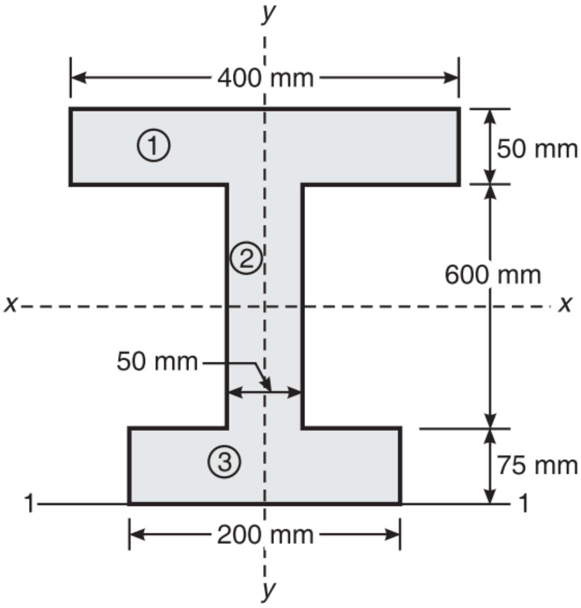
1. Find the moment of inertia along the horizontal axis and vertical axis passing through the centroid of a section shown in Fig



Solution The given figure is symmetrical about the y-axis. Therefore, the centroidal y-axis coincides with the reference y-axis. Hence $\bar{x} = 0$.

Moment of inertia about the centroidal x–x axis

$$I_{1-1} = \bar{I}_x + A\,\bar{y}^2 = \Sigma \bar{I}_x + \Sigma A y^2$$

or
$$I_{1-1} - A\,\bar{y}^2 = \bar{I}_x$$

$$I_{2-2} = \bar{I}_y + A\,\bar{x}^2$$

Comp.	*Area* (mm^2)	*y*	*Ay* (mm^3)	*Ay*^{2} (mm^4)	$\bar{I}_x$ (mm^4)	$\bar{I}_y$ (mm^4)
1.	400×50 $= 20{,}000$	$75 + 600 + \frac{50}{2}$ $= 700$	14,000,000 $= 14 \times 10^6$	9.8×10^9	$\frac{400(50)^3}{12}$ $= 4.167 \times 10^6$	$\frac{50(400)^3}{12}$ $= 266{,}666{,}666.7$
2.	50×600 $= 30{,}000$	$75 + \frac{600}{2}$ $= 375$	11,250,000 $= 11.25 \times 10^6$	4.219×10^9	$\frac{50 \times 600^3}{12}$ $= 900 \times 10^6$	$\frac{600 \times 50^3}{12}$ $= 6{,}250{,}000$
3.	200×75 $= 15{,}000$	$\frac{75}{2} = 37.5$	562,500	21.09×10^6	$\frac{200 \times 75^3}{12}$ $= 7.03 \times 10^6$	$\frac{75 \times 200^3}{12}$ $= 5{,}00{,}00{,}000$
Σ	65,000		25.812×10^6	1.404×10^{10}	911.197×10^6	322,916,666.7

$$\bar{y} = \frac{\Sigma A y}{\Sigma A} = \frac{25.812 \times 10^6}{65{,}000} = 397.108 \text{ mm}$$

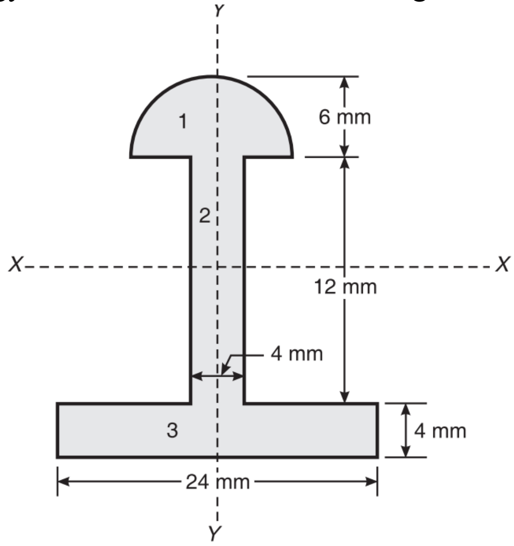
$$\bar{I}_{1-1} = \bar{I}_x + A\,\bar{y}^2 = \Sigma \bar{I}_x + \Sigma A y^2 = 1.495 \times 10^{10} \text{ mm}^4$$

$$\bar{I}_x = I_{1-1} - A\,\bar{y}^2 = 1.495 \times 10^{10} - 65000 \times (397.108)^2 = 4.691 \times 10^9 \text{ mm}^4$$ **Ans.**

When the moment of inertia is required on a symmetrical axis, then

$$\bar{I}_y = \Sigma \bar{I}_y = 329{,}216{,}666.7 \text{ mm}^4$$ **Ans.**

2. Find the polar radius of gyration for the area shown in Fig



Solution

Component	*Area* (mm^2)	*y* (mm)	*Ay* (mm^3)	*Ay*^{2} (mm^4)
Semicircle 1	$\frac{\pi \times 6^2}{2} = 56.549$	$16 + \frac{4 \times 6}{3\pi} = 18.546$	1.049×10^3	19.45×10^3
Rectangle 2	$4 \times 12 = 48$	$4 + \frac{12}{2} = 10$	0.48×10^3	4.8×10^3
Rectangle 3	$24 \times 4 = 96$	$\frac{4}{2} = 2$	0.192×10^3	0.384×10^3
Sum	224.549		1.721×10^3	24.634×10^3

Component	$\overline{I}_x$ (mm^4)	$\overline{I}_{gy}$ (mm^4)
Semicircle 1	$0.11 \times 6^4 = 142.56$	$\frac{\pi \times 6^4}{8} = 508.9376$
Rectangle 2	$\frac{b^2 d_2^3}{12} = \frac{6 \times 12^3}{12} = 864$	$\frac{12 \times 6^3}{12} = 216$
Rectangle 3	$\frac{b_3 d_3^2}{12} = \frac{24 \times 4^3}{12} = 128$	$\frac{4 \times 24^3}{12} = 4608$
Sum	1134.56	5332.938

$$\overline{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{1960.75}{224.549} = 8.731 \text{ mm}$$

$$I_{1-1} = \Sigma \overline{I}_x + \Sigma Ay^2 = 1134.56 + 27{,}034.26 = 28.1688 \times 10^3 \text{ mm}^4$$

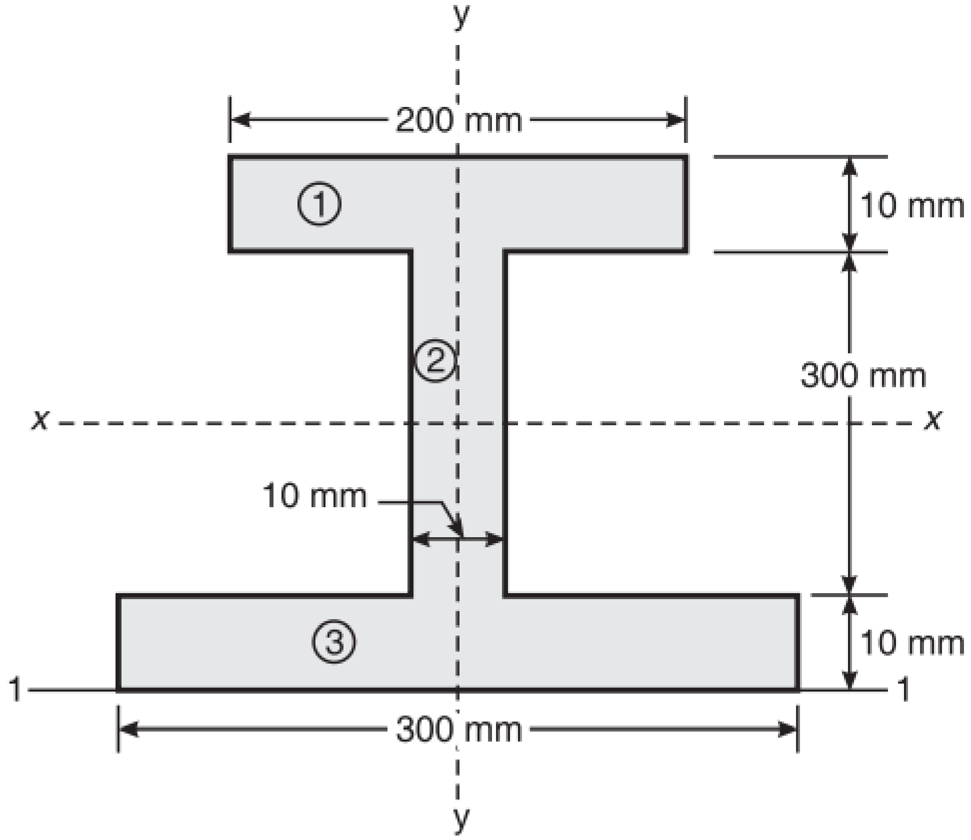
$$\overline{I}_x = I_{1-1} - A\,\overline{y}^2 = 28.1688 \times 10^3 - 224.549 \times (8.731)^2 = 11051.349 \text{ mm}^4$$

$$\overline{I}_y = 5332.938 \text{ mm}^4$$

$$\overline{I}_z = \overline{I}_x + \overline{I}_y = 11051.349 + 5332.938 = 16.384 \times 10^3 \text{ mm}^4$$

$$k_z = \sqrt{\frac{\overline{I}_z}{A}} = \sqrt{\frac{16.384 \times 10^3}{224.549}} = \sqrt{72.965} = 8.542 \text{ mm}$$ **Ans.**

3. Determine the moment of inertia of the unequal I-section about its centroidal axes as shown in Fig



Solution

Comp.	*Area* (mm^2)	*y* (mm)	*Ay* (mm^3)	*Ay*^{2} (mm^3)	$\bar{I}_x$ (mm^4)	$\bar{I}_y$ (mm^4)
1.	200×10 $= 2000$	$10 + 300 + \frac{10}{2}$ $= 315$	6.3×10^5	1.98×10^8	$\frac{200(10)^3}{12}$ $= 16{,}666.67$	$\frac{10(200)^3}{12}$ $= 6{,}666{,}666.67$
2.	300×10 $= 3000$	$10 + \frac{300}{2}$ $= 160$	4.8×10^5	0.798×10^8	$\frac{10 \times 300^3}{12}$ $= 225{,}00{,}000$	$\frac{300 \times 10^3}{12}$ $= 25{,}000$
3.	300×10 $= 3000$	$\frac{10}{2} = 5$	0.15×10^5	75×10^3	$\frac{300 \times 10^3}{12}$ $= 25{,}000$	$\frac{10 \times 300^3}{12}$ $= 2{,}25{,}00{,}000$
Σ	8000		11.25×10^5	2.75×10^8	22,541,666.67	29,191,666.7

$$\bar{y} = \frac{\Sigma Ay}{\Sigma a} = \frac{11.25 \times 10^5}{8000} = 140.625 \text{ mm}$$

$$I_{1-1} = \bar{I}_x + A\,\bar{y}^2 = \Sigma \bar{I}_x + \Sigma Ay^2 = 22{,}541{,}666.67 + 2.75 \times 10^8 = 297{,}541{,}666.67 \text{ mm}^4$$

$$\bar{I}_x = I_{1-1} - A\,\bar{y}^2 = 297{,}541{,}666.67 - 8000 \times (140.625)^2 = 139{,}338{,}541.67 \text{ mm}^4$$ **Ans.**

$$\bar{I}_y = 29{,}191{,}666.7 \text{ mm}^4$$ **Ans.**