

# Institutionen för systemteknik

## Department of Electrical Engineering

**Examensarbete**

### **Real time textrendering**

Examensarbete utfört i Informationskodning  
vid Tekniska högskolan vid Linköpings universitet  
av

**Gustav Adamsson**

LiTH-ISY-EX--YY/NNNN--SE

Linköping 2015



**Linköpings universitet**  
**TEKNISKA HÖGSKOLAN**



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**Titel**                    Realtidstextrendering  
Title                        Real time textrendering

**Författare**            Gustav Adamsson  
Author

**Sammanfattning**  
Abstract

Det här som vi har hållit på med är jätteviktigt faktiskt och det vi gjort blev bara sååå bra.  
Kanske inte helt otippat, men det glass är sååå gott!

Förresten har vi blivit bäst på att skriva rapporter, så nu ska ska vi inte gå in närmare  
på några detaljer såhär i sammanfattningen.

**Nyckelord**

Keywords    textrendering, distance field, distance map, distance transform



## **Abstract**

If your thesis is written in English, the primary abstract would go here while the Swedish abstract would be optional.



## Acknowledgments

We think alla har varit så himla goa hela den här långa och tuffa tiden i våra liv.

*Linköping, Januari 2020*  
N N och M M



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# Notation

## NÅGRA MÄNGDER

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Notation	Betydelse
$\mathbb{N}$	Mängden av naturliga tal
$\mathbb{R}$	Mängden av reella tal
$\mathbb{C}$	Mängden av komplexa tal

## FÖRKORTNINGAR

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Förkortning	Betydelse
ARMA	Auto-regressive moving average
PID	Proportional, integral, differential (regulator)



# 1

---

## Introduction

Redan de gamla grekerna och rommarna trodde att...

### 1.1 Problem formulation

This section covers the problem formulations which were set up at the beginning of this thesis work. The problem formulation has functioned as guidelines throughout the work of the thesis.

- How can distance fields be used when rendering text and how does the technique work?
- What distance field size is it necessary to use for the rendered text to get equally good appearance as the text rendered from Visiarcs current text rendering implementation?
- Is it possible to generate distance field in real time fast enough that the user can not tell the difference if it was pre generated or not?
- What special effects can be done with distance fields?
- Is it viable in terms of FPS to use special effects from distance fields on mobile devices?

### 1.2 Report Structure



# 2

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## Background

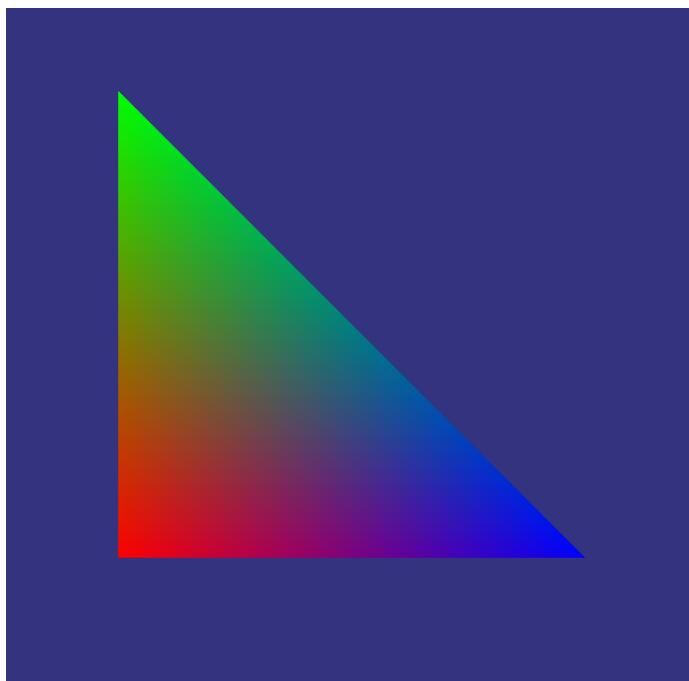
This chapter includes relevant background information about the main topics of this report. The purpose of the chapter is to give the reader some basic knowledge and understanding about the touched topics to facilitate further reading of the report.

### 2.1 Computer graphics

Distance fields and text renderering are subjects closely related to computer graphics. Therefore it is important to understand some basic concepts about computer graphics when reading this report. Two important concept when talking about computer graphics in the context of this thesis are polygons and textures. A polygon is a figure bound together by a finite chain of straight line segments. The corners of the polygon are called vertices and are defined as coordinates in space. A usual representation of a polygon is a list of vertices where the vertices is ordered in a way that every vertice is connected to the next vertice in the list by a line segment. A texture is a representation of an image. The texture is usually represented as a 1 or 2 dimensional array with a 8 or 32 bit value per pixel.

When drawing an image or texture to the screen it has to be mapped to a polygon first. When working in 2 dimensions a useful polygon for mapping textures to is the quad which is the polygon with 4 corners. The process of drawing a texture to a quad on the screen follows. The quad is created by creating a list of vertices. A list of texture coordinates is created. Texture coordinates defines which part of the texture should be drawn on the quad. The texture is uploaded to the GPU together with the list of vertices and the list of texture coordinates. Shaders written by the programmer is run and the texture is drawn on the quad. There are two shaders of importance within the scope of this thesis. It is the vertex shader and the fragment shader. The vertex shader is run one time for every

vertex. In the case with a texture mapped to the quad the vertex shader would just pass through the input values. The next shader run is the fragment shader. The fragment shader is run once for every candidate pixel on the screen. The output data from the vertex shader is interpolated before sent as input to the fragment shader. The interpolation is needed because it is very rare that the vertices map 1 to 1 to the pixels on the screen. Therefore the pixel value needs to be interpolated between several vertex values. The interpolation is done for every variable that is sent to the fragment shader. An schoolbook example of the interpolation is the colored triangle. The 3 vertices of the triangle get three different colors sent to the corresponding vertex shader. The vertex shader pass through the colors and the GPU interpolates the colors to a separate color for each pixel. This will create a color gradient between the 3 corners where each pixel has a unique color.



*Figure 2.1: An example of the interpolation between the vertex and fragment shader on the GPU*

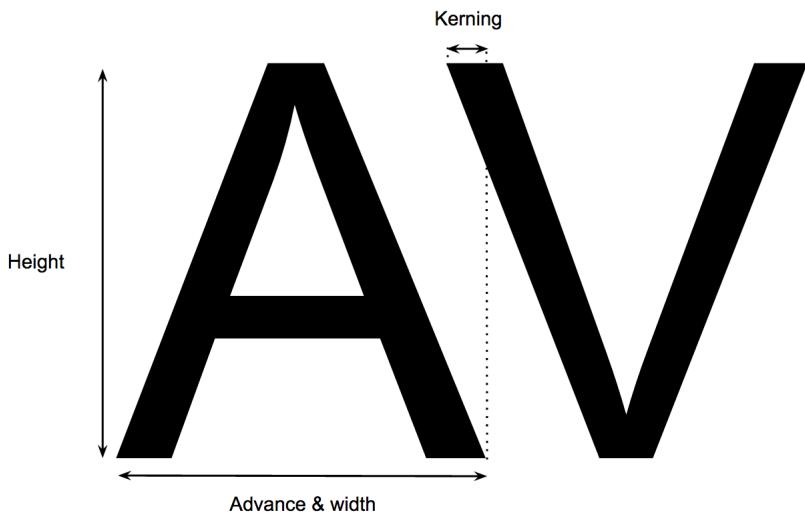
## 2.2 Basic text rendering

Text rendering is a non trivial subject in computer graphics and has been for a long time. There are many different ways to render text to a screen. A common way is to pre render all the glyphs of a font to an image. Characters of this font can then be rendered by using a part of the image as texture for a polygon. Another way is to use some library to render the text to an image and use it to texture

a quad [FreeType, 2014a]. Both of the above mentioned methods has some draw-back, for example the text can not be scaled up without losing the smooth edges and the images takes a lot of memory if you want to have high resolution text.

When rendering a text it is really important that every glyph are positioned to each other as specified in the font file. Every glyph in a font has visual information stored in the font file. For example the OpenType and the TrueType™ font format has information about of advance, kerning, height, width etc [Microsoft Corporation, 2015, Apple Inc., 2015].

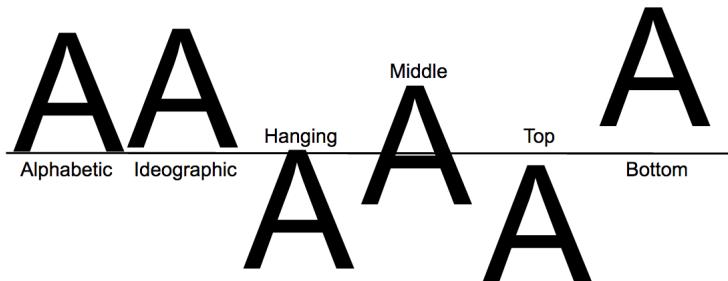
Without kerning it might be hard to get a good looking text if the font is not built in a way that kerning is not a factor. Kerning is displacement along the axis of the advancing direction of the text. A kerning value can be positive and negative and is a function of two parameters, the previous letter and the current letter. A negative kerning value is the most common and it means that the current glyph will be moved backwards relative the advancing direction of the text and a positive value means that it will be moved forward. To get a better understanding of what kerning is take the strings “AV” and “AA” as an example. It is obvious that the left part of “V” is hanging above the “A” but that is not the case in the second string where “A” is followed by an “A”. This is because the kerning table in the font has a negative entry for the combination “AV” but not for the combination “AA”. [FreeType, 2014b]



*Figure 2.2: Illustration of how kerning works*

Another important concepts when rendering text is baseline. A text can be placed on different baselines located on different heights. The baselines used in this thesis work is top, hanging, middle, alphabetic, ideographic and bottom. The most commonly used baseline is the alphabetic baseline which is used when for

example writing english text on a piece of paper.



**Figure 2.3: Some baselines**

## 2.3 Distance fields

Binary images are frequently used in image processing [Ragnemalm, 1993]. A binary image is an image with only one binary value for each pixel. This will limit the image to only contain two values or colors. One example of an image that could be represented as a binary image is a picture of a white letter on a black background. In this case white would be represented as 0 and black as 1 or vice versa.

A distance transform is the transform of an input binary image to an output image called distance field or distance map [Rosenfeld and Pfaltz, 1966]. The values of the distance field pixels represents the closest distance by some distance metric to an arbitrary shape in the input image. The distance metric can differ between different applications but the most commonly used distance metric is the euclidean distance metrics which is also called real distance. If euclidean distance metrics is used when doing the distance transform it is called euclidean distance transform(EDT). Other distance metrics that can be used in distance transforms is the city-block distance metric and the chessboard distance metric. When using city-block distances you are only allowed to travel horizontally or vertically in a grid with the cost of 1. Chessboard distance is an extension of city-block distance where you are also allowed to travel diagonally with the cost of 1.

A distance field can be signed or unsigned. An unsigned distance field only maps the distances either inside the shape or outside the shape while a signed distance field maps the pixels inside the shape with negative distances and the outside of the shape with positive distances or vice versa.

Distance fields has many good advantages compared to other methods for storing shapes like boundary representation. One of the most obvious advantages is that a distance field represents more than just the boundary. For example a signed distance field also represents the environment around the boundary, both the interior and the exterior. This makes it easy to check if a point is inside or

outside a shape by checking the value of that point in the distance field. It is also easy to move the boundary of the object by just changing the threshold value determining where the boundary is located. [Jones et al., 2006]

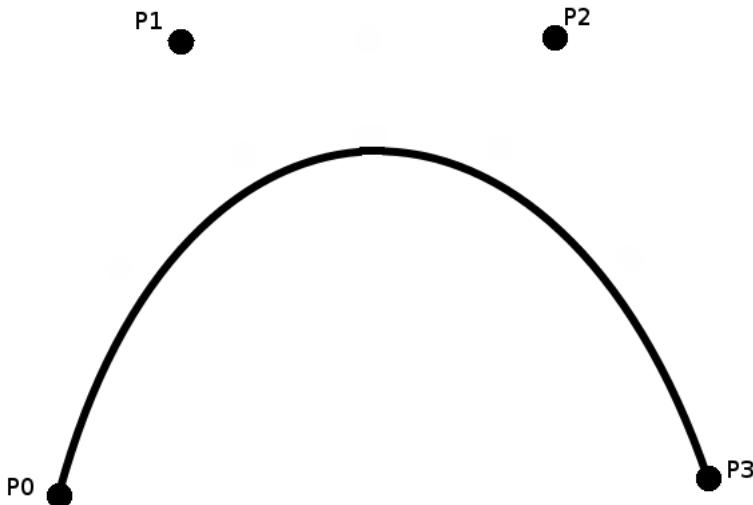
## 2.4 Beziér curves

A beziér curve is defined in space or the plane as two endpoints and a number of control points which is blended together with one blending function per point. The number of control points depends on the degree of the beziér curve. The most commonly used beziér curves are the cubic beziérs with four points and the quadratic beziérs with three points. The quadratic and the cubic beziér is defined as follows.

$$B(t) = (1 - t)^2 P_0 + 2t(1 - t)P_1 + t^2 P_2, t \in [0, 1]$$

$$B(t) = (1 - t)^3 P_0 + 3t(1 - t)^2 P_1 + 3t^2(1 - t)P_2 + t^3 P_3, t \in [0, 1]$$

The quadratic bezier has  $P_0$  and  $P_2$  as endpoints and  $P_1$  as a control point. The cubic bezier has  $P_0$  and  $P_3$  as endpoints and  $P_1$  and  $P_2$  as control points. The sum of the blending functions for both quadratic beziers and cubic beziers is always equal to 1 for any t in the function domain.[Ragnemalm, 2008]



**Figure 2.4:** A cubic beziér curve

One bezier curve can not represent arbitrary curves, therefore it is important to be able to connect several beziers to a path to be able to describe more complex shapes. When connecting curves, continuity between the curves is an important concept to consider. Parametric continuity is a measure that will have grave importance of the appearance of the curve. There are three levels of parametric continuity.

$C^0$  = The curves meet

$C^1$  = The derivative for the both curves are equal where the curves meet

$C^2$  = The second derivative for the both curves are equal where the curves meet

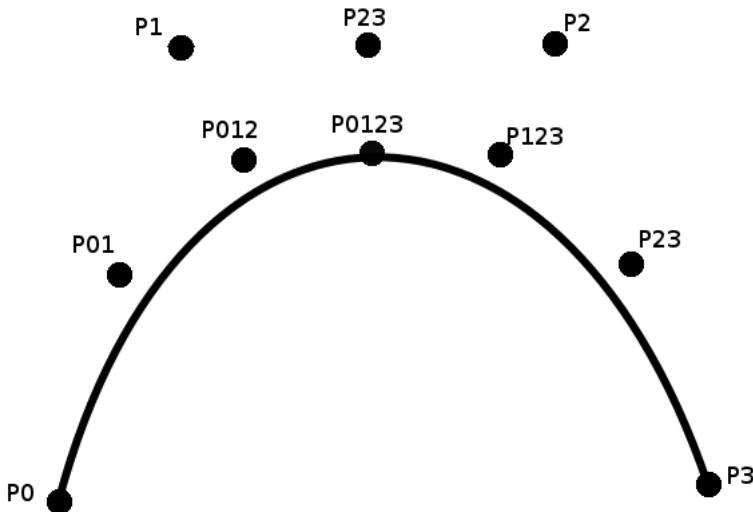
Another measure than can be used for continuity is geometric continuity which is almost the same as parametric continuity. The only difference is that geometric continuity only require the derivatives of  $G^1$  and  $G^2$  to be proportional and not equal. Given two cubic beziér curves p defined by  $p_1, p_2, p_3$  and  $p_4$  and q defined by  $q_1, q_2, q_3$  and  $q_4$ . Assume that p and q is placed in a way that  $p_4 = q_1$ . This will trivially fulfill the requirement for  $C^0$  because they share one endpoint and  $p(1) = q(0)$ .  $C^1$  continuity,  $p'(1) = q'(0)$  will be fulfilled if  $p_3, p_4$  and  $q_1$  are located on the same line.  $C^2$  continuity  $p''(1) = q''(0)$  will be fulfilled if  $p_3, p_4$  and  $q_1$  are located on the same line and  $|p_4 - p_3| = |q_2 - q_1|$  which means that the distance from  $p_3$  to  $p_4$  must be equal to the distance from  $p_4$  to  $q_2$ .

An example of an application of beziérs are fonts. Every glyph in a font is stored as a number of straight lines and bezier curves, either cubic or quadratic.[Phinney, 2001] Even though beziérs are used in fonts it is not a trivial problem to draw a beziér to the screen. Beziér curves are hard to draw because the lowest level graphics hardware can only draw polygons and line segments. This is solved by approximating smooth curves to line segments before drawing[Shreiner et al., 2009]. Another problem with beziér curves is that it is time consuming to find the closest point on a beziér curve from an arbitrary point. To solve this problem  $(p - q(t))q'(t) = 0$ , where  $t \in [0, 1]$ , p is a point in space and q is a beziér curve, has to be solved[Chen et al., 2007]. This implies that a quintic polynomial needs to be solved to find the closest point on a cubic beziér curve given a point in space.

De Casteljau's algorithm can be used to subdivide a bezier curve recursively by using the properties of bezier curves to calculate new beziér points for both the left and the right part of the old beziér.[Fischer, 2000] A special case of the algorithm is to divide the curve at  $t = 0.5$  which gives a first curve  $t \in [0, 0.5]$  and a second curve  $t \in [0.5, 1]$ . An example of this subdivision for a cubic beziér is presented in figure XXX below. The first curve is described by the points  $p_0, p_{01}, p_{012}$  and  $p_{0123}$  and the second curve is described by  $p_{0123}, p_{123}, p_{23}$  and  $p_3$ . The points for the new curves are derived from the initial curve described by  $p_0, p_1, p_2$  and  $p_3$ . The following calculations shows how the new beziér points are computed.

$$\begin{aligned} P_{01} &= \frac{p_0 + p_1}{2} \\ P_{23} &= \frac{p_2 + p_3}{2} \end{aligned}$$

$$\begin{aligned}
 P_{12} &= \frac{P_1 + P_2}{2} \\
 P_{012} &= \frac{P_{01} + P_{12}}{2} \\
 P_{123} &= \frac{P_{12} + P_{23}}{2} \\
 P_{0123} &= \frac{P_{012} + P_{123}}{2}
 \end{aligned}$$

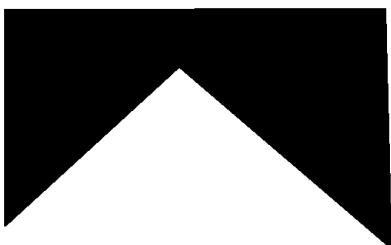


**Figure 2.5:** An illustration of the casteljau's algorithm splitting a cubic bezier at  $t = 0.5$

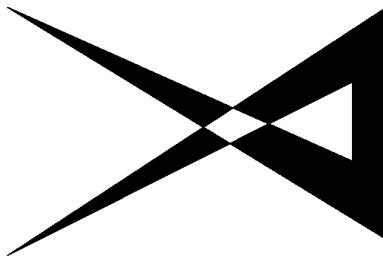
## 2.5 Polygon filling

A polygon is a shape bound together by a finite number of line segments. Drawing polygons is what computer graphics is about but this does not make it a trivial problem. There are several algorithms for doing this where each one of them have their advantages and drawbacks. One algorithm used a lot is the scan-line polygon fill algorithm. The algorithm contains two main components. The first component is a sorted edge table which is an array of linked lists where each linked list corresponds to a row in the image. The second component is an active edge list which is an initially empty linked list of edge references. The x value for the lowest y value of each line segment is inserted into the edge table. The next step in the algorithm is to iterate through the edge table row by row. All edges in the active edge list that ended on the previous line are removed. All remaining values of the active edge list are updated to the intersection between the line seg-

ment and the current row. All values of the current row in the edge table is then inserted into the active edge list and the active edge list is then sorted by x value. The image can now be filled for the current row by using some fill rule. The odd even fill rule is easy to use by just iterating through the active edge list and filling the pixels between every pair of edges starting with an odd edge.



**Figure 2.6:** A case managed by odd even fill



**Figure 2.7:** A case not managed by odd even fill

The odd even fill rule only work if edges can't cross each other. If edges cross each other there might be filled where it should not be filled and vice versa. An example of this can be seen in figureXXX where the odd even fill rule fails to fill the area in the middle. To handle this another fill rule can be used like the non-zero winding number rule. The property of this rule that makes it handle the case in the figure is that it also considers which direction the edges of the polygon are moving and increments a variable with different sign depending on that direction when crossing an edge.

# 3

---

## Related work

### 3.1 Early EDT algorithms

In a very often mentioned article Danielsson [1980] proposed an improved way to generate distance maps by representing the output of the distance transform as a vector, separating the distance of the different dimensions. Danielsson also proposed two sequential algorithms 4SED (4-point Sequential Euclidean Distance mapping) and 8SED (8-point Sequential Euclidean Distance mapping) for calculating the EDT using his representation of distance. Both the 4SED algorithm and the 8SED algorithm consists of 2 consecutive picture scans where they incrementally update pixel values depending on nearby pixels. The 8SED algorithm is described in the following pseudocode.

---

**Algorithm 1** First scan of the 8SED algorithm

```
for j = 1 to N - 1 step 1 do
    for i = 0 to M - 1 step 1 do
        L(i, j) = min(L(i, j), L(i-1, j-1)+(1, 1), L(i, j-1)+(0+1), L(i+1, j-1)+(1,1))
    end for
    for i = 1 to M - 1 step 1 do
        L(i, j) = min(L(i, j), L(i-1, j)+(1, 0))
    end for
    for i = M - 2 to 0 step 1 do
        L(i, j) = min(L(i, j), L(i+1, j)+(1, 0))
    end for
end for
```

---

**Algorithm 2** Second scan of the 8SED algorithm

---

```

for  $j = N - 2$  to  $0$  step  $1$  do
    for  $i = 0$  to  $M - 1$  step  $1$  do
         $L(i, j) = \min(L(i, j), L(i-1, j+1)+(1, 1), L(i, j+1)+(0, 1), L(i+1, j+1)+(1,$ 
         $1))$ 
    end for
    for  $i = 1$  to  $M - 1$  step  $1$  do
         $L(i, j) = \min(L(i, j), L(i-1, j)+(1, 0))$ 
    end for
    for  $i = M - 2$  to  $0$  step  $1$  do
         $L(i, j) = \min(L(i, j), L(i+1, j)+(1, 0))$ 
    end for
end for

```

---

The two scans are very similar. The only difference is that the first scan is done top down evaluating the pixels below and on the sides of the current pixel and the second scan is done bottom up evaluating the pixels above and on the sides of the current pixel[Ragnemalm, 1993]. The 4SED algorithm and the 8SED algorithm are not error free as Danielsson [1980] proves in his article but he also claims that the errors are rare and small and is negligible for practical purposes. In 8SED and 4SED every pixel is visited a constant number of times. This makes the time complexity of 8SED and 4SED trivially  $\mathcal{O}(mn)$ , if m and n are the width respectively the height of the input image.

## 3.2 Exact EDT algorithms

Distance transforms has been used in different applications in many years. The article from Danielsson [1980] was not the first on the subject but the fact that the algorithms he proposed in his article are some of the most widely used[Fabbri et al., 2008] makes it easy to call Danielsson one of the pionjeers on the subject. As mentioned earlier in this report, 8SED and 4SED is not exact. Exact algorithms for EDT has only been around since about the 1990s. In an article Fabbri et al. [2008] compares the execution time of exact EDT algorithms. The 6 algorithms compared in the test is Meijster et al. [2000], Maurer Jr et al. [2001], Eggers [1998], Lotufo and Zampirolli [2001], Cuisenaire and Macq [1999] and Saito and Toriwaki [1994]. The conclusion of the comparison is that the algorithms from Meijster and Maurer are the fastest but Meijster's algorithm is preferred due to slightly better performance and the fact that it is easier to implement than Maurer's algorithm.

Meijster's algorithm consist of two separate phases. The first phase iterates through each column performing a distance propagation in both directions. This will create a new image  $g(i, j)$  which used in the second phase. The second phase iterates through each row left to right and right to left applying a function  $DT(x, y)$  to calculate the output value of each pixel. The function depends

on what distance metric is used. The function used for euclidean distance transform follows.

$$DT(x, y) = \min_{0 \dots m} ((x - i)^2 + g(i)^2)$$

With the same motivation for 4SED and 8SED the time complexity of Meijster's algorithm is  $\mathcal{O}(nm)$ , if m and n are the width respectively the height of the image.

### 3.3 Improved distance measure for EDT algorithms

Calculating a distance transform of large size images can be very time consuming. There is a significant difference in computation time between creating a distance map from a 4096x4096 image and creating a distance map from a 64x64 image. For example, the 8SED and the 4SED algorithms both has time complexity  $O(n)$  where n is the number of pixels. Time complexity  $O(n)$  means that every pixel is visited a constant number of times when transforming the image. Assuming every pixel is visited once and one calculation is done per visit the larger image would require 4096 times more calculations than the smaller image. In an article from Green [2007], distance fields was generated by using a 4096x4096 binary image of a glyph as input to a distance transform. The output from the distance transform was then downsampled to a 64x64 texture. Generating a distance field using a large input image makes the discrete set of possible boundary pixels more dense compared to if a smaller input image is used. The increased density of the possible pixel set helps decrease the calculation error of the distance field assuming subpixel distance measures is not used.

In an article, Gustavson and Strand [2011] showed that the calculation errors of distance transforms can be decreased by using information about the subpixel boundary between the foreground and background pixels. In the article they use an anti-aliased greyscale input image to be able to locate the boundary in the pixels to get more precise distance measures between the pixel and the boundary.

They show that they get approximately the same amount of errors using their method on a 16x16 times smaller input image than by using the method Chris Green proposed in his article. The smaller input image increases the execution time and decreases the memory consumption by a factor of approximately 30 times according to Gustavson and Strand.

Skriv lite om hur algoritmen funkar....



# 4

---

## Method

This chapter will give a detailed description of which method was used to solve the problem real-time text rendering on mobile devices within the scope of this thesis. The chapter is divided into two different parts. The first part gives a detailed description about the implementation of the distance transform module. This part has been reimplemented several times due to poor performance. The second part gives a detailed description about the implementation of the distance field rendering module.

### 4.1 Distance field generation initial attempt

It exists many EDT algorithms for creating distance fields. Most of the EDT algorithms used today runs in  $\mathcal{O}(nm)$ , where n and m are the width respectively the height of the image. Example of algorithms running in  $\mathcal{O}(nm)$  are Danielsson [1980] and Meijster et al. [2000].

The initial implementation of distance field generation in this thesis was built around Gustavson and Strand [2011] article on the subject. A signed version of the 8SED algorithm was used along with the anti-aliased sub-pixel distance measure proposed in the article. A signed distance field has several advantages compared to an unsigned distance field. The most obvious advantages is the increased flexibility it gives and that it facilitates the implementation of proper anti-aliasing around the border of the shape[Gustavson, 2012]. To generate a signed distance field the distance transform was run twice with inverted input representation. The first transforms creates a distance field on the inside of the glyph and the second transform on the outside of the glyph. The resulting distance field was then calculated with the following formula.

$$result(i, j) = inside(i, j) - outside(i, j), \forall (i, j), i \in \{0, \dots, m - 1\}, j \in \{0, \dots, n - 1\}$$

## 4.2 Fast and approximate distance field generation

Fast distance field generation on mobile devices requires some approximations and optimizations to decrease the transformation time. This can be seen in many of the distance field generation implementations done lately. Two examples of this is GLyphy by Behdad Esfahbod and the implementation of distance field generation in Qt. Both the implementations approximate the bezier curves from the font file to some different representation of a glyph that is easier to work with. GLyphy approximates the bezier curves to circular arcs and Qt approximates the bezier curves to line segments. This makes it possible to draw distance fields locally over the outlines of the glyph and a distance transform of the whole image is not needed.

Because the initial implementation of the distance field generation was too slow to meet the requirement another faster implementation was necessary. A variant of the implementation done by Qt was implemented. The implementation can be divided into four steps in the following order, extracting beziers from the given font file, approximating the beziers to line segments, drawing the glyph in an image and drawing distance gradients over the line segments in the image to create a distance field locally over the outlines of the glyph. Extraction of bezier curves from font files can easily be done with different libraries for example freetype and a description of how this is done is not relevant for the proceeding of this report.

### 4.2.1 Bezier to line segment approximations

Bezier to line segment approximations is done alot in computer graphics. A naive way to solve the beziér to line segment problem is to split the Beziér curve into  $n$  smaller curves with a constant length and approximate them to line segments. This would make the line density constant over the whole curve meaning that both the strongly bent parts and the almost straight parts will be represented by equally many line segments. A smarter way to solve this is proposed in an article by Fischer [2000] which involves using a recursive function dividing the bezier into two parts using the de Casteljau's algorithm until a stop condition is met. This will make the line segment density higher where the curve is strongly bent and lower where the curve is almost straight. The following pseudocode is taken from the article by Fischer.

---

**Algorithm 3** Function for approximating beziér to line segment

---

```

procedure flattenCurve(Curve c)
  if isSufficientlyFlat(c) then
    output(c);
  else
    Curve l,r
    subdivide(c,l,r)
    flattenCurve(l)
    flattenCurve(r)
  end if
end procedure

```

---

In the above code Curve is a cubic bezier curve represented by 4 points in the plane. The function isSufficientlyFlat checks the difference between the curve and the line segment from the start point to the endpoint of the curve is small enough to meet the stop condition. This function can be implemented in many different ways. In his article Fischer propose a stop condition initially developed by Roger Willcocks. The pseudocode for this stop condition follows.

---

**Algorithm 4** Stop condition for cubic beziér subdivision

---

```

function isSufficientlyFlat(Curve c)
  double ux = 3.0*c.b1.x - 2.0*c.b0.x - c.b3.x; ux *= ux
  double uy = 3.0*c.b1.y - 2.0*c.b0.y - c.b3.y; uy *= uy
  double vx = 3.0*c.b2.x - 2.0*c.b3.x - c.b0.x; vx *= vx
  double vy = 3.0*c.b2.y - 2.0*c.b3.y - c.b0.y; vy *= vy
  if ux<vx then
    ux = vx
  end if
  if uy<vy then
    uy = vy
  end if
  return (ux+uy ≤ tolerance)
end function

```

---

There are other ways to check if a curve is flat enough to approximate it to a line segment. One solution is proposed by Shemanarev [2005]. The stop condition in his article only depends on the distance between the two points  $P_1$  and  $P_2$  and the line between  $P_0$  and  $P_4$ .

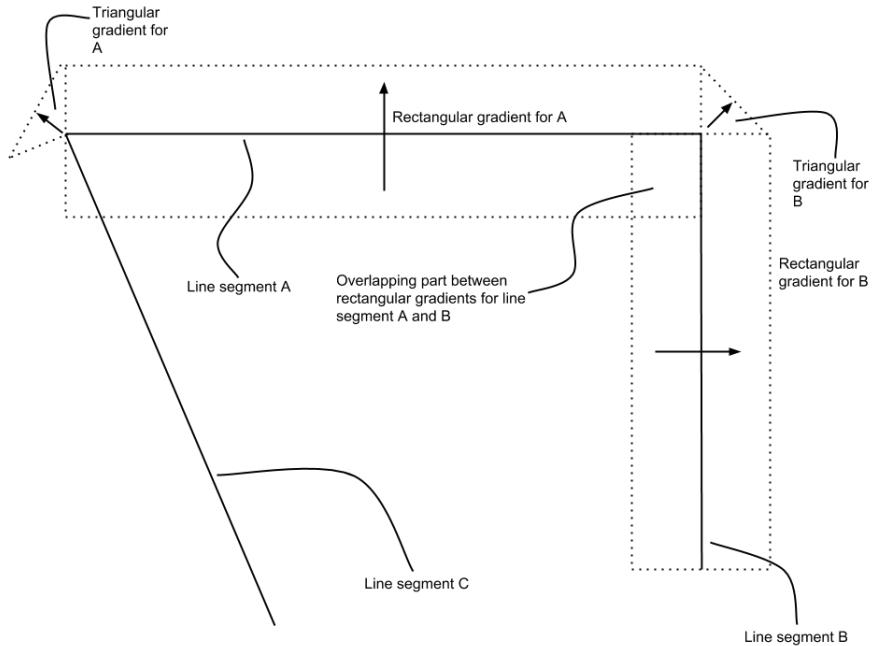
To solve the bezier to line segment approximation in this thesis the function subdivide in the algorithm for approximating beziers to line segments proposed by Fischer is implemented using the special case of de Casteljau's algorithm splitting at  $t = 0.5$ .

## 4.2.2 Drawing the glyph

The distance gradients in the distance field will only be drawn locally within a predefined distance from each outline. This creates a need to draw the glyph in the image before generating the distance field. If the glyph is not drawn before the distance gradients are drawn there will most certainly be parts of the glyph that have the outside color but is located on the inside of the glyph or vice versa. For example the middle of the 'I' might have the same color as pixels outside the glyph. This would lead to a hole in the middle of the 'I' when rendering it which is not desired. The input to this step of the distance field generation is a glyph represented as connected line segments in the shape of a polygon. As mentioned in the background the scan-line algorithm together with the odd even fill rule or the non-zero winding rule can be used to fill a polygon. Because is a very common problem in computer graphics and there are many open source libraries doing this very fast and optimized there was no need to implement this. To solve this the function `drawPath` in the open source 2D graphics library Skia was used. The Skia library was already installed and used in other places within Visiarc's products.

## 4.2.3 Drawing distance gradients

When drawing gradients it is convenient to draw them in primitive shapes like rectangles or triangles. To be able to draw gradients continuously over the outline of the whole glyph two different distance gradients had to be drawn for each line segment. The first gradient is a rectangular gradient and the second gradient is a triangular gradient. The gradients are illustrated in figureXXX.

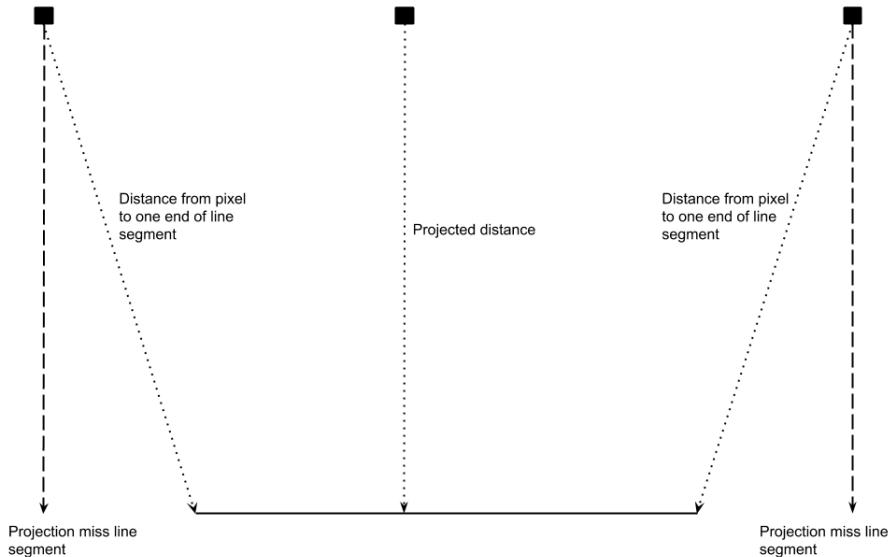


**Figure 4.1: Line segment to pixel distance**

The gradients are drawn in the direction of the arrows in the range from 0 to 255. This will make the inside of the glyph black/grey and the outside of the glyph white/grey. Because the glyph is drawn in the image using the Skia library before drawing the gradients there is no need for the gradients to cover the whole glyph. The triangular gradients are used to repair the distance field when two line segments are not parallel which is almost always the case. As illustrated in the figure it might occur problems in some areas when drawing the gradients. One of the problem areas is the overlap between different gradients. Another problem that might occur is that gradients that should be on the inside of the glyph ends up on the outside or vice versa. This will happen if the angle between two line segments is less than 90 degrees. To solve both the mentioned problems a control is done before drawing in a pixel. A description of the control done follows. If the new and the old value are not both inside values or outside values do not draw. If both the new and the old value are inside values draw if and only if the old value is lower than the new value. If both the new and the old value are outside values, draw if and only if the old value is higher than the new value.

Drawing gradients can be done in many different ways. The initial implementation of this step in this thesis had real measured distances as values in each pixel in the gradient. The pixel to line segment distance was calculated by projecting the pixel coordinate onto the line. If the projection hit the line segment the distance was equal to the projection distance. Otherwise the distance was

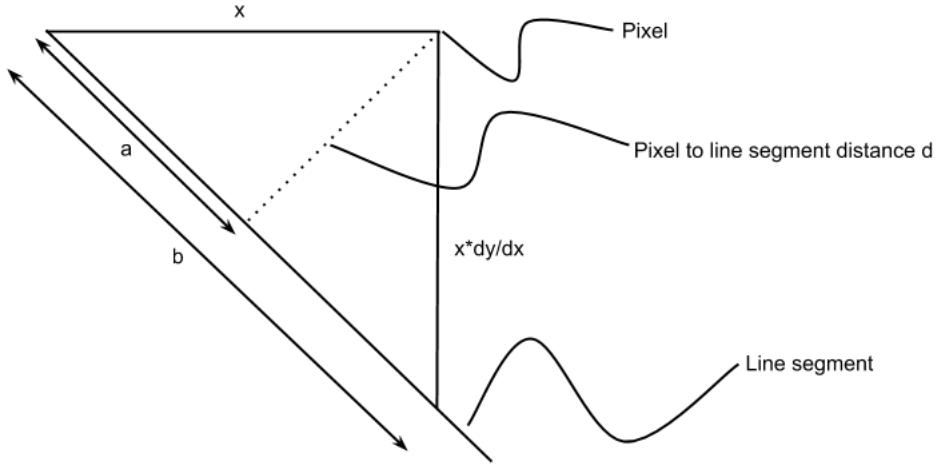
equal to the shortest distance to one of the endpoints of the line segment.



**Figure 4.2: Line segment to pixel distance**

The code for projecting a pixel to a line segment made the distance fields look good because there was few sources of errors drawing the gradients. Every pixel was calculated independently from eachother and if the new distance was not better than the distance already in the pixel the pixel was not updated. The only problem with this approach was the execution time. The code for pixel to line segment distance calculation was located in the innermost loop which means that the code was executed very many times and performance improvements were necessary.

In the distance field generation made by Qt no pixel to line projection is done. The rectangular gradients are drawn directly using trigonometry calculations stepping through the pixels row by row. This influenced how the rectangular gradients are drawn in the final result of this thesis. The problem by using this solution is to find the distance  $d$  in figureXXX.



**Figure 4.3:** Illustration of the calculations done when drawing the rectangular gradients

The line segment is defined between two points in the plane  $p_1$  and  $p_2$ . The derivative of the line can trivially be calculated  $\frac{dy}{dx} = \frac{p_{2y}-p_{1y}}{p_{2x}-p_{1x}}$ . The projection of the pixel to the line segment will always be perpendicular to the line segment which implies that the triangle with sides  $x$ ,  $x \frac{dy}{dx}$  and  $b$  and the triangle with sides  $x$ ,  $a$  and  $d$  are similar triangles. Similarity between two triangles means that the sides of the first triangle are proportional with the same constant to the sides of the second triangle. In mathematic terms it implies that  $\frac{x}{b} = \frac{a}{x \frac{dy}{dx}} = \frac{d}{x}$  is true.

The similarity of the triangles gives two ways to calculate  $d$ , either  $d = \frac{x^2}{b}$  or  $d = \frac{a}{\frac{dy}{dx}}$ . The first solution is used in this thesis and  $b$  is calculated with the

following formula  $b = \sqrt{x^2 + (x * \frac{dy}{dx})^2}$ . Having  $b$  the final formula for calculating  $d$  is  $d = \frac{x^2}{\sqrt{x^2 + (x * \frac{dy}{dx})^2}} = \frac{x}{\sqrt{1 + (\frac{dy}{dx})^2}}$ . The shortest distance from the line segment to the pixel is equal to  $|d|$ .

The rectangular gradients are drawn by stepping through all the pixels covered by the rectangle row by row and calculating the distance  $d$ . The control mentioned previous in this section is used before drawing to remove the overlapping problems. Drawing the triangular gradients is done in the same way with one exception, that is the calculation of the distance  $d$ . The closest point on the line segment to the pixel in the triangular gradient will always be the corresponding endpoint  $p$  of the line segment closest to the triangular gradient. This makes it possible to calculate  $d$  as the distance from the pixel to the point  $p$ .

### 4.3 Distance field rendering

Distance field rendering is not as easy as usual image rendering. When rendering images to the screen with OpenGL the image is loaded into a texture and uploaded to the GPU. On the GPU the image is resized and interpolated to fit the pixels on the screen before it is drawn. The interpolation is a built in operation on the GPU. An image can be drawn to the screen with a basic vertex and fragment shader where the vertex shader pass through the coordinates of the polygon that the texture is mapped to and the fragment shader fetch the color corresponding to its pixel from the texture and output it to the screen. When rendering distance field glyphs a more complex fragment shader has to be used.

The basic idea with distance field rendering is that the interpolation of a greyscale image dont decrease quality of the glyph border as much as if the image was not a greyscale image. This will let the GPU interpolate the image to the screen preserving the information about where the outline of the glyph is located before the fragment shader takes the decision whether the corresponding pixel is inside or outside the glyph. The most basic fragment shader for rendering distance field will set the output pixel transparent if the corresponding lookup in the distance field has a value greater or equal to 0.5 which means outside the outlines of the glyph. This solution is really easy to implement and can be written as one line of code but it will create aliasing artifacts around the border of the glyph. To render good looking text using distance fields some kind of anti-aliasing is needed.

A commonly used function when rendering distance field glyphs is the smoothstep function. The smoothstep function is a built in function in GLSL and it performs a smooth hermite interpolation between 0 and 1 in a range for an input value. The range and the value is parameters to the function. Using this function over an interval around the outline of the glyph will create a smoother edge and decrease the aliasing. This is a pretty cheap way to create descent anti-aliasing which can be modified when changing the interval. A bigger interval for example [0.4, 0.5] will create very blurred edge while a smaller interval for example [0.49, 0.51] will create a sharper edge with the cost off possible aliasing effects. The smoothstep function is not enough aliasing reduction in many cases. There are methods to improve the anti-aliasing. One way used by Gustavson [2012] is to use the glsl functions dFdx and dFdy which returns the derivatives of the distance field for the current pixel in the x and y axis. The fragment shader used in the article follows.

---

#### Algorithm 5 Anti-aliasing function proposed by Gustavson

---

```
procedure aastep(float threshold, float distance)
    float afwidth = 0.7 * length(vec2(dFdx(distance), dFdy(distance)));
    return smoothstep(threshold - afwidth, threshold + afwidth, distance);
end procedure
```

---

All three of the above described fragment shaders was implemented in this thesis. Implementing all three makes it possible to choose shader depending

on the computational power of the GPU. The first mentioned implementation with a simple threshold check is the fastest one and the last mentioned shader containing the `aastep` function is the slowest.



# 5

---

## Resultat

Det här är kapitlet där resultaten presenteras.

### 5.1 Ditten

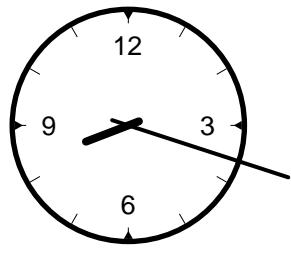
Liksom [?] har vi kommit fram till att glass smakar bäst på sommaren. Kommer  
När vi nu går in på hur glass smakar vid olika tidpunkter under dagen hän- att tänka  
visar vi till figur 5.1, och speciellt till figur 5.1b. Jämför sedan med figur 5.2 för på en  
att se hur det kan bli när man äter glass vid okontrollerade tidpunkter. liten

Veselić, Krešimir (Veselić, Krešimir) skrev en gång en artikel med titeln *Bounds anek-  
for exponentially stable semigroups.* dot...

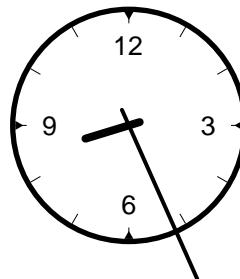
### 5.2 Framtiden

Sen när glassen är uppåten är det bara till att sätta igång och skriva på exjobbet  
igen!

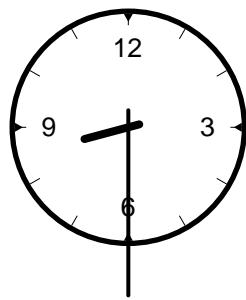
**TODO:**  
Ta bort den löjliga  
anekdoten!



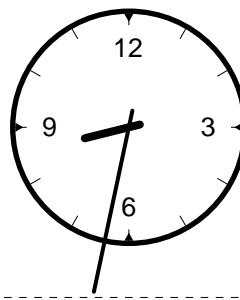
(a) Det här är väl tidigt — din glass hinner smälta innan ditt sällskap dyker upp.



(b) Kiosken stänger snart, men inte nu — perfekt!

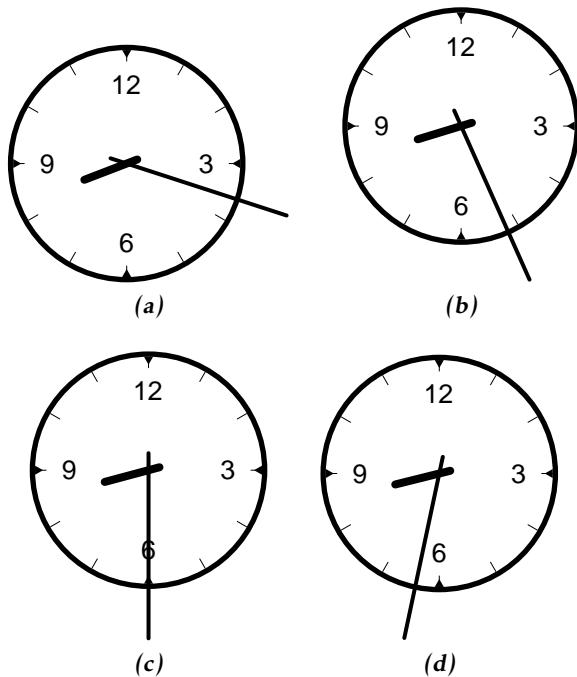


(c) Precis i tid — du får in ett finger i luckan just när kiosken ska stänga. Han som jobbar blir sur, och det blir smolk i bågaren.



(d) Du är sen — kiosken är stängd.

**Figure 5.1:** Illustration av subfloats. Den så kallade bounding boxen visas i (d). Lägg märke till att bounding boxen har satts så att alla bilder har samma storlek, med enhetlig placering av själva innehållet i förhållande till bounding boxen. Antag att du ska träffa en kompis för att äta glass just när kiosken stänger för dagen vid 08:30. När dyker du upp?



**Figure 5.2:** En andra illustration av subfloats. Den här gången har bounding boxen gjorts så liten som möjligt runt själva innehållet. Resultatet är stöksiga placeringar på sidan. Samma sak kan hända med vanliga fyrkantiga figurer när man har text som spretar ut åt lite olika håll från själva rutan med kurvor i.

---

# Appendix

## 5.A Ett par långa bevis

Det här är en appendix-del av det aktuella kapitlet.

# 6

---

## **Avslutande kommentarer**

Sätt av ett kort kapitel sist i rapporten till att avrunda och föreslå rikningar för framtida utveckling av arbetet.



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# **Index**

ARMA

    abbreviation, ix

greker, 1

PID

    abbreviation, ix



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