MODULE - 5 - Notes 1st sem Civil

KINEMATICS AND KINETICS

RECTILINEAR MOTION

Important Definitions

i) Kinematics :

It is the study of motion of bodies and its relation ship with time considered, without considering the forces causing motion It deals with the study of displacement, velocity & acceleration.

ii) Kinetics :

It is the study of motion of bodies and its relation ship with time considered, also considering the forces causing motion It deals with the study of determining the forces required to produce such motion

iii) Displacement :

It is the change in the position of body with respect to time from an arbitrary fixed point. It is a vector quantity

iv) Linear displacement :

It is the displacement of a body along a straight line measured in meters and kilometers.

v) Angular displacement :

It is the displacement of a body along curved or circular path measured in number of revolutions or radians.

vi) Speed :

It is the rate of change of position of body with time irrespective of its direction and thus a scalar quantity.

speed = $\frac{\text{Displacement}}{\text{Time}}$ $\frac{Km}{hr}$ or $\frac{m}{sec}$.

vii) <u>Velocity</u> :

It is the rate of change of displacement in a specific direction. It is a vector quantity.

viii) <u>Uniform velocity</u> :

It is the rate of change of displacement in a specific direction at an uniform rate.

$$\text{Uniform velocity} = V = \frac{s}{t}$$

ix) <u>Non Uniform Velocity</u> :

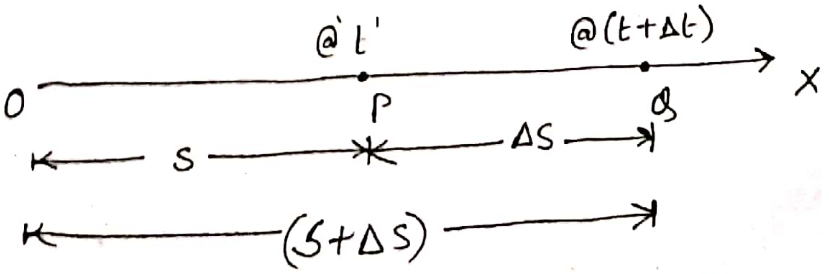
It is the rate of change of displacement in a specific direction at a non uniform rate

$$\text{Non uniform velocity} = V = \frac{ds}{dt}$$

x) <u>Average Velocity</u> :

Considering a body moving in a straight line OX as shown. Let 'P' be the position of body at time 't' & Corresponding displacement be 's'. Let 'Q' be the position of particle after time $(t+\Delta t)$ & Corresponding displacement is $(s+\Delta s)$ then

$$\text{Average velocity} = \frac{\Delta s}{\Delta t}$$



xi) Instantaneous Velocity :

The velocity at any Instant

$$V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

xii) Acceleration :

It is the rate of change of velocity with respect to time.

$$\text{Acceleration} = \frac{\text{change of velocity}}{\text{Time interval}} \qquad \frac{m/sec}{sec} = \frac{m}{sec^2}$$

xiii) Deceleration or Retardation :

The acceleration associated with decreasing velocity is called deceleration.

xiv) Uniform acceleration :

If a body moves in such a way that, the velocity changes equal in magnitude in equal interval of time, it is said to be moving with uniform acceleration $\boxed{a = \frac{v}{t}}$

xv) Variable acceleration :

If a body moves in such a way that, the velocity changes unequal in magnitude in equal interval of time, it is said to be moving with variable acceleration $\boxed{a = \frac{dv}{dt}}$

xvi) Distance traversed :

It is the total distance travelled by a body moving with a uniform velocity (v) in time 'T' secs, given by

$$S = VT \quad (m)$$

Newton's Laws of Motion

Newton's first law of Motion or Law of Inertia :

"Every body continues to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force".

Newton's Second law of Motion :

"The rate of change of momentum is directly proportional to the impressed force and takes place in the same direction in which the force acts".

let m = Mass of body
u = Initial velocity of the body
v = Final velocity of the body
a = Constant acceleration
t = Time to change velocity from u to v
F = force

if Initial Momentum = mu
Final Momentum = mv, then

$$F \propto \frac{mv - mu}{t}$$

$$F \propto \frac{m(v-u)}{t}$$

$$\boxed{F \propto ma}$$

$$F = kma$$

$\therefore$ $\boxed{F = ma}$ where K = Constant of proportionality = 1

Newton's Third law of Motion :

"To every action, there will be an equal and opposite reaction".

Derivation of Equations of Motion

(i) Considering the body with uniform acceleration

(FIRST EQUATION OF MOTION)

$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

Integrating both sides,

$$\int_u^v dv = \int_0^t a\,dt$$

$$[v]_u^v = a[t]_0^t$$

$$v - u = at$$

$$\boxed{v = u + at}$$ ⟶ first equation of Motion

(ii) Equation for Distance travelled,

we know that $v = u + at$ (SECOND EQUATION OF MOT

$$\frac{dx}{dt} = u + at$$

$$dx = (u + at)\,dt$$

$$\int_0^s dx = \int_0^t (u + at)\,dt$$

$$\int_0^s dx = \int_0^t u\,dt + \int_0^t at\,dt$$

$$[x]_0^s = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

⟶ Second equation of Motion

iii) we know that (THIRD EQUATION OF MOTION)

$$v = u + at$$

(Squaring on both sides)

$$v^2 = (u+at)^2$$

$$v^2 = u^2 + 2uat + (at)^2$$

$$v^2 = u^2 + 2a\left[ut + \frac{1}{2}at^2\right]$$

$$\boxed{v^2 = u^2 + 2as}$$ → Third equation of Motion.

Modifications for Equation of Motion

(i) for body freely falling vertically downwards under gravity.

⇒ $v = u + gt$

⇒ $s = ut + \frac{1}{2}gt^2$

⇒ $v^2 = u^2 + 2gs$.

$\boxed{a = g}$

(ii) for body thrown vertically upwards against gravity

⇒ $v = u - gt$

⇒ $s = ut - \frac{1}{2}gt^2$

⇒ $v^2 = u^2 - 2gs$

$\boxed{a = -g}$

(iii) for body just dropped from a height

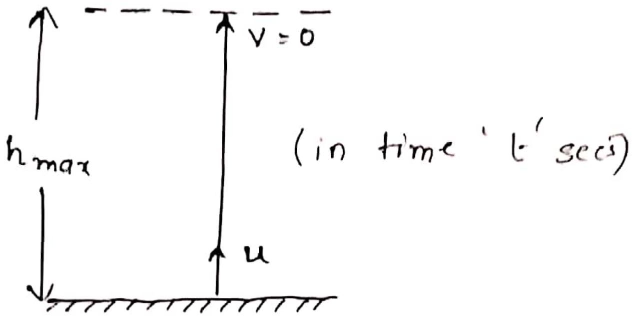
⇒ $v = gt$

⇒ $s = \frac{1}{2}gt^2$

⇒ $v^2 = 2gs$

$\boxed{u = 0}$

Greatest height reached by a body and the time it takes



from eqn $v = u - gt$

$$0 = u - gt$$

$$\boxed{t = \frac{u}{g}}$$ time taken

from eqn $v^2 = u^2 - 2gS$

$$0 = u^2 - 2gS$$

$$u^2 = 2gh_{max} \qquad (v = 0,\ S = h_{max})$$

$$\boxed{h_{max} = \frac{u^2}{2g}}$$ Greatest height

Problems on Rectilinear Motion

1. A body is moving with a velocity of 2m/sec. After 5 seconds the velocity of body reaches 6m/sec. find the acceleration of the body

Given $u = 2 m/s$, $t = 5 sec$ $V = 6 m/sec$

WKT $V = u + at$

$6 = 2 + (a \times 5)$

$5a = 4$

$\boxed{a = 0.8 \ m/s^2}$

2. A car is moving with a velocity of 20 m/sec. The car is brought to rest applying brakes in 6 sec. Determine
   i) Retardation
   ii) Distance travelled by the car after applying the brakes

Given $u = 20 m/sec$ $V = 0$ $t = 6 sec$

i) $V = u + at$

$0 = 20 + a(6)$

$\boxed{a = -3.33 \ m/sec^2}$ (retardation)

ii) let 's' be the distance travelled by the car after applying the brakes

$S = ut + \frac{1}{2}at^2$

$S = 20 \times 6 + \frac{1}{2}(-3.33) \times 6^2$

$\boxed{S = 60 m}$

3. A car is accelerating at a constant rate of $2.5 \, m/sec^2$, If it travels a distance of 700m in 20 secs,

i) What must be its initial velocity?

ii) What must be its final velocity?

Given $a = 2.5 \, m/s^2$ $S = 700 \, m$ $t = 20$ secs

$V = ?$ $u = ?$

i) To find initial velocity

$S = ut + \frac{1}{2}at^2$

$700 = u \times 20 + \frac{1}{2} \times 2.5 \times 20^2$

$\boxed{u = 10 \, m/sec}$

ii) $V = u + at$

$V = 10 + (2.5 \times 20)$

$\boxed{V = 60 \, m/sec}$

4 A motorist is travelling on a straight road at a speed of 10 m/sec, where he observes that the traffic signal 600m ahead of him turns red. The traffic light is timed to stay red for 15 secs. If the motorists wishes to pass the light without stopping just as it turns green again, find

i) The uniform acceleration of the motorist

ii) The speed of the car as it passes the traffic light

Given $u = 10 \, m/sec$ $S = 600 \, m$ $t = 15$ secs

$a = ?$ $V = ?$

i) Uniform acceleration (to pass 600m for 15 secs, maintaining initial velocity 10 m/sec)

$$S = ut + \frac{1}{2}at^2$$

$$600 = (10 \times 15) + \frac{1}{2} a \times 15^2$$

$$\boxed{a = 4 \ m/sec^2}$$

ii) final velocity of the car near traffic signal

$$V = u + at$$

$$= 10 + (4 \times 15)$$

$$\boxed{V = 70 \ m/sec}$$

5. A police officer observes a car approaching at the unlawful speed of 60 kmph. He gets on his motor cycle and starts chasing the car, just as it passes in front of him. After accelerating for 10 secs at a constant rate, the officer reaches his top speed of 75 kmph. How long does it take the officer to over take the car from the time he started?

• <u>Over taking Concept</u>: Distance travelled by both the vehicles will be same.

i.e Distance travelled by Car = Distance travelled by bike

$S_{car} = S_{bike} \longrightarrow$ (A)

Given for car, $u = 60 \ kmph = \frac{60}{3.6} = 16.67 \ m/sec$ (uniform speed)

$a = 0$ (zero acceleration)

for bike, $u = 0$

$V = 75 \ kmph = \frac{75}{3.6} = 20.83 \ m/sec$

(i) Distance travelled by the car (S_{car}) in time 't' secs with Constant speed & zero acceleration is given by

$$S_{car} = ut + \frac{1}{2}at^2$$

$$\boxed{S_{car} = 16.67 \ t} \longrightarrow (1)$$

ii) Total distance travelled by bike with variable and constant acceleration in time 't' secs is given by

S_{bike} = Distance travelled with variable acceleration for 10 seconds (S_1) + Distance travelled with constant acceleration for $(t-10)$ seconds (S_2).

where $S_1 = ut + \frac{1}{2}at^2$ (for $u=0$, $t = 10$ sec, $a = \frac{v-u}{t}$)

where in $v = u + at$

$a = \frac{v-u}{t}$

$\therefore$ $S_1 = 0 + \frac{1}{2}\left(\frac{20.83-0}{10}\right) \times 10^2$

$\boxed{S_1 = 104.15m} \rightarrow (2)$

and $S_2 = ut + \frac{1}{2}at^2$ (for $u = 20.83 m/sec$, $t = (t-10)$ sec, $a = 0$)

final velocity in 10 sec becomes initial velocity for $(t-10)$ seconds.

$\boxed{S_2 = 20.83(t-10)} \rightarrow (3)$

$\therefore$ (1) & (2) & (3) in (A) $\Rightarrow$ $16.67t = 104.15 + 20.83(t-10)$

$\boxed{t = 25.04 \text{ seconds}}$

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6. A burgler's car starts with an acceleration of $2 m/sec^2$. A police van came after 10 seconds & continued to chase the burglar's car with an uniform velocity of $40 m/sec$. find the time taken by the police van to over take the burgler's car.

Over taking Concept : The distance travelled by the police car im 't' secs is equal to the distance travelled by burgler's car im $(t+10)$ secs.

$$S_{\text{police car in 't' seconds}} = S_{\text{burgler's car in } (10+t)} \longrightarrow (A)$$

(i) Distance travelled by police car (S police car) with $u = 40 m/sec$ with zero acceleration $a = 0$ in time 't' secs is given by

$$S = ut + \frac{1}{2}at^2$$

$$\boxed{S = 40t} \longrightarrow (1)$$

(ii) Distance travelled by burgler's car ($S_{\text{burgler's car}}$) with $u = 0$ $a = 2 m/sec^2$ in time $(10+t)$ sec is given by

$$S = 0 + \frac{1}{2}(2)(10+t)^2$$

$$S = 10^2 + 2(10)(t) + t^2$$

$$\boxed{S = 100 + 20t + t^2} \longrightarrow (2)$$

(1) & (2) in (A) $\Rightarrow$ $40t = 100 + 20t + t^2$

$$t^2 - 20t + 100 = 0$$

$$\boxed{t = 10 \text{ secs}}$$

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Rectilinear Motion - Problems with variable Acceleration

7 The motion of a particle moving with a straight line is given by the equation $S = t^3 - 3t^2 + 2t + 5$ where S is displacement in meters ant 't' is time in seconds.

Determine (i) Velocity & acceleration after 4 seconds

(ii) Maximum or minimum velocity & corresponding displacement

iii) time at which velocity is zero.

We know that $V = \frac{ds}{dt}$, $a = \frac{dv}{dt}$

$\left(\text{Velocity} = \frac{\text{distance travelled}}{\text{time taken}}\right)$, $\left(\text{acceleration} = \frac{\text{Velocity}}{\text{time taken}}\right)$

Given $S = t^3 - 3t^2 + 2t + 5$

(i) (a) $V = \frac{ds}{dt} = 3t^2 - 6t + 2$

$V_{@t=4sec} = 3(4)^2 - 6(4) + 2$

$\boxed{V_{@t=4sec} = 26 \text{ m/sec}}$

(i) (b) $a = \frac{dv}{dt} = 6t - 6$

$a_{@t=4sec} = 6 \times (4) - 6$

$\boxed{a_{@t=4sec} = 18 \text{ m/sec}^2}$

ii) (a) Max velocity occurs when $\frac{dv}{dt} = 0$

i.e $\frac{dv}{dt} = 6t - 6$

$0 = 6t - 6$

$\boxed{t = 1 \text{ sec}}$

ii) (b) To find whether maximum or mininum velocity,

@ $t = 1$ sec, $v = 3t^2 - 6t + 2$

$v = 3(1)^2 - 6(1) + 2$

$\boxed{v = -1 \text{ m/sec}}$

$\therefore$ It is minimum velocity.

ii) (c) Corresponding displacement @ $t = 1$ sec is

$S = (1)^3 - 3(1)^2 + 2(1) + 5$

$\boxed{S = 5 \text{ m}}$

iii) Let 't' be the time at which the velocity is zero

$\therefore$ $0 = 3t^2 - 6t + 2$ (By solving)

$\boxed{t = 1.58 \text{ sec or } 0.42 \text{ sec}}$

8. The equation for velocity of a locomotive moving in a straight line is given by the expression

$$V = t^3 - t^2 - 2t + 2$$

The locomotive is found to be at a distance of 4m from station A after 2 secs. Determine

i) Acceleration & displacement after 4 secs

ii) Maximum or Minimum acceleration

Soln: The equation for velocity is given by

$$V = t^3 - t^2 - 2t + 2$$

i) (a) Acceleration at $t = 4$ secs

$$a = \frac{dv}{dt} = 3t^2 - 2t - 2$$

$$a_{@t=4sec} = 3(4)^2 - 2(4) - 2$$

$$\boxed{a_{@t=4sec} = 38\ m/s^2}$$

i) (b) displacement(s) at $t = 4$ secs

WKT $V = \frac{ds}{dt}$

$$\int ds = \int v\,dt$$

$$S = \int v\,dt$$

$$S = \int (t^3 - t^2 - 2t + 2)\,dt$$

$$S = \frac{t^4}{4} - \frac{t^3}{3} - \frac{2t^2}{2} + 2t + c$$

to find c, given @ $S = 4m$, $t = 2$ secs

$$\therefore 4 = \frac{2^4}{4} - \frac{2^3}{3} - \frac{2(2)^2}{2} + 2(2) + c$$

$$\boxed{c = \frac{4}{3}}$$

$\therefore$ $S_{@t=4sec} = \frac{4^4}{4} - \frac{4^3}{3} - \frac{2(4)^2}{2} + 2(4) + \frac{4}{3}$

$$\boxed{S_{@t=4sec} = 36 m}$$

ii) To find Max/Min Acceleration

This occurs when $\frac{da}{dt} = 0$

where $a = \frac{dv}{dt} = 3t^2 - 2t - 2$

$\therefore \frac{da}{dt} = 6t - 2 = 0$

$$\boxed{t = \frac{1}{3} sec}$$

$\therefore$ $a_{@t=\frac{1}{3}sec} = 3\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right) - 2$

$$\boxed{a_{@t=\frac{1}{3}sec} = -2.33 \ m/sec^2}$$ (Retardati

Problem (9)

The acceleration of a particle moving along a straight line is given by the eq^n $a = 25 - 3s^2$ where 'a' is acceleration in m/s^2 and 's' is the displacement in meters. The particle starts from rest. Determine

(i) Velocity when displacement is 2m

(ii) The displacement when velocity is zero

(iii) The displacement at Maximum velocity.

Sol^n Given $a = 25 - 3s^2$

(i) To find velocity @ $s = 2m$

WKT $a = \frac{dv}{dt}$ & $v = \frac{ds}{dt}$

$\therefore dt = \frac{ds}{v}$

$\therefore a = \frac{dv}{\frac{ds}{v}}$

$\therefore a = v\frac{dv}{ds}$

$v\,dv = a\,ds$

$v\,dv = (25 - 3s^2)ds$

$\int v\,dv = \int (25 - 3s)^2 ds$

$\frac{v^2}{2} = 25s - \frac{3s^3}{3} + c$

$\frac{v^2}{2} = 25s - s^3 + c$

To find 'c', applying the condition particle starts from rest

ie $s = 0$, $v = 0$ (at start)

$\therefore \boxed{c = 0}$

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$$\therefore \frac{v^2}{2} = 25S - S^3$$

$$\therefore V = \sqrt{2(25S - S^3)}$$

$$V_{@S=2m} = \sqrt{2(25\times2 - 2^3)}$$

$$= \sqrt{84}$$

$$\boxed{V_{@S=2m} = 9.17\ m/sec}$$

ii) To find displacement when velocity $V = 0$

i.e from $V = \sqrt{2(25S - S^3)}$

$$0 = \sqrt{2(25S - S^3)}$$

$$\therefore 25S - S^3 = 0$$

$$S^2 = 25$$

$$\boxed{S = 5m}$$

iii) To find displacement at maximum velocity

for maximum velocity, $\frac{dv}{dt} = 0$

i.e $a = 0$

$$25 - 3S^2 = 0$$

$$\boxed{S = 2.89\ m}$$

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10 Model Question Paper Problem

A particle, starting from rest, moves in a straight line, whose equation of motion is given by $S = 5t^3 - 3t^2 + 6$. find the displacement velocity & acceleration of the particle after 5 secs

Given $S = 5t^3 - 3t^2 + 6 \longrightarrow (1)$

(i) Displacement @ $t = 5$ sec

$$(1) \Rightarrow S = 5(5)^3 - 3(5)^2 + 6$$

$$\boxed{S = 556\ m} \quad @\ t = 5\ sec$$

(ii) Velocity @ $t = 5$ sec

WKT $V = \frac{ds}{dt}$

$$V = \frac{d}{dt}(5t^3 - 3t^2 + 6)$$

$$V = 5(3t^2) - 3(2t)$$

$$V = 15t^2 - 6t$$

$$V = 15(5)^2 - 6(5)$$

$$\boxed{V = 345\ m/sec} \quad @\ t = 5\ secs$$

(iii) Acceleration @ $t = 5$ec

WKT $a = \frac{dv}{dt}$

$$a = \frac{d}{dt}(15t^2 - 6t)$$

$$a = 30t - 6$$

$$a = 30(5) - 6$$

$$\boxed{a = 144\ m/sec^2} \quad @\ t = 5\ secs$$

11. Model Question Paper problem

A vehicle is moving with variable acceleration & its motion is given by the eqn $S = 25t + 4t^2 - 6t^3$, where 's' is in 'm' & 't' is in seconds. Determine i) The velocity and acceleration at start ii) the time, when vehicle reaches its maximum velocity & iii) Maximum velocity of vehicle

Given $S = 25t + 4t^2 - 6t^3$

(i) (a) To determine Velocity at start $(t=0)$

$$V = \frac{ds}{dt} = \frac{d}{dt}(25t + 4t^2 - 6t^3)$$

$$V = 25 + 8t - 18t^2$$

$$\boxed{V = 25\ m/sec} \quad @\ t = 0$$

(i) (b) To determine acceleration at start $(t=0)$

$$a = \frac{dv}{dt} = \frac{d}{dt}(25 + 8t - 18t^2)$$

$$a = 8 - 36t$$

$$\boxed{a = 8\ m/sec^2} \quad @\ t = 0$$

(ii) Maximum velocity time

it occurs when $\frac{dv}{dt} = 0$

ie $a = 8 - 36t = 0$

$$\boxed{t = 0.22\ sec}$$

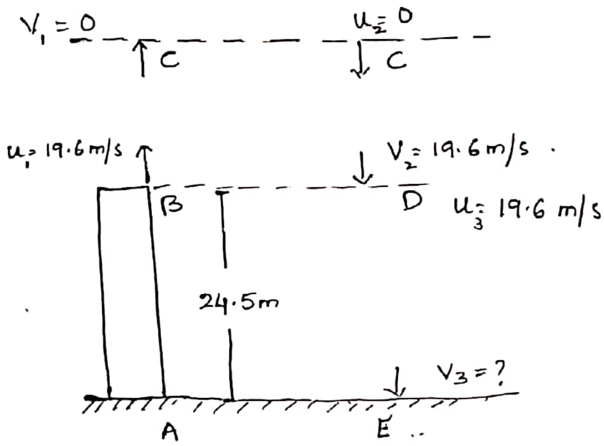
(iii). Maximum velocity @ $t = 0.22$ sec

$$V = 25 + 8(0.22) - 18(0.22)^2$$

$$\boxed{V = 25.88\ m/sec}$$

Problems on Motion due to Gravity ($g = 9.81 m/s^2$)

1. A small steel ball is shot up vertically with a velocity of 19.6 m/sec from the top of a building 24.5m high. Calculate.
   i) Time required for stone to reach maximum height
   ii) How high the ball rise above the building?
   iii) Compute the velocity with which it will strike the ground
   iv) Total time for which the ball is in motion.



(i) Time required for stone to reach Max height (from B to C)

$$t = \frac{u_1}{g}$$

$$= \frac{19.6}{9.81}$$

$$\boxed{t = 2 \text{ secs}}$$

(ii) Maximum height for the ball to rise above building

$$h_{max} = \frac{u_1^2}{2g}$$

$$= \frac{(19.6)^2}{2 \times 9.81}$$

$$\boxed{h_{max} = 19.6 m}$$

(iii) Velocity with which the . . strikes the ground

$$V_3^2 = u_3^2 + 2gS$$

$$V_3 = \sqrt{(19.6)^2 + 2 \times 9.81 \times 24.5}$$

$$\boxed{V_3 = 29.41 \ m/sec}$$

Corresponding time to fall from D to E

$$V_3 = u_3 + gt$$

$$29.41 = 19.6 + 9.81(t)$$

$$\boxed{t = 1 \ sec}$$

(iv) Total time for the ball in motion

$$T = \text{time from B to C} + \text{C to D} + \text{D to E}$$

$$= 2 + 2 + 1$$

$$\boxed{T = 5 \ secs}$$

2. A stone is dropped down a well with no initial velocity and after 4.5 secs, the splash is heard. Then a second stone is thrown downwards with an initial velocity v_0 & the splash is heard in 4 secs. If the velocity of sound is constant at 336 m/sec, determine the initial velocity of the second stone.

First stone: (Initial velocity being zero)

Total time T = Time required for stone to touch the water surface + Time required for splash sound to reach the top

$$4.5 = t + t_s$$

$$\therefore \underline{\underline{t_s = 4.5 - t}}$$

$\therefore$ Distance travelled by the stone from top to bottom in 't' secs = Distance travelled by sound from bottom to top in (4.5 − t) sec

$$ut + \frac{1}{2}gt^2 = \text{Velocity of Sound} \times \text{time taken}$$

$$\frac{1}{2} \times 9.81 \times t^2 = 336 \times (4.5 - t)$$

$$4.905t^2 + 336t - 1512 = 0$$

$$\boxed{t = 4.238 \text{ sec}} \rightarrow \text{Time taken by stone}$$

$\therefore$ Time taken by sound = (4.5 − 4.238)

$$\boxed{t_s = 0.262 \text{ sec}}$$

$\therefore$ Distance travelled by Sound = 336(4.5 − 4.238)

= 88.03m = Distance travelled by first stone

A

for second stone, (with initial velocity V_0)

$\Rightarrow$ Total Time $(T) = 4 = t + t_s$

Distance travelled by stone from top to bottom in 't' sec = Distance travelled by sound in $(4-t)$ sec

$$ut + \frac{1}{2}gt^2 = \text{Velocity of Sound} \times \text{time take}$$

$$V_0 t + \frac{1}{2} \times 9.81 \times t^2 = 336(4-t)$$

where $S = 336(4-t)$

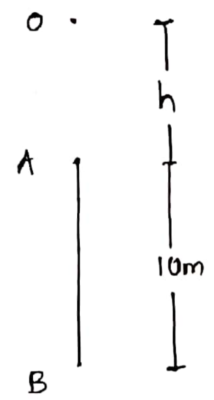
$88.03 = 336(4-t)$

$t = 3.738$ sec

$$\therefore V_0(3.738) + \frac{1}{2} \times 9.81 \times 3.738^2 = 336(4-3.738)$$

$\boxed{V_0 = 5.215 \text{ m/s}}$ $\longrightarrow$ Initial velocity of Second stone.

A particle falling vertically under the action of gravity passes two points 10m apart in 0·2 secs. Find the height from which the particle did start to fall above the higher point.



Let A & B be the two points 10m apart. Let the particle start to fall vertically from 'O' at a height 'h' from point 'A'.

Let 't' be the time taken from O to A so that from O to B, time is $(t+0{\cdot}2)$ secs.

(i) Portion OA

$$s = ut + \frac{1}{2}gt^2 \qquad (u=0,\ g = 9{\cdot}81\ m/s^2)$$
$$s = h$$

$$h = \frac{1}{2} \times 9{\cdot}81 \times t^2$$

$$h = 4{\cdot}905\, t^2 \longrightarrow (1)$$

(ii) Portion OB

$$s = ut + \frac{1}{2}gt^2 \qquad (u=0,\ g = 9{\cdot}81\ m/s^2$$

$$(h+10) = \frac{1}{2} \times 9{\cdot}81 \times (t+0{\cdot}2)^2 \qquad s = (h+10),\ t = (t+0{\cdot}2))$$

$$h + 10 = 4{\cdot}905\,(t+0{\cdot}2)^2 \longrightarrow (2)$$

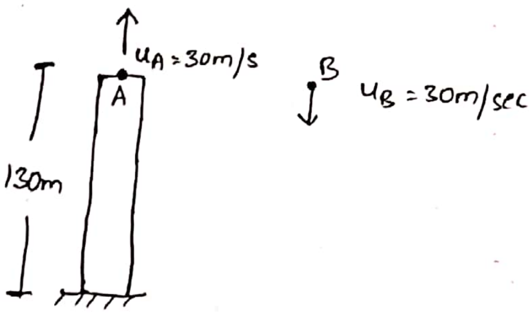
$$4{\cdot}905\, t^2 + 10 = 4{\cdot}905\,(t+0{\cdot}2)^2$$

$$\boxed{t = 5 \text{ sec}}$$

$$\therefore (1) \Rightarrow h = 4{\cdot}905 \times 5^2 = \underline{\underline{122{\cdot}5\, m}}$$

4. Two objects A and B are projected vertically at 130m above the ground. level. A is projected up with a velocity of 30m/sec and 'B' is projected downwards with same velocity find the time taken by each object to reach the ground.

Also find the height from which the object 'A' must be just released from rest in order the two objects hit the ground simultaneously.



(i) <u>Object A</u> (Calculating Total time)

<u>Method 1</u>: (Similar to first problem)

⇒ Time taken for the particle to reach Maximum height

$$t = \frac{u}{g} = \frac{30}{9.81} = 3.06 \text{ sec}$$

⇒ Maximum height reached

$$h_{max} = \frac{u^2}{2g} = \frac{30^2}{2\times 9.81} = 45.87 m$$

⇒ final velocity of particle A on striking the Ground from 130m height with initial velocity u = 30m/sec

$$v^2 = u^2 + 2gs$$

$$v = \sqrt{30^2 + 2\times 9.81\times 130}$$

$$v = 58.742 \text{ m/s}$$

⇒ Time taken by particle A to fall from 130m height to ground

$$v = u + gt$$

$$58.742 = 30 + 9.81t$$

$$t = 2.929 \text{ sec}$$

∴ Total Time = $(2\times 3.06) + 2.929 = 9.05$ sec.

Method 2

If 't_A' is the total time taken by particle A to reach the ground,

$$s = ut_A - \frac{1}{2}gt_A^2$$ (-ve becoz throw against gravity

$$-130 = 30t_A - \frac{1}{2} \times 9.81 \times t_A^2$$

$$4.905t_A^2 - 30t_A - 130 = 0$$

$$\boxed{t_A = 9.05 \text{ sec}}$$

(ii) Object - B

'if 't_B' is the total time taken by particle B to reach the ground,

$$s = ut_B + \frac{1}{2}gt_B^2$$

$$130 = 30t_B + \frac{1}{2} \times 9.81 \times t_B^2$$

$$4.905t_B^2 + 30t_B - 130 = 0$$

$$\boxed{t_B = 2.929 \text{ sec}}$$

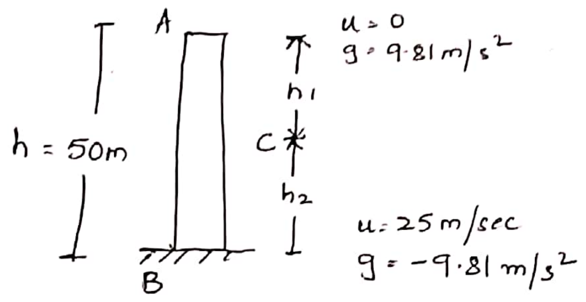
(iii) let 'h' be the height of particle A to be released at rest so as to reach in time with particle B ie in 2.929 secs

$$s = ut + \frac{1}{2}gt^2$$

$$h = 0 + \frac{1}{2} \times 9.81 \times 2.929^2$$

$$\boxed{h = 42.04 \text{ m}}$$

5. A stone 'A' is dropped from top of a tower 50m height. At the same time another stone 'B' is thrown up from the foot of the tower with velocity of 25 m/sec. At what distance from the top and after how much time the two stones will cross each other?



let 'c' be the point above the ground where two stones meet at a distance h_2 from bottom & h_1 from top

$$\therefore \quad h_1 + h_2 = 50 \longrightarrow (1)$$

(i) for stone A,

$$S = ut + \frac{1}{2} g t^2$$

$$h_1 = \frac{1}{2} \times 9.81 \times t^2 \longrightarrow (2)$$

(ii) for stone B,

$$S = ut - \frac{1}{2} g t^2$$

$$h_2 = 25t - \frac{1}{2} \times 9.81 \times t^2 \longrightarrow (3)$$

(1) = (2) + (3) $\Rightarrow$ $50 = \frac{1}{2}\cancel{9.81}t^2 + 25t - \frac{1}{2}\cancel{9.81}t^2$

$$\boxed{t = 2\ secs}$$

(2) $\Rightarrow \therefore$ $h_1 = \frac{1}{2} \times 9.81 \times 2^2 = 19.6\ m$

$h_2 = 50 - 19.6 = 30.4\ m$

D' Alembert's Principle – KINETICS

The main concept of principle of D' Alembert's principle is that conversion of dynamic problems into static equilibrium problem by introducing an additional force for equillibrium condition

D' Alembert's Principle

Newton's second law states that the rate of change of Momentum is directly proportional to the impressed force & takes place in the same direction in which the force acts.

i.e $F = ma$

where F = force
m = Mass
a = acceleration

if the impressed force is considered as a single resultant force R, then $R = ma$

$$R - ma = 0$$

i.e By applying a force of '$-ma$' to any dynamic problem, can be converted in to dynamic equilibrium problem

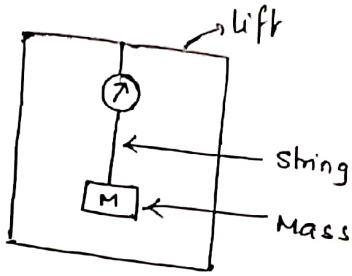
D' Alembert Principle states that "The resultant of system of forces acting on a body of mass 'm' & with acceleration 'a' is in dynamic equillibrium with inertia force 'ma' applied in reverse direction of motion".

i.e $R - ma = 0$

$$(\text{Net accelerating force}) - (\text{inertia force}) = 0$$

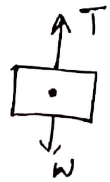
Analysis of Lift Motion (Elevator)

Considering an object of mass 'm' suspended by a from the ceiling of the lift.



Case (i) Lift at rest

$\Sigma V = 0 \qquad T = W$

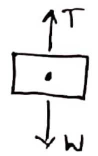


(No motion)

Hence tension in string (T) will be equal to weight of object (W).

Case (ii) Lift moving with constant velocity $(a=0)$

$T = W$



(No Motion)

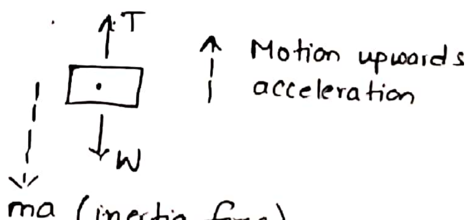
Case (iii) Lift accelerating upwards

Let 'a' be the acceleration of lift upwards (inertia force 'ma' opposes motion i e downwards)

$\Sigma V = 0 \quad T - W - ma = 0$

$T = W + ma$

$T = W\left(1 + \frac{a}{g}\right) \qquad \left(m = \frac{W}{g}\right)$

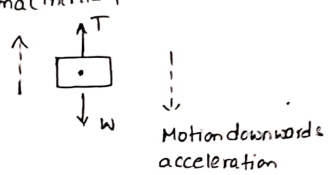


Hence, apparent weight is more,

- Person while moving upwards in lift exerts more force

Case (iv) lift acceleration downwards

ma (inertia force)



$T - W + ma = 0$

$T = W - ma$

$T = W\left(1 - \frac{a}{g}\right)$ $\left(m = \frac{W}{g}\right)$

Hence, apparent weight is less

∴ Person while moving down wards in lift exerts less force

Problems on lift

1. A lift carries a man of weight 4000 N and is moving with a uniform acceleration of $3.5\ m/sec^2$. Determine the tension in the cable.
   i) When lift is moving upwards
   ii) When lift is moving downwards

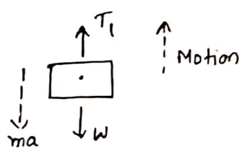
$R = m(g+a)$

$= \frac{4000}{9.81}(9.81 + 3.5)$

Sol^n $W = 4000\,N$, $a = 3.5\,m/s^2$

Let T be the tension in rope

Case (i) lift moving upwards



$T_1 - W - ma = 0$

$T_1 = W + ma$

$= W\left(1 + \frac{a}{g}\right)$

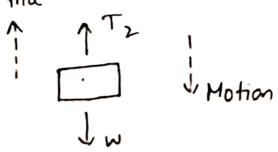
$= 4000\left(1 + \frac{3.5}{9.81}\right)$

$\boxed{T_1 = 5427.12\,N}$

$R = m(g - a)$

$= \frac{4000}{9.81}(9.81 - 3.5)$

ma



Case (ii) lift moving down wards

$T_2 - W + ma = 0$

$T_2 = W - ma$

$T_2 = W\left(1 - \frac{a}{g}\right)$

$= 4000\left(1 - \frac{3.5}{9.81}\right)$

$\boxed{T_2 = 2572.89\,N}$

2. A lift has an upward acceleration of $2.5 m/sec^2$. What pressure will a man weighing 750 N exert on the floor of the lift? What pressure would be exerted if the lift has an acceleration of $2.5 m/s^2$ downwards? What acceleration would cause this weight to exert a pressure of 1000 N on the floor?

Given $a = 2.5 m/s^2$ $W = 750 N$

Pressure on floor = Tension in cable due to his weight

(i) Upward acceleration

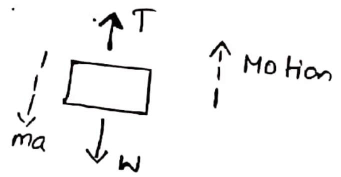
$$T - W - ma = 0$$

$$T = W + ma$$

$$T = W\left(1 + \frac{a}{g}\right)$$

$$= 750\left(1 + \frac{2.5}{9.81}\right)$$

$$\boxed{T = 941.13 N}$$



$R = m(g + a)$

$= \frac{750}{9.81}(9.81 + 2.5) = 941.1$

(ii) Downward acceleration

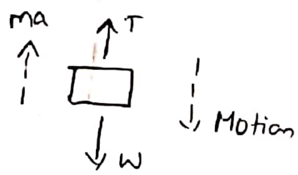
$$T + ma - W = 0$$

$$T = W - ma$$

$$T = W\left[1 - \frac{a}{g}\right]$$

$$= 750\left[1 - \frac{2.5}{9.81}\right]$$

$$\boxed{T = 558.87 N}$$



$R = m(g - a)$

$= \frac{750}{9.81}(9.81 - 2.5)$

$= 558.86 N$

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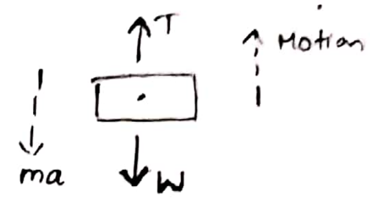
(iii) · Let 'a' be the acceleration required to cause pressure of 1000N on floor (T = 1000 N)

$$T = W + ma$$

$$= W\left[1 + \frac{a}{g}\right]$$



$$1000 = 750\left[1 - \frac{a}{9.81}\right]$$

$$1 - \frac{a}{9.81} = 1.333$$

$$a = 3.27\ m/s^2$$

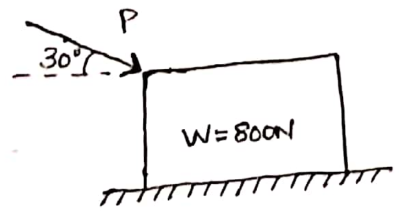
$P = m(g + a)$

$1000 = \frac{750}{9.81}(9.81 + a)$

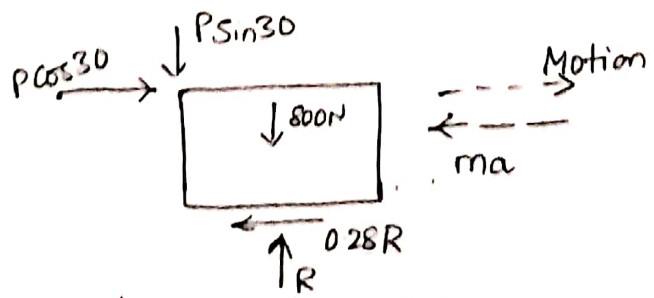
$= 750 + .$

$a = 3.27\ m/s^2$

3 A block weighing 800N rests on a horizontal plane as shown in figure. Find the magnitude of force 'P' required to give the block an acceleration of $a = 3 m/s^2$ to the right. The coefficient of friction b/w the block and the plane is 0.28



Given W = 800N, $a = 3 m/s^2$, $\mu = 0.28$



$F_r = \mu N.$

Considering dynamic equillibrium

$\Sigma V = 0$ $\quad R - 800 - P\sin 30 = 0$

$+\uparrow \downarrow^-$ $\quad R = 800 + 0.5P \longrightarrow (1)$

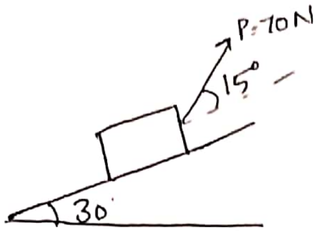
$\Sigma H = 0 \quad \xleftarrow{-} \xrightarrow{+}$

$P\cos 30 - 0.28R - ma = 0$

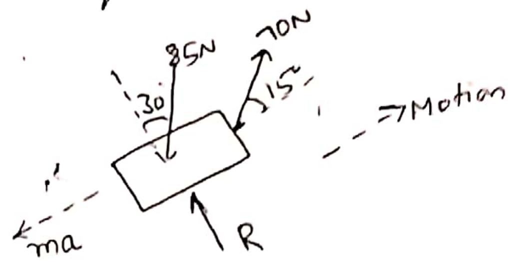
$P\cos 30 - 0.28(800 + 0.5P) - \left(\frac{800}{9.81} \times 3\right) = 0$ from (1) $m = \frac{W}{g}$

$P = 645.52\, N$

4 A block 85N is pulled up the smooth plane by a force of 70N as shown. Determine the acceleration along the plane.



Considering FBD of block in dynamic equilibrium (smooth plane $- \mu = 0 -$ No friction)



$\Sigma Y = 0$
$+\uparrow \downarrow -$

$+R - 85\cos 30 + 70\sin 15 = 0$

$R = 55.495\, N$

$\Sigma H = 0$
$\xleftarrow{-} \xrightarrow{+}$

$-ma - 85\sin 30 + 70\cos 15 = 0$

$ma = 25.115$

$\frac{W}{g} a = 25.115$

$\frac{85}{9.81} \times a = 25.115$

$a = 2.899\, m/sec^2$

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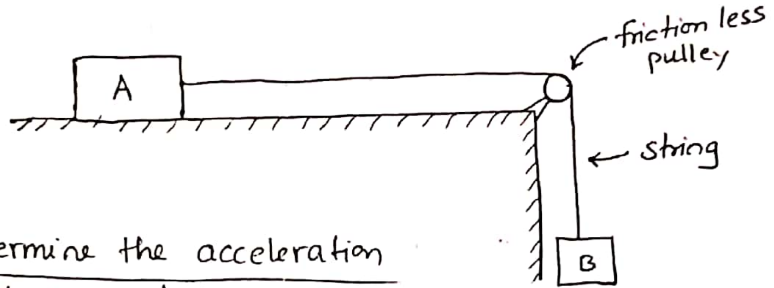
D'Alemberts principle on Motion of Connected bodies
(By strings and pulleys)

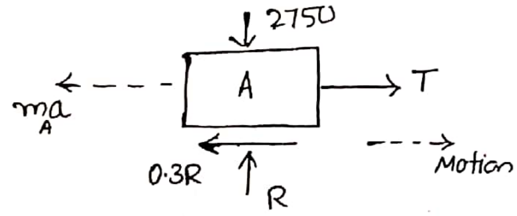
1. Two blocks 'A' & 'B' of weight 2750 N & 4150 N respectively are joined by an inextensible string as shown. Assume the pulley is frictionless and the coefficient of friction b/w block A & surface is 0.3. The system is initially at rest.

a) Determine the acceleration of block A
b) Velocity after it has moved 3.5 m
c) Velocity after 1.75 seconds.



a) <u>To determine the acceleration</u>

Considering dynamic equillibrium of Block A



$\Sigma V = 0$ $\quad$ $R = 2750\,N$

$\uparrow + \downarrow -$

$\Sigma H = 0$ $\quad$ $T - m_A a - 0.3R = 0$

$\leftarrow - \rightarrow +$

$$T = \frac{W}{g} a + 0.3(2750)$$

$$T = \frac{2750}{9.81} a + 825$$

$$T = 280.33\,a + 825 \rightarrow (1)$$

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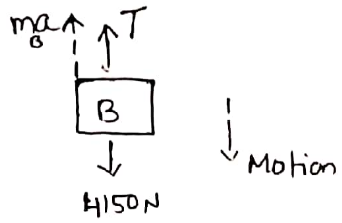
Considering dynamic equillibrium of block B



$\Sigma V = 0$

$\uparrow + \downarrow -$

$$T + m a_B - 4150 = 0$$

$$T = 4150 - \frac{W}{g} a$$

$$T = 4150 - \frac{4150}{9.81} a$$

$$T = 4150 - 423.03\,a \longrightarrow (2)$$

$(1) = (2) \Rightarrow \quad 28033\,a + 825 = 4150 - 423.03\,a$

$$\boxed{a = 4.727 \text{ m/sec}^2}$$

b) <u>To determine the velocity after it has moved by 3.5m</u>

$$V_1^2 = u^2 + 2aS \qquad (u = 0)$$

$$V_1 = \sqrt{2 \times 4.727 \times 3.5}$$

$$\boxed{V_1 = 5.75 \text{ m/sec}}$$

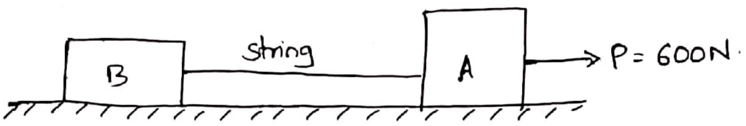
c) <u>To determine velocity after 1.75 seconds</u>

$$V_2 = u + at \qquad (u = 0)$$

$$V_2 = 4.727 \times 1.75$$

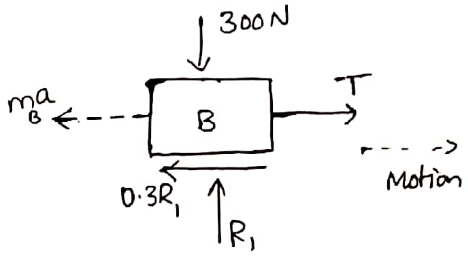
$$\boxed{V_2 = 8.27 \text{ m/sec}}$$

Two blocks A & B weighing 1200N & 300N are connected by a string and move along a horizontal rough plane by a horizontal force of 600N as shown. The coefficient of friction for block A is 0.25 & for block B is 0.3. Determine the tension in the string & acceleration of the weight applying D'Alembert's principle



Let T be the string tension & 'a' be the acceleration of weights

Considering dynamic equillibrium of block B



$\Sigma V = 0$ $\quad \uparrow + \downarrow -$

$R_1 - 300 = 0$

$R_1 = 300\,N$

$\Sigma H = 0$ $\quad \leftarrow - \rightarrow +$

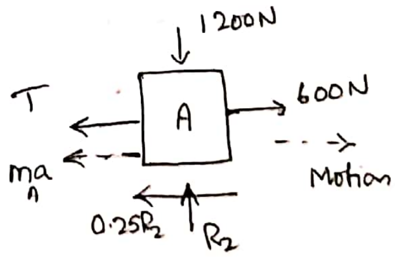
$T - 0.3R_1 - ma_B = 0$

$T = 0.3(300) + \frac{W}{g}a$

$T = 90 + \frac{300}{9.81}a$

$T = 90 + 30.58a \longrightarrow (1)$

Considering dynamic equillibrium of block A



$\Sigma V = 0$ $\quad \uparrow + \downarrow -$

$R_2 - 1200 = 0$

$R_2 = 1200\,N$

$\Sigma H = 0$ $\quad \leftarrow - \rightarrow +$

$-T - ma_A - 0.25R_2 + 600 = 0$

$T = 600 - 0.25(1200) - \frac{W}{g}a$

$T = 600 - 300 - \frac{1200}{9.81}a$

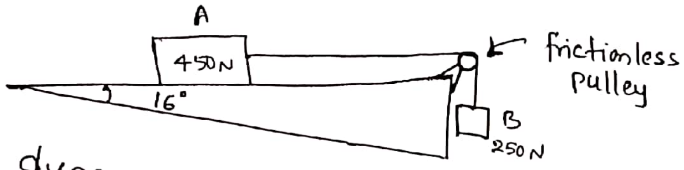
$T = 300 - 122.32a \longrightarrow (2)$

(1) = (2) $\Rightarrow$ $90 + 30.58a = 300 - 122.32a$

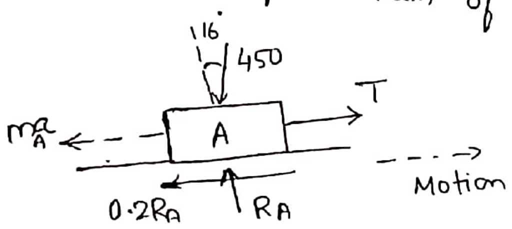
$\boxed{a = 1.373\ m/sec}$

$\therefore$ (1) $\Rightarrow$ $T = 90 + 30.58(1.373) = 131.99\ N$

3. Two blocks A & B weighing 450 N & 250 N respectively are Connected by a string passing over a pulley as shown. Block A is Placed on an inclined plane making an angle of 16° with horizontal and Coefficient of friction 0.2 & block B is hanging. If the block A moves up the plane, (i) Determine Tension in string ii) Determine acceleration of the body (iii) Determine the distance moved by the block in 2.5 secs starting from rest.



(i) Considering dynamic equillibrium of block 'A'



$\Sigma V = 0$

$\uparrow + \downarrow -$

$$-450 \cos 16 + R_A = 0$$

$$R_A = 432.56 \text{ N}$$

$\Sigma H = 0$

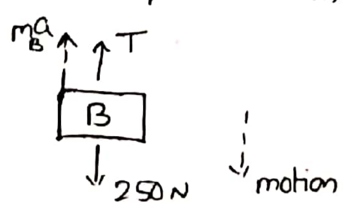
$\overset{-}{\leftarrow} \overset{+}{\rightarrow}$

$$-m a_A + T - 0.2 R_A - 450 \sin 16 = 0$$

$$T = 0.2(432.56) + 124.04 + m a_A$$

$$T = 210.55 + m a_A \longrightarrow (1)$$

(ii) Considering dynamic equillibrium of block 'B'



$\Sigma V = 0$

$\uparrow + \downarrow -$

$$T + m a_B - 250 = 0$$

$$T = 250 - m a_B \longrightarrow (2)$$

① = ② $\Rightarrow$ $210.55 + \frac{450}{9.81}a = 250 - \frac{250}{9.81}a$

$\boxed{a = 0.553 \ m/sec^2}$ $\longrightarrow$ acceleration of weight

$\therefore$ ① $\Rightarrow T = 210.55 + \frac{450}{9.81} \times 0.553$

$\boxed{T = 235.91 \ N}$ $\longrightarrow$ Tension in string

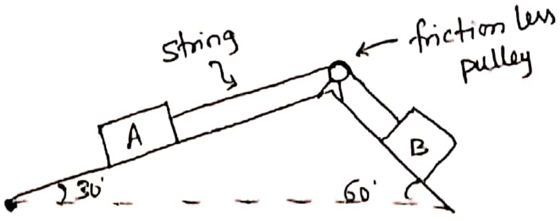
Let 's' be the distance travelled by block in time 2.5 secs from rest.

$S = ut + \frac{1}{2}at^2$ $(u = 0)$

$S = \frac{1}{2} \times 0.553 \times 2.5^2$

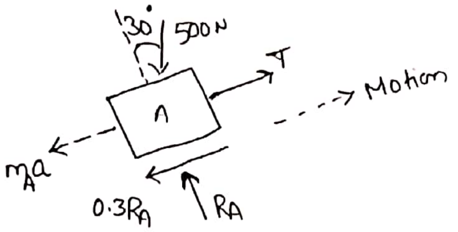
$\boxed{S = 1.728 \ m}$

4. Two blocks A & B weighing 500N & 1500 N respectively are connected by a weightless string passing over a frictionless pulley lying on planes making angles 30° & 60° as shown. Determine the resulting acceleration & tension in string. Also determine the velocity of the system 4 seconds starting from rest. Assume coefficient of friction to be 0.3 for all contact surfaces.



Let 'T' be the tension in string & 'a' be the acceleration of the weights

Considering dynamic equilibrium of block A



$\Sigma V = 0$
$\uparrow + \downarrow -$

$R_A - 500 \cos 30 = 0$

$R_A = 433.02\ N$

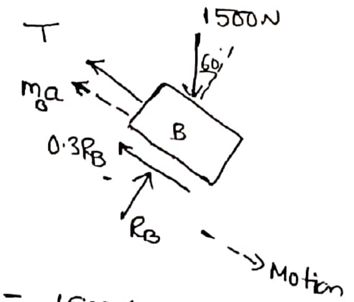
$\Sigma H = 0$
$\leftarrow - \rightarrow +$

$T - m_A a - 0.3R_A - 500 \sin 30 = 0$

$T = 0.3(433.02) + 250 + m_A a$

$T = 379.906 + m_A a \rightarrow (1)$

Considering dynamic equilibrium of block B



$\Sigma V = 0$
$\uparrow + \downarrow -$

$R_B - 1500 \cos 60 = 0$

$R_B = 750\ N$

$\Sigma H = 0$
$\leftarrow - \rightarrow +$

$-T - m_B a - 0.3 R_B + 1500 \sin 60 = 0$

$T = 1299.04 - 0.3(750) - m_B a$

$T = 1074.04 - m_B a \rightarrow (2)$

(1) = (2) $\Rightarrow$ $379.906 + \frac{500}{9.81} a = 1074.04 - \frac{1500}{9.81} a$

$\boxed{a = 3.405\ m/s^2}$

$\therefore$ (1) $\Rightarrow$ $T = 379.906 + \frac{500}{9.81} \times 3.405$

$\boxed{T = 553.45\ N}$

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let V_1 be the velocity after 4 seconds from rest

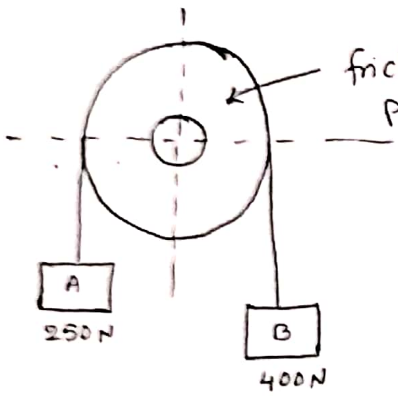
$\therefore$ $V_1 = u + at$

$V_1 = at$ $\qquad (u = 0)$

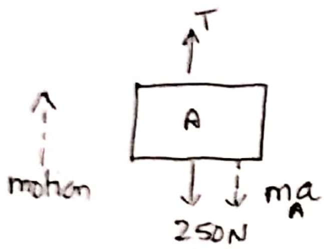
$V_1 = 3.405 \times 4$

$\boxed{V_1 = 13.62 \text{ m/sec}}$

5 Two bodies A and B hung at the ends of a rope passing over a frictionless pulley. Body A weighs 250N & body B weighs 400N as shown. Determine the acceleration with which the heavy body comes down. Also determine, the tension in string



Considering the dynamic equillibrium of body A 250N (moving up)

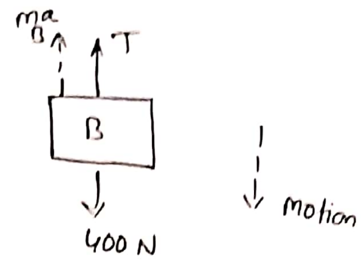


$\Sigma V = 0$
$\uparrow + \downarrow -$

$$T - m_A a - 250 = 0$$

$$T = 250 + m_A a \longrightarrow ①$$

Considering equillibrium of block B



$\Sigma V = 0$

$\uparrow + \downarrow -$ $\quad T + m_B a - 400 = 0$

$T = 400 - m_B a \longrightarrow (2)$

(1) = (2) $\quad 250 + m_A a = 400 - m_B a$

$$250 + \frac{250}{9.81} a = 400 - \frac{400}{9.81} a$$

$$\boxed{a = 2.264 \ m/sec^2}$$

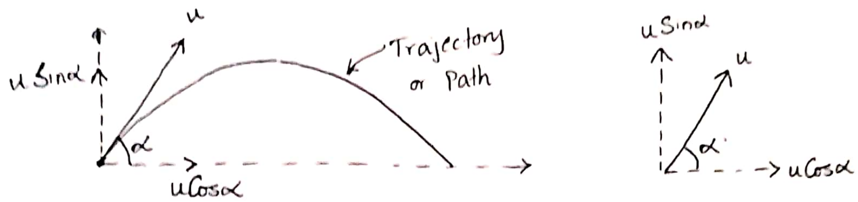
(1) => $\quad T = 250 + \frac{250}{9.81}(2.264)$

$$\boxed{T = 307.69 \ N}$$

PROJECTILES

A particle projected upwards at certain angle, moving under the combined effect of vertical and Horizontal components of velocity is called a projectile

Considering a particle projected with an initial velocity 'u' as shown.



Important Definitions

1 Trajectory :

Path traced by the projectile in space is known as trajectory.

2 Velocity of Projection :

The velocity 'u' with which a projectile is projected into space is called velocity of projection.

It has both vertical component (u Sinα) and horizontal component (u Cosα)

3 Angle of Projection :

It is the angle 'α' with the horizontal at which the projectile is projected.

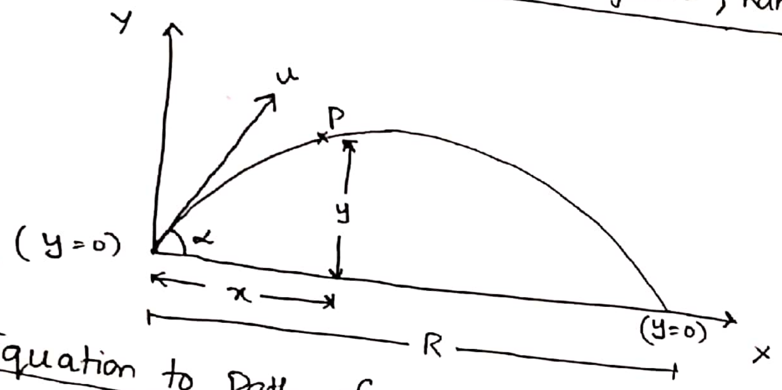
4 Time of flight (T) :

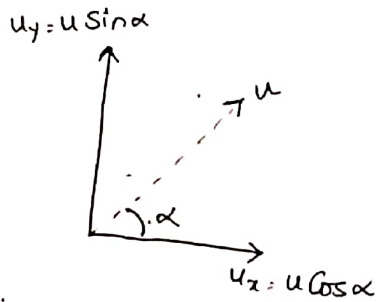
It is the total time 'T' taken from projection, to reach maximum height & return back to the ground. is known as Time of flight.

5. **Range :**

The distance between the point of projection and the point where projectile strikes the ground is known as range

Equation to the path of projectile, Range, Time of flight, Max Height





(i) **Equation to Path of projectile :**

The Horizontal component of $u = u\cos\alpha = u_x$

The vertical component of $u = u\sin\alpha = u_y$

WKT horizontal distance $x = u_x t$

$$x = (u\cos\alpha)t \longrightarrow ①$$

vertical distance $y = u_y t - \frac{1}{2}gt^2$

$$y = (u\sin\alpha)t - \frac{1}{2}gt^2 \longrightarrow ②$$

Substituting $t = \dfrac{x}{u\cos\alpha}$ from ① in ② gives

$$y = u\sin\alpha\left(\frac{x}{u\cos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$$

$$\boxed{y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}} \longrightarrow \text{equation for parabola}$$

ii) Equation for horizontal range

from the eq^n $y = x\tan\alpha - \dfrac{gx^2}{2u^2\cos^2\alpha}$

Substituting $x = R$ as $y = 0$

$$0 = R\tan\alpha - \frac{gR^2}{2u^2\cos^2\alpha}$$

$$\frac{gR}{2u^2\cos^2\alpha} = \tan\alpha$$

$$R = \frac{2u^2\cos^2\alpha\tan\alpha}{g}$$

$$R = \frac{2u^2\cos^2\alpha\,\dfrac{\sin\alpha}{\cos\alpha}}{g}$$

$$R = \frac{2u^2\cos\alpha\sin\alpha}{g}$$

$$\boxed{R = \frac{u^2\sin 2\alpha}{g}}$$ → equation for range of projectile.

When $\sin 2\alpha = 1$

i.e $2\alpha = 90^\circ$

$\alpha = 45^\circ$, the range will be Maximum.

i.e $\boxed{R_{max} = \dfrac{u^2}{g}}$

(iii) Equation for time of flight

Substituting $y=0$ in eq^n (2)

$$y = (u \sin\alpha)t - \frac{1}{2}gt^2$$

$$0 = (u \sin\alpha)t - \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 = (u \sin\alpha)t$$

$$\frac{gt}{2} = u \sin\alpha$$

$$\boxed{t = \frac{2u \sin\alpha}{g}}$$

(iv) Maximum height of a projectile

W.K.T, $v^2 = u^2 + 2as$

In Y direction, $v_y^2 = u_y^2 + 2as \rightarrow$ (1)

When, $Y = Y_{max}$, $v_y = 0$

& also, $a = -g$, $u_y = u\sin\alpha$

$S = H$, $u_y^2 = u^2 \sin^2\alpha$

where, g is acceleration due to gravity

H is maximum height of a projectile

then substituting in eqn (1)

$$0 = u^2 \sin^2\alpha - 2gH$$

$$2gH = u^2 \sin^2\alpha$$

$$\boxed{H = \frac{u^2 \sin^2\alpha}{2g}}$$

Problems on Projectile

A. projectile is fired with an initial velocity of 40m/sec at an angle of 25° with the horizontal. Determine

i) Horizontal Range
ii) Maximum height attained by the particle
iii) Time of flight

i) Horizontal Range

$$R = \frac{u^2 \sin 2\alpha}{g}$$

$$= \frac{40^2 \sin(2 \times 25)}{9.81}$$

$$\boxed{R = 124.94\ m}$$

ii) Maximum height

$$H_{max} = \frac{u^2 \sin^2\alpha}{2g}$$

$$= \frac{40^2 (\sin 25)^2}{2 \times 9.81}$$

$$\boxed{H_{max} = } \quad 14.57\ m$$

iii) Time of flight

$$T = \frac{2u \sin\alpha}{g}$$

$$= \frac{2 \times 40 \times \sin 25}{9.81}$$

$$\boxed{T = 3.446\ sec}$$

2. A projectile is fired at certain angle with the horizontal has a horizontal range of 3.5 km. If the maximum height is 500m, what is the angle of elevation of the cannon? what is the muzzle velocity of projectile?

Given $R = 3500m$, $H_{max} = 500m$, $\alpha = ?$ $u = ?$

WKT $R = \frac{u^2 \sin 2\alpha}{g} = 3500 \longrightarrow (1)$

$H_{max} = \frac{u^2 \sin^2\alpha}{2g} = 500 \longrightarrow (2)$

from (2), $u^2 = \frac{2 \times 500 g}{\sin^2\alpha} \longrightarrow (3)$

(3) in (1) $\Rightarrow$ $\frac{\left(\frac{1000 \cdot}{\sin^2\alpha}\right) \sin 2\alpha}{g} = 3500$

$\frac{1000 \cancel{g} \times \sin 2\alpha}{\sin^2\alpha \times \cancel{g}} = 3500$

$\frac{1000 (2 \cancel{\sin\alpha} \cos\alpha)}{\cancel{\sin\alpha} \sin\alpha} = 3500$

$\frac{2 \times 1000}{\tan\alpha} = 3500$

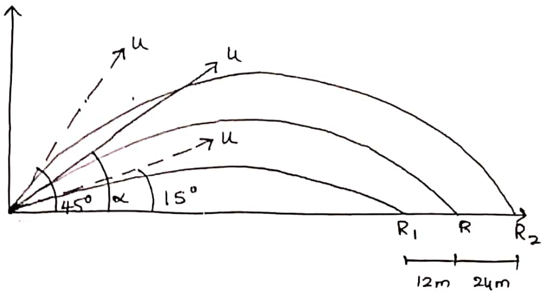
$\tan\alpha = 0.57$

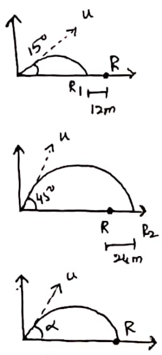
$\boxed{\alpha =}$ $29.74°$

$\therefore$ (3) $\Rightarrow$ $u^2 = \frac{2 \times 500 \times 9.81}{\sin^2 \; 29.74°}$

$\boxed{u = \quad m/sec}$
199.66

3. A projectile is aimed at a mark on a horizontal plane through the point of projection. It falls 12m short when the angle of projection is 15°, while it over shoots the mark by 24m. when the angle is 45°. Find the angle of projection to hit the mark. Assume no air resistance & Same velocity





Let 'R' be the range of mark with angle of projection α

let 'R_1' be the range for $\alpha_1 = 15°$

let 'R_2' be the range for $\alpha_2 = 45°$

Then $R_1 = R - 12$; $\alpha_1 = 15°$

$R_2 = R + 24$; $\alpha_2 = 45°$

$$\therefore \quad R_1 = \frac{u^2 \sin 2\alpha_1}{g} \Rightarrow (R-12) = \frac{u^2 \sin(2\times 15)}{g} \longrightarrow (1)$$

$$R_2 = \frac{u^2 \sin 2\alpha_2}{g} \Rightarrow (R+24) = \frac{u^2 \sin(2\times 45)}{g} \longrightarrow (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{R-12}{R+24} = \frac{\sin 30}{\sin 90}$$

$$\therefore (R-12) = 0.5\,(R+24)$$

$$\boxed{R = 48\ m}$$

$\therefore$ ① $\Rightarrow$ $R - 12 = \dfrac{u^2 \times \sin 30}{g}$

$48 - 12 = \dfrac{u^2 \times 0.5}{9.81}$

$\boxed{u = 26.57 \text{ m/sec}}$

$\therefore$ $R = \dfrac{u^2 \sin 2\alpha}{g}$

$48 = \dfrac{(26.57)^2 \sin 2\alpha}{9.81}$

$\boxed{\alpha = 20.92^\circ}$

Model Question Paper problem

4. A particle is projected with a velocity of 20m/s in air at an angle 'a' with the horizontal. The x & y Coordinates of a point lying on the trajectory of the particle with respect to point of projection are 20m & 8m respectively find the angle of projection of particle

Given $u = 20$ m/sec $\alpha = a = ?$ $x = 20$m $y = 8$m

WKT $y = x \tan\alpha - \dfrac{gx^2}{2u^2 \cos^2\alpha}$

$8 = 20 \tan(a) - \dfrac{9.81 \times 20^2}{2 \times 20^2 \times \cos^2(a)}$

$8 + \dfrac{4.905}{\cos^2(a)} = 20 \tan(a)$

$$8 + \frac{4.905}{\cos^2 a} = 20 \frac{\sin a}{\cos a}$$

$$\frac{8\cos^2 a + 4.905}{\cos^2 a} = 20 \frac{\sin a}{\cancel{\cos a}}$$

$$8\cos^2 a + 4.905 = 20 \sin a \cos a$$

$$8\cos^2 a + 4.905 = 20\left(\sqrt{1-\cos^2 a}\right)\cos a$$

$(\because \sin^2 a + \cos^2 a = 1)$

Squaring on both sides

$$(8\cos^2 a + 4.905)^2 = \left(20\sqrt{1-\cos^2 a}\cdot \cos a\right)^2$$

$$(8\cos^2 a + 4.905)^2 = \left(\sqrt{(1-\cos^2 a)\times 400\cos^2 a}\right)^{\cancel{2}}$$

$$64(\cos^2 a)^2 + 24.06 + 78.48\cos^2 a = 400\cos^2 a - 400(\cos^2 a)^2$$

$$464(\cos^2 a)^2 - 321.52\cos^2 a + 24.06 = 0$$

$$\cos^2 a = 0.607 \text{ (or) } \cos^2 a = 0.085$$

$$\cos a = \sqrt{0.607} \text{ (or) } \cos a = \sqrt{0.085}$$

$$\boxed{a = 38.82^\circ} \text{ (or) } \boxed{a = 73.05^\circ}$$

Model Question Paper problem

5. A particle is projected in air with a velocity of 120m/sec at an angle of 30° with the horizontal. Determine
   i) the horizontal range
   ii) the maximum height by the particle
   iii) the time of flight.

Given $u = 120\,m/sec$ $\alpha = 30°$ $R = ?$ $H_{max} = ?$ $t = ?$

WKT (i) $R = \frac{u^2 \sin 2\alpha}{g}$

$= \frac{(120)^2 \sin(2 \times 30)}{9.81}$

$\boxed{R = 1271.22\ m}$

(ii) $H_{max} = \frac{u^2 \sin^2 \alpha}{2g}$

$= \frac{(120)^2 \sin^2 (30)}{2 \times 9.81}$

$\boxed{H_{max} = \quad m}$

183.49

(iii) time of flight $t = \frac{2u \sin\alpha}{g}$

$= \frac{2 \times 120 \times \sin 30°}{9.81}$

$\boxed{t = 12.23\ sec}$