Module: 02

Equilibrium:

⇒ Equilibrium of forces:

Any system of forces acting on a body are said to be in equilibrium when the resultant of all forces is zero & algebraic sum of moments of all the forces is zero.

⇒ Condition of equilibrium:

A system of forces is in equilibrium when ?

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 0.$$

& $\Sigma M = 0$ — $\{ M = F \times d$ — ⊥ distⁿ, mag. of force

i.e; when
1) $\Sigma H = 0$
2) $\Sigma V = 0$ → Remember
also 3) $\Sigma M = 0$

where,

ΣH = Algebraic sum of H'al component of forces.

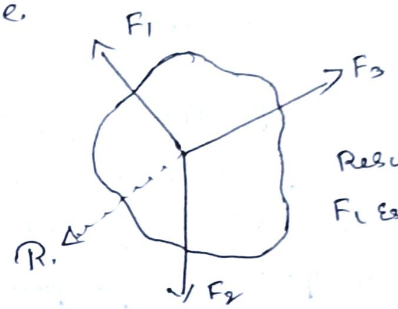
ΣV = Algebraic sum of V'le component of forces

ΣM = Algebraic sum of moments of forces about any point.

⇒ principles of equilibrium for different force systems.

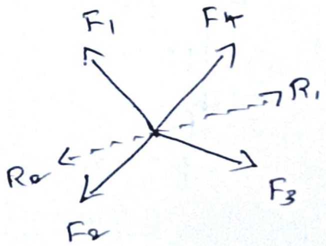
① Two force system: If a body is acted upon by two forces, then for equilibrium they must be equal in magnitude opposite & collinear (same line).

② Three force system: If a body is acted upon by three forces then for equilibrium the resultant of any two forces must be equal opposite & collinear with third force.



Resultant of F_1 & $F_2 = R = F_3$

③ Four force system: If a body is acted upon by four forces, then for equilibrium the resultant of any two forces must be equal, opposite & collinear with the resultant of remaining two forces.



Resultant of F_1 & $F_2 = R_1$

Resultant of F_3 & $F_4 = R_2$

⇒ Equilibrant:

An equilibrant is a force equal in magnitude opposite in direction & collinear with the resultant.

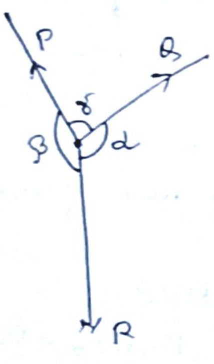
If an equilibrant is added to a concurrent system of forces then the system will be in equilibrium.

⇒ Law of Super position:

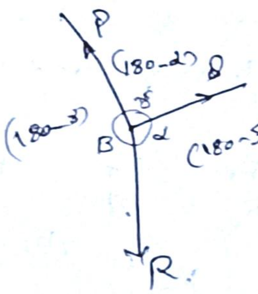
The action of given System of forces on a rigid body will in no way change, if we add to (or) substract from them another System of forces in Equilibrium.

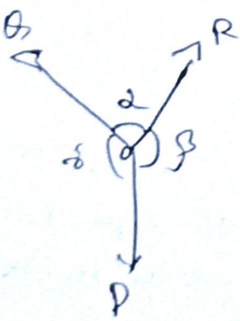
⇒ Lami's Theorem:

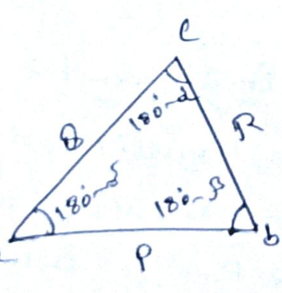
Statement: "If three coplanar forces acting simultaneously at a point be in equilibrium, then each force is proportional to the sine of the angle b/w the other two forces".



fig(a)







Let P, Q, R be the three forces acting at a point 'O' & let α, β, γ be the angles b/w R & Q, P & R, P & Q respectively.

applying sine rule for Δ'e abc,

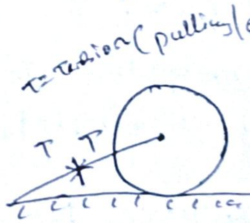
$$\frac{P}{\sin(180-\alpha)} = \frac{Q}{\sin(180-\beta)} = \frac{R}{\sin(180-\gamma)}$$

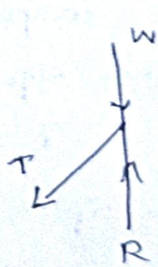
$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

it is possible to apply the lami's theorem, if only three forces are acting on a particle (or) at a point

⇒ Free body diagram:

A free body diagram which represents the various forces acting on the body.

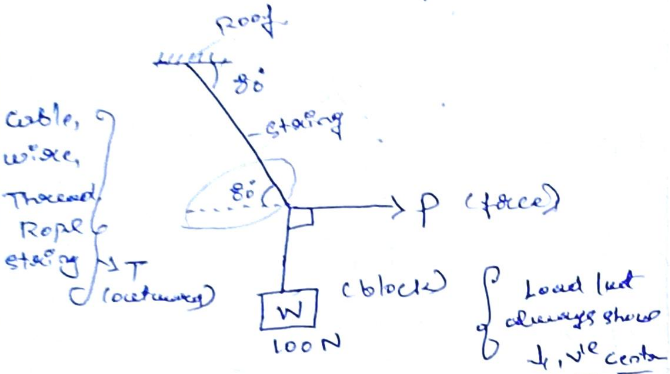




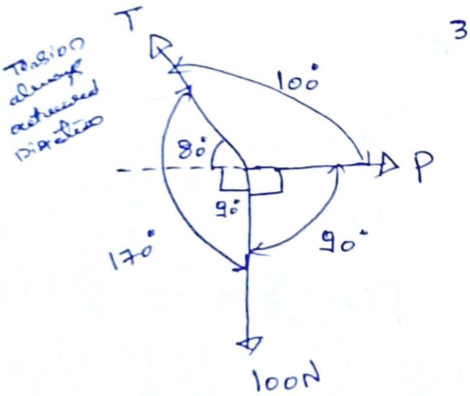
Let us consider a spherical ball of mass 'm', placed on a Hzl plane & tied to the plane by a string as shown in above fig(a).

Problems on Lami's theorem in Equilibrium:

① A Horizontal force 'P' is a shown in figure keeps the weight of 100N in the Equilibrium. find the magnitude of force 'P' & Tension in the string.



Solu: FBD of point:



$360 - 170 - 90$
$= 100°$

Lami's Theorem three conditions:

coplanar

① Must Three concurrent, forces

② Three direction same (outward)

③ Three angle

④ one known value (weight)

By Lami's theorem,

$$\boxed{\frac{P}{\sin 170°} = \frac{100}{\sin 100} = \frac{T}{\sin 90°}}$$

Select, $\frac{P}{\sin 170°} = \frac{100}{\sin 100}$

$P \sin 100 = 100 \sin 170$

$P = \frac{100 \sin 170}{\sin 100}$ $\boxed{P = 17.638N}$

Hz force

Select,

$\frac{100}{\sin 100} = \frac{T}{\sin 90}$

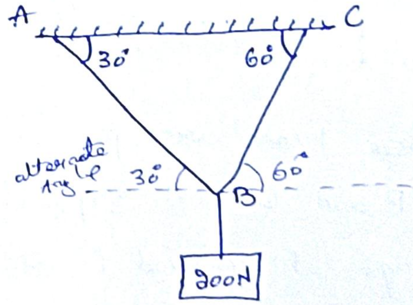
$100 \sin 90 = T \sin 100$

$\therefore T = \frac{100 \sin 90}{\sin 100}$

$\boxed{T = 101.542N}$

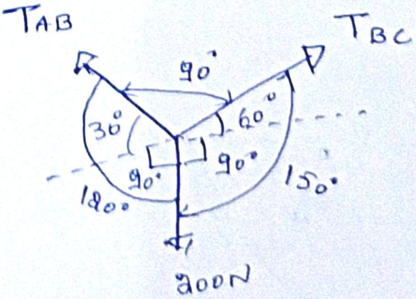
Tension

② Calculate the Tension in the string AB & BC, if the weight of 200N is attached by the two string as shown in fig.



Solu: FBD of joint 'B'

$360 - 120 - 150$
$= 90°$



By Lami's theorem,

$$\frac{T_{AB}}{\sin 150^\circ} = \frac{200}{\sin 90^\circ} = \frac{T_{BC}}{\sin 120^\circ}$$

Select,

$$\frac{T_{AB}}{\sin 150^\circ} = \frac{200}{\sin 90^\circ}$$

$$T_{AB} \sin 90^\circ = 200 \sin 150^\circ$$

$$T_{AB} = \frac{200 \sin 150^\circ}{\sin 90^\circ}$$

$$\boxed{T_{AB} = 100 N}$$

Select,

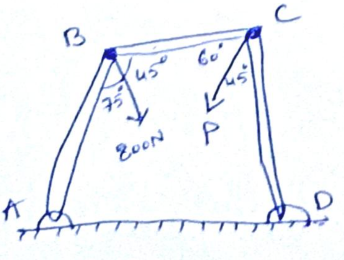
$$\frac{200}{\sin 90^\circ} = \frac{T_{BC}}{\sin 120^\circ}$$

$$200 \sin 120^\circ = T_{BC} \sin 90^\circ$$

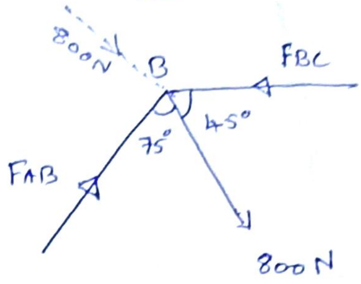
$$T_{BC} = \frac{200 \sin 120^\circ}{\sin 90^\circ}$$

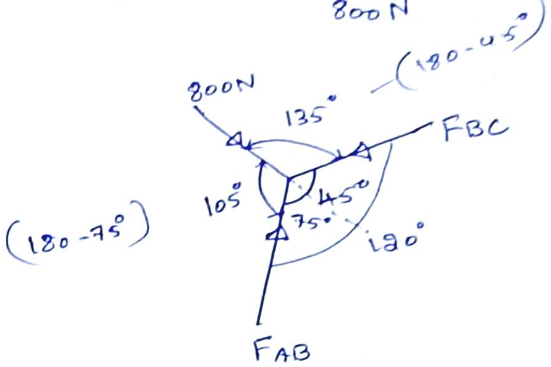
$$\boxed{T_{BC} = 173.205 N}$$

③ Three bars are pinned together at B and C and supported by a hinge at A and D as shown in fig to form a four link mechanism. Determine the value of force 'P' that will prevent the motion.



Solu: FBD of joint (B)





By Lami's theorem,

$$\boxed{\frac{F_{AB}}{\sin 135^\circ} = \frac{800}{\sin 120^\circ} = \frac{F_{BC}}{\sin 105^\circ}}$$

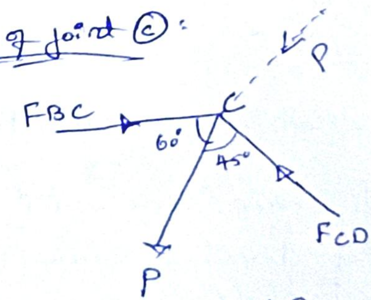
Select

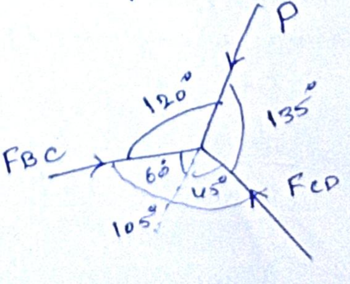
$$\frac{800}{\sin 120^\circ} = \frac{F_{BC}}{\sin 105^\circ}$$

$$F_{BC} = \frac{800 \sin 105^\circ}{\sin 120^\circ}$$

$$\boxed{F_{BC} = 892.284 N}$$

FBD of joint (c):





continue ⇒

$$\boxed{\frac{P}{\sin 105^\circ} = \frac{F_{BC}}{\sin 135} = \frac{F_{CD}}{\sin 180}}$$

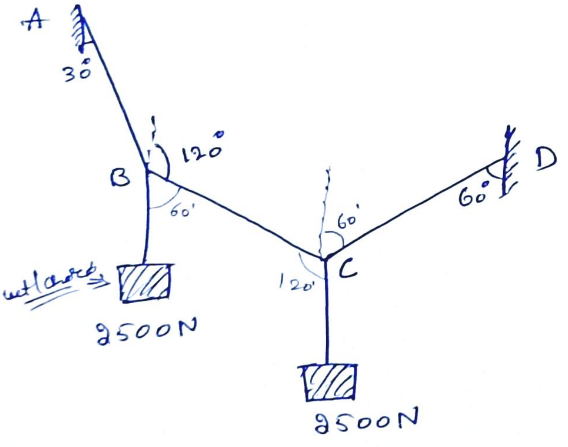
Select,

$$\frac{P}{\sin 105^\circ} = \frac{892.284}{\sin 135^\circ}$$

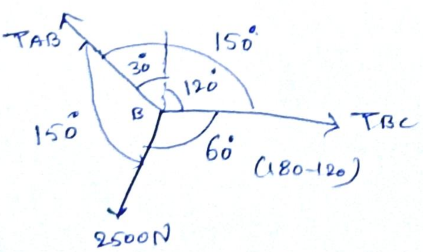
$$P = \frac{892.284 \cdot \sin 105^\circ}{\sin 135^\circ}$$

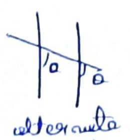
$$\boxed{P = 1218.888\,N}$$

(4) Two equal loads of 2500N are supported by a flexible string ABCD at point B and C. Find the tension in the portions AB, BC & CD of the string.



Solu^n: FBD of joint B





By Lami's theorem,

$$\boxed{\frac{T_{AB}}{\sin 60} = \frac{2500}{\sin 150} = \frac{T_{BC}}{\sin 150}}$$

Select,

$$\frac{T_{AB}}{\sin 60} = \frac{2500}{\sin 150}$$

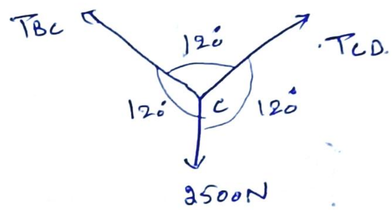
$$\boxed{T_{AB} = 4330.127\,N}$$

Select,

$$\frac{2500}{\sin 150} = \frac{T_{BC}}{\sin 150}$$

$$\boxed{T_{BC} = 2500\,N}$$

FBD of joint C



By Lami's theorem,

$$\frac{T_{CD}}{\sin 120} = \frac{2500}{\sin 120} = \frac{T_{BC}}{\sin 120}$$

$$\therefore T_{CD} = 2500 = T_{BC}$$

$$\boxed{T_{CD} = 2500\,N}$$