0.9090\text{m}\pm0.0005\text{m}})}}$$

$$t = 1.2666\text{s}$$

Calculation of Δt

$$\frac{\Delta t}{t} = \sum \frac{\Delta x}{x}$$

where $\Delta t =$ uncertainty value of the predicted time /s

$t =$ calculated predicted value of time /s

$\Delta x =$ uncertainty value of a value in the equation with a uncertainty

$x =$ a value in the equation with an uncertainty

$$\frac{\Delta t}{1.2666\text{s}} = \frac{0.0005\text{m}}{0.9090\text{m}} + \frac{0.0005\text{m}}{0.1470\text{m}} + \frac{0.0005\text{m}}{0.9090\text{m}}$$

$$\Delta t = 0.00570\text{s}$$

$$= 0.006\text{s}$$

$$t = 1.267\text{s} \pm 0.006\text{s}$$

5 Comparing experimental times to predicted times

Table 2: Summary of the experimental and predicted times

	Hollow cylinder	Solid sphere
Predicted Value	$1.514\text{s} \pm 0.007\text{s}$	$1.267\text{s} \pm 0.006\text{s}$
Experimental Value	$1.7\text{s} \pm 0.3\text{s}$	$1.5\text{s} \pm 0.2\text{s}$

A summary of the experimental and predicted times for the hollow cylinder and the solid sphere to roll down the ramp is presented in Table 2.

5.1 Evaluation of the results for the hollow cylinder

Given that the hollow cylinder was experimentally determined to take $1.7\text{s} \pm 0.3\text{s}$ to roll down the ramp and was predicted to take $1.514\text{s} \pm 0.007\text{s}$, it can be concluded that the experimental values for this investigation are in agreement with the predicted values, as the range given by the predicted value is a subset of the range given by the experimental value.

The percent error can also be calculated as a mathematical reference as shown below:

$$\begin{aligned}
\%_{err} &= \left| \frac{experimental - predicted}{predicted} \right| \times 100\% \\
&= \left| \frac{1.7\text{s} - 1.514\text{s}}{1.514\text{s}} \right| \times 100\% \\
&= 12\%
\end{aligned}$$

5.2 Evaluation of the results for the solid sphere

Given that the solid sphere was experimentally determined to take $1.5\text{s} \pm 0.2\text{s}$ to roll down the ramp and was predicted to take $1.267\text{s} \pm 0.006\text{s}$, it can be concluded that the experimental values for this investigation are not in agreement with the predicted values, as the range given by the predicted value does not intersect with the range of the experimental value (the maximum of the predicted value is 1.273s, while the minimum of the experimental value is 1.3s).

The percent error can also be calculated as a mathematical reference as shown below:

$$\begin{aligned}
\%_{err} &= \left| \frac{experimental - predicted}{predicted} \right| \times 100\% \\
&= \left| \frac{1.5\text{s} - 1.267\text{s}}{1.267\text{s}} \right| \times 100\% \\
&= 18\%
\end{aligned}$$