Two Objects Rolling Down an Inclined Plane
IB Physics SL

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1 Background information

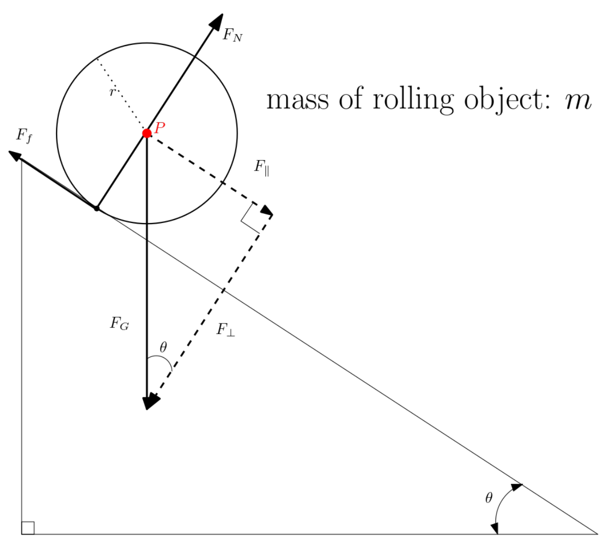

Figure 1: Diagram of rolling object on inclined ramp labelled with forces

Referring to Figure 1, the translational F_{net} is given by

$$F_{net} = F_\parallel - F_f$$

as every force in the system other than frictional force (F_f) and the force component of gravitational force parallel to the incline of the ramp ($F_\parallel$) is being cancelled out by some other force.

The rolling object is also experiencing a torque relative to the point P in Figure 1. This torque is only arising from frictional force, as while F_N passes through point P and F_G is originating from point P, F_f is the only force that is creating a force that originates from some point other than P and is perpendicular to the line of the force's own origin to P.

Using this information, the formula for the translational acceleration of the two rolling objects (a hollow cylinder and a solid sphere) can be derived.

1.1 Derivation of the formula for translational acceleration of the hollow cylinder

The moment of inertia for a hollow cylinder is $I = mr^2$. The derivation for the translational acceleration of this cylinder is shown below.

$$
\begin{aligned}
&\Gamma = I\alpha,\ \Gamma = F_f r,\ \alpha = \frac{a}{r}\\
&F_f r = mr^2\left(\frac{a}{r}\right)\\
&F_f = ma
\end{aligned}
$$

$$
\begin{aligned}
&\sin(\theta) = \frac{F_\parallel}{F_G},\ F_G = mg\\
&F_\parallel = mg\sin(\theta)
\end{aligned}
$$

$$
\begin{aligned}
&F_{net} = ma\\
&F_\parallel - F_f = ma\\
&mg\sin(\theta) - ma = ma\\
&a = \frac{1}{2}g\sin\theta
\end{aligned}
$$

1.2 Derivation of the formula for translational acceleration of the solid sphere

The moment of inertia for a hollow cylinder is $I = \frac{2}{5}mr^2$. The derivation for the translational acceleration of this cylinder is shown below.

$$
\begin{aligned}
&\Gamma = I\alpha,\ \Gamma = F_f r,\ \alpha = \frac{a}{r}\\
&F_f r = \frac{2}{5}mr^2\left(\frac{a}{r}\right)\\
&F_f = \frac{2}{5}ma
\end{aligned}
$$

$$
\begin{aligned}
&\sin(\theta) = \frac{F_\parallel}{F_G},\ F_G = mg\\
&F_\parallel = mg\sin(\theta)
\end{aligned}
$$

$$
\begin{aligned}
&F_{net} = ma\\
&F_\parallel - F_f = ma\\
&mg\sin(\theta) - \frac{2}{5}ma = ma\\
&a = \frac{5}{7}g\sin\theta
\end{aligned}
$$

1.3 How the predicted time to reach the end of the ramp is calculated

Given that we will be able to calculated the angle of inclination of the ramp using the length and height of the ramp, we will be able to find what the translational acceleration of the rolling object will be. Additionally, we know the incline length of the ramp and that the initial translational velocity of the rolling object will be zero. Since we know the values of a, u, s, then we can calculate the predicted time to reach the end of the ramp.

$$
\begin{aligned}
&s = ut + \frac{1}{2}at^2 \\
&\text{Because } u = 0 \\
&s = \frac{1}{2}at^2 \\
&at^2 = 2s \\
&t^2 = \frac{2s}{a} \\
&t = \sqrt{\frac{2s}{a}}
\end{aligned}
$$

2 Raw data

2.1 Qualitative observations

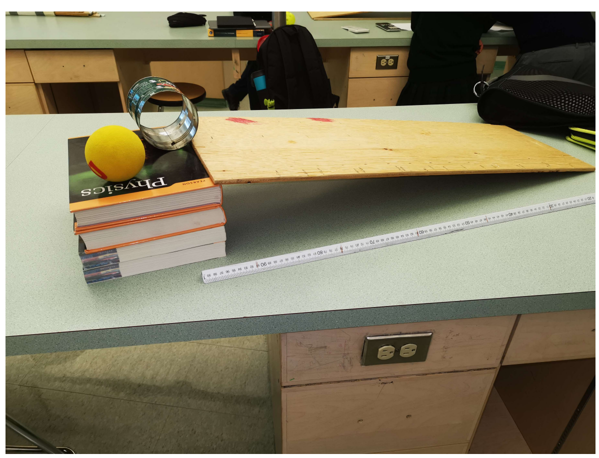

Figure 2: Photo of the sphere, cylinder and ramp

2.1.1 Hollow cylinder

Upon close inspection, the hollow cylinder does not appear to be completely circular and is rather in a slight elliptical shape. This is assumed to be as a result of the cylinder's malleable material.

Additionally, the cylinder has grooves and holes along the exterior of it. This may or may not affect the frictional force of the cylinder moving down the ramp.

2.1.2 Solid sphere

The solid sphere, being made of foam, is soft and squishy. This may affect the normal force of the sphere, as rather than only having on point of contact for the normal force to originate from, the sphere will have a considerable area of contact, in which various parts of the area of contact will have different magnitudes of normal force.

When conducting the trials, the sphere was found to sometimes roll diagonally down the ramp. This may be as a result of the sphere being soft and squishy, causing a larger area of contact and potentially a normal force directed to one side or another.

The material of foam may also have a slight affect on the moment of inertia of the sphere, as the air in the sphere may cause a slightly different distribution of mass in the sphere.

2.2 Quantitative data

Time for hollow cylinder to roll down ramp /s $\Delta t \pm 0.3$s	Time for solid sphere to roll down ramp /s $\Delta t \pm 0.2$s
1.4	1.2
1.4	1.5
1.5	1.6
1.6	1.5
2.0	1.6
1.6	1.3
1.7	1.4
1.6	1.3
1.7	1.3
1.7	1.5
1.7	1.5
1.6	1.6
1.6	1.5
1.7	1.4
1.8	1.5
1.6	1.5
2.0	1.4
1.7	1.6
1.7	1.4
1.7	1.5

Table 1: Raw data of time to roll down ramp for both hollow cylinder and solid sphere

2.2.1 Calculation of uncertainty from human error

The uncertainty for both the timing of the hollow cylinder and the solid sphere is calculated using the following formula.

$$err = \frac{MAX - MIN}{2}$$

The hollow cylinder had a maximum time of 1.99 s and a minimum time of 1.36 s. The calculation for the human error uncertainty of the hollow cylinder is shown below.

$$
\begin{aligned}
err &= \frac{MAX - MIN}{2} \\
err &= \frac{1.99\text{s} - 1.36\text{s}}{2} \\
err &= 0.315\text{s} \\
err &= 0.3\text{s}
\end{aligned}
$$

The solid sphere had a maximum time of 1.64 s and a minimum time of 1.24 s. The calculation for the human error uncertainty of the solid sphere is shown below.

$$
\begin{aligned}
err &= \frac{MAX - MIN}{2} \\
err &= \frac{1.64\text{s} - 1.24\text{s}}{2} \\
err &= 0.2\text{s}
\end{aligned}
$$

3 Processed data

4 Comparing experimental times to predicted times