DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
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Impact of Power Grid Structure on Energy Storage Performance

(Final Thesis)

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ABSTRACT

With an increasing need to install variable renewable energy sources to the grid, energy storage (ES) has become a key component for electrical grids to maintain stability. Various methods have been developed to optimize siting of ES systems to benefit a certain actor in the power grid, such as consumers, system operators, or power generators, or to optimize a particular variable, such as frequency, voltage, investment profits, or operator costs. In this project, a new method has been developed to maximize the efficiency of the grid by determining optimal ES siting and to evaluate the impact of grid structure on ES performance. The method is able to decouple the impact of grid structure on grid performance from other factors to show which grid structure is best for ES performance. Inspired by a centrality concept known as net ability that determines the importance of transmission lines, the net ability equation was modified to include ES within the grid. This paper introduces a weighted topological method called ES net ability that considers capacity, impedance, power transfer distribution factor (PTDF), and node type. ES net ability was also compared to another centrality concept known as betweenness. Using a linear programming method based on DC power flow, a Matlab program was written to determine ES optimal siting for bus test systems IEEE 30, IEEE 118, and IEEE 300. The ES net ability and betweenness of each bus was calculated and compared to determine which bus has the greatest impact on energy storage performance and efficiency. The rankings of buses could be used to compare buses within a network to determine which node provides the greatest benefit to ES performance and to the grid. The average ES net ability of each IEEE test system was calculated to determine which grid structure is best suited for ES. Grid structure was further broken down into different factors, such as line impedance, line capacity, number of lines per node, number of connections per node, and node location. It was shown that decreasing line impedance and increasing line capacity increases the overall impact of grid structure on ES performance. Decreasing the average distance between nodes and increasing the number of connections leads to a higher ES performance for individual nodes. Increasing the ratio of lines per node may improve ES performance.

NOMENCLATURE

A_y	Net ability
$A(e)_y$	ES net ability matrix
B	Susceptance
B_d	Susceptance matrix
B_L	Diagonal susceptance matrix
C	Capacity matrix
C_{es}	ES capacity matrix
D	Load d
d	distance between two nodes
DF	Distribution factor matrix
E	ES node e
ES	Energy storage
G	Generator node g
G_A	Grid ES net ability
G_T	Grid betweenness
g_t	total number of nodes
i	Node i
j	Node j
l	Line l
N	Node n
N_D	Total number of load nodes
N_G	Total number of generator nodes
$PTDF$	Power transfer distribution factor matrix
P_L	Power line flow equation
p_L	Power line flow matrix
P_N	Power node flow equation
p_N	Power node flow matrix
T	Betweenness
t_c	Charge time
t_d	Discharge time
V	Voltage
X	Reactance
X	Correlation factor
Y	Admittance matrix
Z	Impedance matrix
Z_e	Equivalent impedance matrix
Z_{es}	ES impedance matrix
δ	Voltage angle
σ_{ij}	Geodesic path from node i to node j

1. INTRODUCTION

1.1 Importance of ES within Electrical Grid Networks

The world's leaders ratified the Paris Agreement in 2016 and decided that drastic climate initiatives needed to be formulated and implemented to avoid catastrophic environmental and economic consequences. Current trends indicate that the world will double carbon dioxide emissions if no actions are taken [1]. One of the main focuses of reducing carbon emissions will be in the electricity sector. As of 2015, electricity generation composed 18% of the total energy production in the world, with two-thirds coming from fossil fuels [2]. It is imperative that the electricity sector shifts generation away from fossil fuels to sustainable and efficient energy sources. Solar and wind energy will be at the center of the shift from fossil fuels to a more sustainable energy infrastructure [3], and currently are the fastest growing forms of electricity in the world [4].

The promising future of renewables comes with some caveats. In order for any electrical grid to function properly, the generation and load must be in balance [5]. Any increase in load means that the generation must be increased instantaneously, and any increase in generation requires that it be used immediately in order to maintain balance within the grid. Due to the variability and rather unpredictable nature of wind and solar, a grid composed mainly of variable renewable energy sources will need to be complemented with infrastructure to accommodate solar and wind. In the United States, it is anticipated that the western and eastern grids could handle between 30% and 35% of their electricity coming from renewables before major modifications or additions are required [6][7]. Other studies show grids typically can handle 20-30% of electricity coming from renewables, with more integration requiring larger investments in peaker plants or transmission [8]. A technique to avoid these issues is to install energy storage (ES) systems to store energy during times of high generation, and discharge the energy when renewables cannot support the demand. ES is currently the only technology that can store energy when generation exceeds demand, and discharge during periods of peak demand [5], essentially allowing generation and load to become decoupled [1].

ES has several benefits that help support the grid. These benefits include arbitrage, improved power quality and efficiency, reducing installed capacity, increasing energy access for rural communities, and the ultimate goal of reducing carbon emissions [1][5][9]. The two main benefits ES provides grids is reducing installed capacity and shifting electricity to meet peak demand. Grids that do not have ES must be sized to accommodate the largest demand faced annually, even if this demand only occurs a few times a year. In the United States, this demand problem is generally solved using inefficient peaker plants that operate 2%-7% of the year in order to meet peak demand [5]. ES will allow grids to eliminate costly peaker plants from operation while at the same time reducing overall costs and improving efficiency [5]. Another important benefit from installing ES, as mentioned previously, is that it can shift renewable electricity generation from periods of excess electricity to periods lacking enough electricity. By storing energy and discharging during peak demand, less efficient plants do not need to operate, and plants already supplying base demand do not need to inefficiently ramp up to meet demand [5][9].

This problem is currently being dealt with in California. State government incentives and policies has led to a massive increase in solar and wind energy in the state, comprising 19.6% of the installed capacity and 16.8% of electricity generation in California in 2016 [11]. Solar generation peaks during the day when demand is low, and drops significantly when demand peaks in the evening, when it is needed the most. This has led to non-renewables generating a small amount during the daytime, and having to drastically ramp up their power generation during the evening, increasing electricity costs and carbon dioxide emissions [5]. It is estimated that 20% of the CO_2 emissions removed by solar and wind will be added simply due to ramping [5]. In order to avoid the unforeseen problems of installing renewables to the grid, California has mandated the state's largest utility companies to install 1.3 GW of ES by 2020 [10][12]. As the shift from fossil fuels to solar and wind occurs, issues with integration such as increased curtailment and load ramping will become more prominent [5]. To deal with these issues, ES will be extremely valuable and will be an important system in improving grid efficiency, stability, and reliability, as well as reducing carbon emissions [1].

1.2 Current Methods to Determine Optimal Siting of ES

Since ES will be an important component of the future grid infrastructure, ES location and capacity has been the focus of extensive and ongoing research. Optimal ES siting, as it is called in current research, aims at finding the optimum location for ES within an electrical network [13]. The economic problems for electrical grids surround the transmission costs and electricity across a network. The more efficient a network is in transmitting electricity, the more power that can be delivered from power plant to customer. Various methods have been developed to determine the siting of ES by focusing on a variable of power flow, such as improving grid reliability, regulating voltage, or by minimizing operating or investment costs [13-17]. Zidar et al. provides a comprehensive review of different methods to determine the size and location and details the different factors that influence the integration of ES into the grid. There is a wide range of optimization goals for ES, and can focus on benefitting generators, customers, or transmission and distribution network services [18]. The optimization goals can include arbitrage, avoiding curtailment, voltage regulation, and minimizing investment and operating costs [18]. Most of the papers reviewed by Zidar et al. only focus on the size of ESS, with few addressing the issue of optimizing the location. The techniques used to solve the issue of ES optimization include analytical methods (AM), mathematical programming (MP), exhaustive search (ES) and heuristic methods (HM) [18]. Since the publishing of [18], there have been several transaction papers on the optimal siting and sizing of ES. Fernandez-Blanco et al. uses a linear programming method to minimize the operating and investment costs while performing arbitrage and determining the optimal size of ES [13]. Their method was applied to the Western Interconnection Western Electricity Coordinating Council (WECC) interconnection and their analysis determined that the most congested buses were the best candidates to install ESS. The congestion was typically a result of renewable energy generation or non-dispatchable generation installed near congested buses. This was a thorough analysis and method to determine optimal ES siting, and their technique can be adjusted to reflect changes in economic or governmental policies.

Many of these techniques require vast computing power and various inputs and formulas in order to determine the optimal size and siting. There has been a movement to determine the optimal site of ES by calculating the centrality of a node within a system. Centrality can be thought of as how important or vital a node is for transferring information within a network. There are numerous methods to determine the centrality of various networks, including electrical grid networks, thus the centrality of buses within a grid can be determined. Several methods have developed an unweighted method, which does not account for line impedance or capacity, to determine the centrality of the buses within a grid. Fiorini et al. and Bompard et al. state that an unweighted, or pure topological approach, is possible by considering only the connections of the nodes, but this method disregards the impedance of transmission lines, thus ignoring the physics of power flow in electrical grids [14][19][20]. This shows that a weighted graph of an electrical grid is superior to a pure topological approach [14][19][20]. Fiorini et al. discusses how centrality can be adjusted to account for and provide a better representation of power flow, but their method focuses on minimizing production costs to determine the optimal siting of ES, as opposed to optimizing the efficiency of the entire grid. Bompard et al. proposed a method to determine the vulnerability, or importance, of each transmission line in a network called net ability, and a centrality concept called betweenness [19][20][21]. These techniques account for the capacity as well as either the impedance of each transmission line, or the power distribution within the grid to produce a weighted graph of the electrical network in order to determine the importance of each transmission line [19][20][21]. They liken their novel methods of net ability and betweenness to measuring the efficiency of the grid, and can determine the importance of each transmission line by removing said line from the grid. This paper will apply betweenness to determine optimal ES site, propose a modified net ability equation based on [19], [20], and [21] to determine optimal ES siting.

1.3 Concept of Centrality and How Net Ability Can be Used for Optimal ES Siting

The concept of centrality was created by Bavelas in 1948 as a way to compare the importance of each node within a social network to investigate the different facets of it, such as efficiency, job satisfaction, and time management [22]. Since then, centrality has been used to solve a host of problems, such as the political administration of India, urban development in medieval Russian cities, and adoption of technological innovations in the steel industry [23]. A general definition of centrality is that it determines the most important or central nodes within a network. It is universally agreed upon in academia that the basic unit of centrality is a node [22], and that centrality is an essential concept in networks [23], but the similarities between different networks and the methods to evaluate centrality typically end there. Due to networks having different properties, such as different flow patterns, it has been difficult to aggregate a single equation or method to evaluate the centrality of nodes in a network. In an attempt to categorize the different types of networks, Borgatti differentiated networks by their flow path type and the method of spread [24]. He notes that off-the-shelf formulas can be applied to the problem of centrality, but it is crucial to determine the correct flow process or else the results will be incorrect [24]. The type of flow process in electrical grids as indicated by various papers have shown that electrical grid networks follow the patterns of small-world networks [19]. A small-world network is a collection of nodes where each node only has a few connections, but most nodes within the network can be reached by travelling through a small number of nodes. Latora et al. was able to further the concept of small world networks by measuring the efficiency of a network as the ability to exchange information through the network [25]. The efficiency of a network, as defined by Latora et al., is proportional to the sum of the inverse of the shortest distance for all node pairs inverse in the network. But as noted by Latora et al., the network model can be transformed from an unweighted model to a weighted model. A weighted model will be able to account for the different physical properties that influence the transfer of information within the network.

Previous models have used an unweighted method, also known as a pure topological approach, to determine the centrality or importance of buses or lines within a network. Freeman introduced a method called betweenness that determines the number of geodesic paths, or the shortest path between two nodes, that pass through each node or line [26]. The more geodesic paths that pass through a node, the more control the node has within the network and the more central or vital it is to the grid. Freeman reasons that a node with more geodesics is more important in the network because more information passes through the node, thus it has more control over the network. This is a general concept that can be applied to communication within a network, such as transportation, and biology. Since Freeman's definition of betweenness is more general and can be applied to networks in different fields of study, it needs to be modified to account for the physics of power flow within an electrical grid to determine the most vital nodes or lines within a grid. In order to apply the concept of centrality, while at the same time modeling a precise representation of the power flow in a network, a weighted model is used. Bompard et al. proposed a concept called extended betweenness that accounts for the power distribution of an electrical grid, which provides a weight to the model [19][20]. Freeman uses geodesics, which is a single path between two nodes, but electricity can flow along an inordinate number of paths between a generator and a load, especially when there are numerous nodes and lines. Bompard et al. used the capacity of the network, and the power transfer distribution factor to determine the most vital nodes and lines in a network [19]. The PTDF mathematically represents how the power is distributed throughout the grid, and will be used to show how much electricity flows through each node. The capacity of the network was also included in Bompard et al.'s definition of extended betweenness because the capacity of any transmission line in the network cannot be exceeded [19]. The extended betweenness concept and equation developed by Bompard et al. to determine the most vital components in a grid will be revised to determine the optimal ES site within a network, and then further applied to determine the effect of structure on the electrical network. No modifications will be made to the equation, and will be discussed further in the section 2.6 about the theory of betweenness. According to the equation of extended betweenness, the most optimal ES sites are the nodes with the highest power flow in the network.

A second method will be modified in this paper called net ability to determine optimal ES siting, a centrality concept proposed by Bompard et al and Arianos et al. that combines electrical distance (impedance), line flow limits (capacity), and PTDF to determine the most vulnerable transmission lines within an electrical grid [19]. This method is called net ability, and they use it to determine the most vital or important transmission lines within a network [19][20][21]. Net ability is a way to measure the overall performance of the grid, and is a distinct concept from centrality. The concept of centrality differs in that it determines the most central or vital nodes in a network, yet net ability is a measure of grid performance. By systematically removing each transmission line from the network and measuring the network's net ability after line removal, the importance of each line can be measured and ranked. In order to measure net ability, the impedance of the transmission lines and the node type are used to measure the power flow within the network, as well as to provide a weight to the model. Using a pure topological approach, the model is not weighted, but when impedance is considered, it provides a much better representation of power flow in a network. Capacity is also considered in the net ability equation because the electrical grid cannot exceed the power capacity of transmission lines. The model will be set to the transmission line that reaches rated capacity first, and once this occurs the grid will be at maximum rated power. Ultimately, capacity and impedance are the most important factors when determining the flow rate through transmission lines. Impedance will produce a weighted graph, providing a weight towards buses or lines that transmit more power, and capacity will set the maximum power transfer through the network.

Although the net ability equation was originally intended to calculate the most vulnerable transmission lines within a network, it will be applied to find the optimal bus for ES siting. Arianos et al. states the efficiency of the network is akin to the ability to send information between two nodes, and is proportional to the inverse of the distance between said nodes [21]. Bompard et al. states that the units for net ability are MW/Ω, showing that the equation's units match efficiency [19][20]. This shows that a modified net ability equation to find the optimal ES site will be improving the efficiency of the grid, as opposed to optimizing arbitrage or regulating grid voltage.

The modified net ability equation will be called ES net ability, and it is a concept based on net ability, but it is a form of centrality that is used to measure the grid performance of ES at each node. Centrality, as previously stated, is a way to measure the importance of each bus, but net ability is a way to measure the overall performance of the grid. ES net ability adds ES to each node and using the same method as net ability, measures the overall grid performance. In effect it is form of centrality, since it is measuring the performance of ES at each node, while at the same time measuring grid performance. By comparing the performance of ES at each node, one can identify the optimum site for ES. This new method is a way to measure the impact of grid structure on the performance of ES. ES net ability differs from other methods as this is a way to decouple grid structure from other factors, such as power injection and removal, and economic or financial costs.

At the same time, ES net ability will be able to measure the efficiency, or performance of ES in the grid. Enhancing grid efficiency benefits all users of the grid, whereas optimizing other aspects of the grid may only benefit certain players in the network. The theory section will discuss in detail how to calculate impedance, capacity, and how the net ability equation will be modified for calculating optimal ES siting.

2. METHODOLOGY

This section will briefly explain the theory of net ability and how to modify the net ability equation for calculating optimal ES siting. The basic equations to calculate the impedance and capacity between any generator-ES pair and ES-load pair will be introduced and explained. The DC power flow section is based on work from Van den Bergh et al. [27]. Even though the power flow in standard power grids is AC, the power flow model will be simplified down to DC to reduce computational burden.

2.1 DC Power Flow

To calculate the power flow distribution in this paper, it will be simplified down to a DC power flow. Even though this project will use AC power networks, DC power flow equations will be used to evaluate power flow in the network. Simplifying grid power flow by assuming DC power flow provides a simplified yet accurate method. For a grid with N nodes, calculating AC power flow for each node will require 2 equations with 4 unknowns (2N equations with 4N unknowns) [27]. In order to solve these equations, 2N will need to be known prior to calculation. In order to circumvent this problem, an iterative method is required to determine the remaining 2N variables. This process is highly accurate, but it has been argued that the accuracy obtained through iteration may not outweigh the computational effort [27]. Linearization of the power flow analysis is performed by assuming power flow within the grid is DC.

In order for DC power flow to be a viable option to model AC power flow, a few assumptions are made. The voltage losses through the transmission are assumed to be zero, therefore the resistance in the transmission line is also assumed to be zero.

$$Y_l = \frac{1}{Z_l} = \frac{1}{R_l + jX_l} \approx \frac{1}{jX_l} \tag{1}$$

$$G_L = \frac{R_L}{{R_L}^2 + {X_L}^2} \approx 0 \tag{2}$$

$$B_L = \frac{X_L}{{R_L}^2 + {X_L}^2} \approx \frac{1}{X_L} \tag{3}$$

$$Y_L = G_L + jB_L \approx jB_L \tag{4}$$

The admittance of each transmission line can be simplified to the inverse of the reactance. The conductance can be simplified to zero, and the susceptance is simplified to the inverse of the reactance. This assumption will be used to turn the diagonal admittance matrix into a diagonal susceptance matrix [27], as shown in equation 4.

The second assumption to be made is that all voltages are equal. This reduces all node voltages to a per unit value of 1.

$$V_i \approx 1\, p.u. \tag{5}$$

The third assumption is that the difference between voltage angles of adjacent nodes is small.

$$\sin(\delta_i - \delta_j) \approx \delta_i - \delta_j \tag{6}$$

Since it is assumed that the voltage angle between adjacent nodes is small, the sine of the difference in voltage angles is the same as the difference in voltage angles. These two assumptions will simplify the following power flow equations.

$$P_L = \frac{|V_i||V_j|}{X_L}\sin(\delta_i - \delta_j) \tag{7}$$

Equation 7 is a power flow equation for lossless transmission lines, and using the assumptions shown in equations 3, 5, and 6, equation 7 can be simplified to the following equation. Equations 7 and 8 are for one single line, but equation 9 is in matrix form for all lines of the electrical grid.

$$P_L = B_L\left(\delta_i - \delta_j\right) \tag{8}$$

$$p_L = B_d \cdot A \cdot \delta_i \tag{9}$$

Power line flow is a LN matrix $\boldsymbol{p_L}$. The power line flow matrix for consists of the susceptance matrix (LL matrix $\boldsymbol{B_d}$**)**, incidence matrix (LN matrix $\boldsymbol{A}$**)**, and the voltage angle for node i. The susceptance matrix is a diagonal matrix of line susceptance. Line susceptance is simplified to the inverse of line reactance, as shown in equation 3. The incidence matrix describes the connections within the grid. For row *l*, if a node is the "from bus" for line *l*, element A_{ln} equals 1. For the "to bus" the element equals -1. For all other elements in row *l*, they equal 0. An element with the value of 0 means the node is not attached to the line. The same process of using the above simplifications are used to simply the power node flow equation below.

$$P_N = \sum_j B_L(\delta_i - \delta_j) \tag{10}$$

$$p_N = A^T \cdot B_d \cdot A \cdot \delta_i \tag{11}$$

Power node flow is a NN matrix $\boldsymbol{p_N}$. Equations 9 and 11 will be combined in the power transfer distribution factor section to produce the equation to calculate the power transfer distribution factor (PTDF).

2.2 Power Transfer Distribution Factor

Any electrical grid will have the same basic features; generator buses, load buses, and the transmission lines connecting each bus. A grid can have multiple generators and loads, and to discern the contribution or effect of power flow from any generator or load can be difficult to ascertain due to the multiple generators, loads, and possible paths the electricity can take within the grid. Fortunately, the contribution of each transmission line in relation to the energy removal or injection by each bus can be computed, known as the power transfer distribution factor (PTDF). For any injection or removal of energy within the grid by any generator-load pair, the PTDF in effect is able to mathematically represent the power flow distribution within the network. The following equation combines equations 9 and 11 to form the PTDF matrix.

$$PTDF = (B_d \cdot A) \cdot (A^T \cdot B_d \cdot A)^{-1} \tag{12}$$

The PTDF matrix is a LN matrix, and shows the power change within any transmission line for any injection or removal of power at any node. Equation 11 is not linearly dependent, which means the inverse of the matrix cannot be calculated. Therefore, one column has to be removed, referred to as the reference node. Generally, in power flow analysis, the slack bus is designated as the reference node. For matrix $\boldsymbol{p_L}$, the slack bus column is removed, forming a LN-1 (line x node – 1) matrix. In order to multiply the node flow ($\boldsymbol{p_N}$) and line flow ($\boldsymbol{p_L}$) matrices, the slack bus column and row in the $\boldsymbol{p_N}$ matrix also has to be removed. Once the PTDF matrix has been calculated, it will be a LN-1 matrix. The slack bus will be added to the PTDF matrix as a column of zeros. This process of calculating the PTDF can be more visually apparent and will be more clearly shown in figure 5 in the method section.

2.3 Capacity

The elements in the PTDF matrix can also be used to calculate a matrix called the distribution factor (DF), composed of elements a_l^{gd} which represent the power change in transmission line l due to power injection at generator g and power removal at load d. Since each element in the PTDF matrix shows the change in power within a line due to a power change at any node, the difference in power change in a line due to two nodes can be calculated. This means that the contribution of power in a line due to any generator- load pair can be calculated. This concept is illustrated in equations 13 and 14.

$$a_l^{gd} = a_{lg} - a_{ld} \tag{13}$$

$$a_l^{ij} = a_{li} - a_{lj} \tag{14}$$

According to Bompard et al, the DF matrix is a GDL (generator x load x line) matrix, but this paper will use equation 14 to calculate the DF matrix, which will create an NNL (node x node x line) matrix. Bompard et al.'s DF matrix describes the power change in each line for power removal and injection for each generator-load pair, but this paper's DF matrix is performed for each node-node pair. The new DF matrix can still evaluate any generator-load pair, but there will be situations where a generator-generator pair or load-load pair will need to be evaluated, therefore the DF matrix will need to be adjusted and expanded. This change to use NNL dimensions for the DF matrix is a necessary adjustment for calculating net ability, and will be further explained in the net ability section. With the DF matrix calculated, the capacity of the grid for each node-node pair can be calculated.

The capacity or line flow limit is an important consideration and vital to the stability of the network. For each node-node pair, the capacity of each line is divided by the DF for said line, and the line with the smallest value will be the first line to reach capacity. In other words, in a power grid with an ever-increasing amount of energy, the line to reach capacity first will determine the capacity of the entire network, since capacity for any line cannot be exceeded. For each node-node pair, this line will dictate the capacity of the network. The capacity matrix is a NN matrix **C**, and the expression to calculate capacity for each node-node pair is shown below.

$$C_i^j = MIN(\frac{P_1^{max}}{\|a_1^{ij}\|}, \cdots, \frac{P_l^{max}}{\|a_l^{ij}\|}, \cdots, \frac{P_L^{max}}{\|a_L^{ij}\|}) \tag{15}$$

In equation 15, l represents any line l, and L represents the line with the highest serial number. As mentioned above and shown in equation 15, the capacity for each node-node pair is based on the line to reach its line flow line first.

2.4 Equivalent Impedance

The impedance used by Bompard et al. to calculate net ability is called equivalent impedance, and is calculated from the impedance matrix. The equivalent impedance will be used as the weight to turn the net ability model from a pure topological model to a weighted topological model. To calculate the equivalent impedance, the admittance matrix is formed from only the impedance, and the resistance is assumed to be zero. The simplification for calculating the admittance is shown below.

$$Y_l = \frac{1}{Z_l} = \frac{1}{R_l + jX_l} \approx \frac{1}{jX_l} \quad (16)$$

In high voltage transmission lines, the resistance does not have a significant impact on the voltage drop, therefore the resistance is ignored. The reactance is considered the only factor in the voltage drop through a transmission line, therefore the admittance for each line can be simplified to the inverse of its reactance. The admittance is a NN (node x node) matrix **Y**, and the following expressions summarize the admittance matrix.

$$Y_{ij} = -\frac{1}{X_l^{ij}} \; if\ i\ and\ j\ are\ connected \quad (17)$$

$$Y_{ij} = 0 \; if\ i\ and\ j\ are\ not\ connected \quad (18)$$

$$Y_{ii} = \frac{1}{X_l^{i1}} + \cdots + \frac{1}{X_l^{iN}} = Y_{i1} + \cdots + Y_{iN} \quad (19)$$

To calculate the impedance matrix of an electrical grid, the inverse of the admittance matrix is performed, as shown below.

$$Z = Y^{-1} \quad (20)$$

The impedance is a NN matrix **Z**, but the goal is to find the equivalent impedance. Bompard et al states that the equivalent impedance is equal to electrical distance within a network. This is the weighted portion of the model. Equivalent impedance is a NN matrix $\mathbf{Z_e}$, and the equation to calculate the equivalent impedance is shown below.

$$Z_e^{ij} = \left(Z_{ii} - Z_{ij}\right) - \left(Z_{ij} - Z_{jj}\right) = Z_{ii} - 2Z_{ij} + Z_{jj} \quad (21)$$

2.5 ES Net Ability

The original net ability equation developed by Bompard et al. and Arianos et al. was created to find the most vulnerable transmission lines in a network, but this paper will modify the net ability equation to find the optimal site for ES. Net ability, as explained by Bompard et al., is a way to measure its ability to function under normal operating conditions, and is influenced by the impedance of the transmission lines, the power distribution within the network, and the power flow limits of the transmission lines. According to Bompard et al., net ability is a method to measure the efficiency of the network, thus net ability may provide significant implications for optimal ES siting. Below is the net ability equation created by Bompard et al. and Arianos et al.

$$A_Y = \frac{1}{N_G N_D} \sum_{g \in G} \sum_{d(d \neq g) \in D} C_{dg} \frac{1}{Z_{dg}} \tag{22}$$

Equation 22 calculates the net ability for an entire network by calculating the capacity and equivalent impedance for each generator-load pair, and then summing the results together to for the result, but this paper will need to account for the additional ES bus in the network. The reasoning behind the specific modifications for net ability in relation to optimal ES siting are summarized below.

As mentioned previously, an electrical network has multiple paths that electricity can take from generator to load, and the specific power flow distribution can be mathematically calculated through the PTDF. The PTDF can be further broken down to show the contribution of power flow distribution due to each generator-load pair in a network, therefore between each generator-load pair there must be a specific equivalent impedance and capacity that dictates the power flow distribution between said pair. Figure 1 below is a representation of how the grid between each generator-load pair can be simplified and represented as a line with a specific and quantifiable equivalent impedance and capacity.

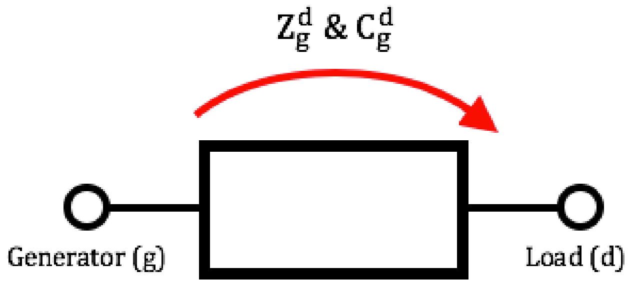

Figure 1: Equivalent impedance and capacity of typical grid from generator to load

The process to calculate the equivalent impedance and capacity for each generator-load pair was summarized above, and these results are used to calculate the net ability contribution for each pair. The issue now is there is no current method to calculate the net ability with ES attached to a bus within the network. Adding ES to the grid will change the characteristics of the grid, and the equivalent impedance and capacity shown in figure 1 cannot be used. For grids with ES, the capacity and equivalent impedance of each generator-ES pair and ES-load pair will be applied, as opposed to evaluating the capacity and equivalent impedance for each generator-load pair. Figure 2, shown below, shows the equivalent impedance and capacity of the grid with the addition of ES to the grid.

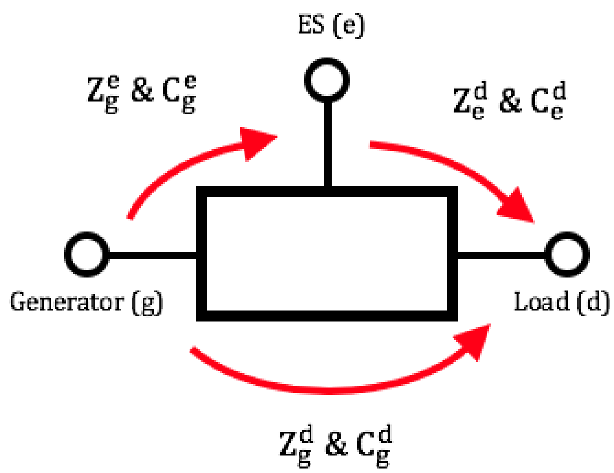

Figure 2: Equivalent impedance and capacity of typical grid from generator to ES to load

The grid will of course have power flow directly from the generator to the load, but for this project, ES net ability will only compare the added efficiency from the placement of ES at each bus. The purpose of this paper is to compare the centrality of each bus and only evaluate the efficiency of the power flow from generator to ES to load, hence it is not necessary to include the power that flows directly from generator to load. Since this power flow can be ignored, the equivalent impedance and capacity of each generator-ES pair and ES-load pair is required for evaluating the optimal ES site. It should be noted that an ES bus may also have either a generator or load connected to the same bus. This means that since ES may be connected to a generator or load bus, the capacity for a generator-ES pair (C_g^e) or ES-pair (C_e^d) can be represented as C_{g1}^{g2} or C_{d1}^{d2}, respectively. Any time ES is connected to a generator, the distribution factor between generator and ES will be equivalent to C_{g1}^{g2}, since ES is connected to generator 2 (*g2*). The same is true if ES is connected to a load bus, the distribution factor between ES and load will be C_{d1}^{d2}, since ES is connected to load 1 (*d1*). This will cause problems when calculating the capacity of each generator-ES or ES-load pair, since the DF matrix is a GDL matrix. The DF calculated by Bompard et al is a GDL matrix, which means that any generator-generator or load-load capacity is not included in the matrix. This is why the calculation for DF was expanded to an NNL matrix to include every node-node pair. This does not mean that every element in the DF matrix will be used, since there are transmission buses within the NNL matrix. For example, a transmission-transmission pair will exist in the DF matrix, but it will never be used. Each time the ES net ability is calculated, the capacity for each generator-ES pair and ES-load pair will be used, and the capacity for each transmission-ES pair and ES-transmission pair will be skipped, even though these values are in the DF matrix. The Matlab code will account for this, and can be seen in the appendix section. Figure 3 shows the concept of ES net ability to show that power flow directly from generator to load will not be considered.

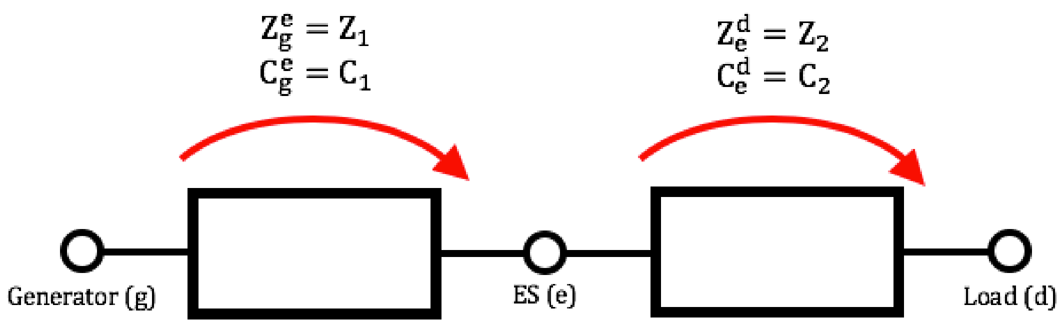

Figure 3: Equivalent impedance and capacity from generator to ES to load with flow directly from generator to load to be ignored.

For each calculation for the contribution of net ability by each generator-load pair, there will be two values of equivalent impedance and capacity each, as opposed to one for the original net ability equation. The problem arises as to how to combine the equivalent impedance and capacity for a generator-ES pair and ES-load pair. The impedance for two elements in series can simply be added together, as shown below. The power flows first from generator to ES, then from ES to load, as shown in figure 3. Since this is the case, it can be considered as two systems in series, and can be added together, as shown in equation 23. This new matrix will be called ES equivalent impedance.

$$Z_{es}^{ged} = Z_1 + Z_2 = Z_g^e + Z_e^d \tag{23}$$

Equivalent impedance, as mentioned above, is a NN matrix, which calculates the equivalent impedance for every node-node pair in the grid. The ES equivalent impedance is a GDN (generator x load x node) matrix $\boldsymbol{Z_{es}}$.

To evaluate the ES capacity between any generator-ES pair and ES-load pair, the capacity cannot be added like the equivalent impedance. An analogy between the capacity of a line or electrical network is equal to power, which can be broken down into energy divided by time. The capacity of a transmission line is equal to the power limit of the line, which can be measured in kW. The power limit, or capacity of the transmission line can be broken down into the energy delivered through the line divided by time. When only considering the capacity of a generator-ES-load combination, the summation of the time to charge the battery and the time to discharge the battery will be equal to the time for the electricity to travel from generator to ES, then from ES to load. The charge and discharge time of a battery or ES can be thought of as the energy entering or exiting the battery divided by the capacity of the battery. This can be more clearly shown in equation 24, shown below.

$$C = P = \frac{E}{t} = \frac{E}{t_c + t_d} = \frac{E}{\frac{E}{C_1} + \frac{E}{C_2}} \tag{24}$$

Since ES will be added to the network, time can be thought of as the amount of time it takes energy to charge as well as discharge. In equation 24, t_c is charging time and t_d is discharging time. The total time (t) can be thought of as the time it takes for the battery to charge (t_c) and discharge (t_d). The charge and discharge time can be further broken into energy divided by capacity, where charge capacity (C_1) will be the generator-ES capacity, and discharge capacity (C_2) will be ES-load capacity. Equation 25 below shows the simplification of the result from equation 24 to obtain the capacity for any generator-ES-load combination.

$$C_{es}^{ged} = \frac{E}{\frac{E}{C_1}+\frac{E}{C_2}} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{C_1C_2}{C_1+C_2} \tag{25}$$

The ES capacity is a GDN matrix $\boldsymbol{C_{es}}$, the same dimensions as ES equivalent impedance. With the capacity and equivalent impedance adjusted and calculated for the addition of ES, the following equation shows the net ability.

$$A(e)_Y = \frac{1}{N_G N_D} \sum_{g \in G} \sum_{d(d \neq g) \in D} C_{es}^{ged} \frac{1}{Z_{es}^{ged}} \tag{26}$$

The ES net ability equation for calculating optimal ES siting will calculate the net ability for ES at each node e. These results can then be used to compare the efficiency of ES at each node to determine the optimal site for ES.

2.6 Betweenness

Just like the net ability equation, the betweenness equation was originally created by Bompard et al. to determine the most crucial transmission lines in an electrical network, but will be modified to determine the optimal ES site in an electrical grid. The concept of betweenness centrality was originally developed by Freeman and is based on the shortest path between two nodes. The shortest path between a pair of nodes is known as a geodesic path. If a node is located on a geodesic, then it is considered vital to the transmission of information between a pair of nodes. The more geodesics that pass through a node, then the more central the node is in the network. Freeman developed an equation to measure the betweenness centrality of a node, and is shown below.

$$g(n) = \frac{\sigma_{ij}(n)}{\sigma_{ij}} \tag{27}$$

The above equation calculates the geodesics that pass-through node n and determine the likelihood node n is along line σ_{ij}. Another way of defining Freeman's definition of betweenness of a node is to say the probability of a geodesic passing through a node.

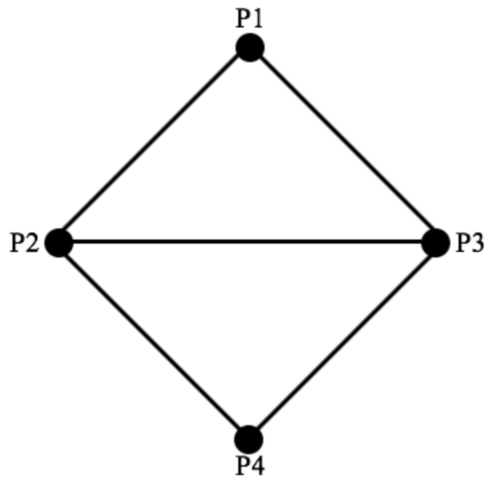

Figure 4: Diagram to explain betweenness

In figure 4, assuming each line has an equal distance, points P1 and P4 has two geodesic paths. Points P2 and P3 are located along one of the geodesics. According to equation 27 and since there are a total of two geodesics, points P2 and P3 have the same betweenness value of 0.5. From an intuitive sense, P2 and P3 are vital in the communication between points P1 and P4. They also have the same value of betweenness because the distance between each node is equal. The value of the distance between two nodes needs to be reconsidered since it does not reflect the electrical distance. The electrical distance can be applied in a way to model the flow of electricity which provides a weight to the model. Freeman's definition of betweenness does not consider any of the electrical properties of the grid, therefore betweenness needs to be adjusted to account for the physical characteristics of the electrical grid.

Bompard et al. proposed a new betweenness centrality to determine the most crucial nodes and buses within an electrical network [19][20]. The main difference between the pure topological approach introduced by Freeman and weighted betweenness centrality proposed by Bompard et al. is the geodesic paths. From a topological perspective, there is a distinct number of geodesics between two points that can be easily measured. Said in another way, there can only be a fixed number of routes between two nodes. For a weighted betweenness, this method must consider the flow of electricity in a network. There are an inordinate number of paths that electricity can flow from a generator to load. In order to model this flow of electricity, the PTDF was used to weight the model because it mathematically represents the flow distribution of electricity in an electrical grid. As mentioned earlier, the PTDF can be modified to show the flow in any line due to flow of electricity between any generator-load pair, called the distribution factor (DF). An element of a PTDF matrix represents the change in power flow in a line due to the injection or removal of power at any node. The difference between any two elements in the PTDF matrix will provide the flow along the corresponding line due to the power flow between any generator-load pair. The DF will be used because the PTDF only shows the change in power due to a unit change in power for a node, while DF shows the power flow distribution in a line for a generator-load pair. To further explain extended betweenness, Bompard et al.'s equation is shown below in equation 28. The betweenness equation that will be used for this project will be modified, and is shown in equation 29.

$$T = \frac{1}{2} \sum_{g \in G} \sum_{d \in D} C_g^d \sum_{l \in L} \left| a_l^{gd} \right| \qquad (28)$$

The last term of equation 28 is the sum of the DFs of all the lines connected to bus v when power is added [19][20]. The summation is equivalent to the power in and power out for each node, just like summing the currents for a node using Kirchhoff's current law. Adding the total power flowing into and out of the node will be double the electricity flowing through the node, therefore dividing the summation by two gives the electricity that flows through the node. The capacity for each generator-load pair is multiplied into the equation to account for the fact that the network cannot exceed its own capacity. The factor of the summation of DF for each node and the capacity of the respective generator-load pair is summed together to produce the betweenness of each node.

The betweenness equation will be modified in order to compare the grid betweenness of different grids. Grid betweenness is the average betweenness of every node in a network, and will be explained in further detail in section 3.4. Just as in the ES net ability equation, the betweenness equation will account for the number of generators and loads in the network by dividing by the product of the number of generators and loads in the network. The betweenness equation to be used is shown below in equation 29.

$$T = \frac{1}{2N_G N_D} \sum_{g \in G} \sum_{d \in D} C_g^d \sum_{l \in L} \left| a_l^{gd} \right| \tag{29}$$

3. CASE STUDY

The ES net ability and betweenness of IEEE 30, 118, and 300 bus systems will be calculated by writing a code in Matlab. IEEE bus system test data was provided by the University of Washington [28], and the data files include data about each IEEE bus system, including but not limited to bus type, line resistance and reactance, and to and from bus for each line. The IEEE bus system data has various matrices, but only two will be used, the bus matrix and branch matrix. Table 1 below shows the information required by the IEEE bus system test data to input into the Matlab program.

Table 1: Information to input into Matlab code

Information		Purpose
Branch Matrix	Line Reactance	Calculate line impedance
	To and From Bus	Calculate equivalent impedance and capacity Determine parallel lines
	Number of Lines	Determine size of for loops
Bus Matrix	Bus Number	Calculate and compare net ability for each bus. Save list of original bus numbers (Bus numbers for IEEE 300 are not continuous. A list of continuous bus numbers is used to run the Matlab program. Original bus numbers will replace continuous bus numbers at end of code)
	Number of Buses	Determine size of for loops
	Bus Type	Determine slack bus Determine generator buses Determine load and transmission buses
	Real and Reactive Power	The only way to tell the difference between transmission and load nodes is by the real and reactive power. If there is no power, then it is a transmission node. If there is power, then it is a load node.

For the Matlab program to evaluate the net ability for other grids, it is assumed that the above information is available, and in the same format as in the IEEE bus system data files provided by the University of Washington. For bus systems IEEE 30 and 118, the bus numbers are continuous, meaning the bus numbering system does not skip numbers. The IEEE 300 system skips numbers; therefore, this must be considered when designing the code to calculate net ability.

The bus data files that will be used to calculate the ES net ability for each grid network has several matrices, but for this paper, only two were needed to run the Matlab code. The two matrices are called bus and branch, and below summarizes the contents of each matrix and its usefulness to calculating ES net ability.

1. **Bus matrix.** The two columns used in this array for the Matlab code are the bus number and bus type. Using a numerical method, the bus type array describes whether the bus is a slack bus, generator bus or load bus.
2. **Branch matrix.** This matrix describes the from bus, to bus, and reactance of each line in the grid.

Prior to the formation or calculation of any matrices, the data provided by the bus and branch matrices must be adjusted or formatted in a way that will be used to calculate the initial matrices of the code. Below is a list providing these details.

1. **Convert reactance of parallel lines to equivalent reactance.** Certain grid systems have parallel lines, where two or more lines are connected to the same two nodes. The equivalent reactance for each set of parallel lines will be calculated. Since a new equivalent transmission line will represent and replace each set of parallel lines, one fewer line will exist in the grid, therefore one row in the branch matrix will be deleted. For this paper, it will be assumed that there can be no more than two parallel lines. If there is a set of three or more parallel lines, the code will not calculate the correct equivalent reactance.
2. **Save original bus numbers and create array of continuous bus numbers.** The bus numbers in certain grids are not continuous. The Matlab code assumes that the bus numbers are continuous, which means the bus numbers need to be adjusted. An array of the original bus numbers is saved in the code, and a new array of continuous numbers running from 1 to N, where N represents the total number of nodes or buses in the grid, will replace the old bus numbers. The bus numbers in the from bus and to

bus columns in the branch matrix are also replaced with the corresponding new bus numbers. After the ES net ability has been calculated, the bus numbers are replaced with the original bus numbers.

3. **Calculate line impedance array.** The branch matrix has the reactance of each line, which is used to calculate line susceptance and impedance. As noted in the theory section, using the assumption that line resistance is zero, the line susceptance and impedance is the inverse of line reactance. This array will be used to calculate the admittance matrix ($\boldsymbol{Y}$) and the diagonal line susceptance matrix ($\boldsymbol{B_d}$).

The rest of the method section will outline the process of calculating the ES net ability and betweenness through the use of flowcharts, and a brief explanation of the process is included. A more detailed explanation of the calculation for each matrix is included in the appendix, including the Matlab code used to create each matrix.

3.1 Power Transfer Distribution Factor

Calculating the PTDF will be based on the equations 9 and 11 to calculate the power line flow and power node flow matrices from the susceptance and incidence matrices. The two power flow matrices will be used to calculate the PTDF. The process can be seen clearly in figure 5.

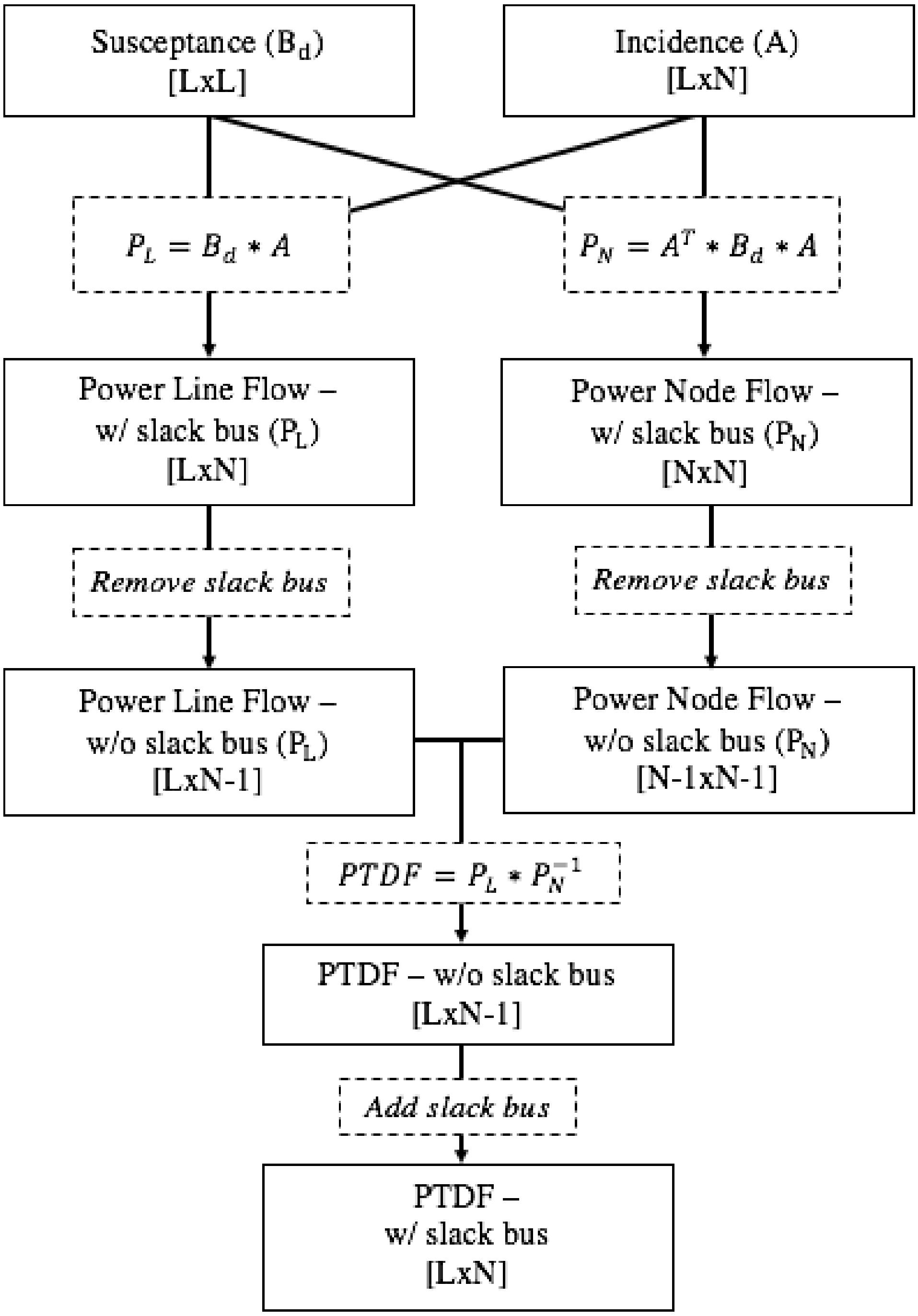

Figure 5: Flowchart to calculate PTDF

The calculation of the PTDF begins with the formation of the diagonal susceptance matrix, and the incidence matrix. The susceptance matrix is a diagonal LL matrix representing the susceptance of each line. As mentioned in the theory section, the line susceptance has been simplified to equal the inverse of each line. The incidence matrix is a LN matrix that represents the connections between nodes and lines in a grid. These two matrices are used to calculate the power line flow and power node flow matrices. Since the power line flow matrix is linearly dependent, the inverse of the matrix does not exist. The slack bus column in the power line flow matrix is removed, and the slack bus column and row in the power node flow matrix is also removed. After the PTDF has been calculated, the slack bus column is added to the PTDF as a column of zeros. Now that the PTDF has been calculated, the ES net ability can be calculated. The PTDF result will be used to continue the calculation for ES net ability, and is outlined in figure 5.

3.2 ES Net Ability

The calculations for the ES net ability are summarized in the theory section, and the equations used are equations 14, 15, 20, 21, 23, 25, and 26. The ES net ability calculation begins with the calculation for PTDF, shown in figure 5, and the admittance matrix. Figure 6 below shows the process to calculate ES net ability.

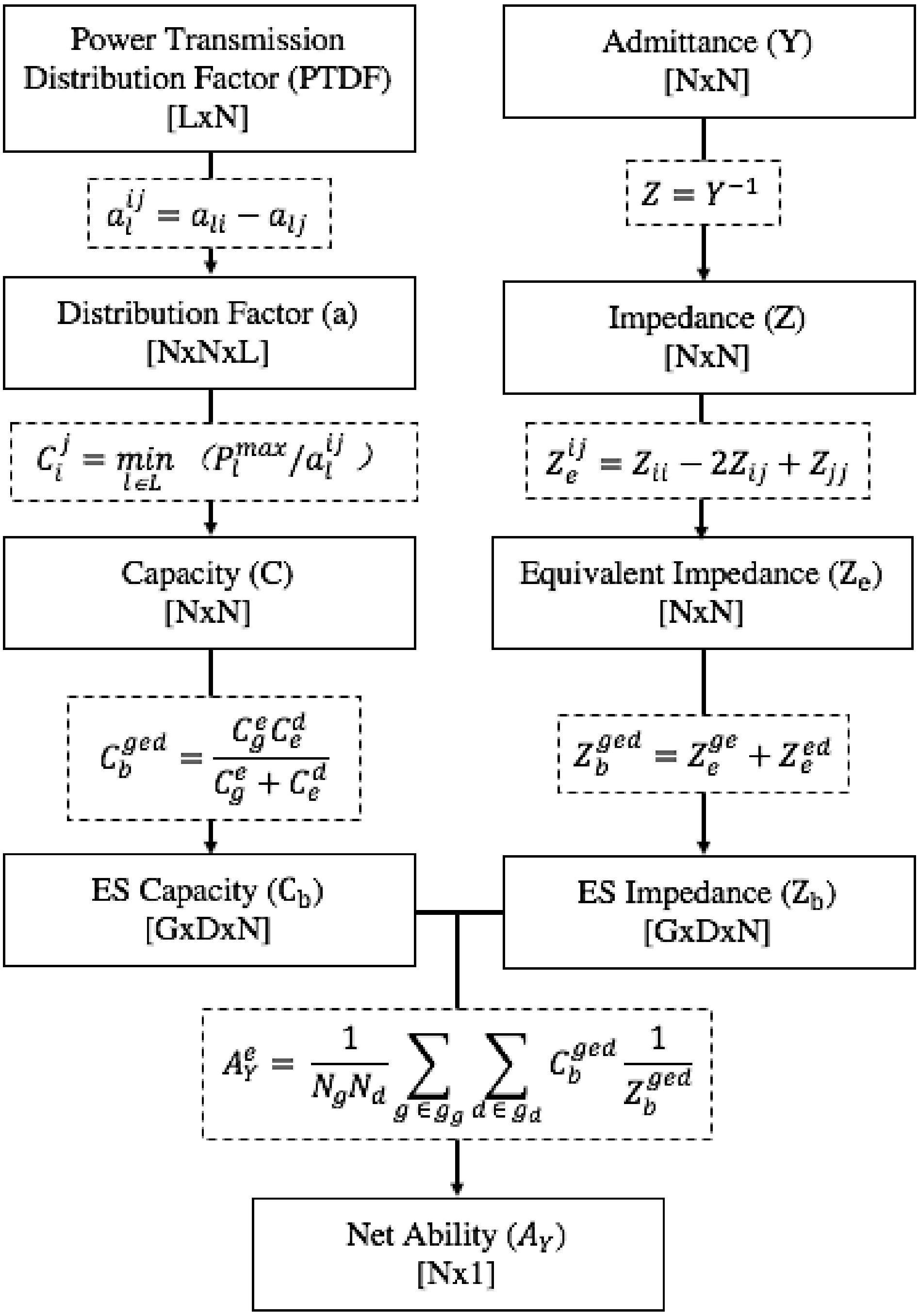

Figure 6: Flowchart to calculate ES net ability

The calculation for ES net ability begins with the PTDF and admittance matrices. After formation for the PTDF, which is a LN matrix, it is used to calculate the distribution factor. By definition, the PTDF represents the change in power in a line due to the injection or removal of power at any node. The difference of the contribution of power between two nodes on the same line can be calculated, which is the distribution factor (DF). The DF is an NNL matrix $\boldsymbol{a}$. Each element represents the change in power as a result of power injection and equal power removal between any two nodes. The distribution factor has to be an NNL matrix, and not a GDL matrix because a matrix of size GDL cannot be used to calculate ES capacity.

The capacity between each node-node pair is calculated by calculating the line with the minimum capacity, and creates a NN matrix $\boldsymbol{C}$. The max power capacity for each line is assumed to be equal, and is set to 100 MW. Once the capacity between each node-node pair is known, the ES capacity for every generator-node-load pair is calculated, which will be used to calculate the ES net ability. The ES capacity is a GDN matrix $\boldsymbol{C_b}$.

The other side of the ES net ability calculation begins with the admittance matrix, and each element is based on the inverse of the reactance. The admittance matrix is an NN matrix $\boldsymbol{Y}$, and the formation of this matrix is explained in greater detail in the appendix. The impedance matrix is calculated from the inverse of the admittance matrix, and is an NN matrix $\mathbf{Z}$. To calculate the equivalent impedance, the voltage difference between two nodes is equal to the equation in figure 6, and is a NN matrix $\boldsymbol{Z_e}$. The ES impedance for any generator-node-load combination can be calculated by adding the generator-node and node-load impedances together since the impedances are in series. The ES impedance is a GDN matrix $\boldsymbol{Z_b}$.

The ES impedance and ES capacity matrices are of the same dimensions. The net ability equation has been modified to calculate the ES net ability, and can be calculated according to the equation in figure 6.

3.3 Betweenness

The equations used to calculate betweenness include equations 14, 15 and 28. he calculation for betweenness begins with the PTDF matrix. The calculation for the PTDF is shown in figure 5. Figure 7 shows the flowchart to calculate betweenness.

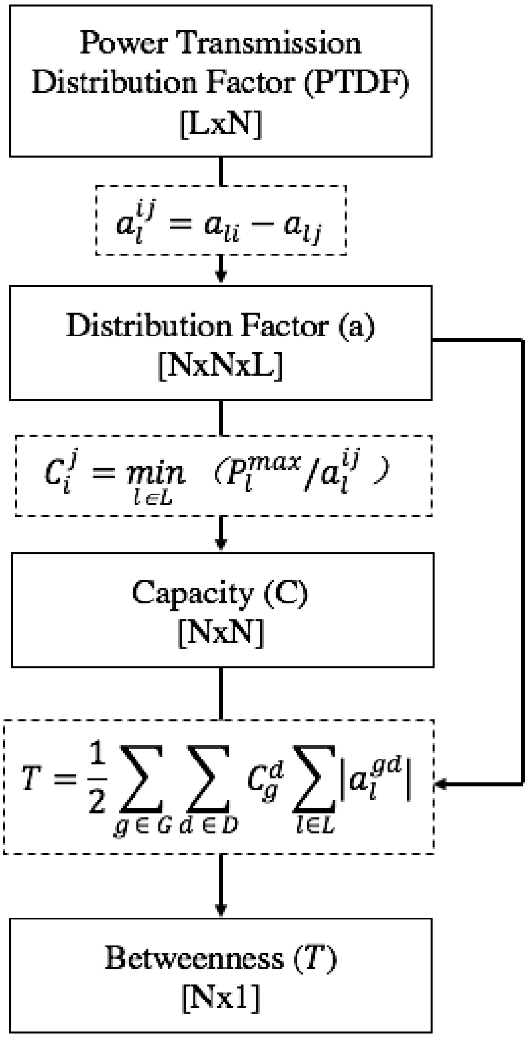

Figure 7: Flowchart to calculate betweenness

Just like the calculation for ES net ability, betweenness begins with the PTDF matrix, but does not need the admittance matrix. An explanation to calculate the distribution factor (DF) and the capacity can be found in sections 2.3 and 3.2.

The DF matrix and capacity matrix are NNL and NN matrices, respectively. This means that matrices include every node-node pair in the matrix, as opposed to only having every generator-load pair. The equation for betweenness only needs generator-load pairs because electricity will only flow from generators to loads, and not from node to node. One might consider that since betweenness is calculating the optimal ES site, thus electricity will be flowing from generator to ES and then from ES to load that betweenness should consider every generator-ES-load combination, but that is not the nature of the betweenness equation. In order for betweenness to calculate the electricity flow through each node without using the capacity or DF for every node-node is that the DF consider the electricity flow through a line for a certain generator-load pair. The flow in the line must flow through the two nodes that are connected to the line, thus giving the betweenness equation the knowledge of the electricity flow through every node for every generator-load combination.

3.4 Grid Structure Analysis

To determine the overall grid structure's effect on ES performance, the ES net ability and betweenness of each node in a grid will be averaged to give the average ES net ability and average betweenness for each grid. The average ES net ability and betweenness will be referred to as grid ES net ability and grid betweenness. The grid ES net ability and grid betweenness will be compared to determine which grid has a superior grid structure for not only installing ES, but for the overall performance of the grid. This will provide an indication as to the structure's effect on the performance of electrical grids. The equations for grid betweenness and grid ES net ability are shown below.

$$G_A = \frac{1}{n_t} \sum_{n \in N} A_Y \tag{30}$$

$$G_T = \frac{1}{n_t} \sum_{n \in N} T \tag{31}$$

To calculate grid ES net ability, sum the ES net ability of every node in the network, and divide the total by the number of nodes in the system. The same method is used for grid betweenness, sum the betweenness of every node and divide by the total number of nodes in the system. The grid betweenness and grid ES net ability will be calculated and compared for IEEE bus systems 14, 30, 33, 57, 118, and 300.

Grid structure will be broken down further to determine which factors influence ES net ability and betweenness. Listed below are the three factors to be tested.

- Line Impedance
- Line Capacity
- Node Location

Line impedance and line capacity will be compared to grid ES net ability and grid betweenness. Since impedance and capacity are characteristics of lines, it is difficult to determine its effect on different nodes, therefore impedance and capacity will be used to evaluate its effect on the entire grid, instead of individual nodes. In contrast, location will be compared to the ES net ability and betweenness of each node in a system. Since location is a characteristic of a node, the location of a node can be compared to the ES net ability and betweenness of said node.

The line impedances in every grid are not the same. In order to determine its effect on grid ES net ability and grid betweenness, the line impedances will all be set to the same value, and the grid ES net ability and grid betweenness of a bus system will be measured. Grid ES net ability and grid betweenness will then be measured for different values of impedance. Each time the grid ES net ability and grid betweenness are measured, the line impedances in the grid have the same value. This experiment will determine the effect of impedance on ES net ability and betweenness. The same experiment will be used to evaluate the effect of capacity on grid ES net ability and grid betweenness. The line capacities will be set to the same value, and the grid ES net ability and grid betweenness will be measured for different capacities. Line capacity and impedance will be plotted against grid ES net ability and grid betweenness to determine its effect on ES performance.

To measure the effect of location on a node, one needs to quantify its location. A nodes location will be defined as its position relative to the other nodes in the network. The closer a node is to another node, the higher its value, and the further away a node is from another node, the lower its value. One branch or line between any two nodes will be set to a value of one unit, which is the basic unit of distance for this experiment. The distance between any two nodes will be defined as the shortest path between said nodes. To explain this concept further, figure 8 will be used.

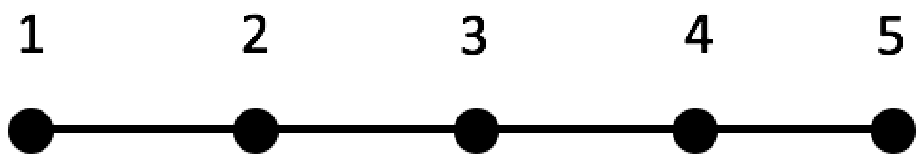

Figure 8: 5-node system to explain definition of distance to calculate location

The distance between node 1 and node 2 is 1 unit, since the shortest path is one line. The distance between nodes 1 and 5 is 4 units, since the shortest path between 5 and 1 consists of 4 lines. The location of a node will be a function of the sum of the distances between the other nodes of the system. Below are the equations to be used to determine location of a node in a network, where d is distance between two nodes, and x is an exponent that will be calculated that gives the highest correlation between location and ES net ability and betweenness. To simply the process, x will only be an integer.

$$L(d) = \sum_{n \in N} d^x \tag{32}$$

It is also unknown what value of x will produce a higher correlation, therefore multiple values of x will be tested. The location of nodes in a grid will be plotted against their ES net ability and betweenness, and the value of x that produces the highest coefficient of determination will be the optimum value.

4. RESULTS

Summarized below are the results for the IEEE 30, 118, and 300 bus systems test cases. For each IEEE bus system, there is a list of the 10 buses with the highest and lowest ES net ability, a net ability vs. connections graph detailing the positive relationship between the number of connections a bus has and its net ability, and a diagram of each bus system indicating the bus with the highest ES net ability. For the net ability vs. connections graph, each parallel set of lines is treated as one connection, since the set will be simplified down to a single equivalent line. A regression line is included in the graph, and the coefficient of determination (R^2) is shown in the legend. The IEEE bus systems diagrams were provided by the Russia Department of Energy [29].

4.1 IEEE 30 Results

Table 2: IEEE 30 bus system ES net ability results

IEEE 30 Bus System – ES Net Ability			
Top 10 Buses		Bottom 10 Buses	
Bus Number	ES Net Ability (MW/Ω)	Bus Number	ES Net Ability (MW/Ω)
6	243.3	23	99.8
4	218.5	19	99.4
12	175.6	18	98.8
28	165.7	27	91.4
3	159.9	25	88.4
2	153.0	13	64.5
10	150.7	30	45.3
22	145.5	29	44.8
15	141.5	11	40.8
7	136.2	26	29.9

Table 3: IEEE 30 bus system betweenness results

IEEE 30 Bus System - Betweenness			
Top 10 Buses		Bottom 10 Buses	
Bus Number	Betweenness	Bus Number	Betweenness
6	66.7	19	16.8
4	66.5	23	14.8
12	65.9	21	14.6
10	47.8	8	13.2
2	42.2	14	10.6
15	36.1	13	8.3
9	26.1	11	8.3
27	25.5	29	6.5
24	25.2	30	5.7
7	23.1	26	2.8

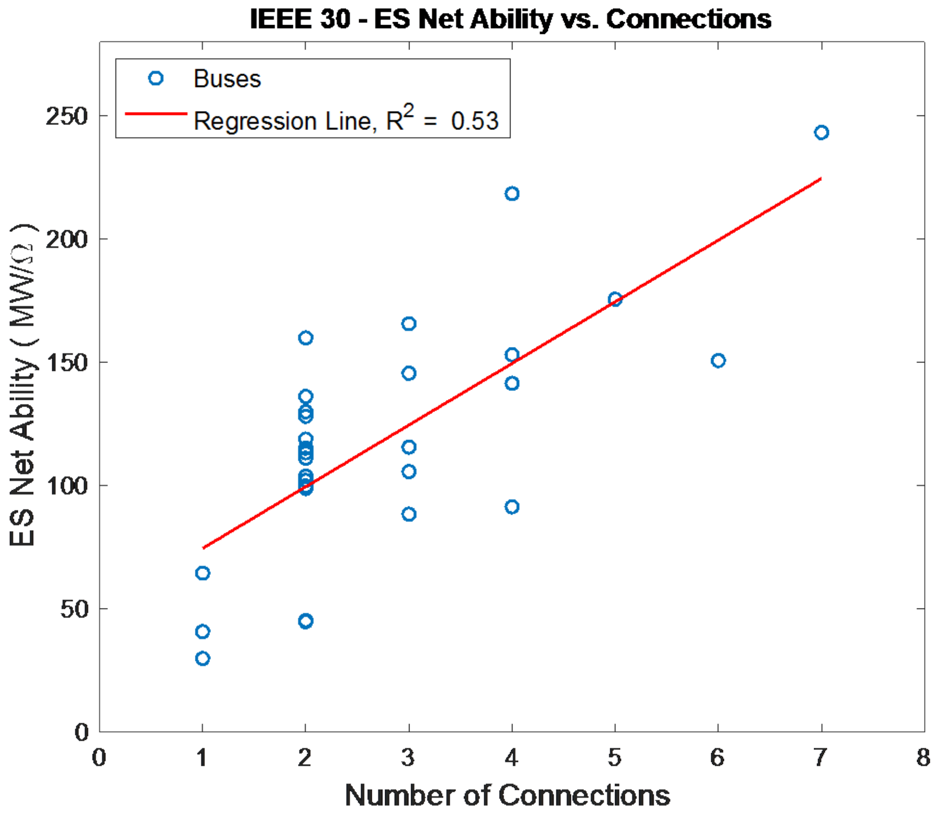

Figure 9: IEEE 30 – Net ability vs. connections

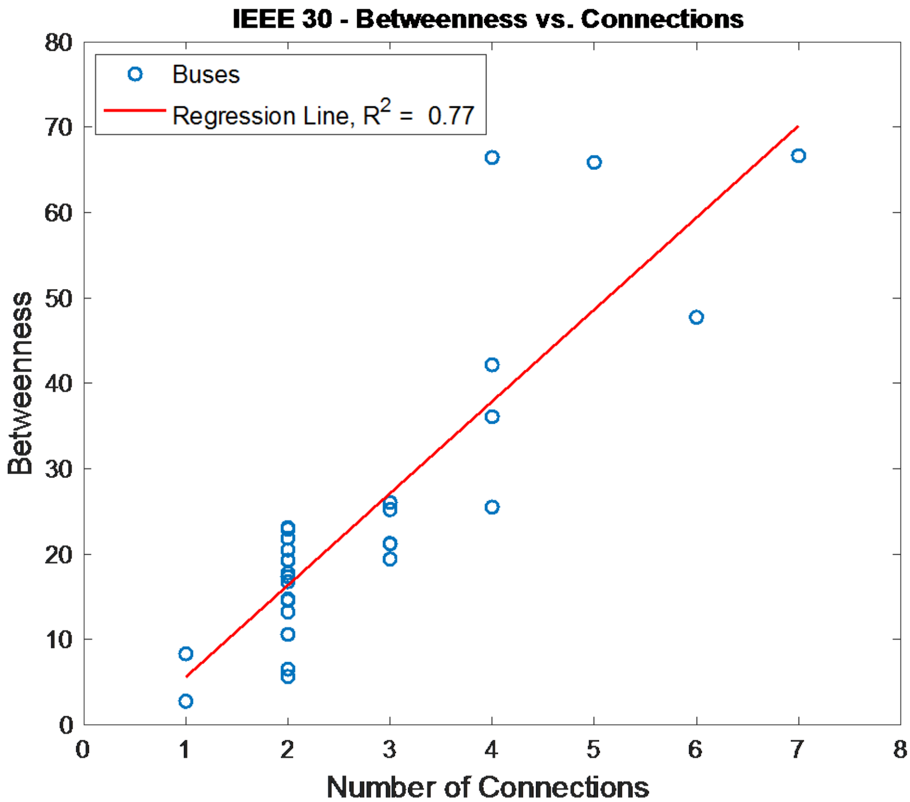

Figure 10: IEEE 30 – Betweenness vs. connections

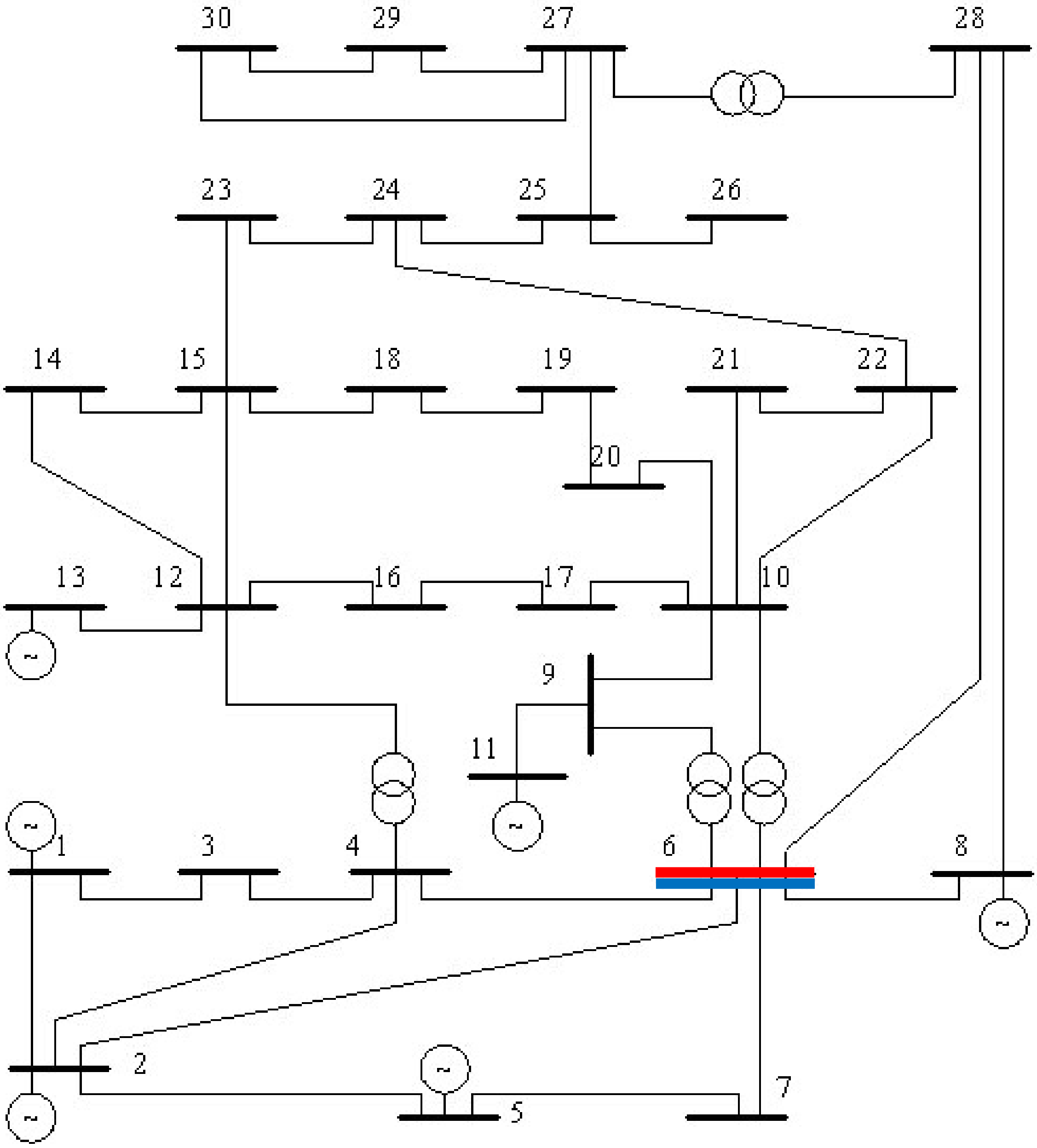

Figure 11: Location of optimal bus in IEEE 30 bus system according to ES net ability (red) and betweenness (blue)[29]

4.2 IEEE 118 Results

Table 4: IEEE 118 bus system ES net ability results

IEEE 118 Bus System – ES Net Ability			
Top 10 Buses		Bottom 10 Buses	
Bus Number	ES Net Ability (MW/Ω)	Bus Number	ES Net Ability (MW/Ω)
69	294.3	110	124.1
80	288.9	108	122.7
49	286.1	107	116.1
77	285.0	10	104.0
65	282.4	73	102.5
68	282.2	117	75.6
37	272.9	86	73.3
30	270.8	112	69.9
17	268.6	111	67.6
66	265.1	87	45.2

Table 5: IEEE 118 bus system betweenness results

IEEE 118 Bus System - Betweenness			
Top 10 Buses		Bottom 10 Buses	
Bus Number	Betweenness	Bus Number	Betweenness
65	59.9	86	2.7
68	50.4	107	2.1
80	49.0	9	1.9
38	41.6	10	0.9
30	40.0	116	0.9
49	39.7	117	0.9
77	37.9	73	0.9
81	37.2	112	0.9
100	37.0	111	0.9
69	33.3	87	0.9

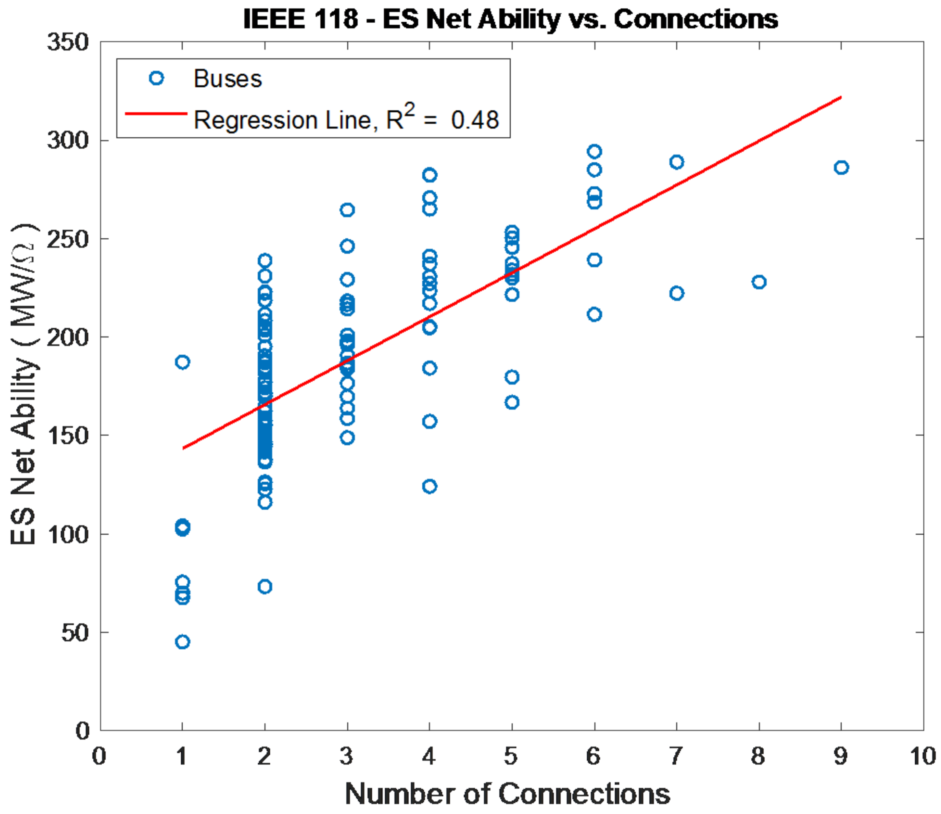

Figure 12: IEEE 118 – ES net ability vs. connections

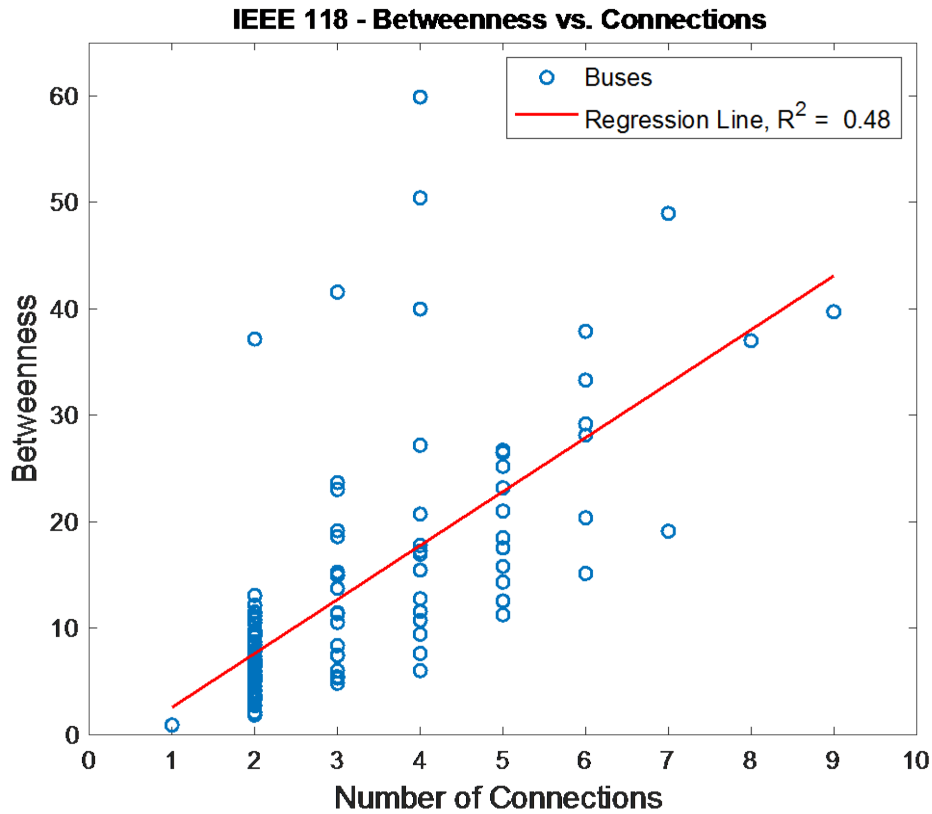

Figure 13: IEEE 118 – Betweenness vs. connections

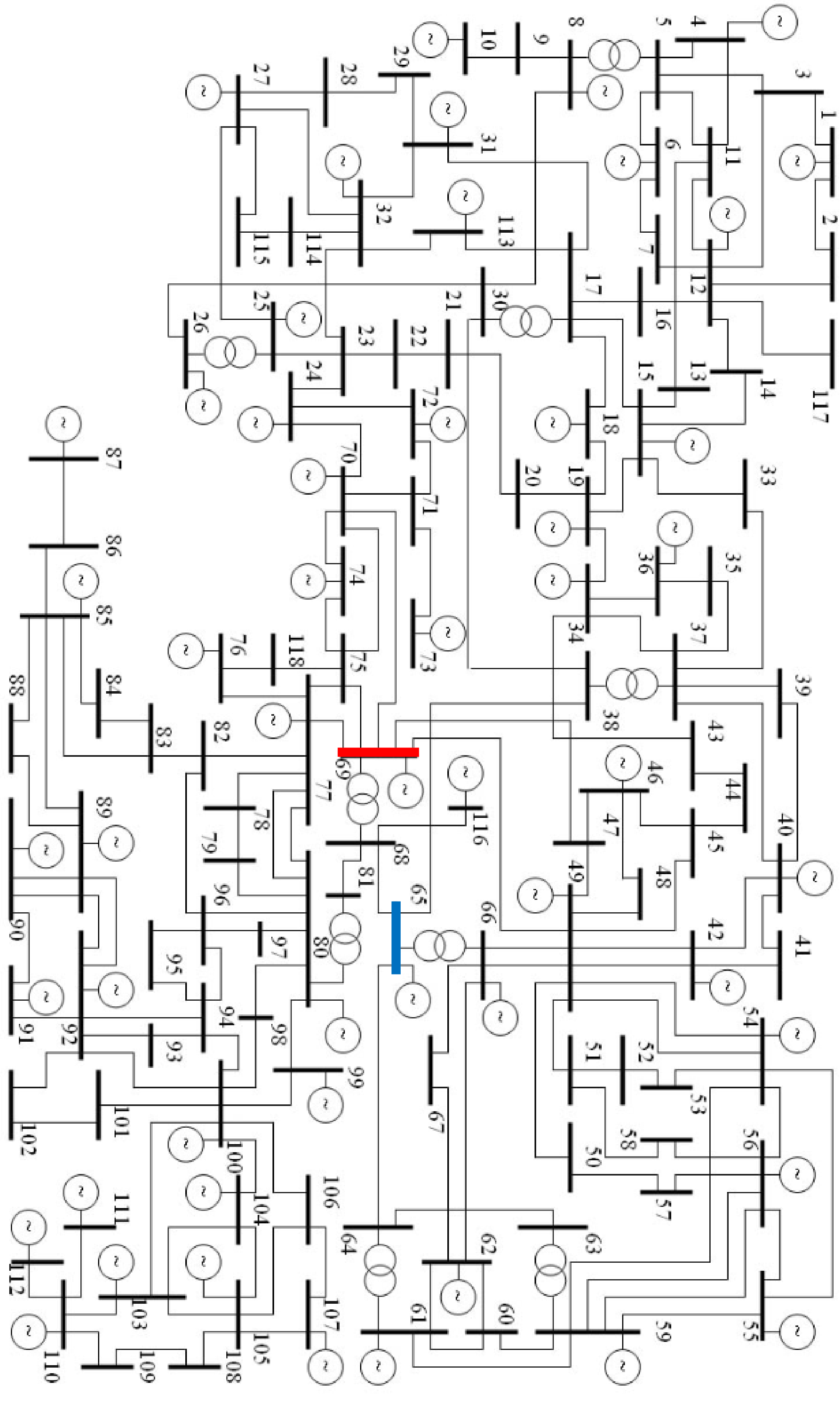

Figure 14: Location of optimal bus in IEEE 118 bus system according to ES net ability (red) and betweenness (blue) [29]

4.3 IEEE 300 Results

Table 6: IEEE 300 bus system ES net ability results

IEEE 300 Bus System – ES Net Ability			
Top 10 Buses		Bottom 10 Buses	
Bus Number	ES Net Ability (MW/Ω)	Bus Number	ES Net Ability (MW/Ω)
3	267.8	9071	6.5
7	266.3	9022	6.2
130	265.2	9035	5.8
42	254.8	9024	5.7
45	252.6	9031	4.5
133	250.7	9033	4.4
46	248.1	9032	4.3
16	247.7	9026	4.3
4	241.8	9025	4.1
41	240.9	9042	3.9

Table 7: IEEE 300 bus system betweenness results

IEEE 300 Bus System - Betweenness			
Top 10 Buses		Bottom 10 Buses	
Bus Number	Betweenness	Bus Number	Betweenness
42	44.4	9041	0.3
130	40.7	9022	0.3
3	40.6	9042	0.3
46	36.0	9043	0.3
16	35.6	9025	0.3
137	35.1	9042	0.3
81	34.6	9121	0.3
4	31.5	9026	0.3
37	29.1	9533	0.3
133	29.1	1201	0.1

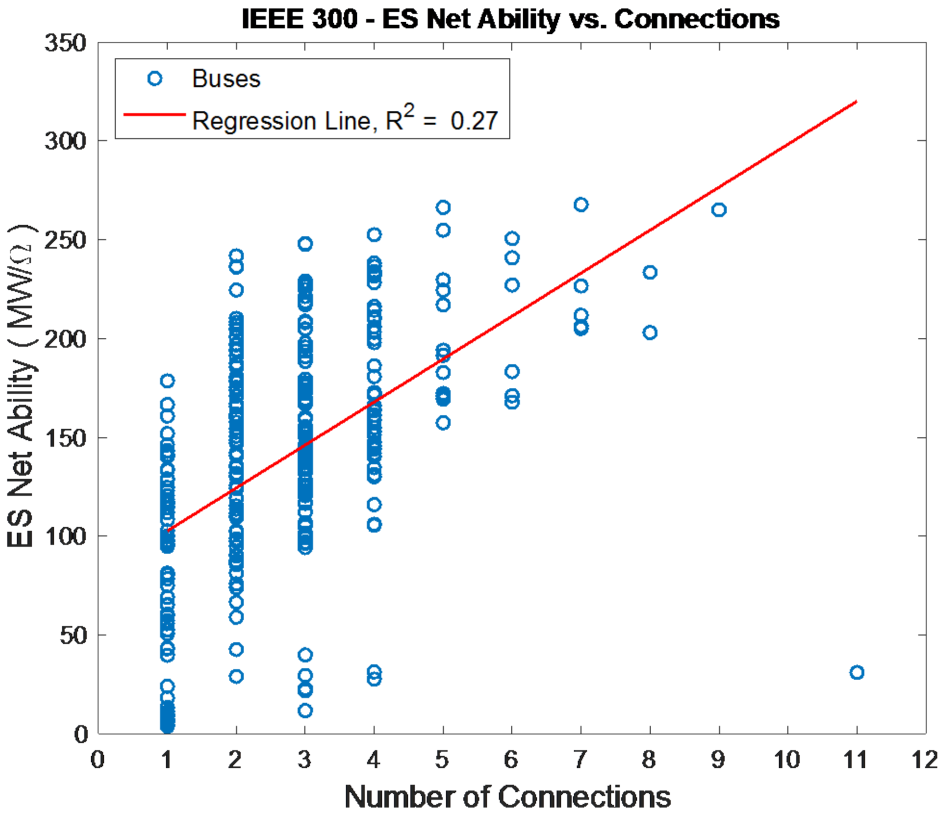

Figure 15: IEEE 300 – ES net ability vs. connections

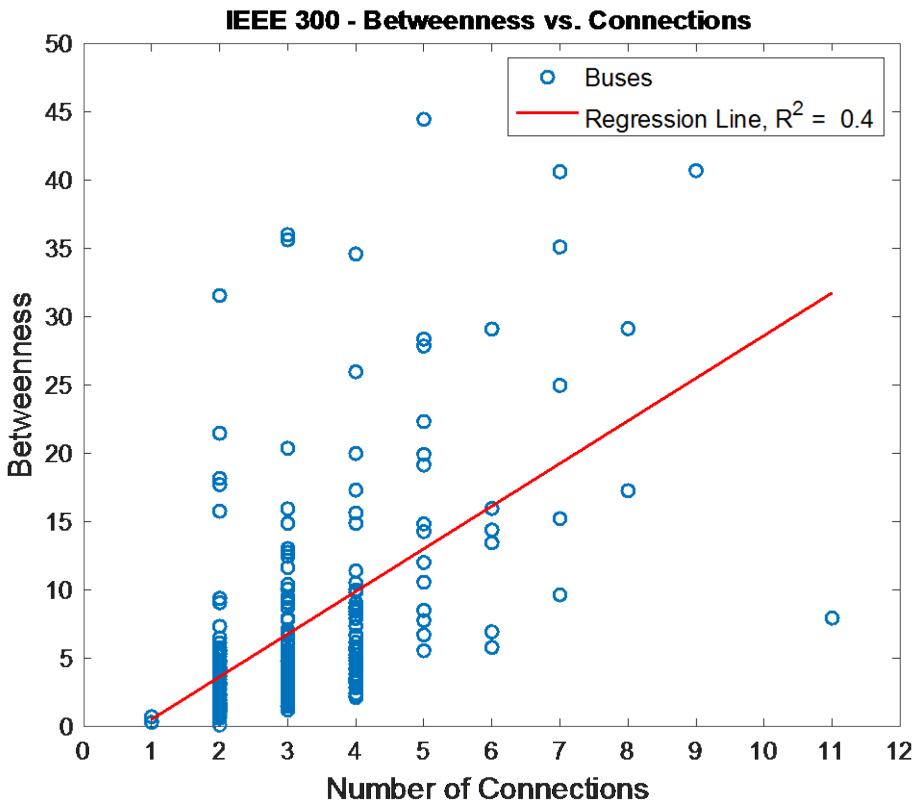

Figure 16: IEEE 300 – Betweenness vs. connections

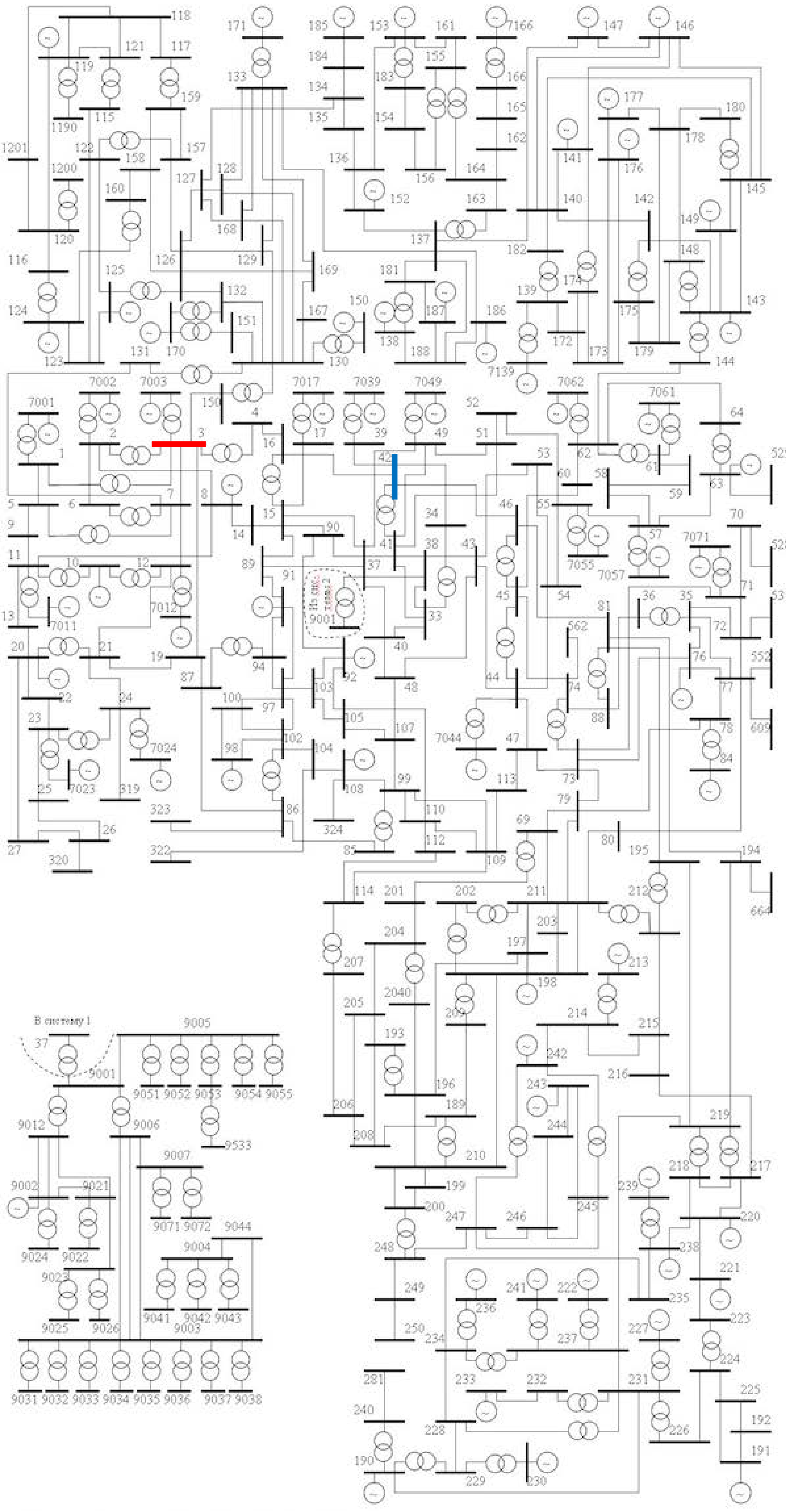

Figure 17: Location of Optimal Bus in IEEE 300 Bus System according to ES net ability (red) and betweenness (blue) [29]

4.4 Review of Optimum Bus Site for Each Bus System

As mentioned previously, the units for net ability as calculated by Bompard et al. are MW/Ω, which is equal to efficiency. As such, these results show the efficiency or performance of ES siting for each bus. ES net ability results for buses within a network, as well buses from other networks can be compared. The results show that the buses with the highest ES net ability for IEEE 30, 118, and 300 are buses 6, 69, and 3, respectively. For betweenness, the method is also applied to find the optimum bus for ES that gives the grid the highest performance. The results for betweenness show that in IEEE 30, 118, and 300, the buses with the highest betweenness are buses 6, 65, and 42, respectively.

When comparing the ranking of buses for ES net ability and betweenness, it is clear that the order is not the same, but the ranking of the buses is quite similar. Of the ten buses listed in the top 10 for each method, seven are listed in both lists for IEEE 30, 118, and 300. Although the two equations for ES net ability and betweenness are different, they produced similar results in terms of how they ranked the buses.

In the IEEE 30 bus system, bus 6 has the highest ES net ability and betweenness due to its high number of connections, its proximity to other important nodes, and as will be shown later, its location. Bus 6 has the highest number of connections in IEEE 30, following the trend of more connections leads to a higher ES net ability and betweenness. It is also directly connected to buses 4, 2, and 10, all of which are listed in the top 10 of buses with the highest ES net ability and betweenness. Buses 10 and 12 also have a high number of connections, but most of the connections are to buses with a low ES net ability and betweenness. Bus 6 is also connected to various buses that are located on the edge of the bus system, including buses 2, 7, 8, and 28. It may be beneficial for a bus to be connected to various buses at geographically distant locations is order to be able to transport electricity throughout the network more easily. As will be shown in section 4.6, the location of a node is a strong indicator of its ES net ability, therefore a central node that can distribute electricity directly to buses This claim is not corroborated by the data from the IEEE 118 and IEEE 300 bus systems, as all of the buses in these systems are connected to buses that are geographically close, as opposed to buses that are on the other side of the network.

In the IEEE 118 bus system, bus 69 has the highest ES net ability for having a relatively large number of connections and is directly connected to buses with a high ES net ability. Bus 69 does not have the most connections, but it is connected directly to buses 77 and 49, which have the third and fourth highest ES net ability in the network, and is one step away from the bus with the second highest ES net ability, bus 80. Buses 77 and 80 are directly connected, but do not have a direct connection to 49. Bus 69 is centrally located between buses 49, 77, and 80, and this may be why it has the highest ES net ability despite not having the most connections.

In IEEE 118, bus 65 has the highest betweenness, different from the ES net ability result. Although bus 65 does not have many connections, it is centrally located between several buses with high betweenness, such as buses 66 and 68, and is only one bus away from bus 49. Although this has not been proven, it also has connections to buses that are located in geographically distant, and may be a reason why certain nodes have a higher than expected betweenness or ES net ability.

Bus 3 in the IEEE 300 bus system has the highest net ability for having a relatively large number of connections, and for being one of the crucial connection points between two different subsystems. In the IEEE 300 system, there are three subsystems, with only a few connections between each subsystem. This means that if electricity needs to flow between two subsystems, thus a substantial amount of electricity needs to flow through a small number of crucial buses, resulting in these buses being vital to the operation of the network. Bus 3 is one of these buses, forming one of only three connections between two of the three subsystems. It is also located between buses 7 and 130, which have the second and third highest ES net abilities in the network.

Bus 42 in IEEE 300 has the highest betweenness, and this may be due to its central location within the network. As described above, there are three subsystems. Bus 42 is located in subsystem 1, which is between subsystems 2 and 3, and is the largest of the three subsystems. Bus 42 is not located in the middle of subsystem 1, but is shifted closer to subsystem 2, which is the larger than subsystem 3. Since bus 42 is located closer to the larger of the other two subsystems, it may give bus 42 an advantage in distributing electricity efficiently in the network.

4.5 Comparison of Grid Structure

Table 8 shows the average ES net ability for each bus system in order from highest to lowest. New England 118 has the highest average ES net ability, while IEEE 33 has the lowest. The average ES net ability shows how the structure of each bus system affects energy storage performance. By comparing the average ES net ability of each grid, the grid structure's effect on energy storage performance can be decoupled from other aspects, and measured as a single factor.

Table 8: Ranking of bus systems based on average ES net ability

Rank	Bus System	Grid ES Net Ability (MW/Ω)
1	IEEE 118	188.7
2	IEEE 57	161.9
3	IEEE 14	151.8
4	IEEE 300	140.2
5	IEEE 30	117.9
6	IEEE 33	21.6

IEEE 118 may have the highest grid ES net ability because it has the highest ratio of lines per node, allowing the network to more easily distribute electricity through the network. In section 4.6, it will be shown there may be a correlation between lines per node and grid ES net ability, but due to the small sample size, this relationship is not conclusive. Another reason for the high ES net ability is the low line impedances in IEEE 118. In section 4.6, it will be shown that line impedance plays an important role in influencing ES net ability, as they are inversely related. The line impedances will be set to the same value, and grid ES net ability will be recalculated to show how impedance effects ES net.

Table 9: Ranking of bus systems based on average betweenness

Rank	Bus System	Grid Betweenness
1	IEEE 33	25.29
2	IEEE 14	14.39
3	IEEE 30	6.74
4	IEEE 57	4.56
5	IEEE 118	0.52
6	IEEE 300	0.15

Table 9 shows the average betweenness of the evaluated grids, and a fairly consistent pattern of a higher number of nodes leads to a higher average betweenness. The exception is of course the IEEE 33 system, which is shown as the grid with the highest betweenness. This leads to the conclusion that the betweenness equation cannot be used to compare the grid structure, and can only be used to compare buses from the same grid.

4.6 Investigation of Additional Grid ES Net Ability Factors

In order to separate the factors that influence the ES net ability and betweenness of electrical grids, the capacity and impedance of the transmission lines were changed in the IEEE 30 bus system. In figure 18, the capacities of every line were all adjusted to the same capacity. The original capacity equation to calculate ES net ability and betweenness had all of the capacities of the lines set to the same value, which was 100 MW. A simulation was performed where all the line capacities in the grid were multiplied by a factor, and the resulting ES net ability and betweenness was recorded. The results for the ES net ability and betweenness for IEEE 30 are shown in figure 18a and figure 18b, respectively.

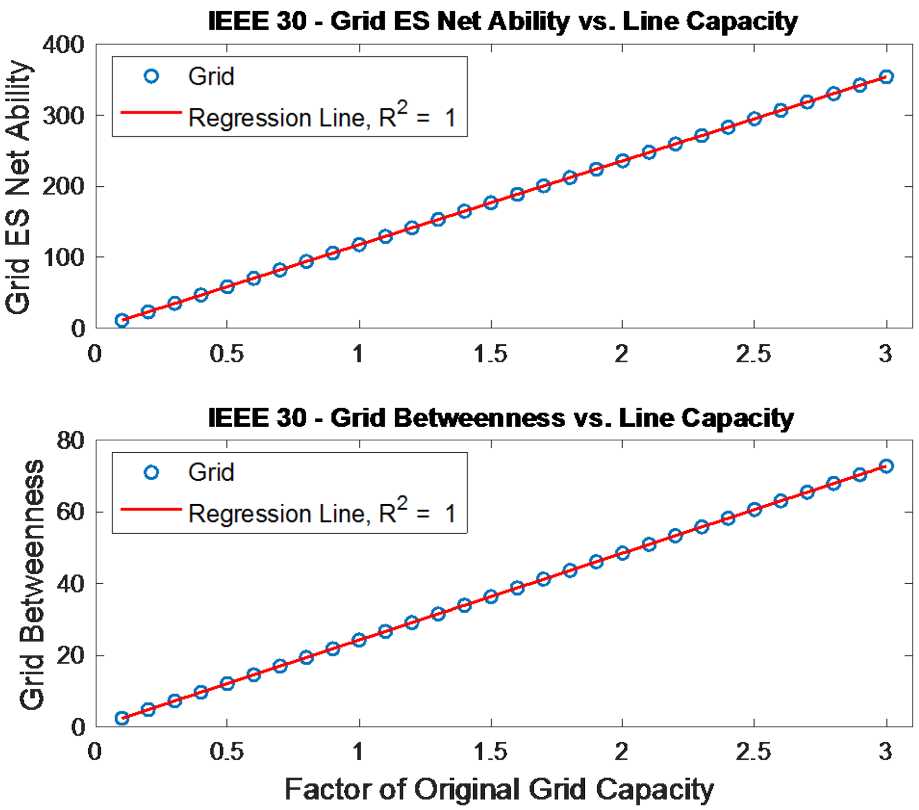

Figure 18: Comparing (a) grid ES net ability and (b) grid betweenness to line capacity

Figure 18 shows a linear relationship between increasing the capacity of the lines of a network, which means that for every increase in capacity of transmission lines in a network, the performance of ES in a network increases proportionally.

Figure 19 performs the same experiment for line reactance, where every line impedance is changed to the same value, and is adjusted accordingly. The input values for the line impedances has a low value of 0.1 Ω and a high value of 3 Ω. For every impedance value, the corresponding ES net ability and betweenness were recorded and plotted. Figure 19a and figure 19b show the results for the ES net ability and betweenness of the IEEE 30 bus system, respectively.

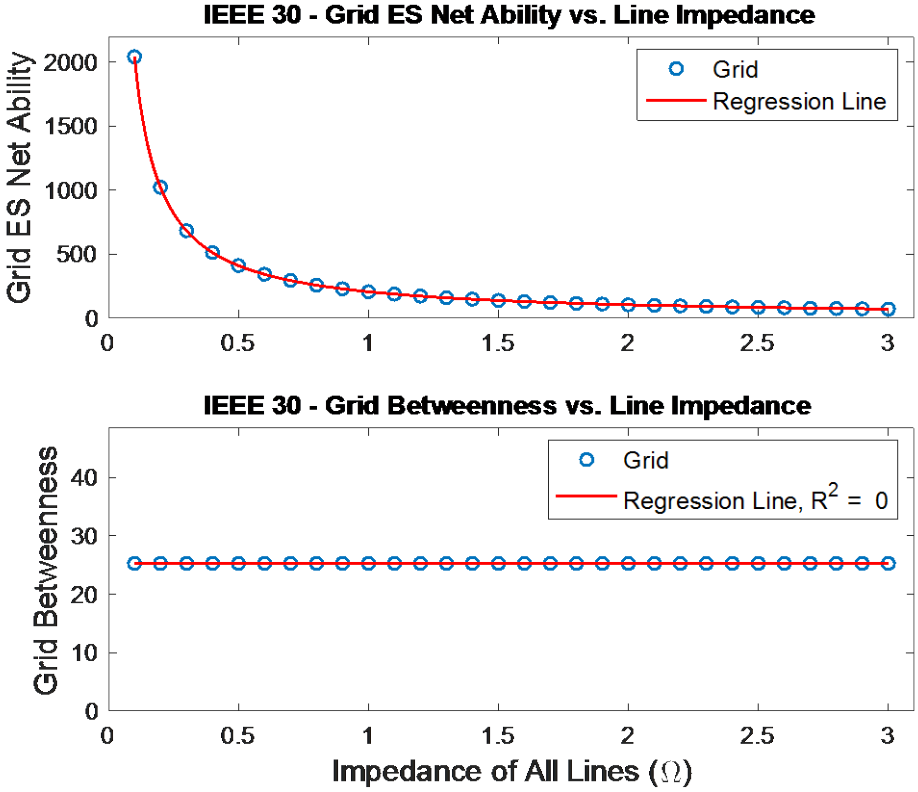

Figure 19: Comparing (a) grid ES net ability and (b) grid betweenness to line impedance

Figure 19 shows how changing the line impedances has a different effect on the measured values of betweenness and ES net ability. As shown in figure 19a, the ES net ability and line impedance are inversely proportional.

Table 10: Influence of Line Impedance on Grid ES Net Ability

Bus System	Grid ES Net Ability		Change in ES	Average Original Line Impedance (Ω)
	Original Line Impedances	Identical Line Impedances		
IEEE 14	151.9	197.8	+ 45.9	0.20
IEEE 30	117.9	136.2	+ 19.2	0.22
IEEE 33	21.6	64.1	+ 42.5	0.67
IEEE 57	161.9	159.2	- 2.7	0.24
IEEE 118	**188.7**	**125.7**	**- 63.0**	**0.11**
IEEE 300	140.2	56.9	- 83.3	0.24

The reason for grid ES net ability and line impedance being inversely proportional can be explained through the ES net ability equation. In the ES net ability equation, the denominator is composed of the ES impedance, which is directly related to the impedance of the transmission lines. Since ES impedance is in the denominator, ES net ability is inversely proportional to line impedance.

Since it was shown that changing the impedances of all lines in a grid would drastically change the ES net ability, the line impedances for in all the grids were changed to the same value of 0.15 Ω, and the grid ES net ability for each grid was calculated. The change in ES net ability is not important, it is simply to remove a variable and to see what other variables concerned with grid structure impact ES performance. Figures 20-26 test the relationship between a grids ES net ability and different aspects of a grid, such as lines per node, and node location, and number of connections.

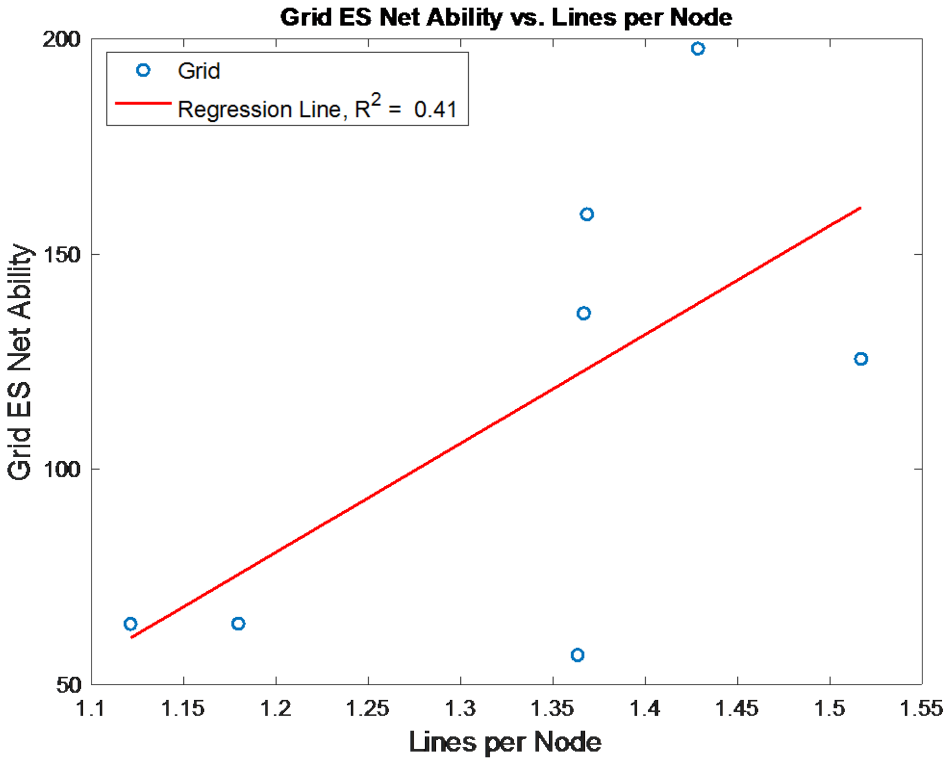

Figure 20: Grid ES net ability vs. lines per node with identical line impedances

Figure 20 shows the relationship between grid ES net ability and the ration of lines per node. It appears that a higher ratio of lines per node may lead to higher ES net ability, but due to the small sample size, and also the relatively low coefficient of determination, it is difficult to make conclusions from this graph. It was previously shown in this paper that number of connections for a node leads to a higher ES net ability, which means that if there are more connections for a node, then the ratio of lines per node would also be higher, and possibly leading to a higher grid ES net ability.

The remaining factor of ES net ability that was investigated was location. A nodes location will be defined as its position relative to the other nodes in the network. In order to quantify a nodes location, an equation was developed that uses the distance between any node and the other nodes in a network. The distances between a node and the other nodes in the network are exponentiated by a value x, and then summed. Below is the equation to calculate the location of a node in a network, where x is an exponent that gives the highest correlation between location and ES net ability.

$$L(d) = \sum_{n \in N} d^x \tag{33}$$

The value of x that provided the highest correlation between location and ES net ability was -2. Equation 33 was rewritten to include the value of x in equation 34.

$$L(d) = \sum_{n \in N} 1/_{d^2} \tag{34}$$

Figures 21, 23, and 25 show the comparison of node location and ES net ability for IEEE 30, 188, and 300. For all three graphs, the line impedances were set to identical values of 0.15 Ω, and the ES net ability was calculated to remove line impedance as a possible variable. For comparison, figures 22, 24, and 26 plotting ES net ability and number of connections for IEEE 30, 118, and 300. Figures 21-26 will show which method, node location or number of connections, is a better predictor of ES net ability. In order to remove all variables, all line impedances in figures 21-26 were set to the same value.

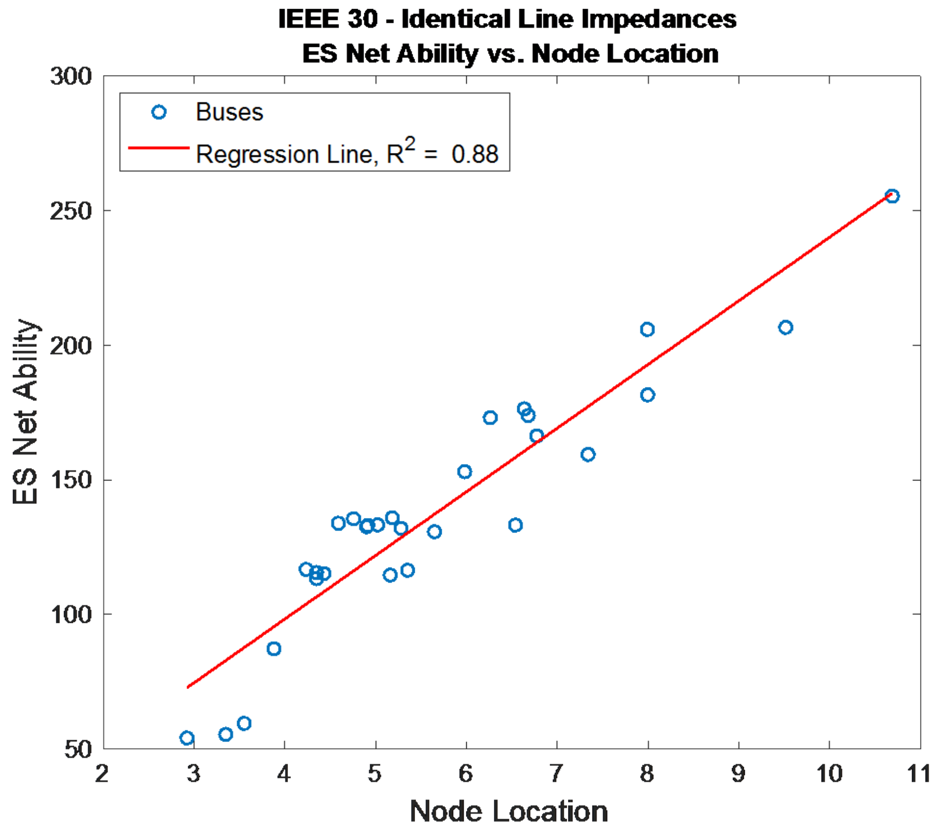

Figure 21: IEEE 30 - ES net ability vs. node location (identical line impedances)

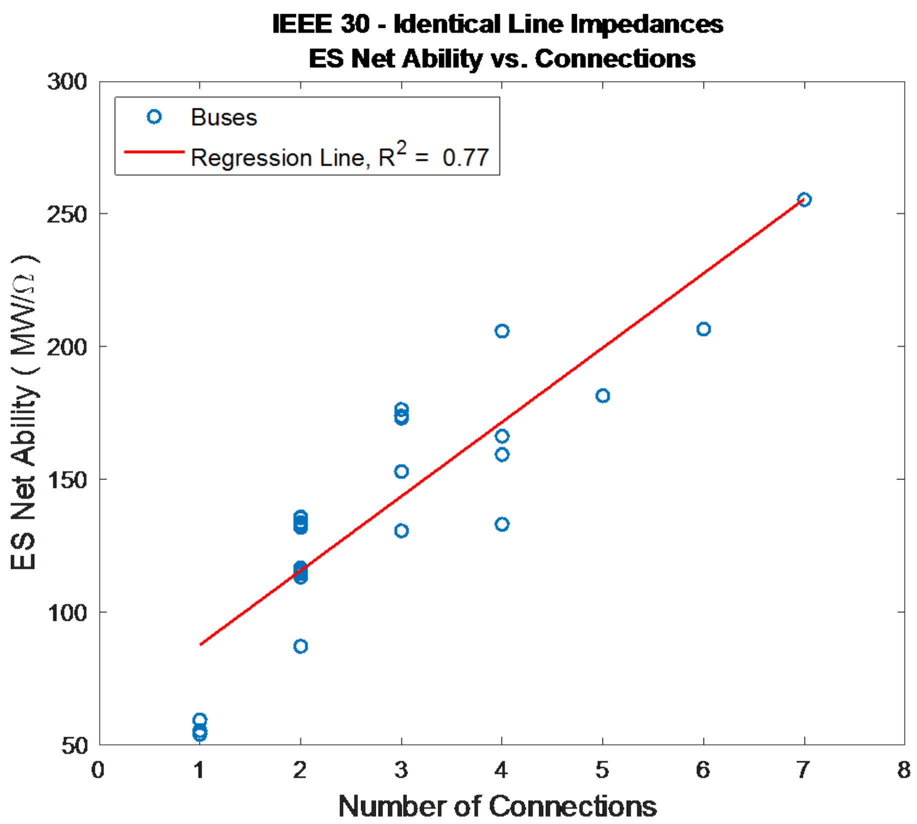

Figure 22: IEEE 30 - ES net ability vs. number of connections (identical line impedances)

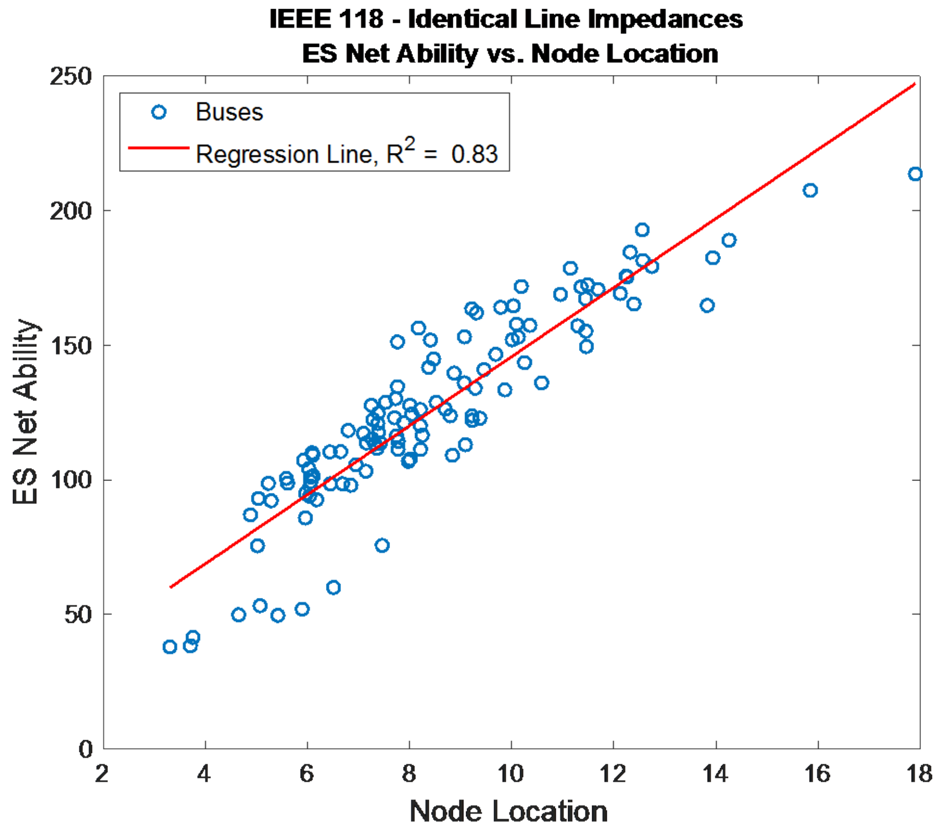

Figure 23: IEEE 118 - ES net ability vs. node location (identical line impedances)

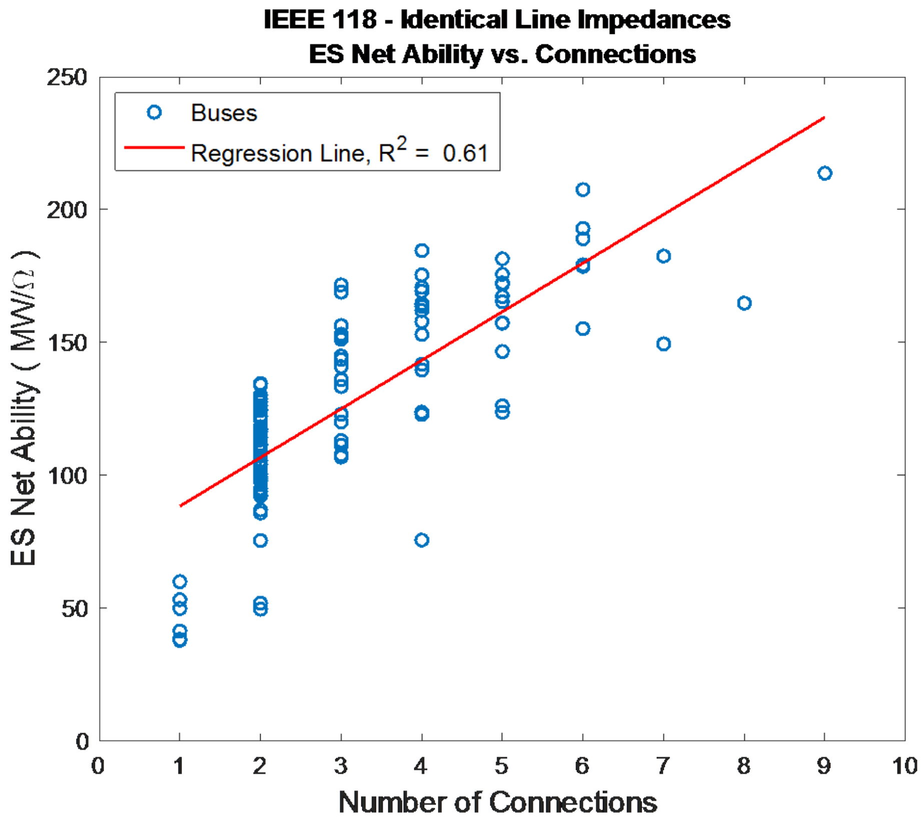

Figure 24: IEEE 118 - ES net ability vs. number of connections (identical line impedances)

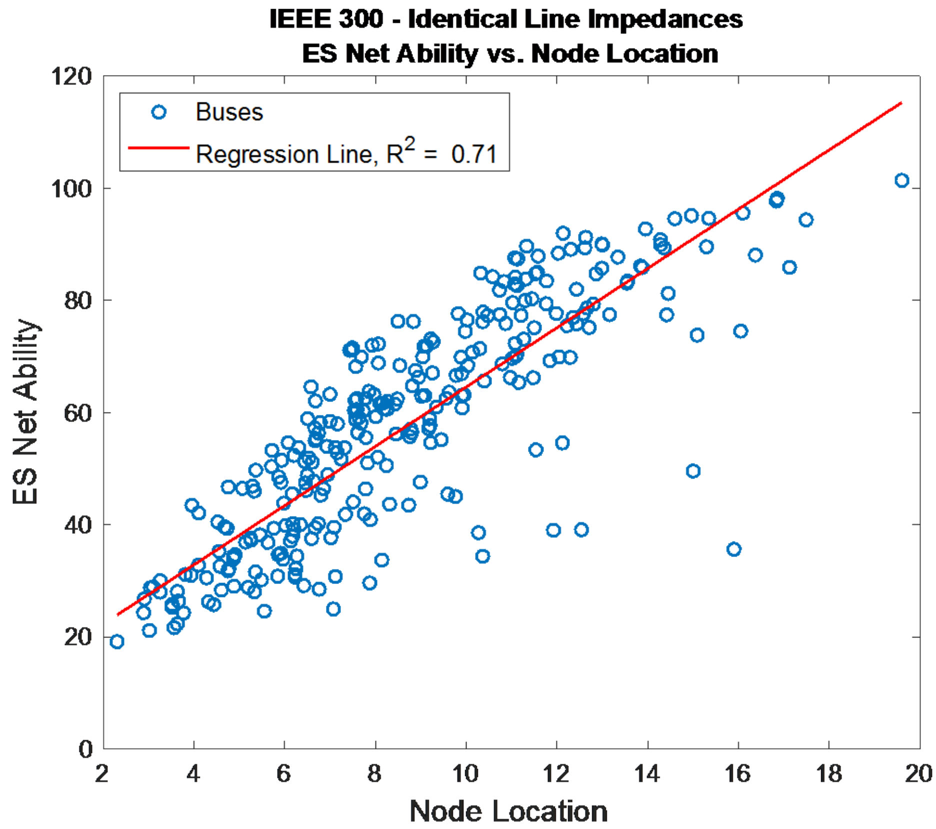

Figure 25: IEEE 300 - ES net ability vs. node location (identical line impedances)

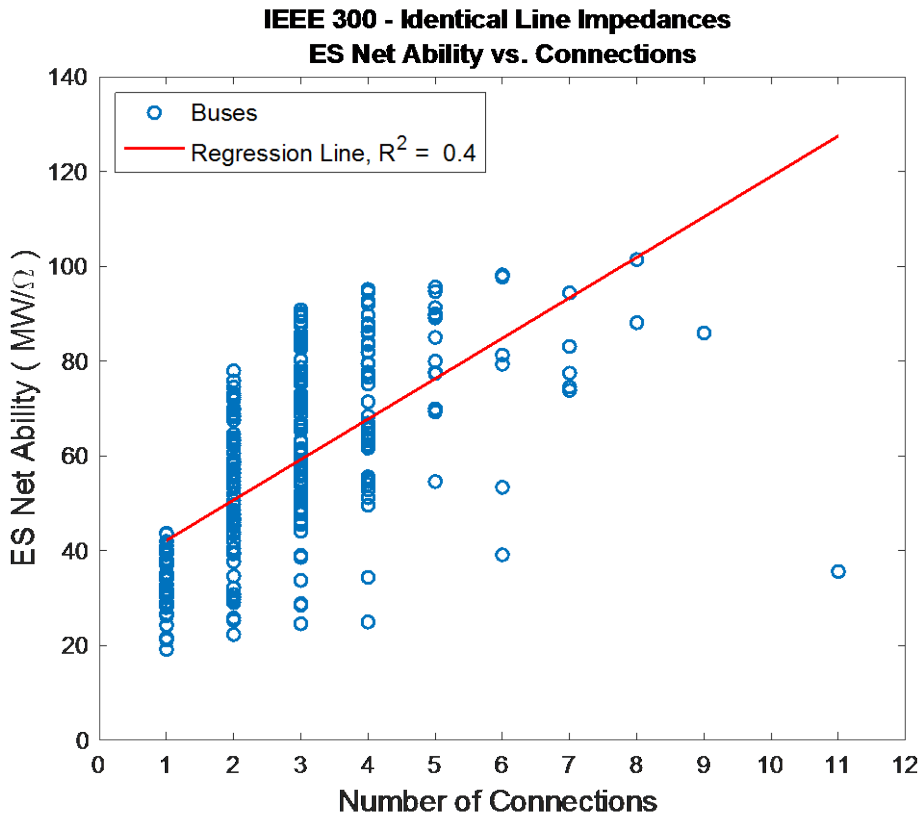

Figure 26: IEEE 300 - ES net ability vs. number of connections (identical line impedances)

From figures 20-26, it is clear that node location is a superior predictor of ES net ability. The coefficients of determination for node location are much higher, and the data points are much closer to the line of best fit than number of connections. The high coefficient of determinations for all three graphs are relatively high, showing the strong connection between location and ES net ability. Table 11 is a compilation of the coefficient of determinations for different graphs to further show the superior correlation between location and ES net ability. Table 11 also includes the correlation between number of connections and ES net ability for comparison purposes. It should also be noted that the all the line impedances for all bus systems were set to the same value in order to remove the effect of line impedance on ES net ability.

Table 11: Comparison of correlation between location and ES net ability, and connections and ES net ability. Each bus system has identical line impedances

Bus System	Coefficient of Determination (R^2)	
	Node Location	Number of Connections
IEEE 14	0.85	0.81
IEEE 30	0.88	0.77
IEEE 33	0.47	0.42
New England 39	0.87	0.71
IEEE 57	0.89	0.74
IEEE 118	0.83	0.61
IEEE 300	0.71	0.40

Table 11 continues the trend that node location is a better predictor of ES net ability than number of connections. For all of the bus systems, the coefficients of determination for node location are all higher than number of connections, further proving that node location is a better predictor of ES net ability. Most of the bus systems have high correlations between location and ES net ability. The exception is IEEE 33, which is a distribution network. It was predicted that ES net ability and location of nodes in IEEE 33 would be highly correlated, yet it is unclear why the correlation is not as high as expected. Even though the correlation is not a high as expected, it is still much higher than the correlation for number of connections and ES net ability for IEEE 33.

Table 11 shows the correlation between a nodes location and ES net ability, but it does not show the correlation between grid ES net ability and average node location. The method used to determine average node location was to sum the location value of every node in the grid, and then divide it by the number of nodes in the grid. Unfortunately, this method did not work. Since all of the grids are of different sizes, the average node location cannot be compared between grids. For example, the nodes in a smaller grid will have a shorter distance to travel to the other nodes, whereas in a larger network, the nodes will be further apart, causing the average distance to increase. Due to this issue, the average node location cannot be used to compare to grid ES net ability.

As a review, line impedance and line capacity were used to show a correlation with grid ES net ability. Since line impedance and capacity effect the grid ES net ability, it is certain that they effect the ES net ability of individual nodes. Line impedance has an inverse correlation with grid ES net ability, and line capacity has a positive correlation with grid ES net ability. It is unclear what the impact of changing one lines impedance or capacity has on the ES net ability of individual nodes, as this was not tested.

Also, node location and number of connections was also tested, and both have a positive correlation with ES net ability. Node location is a much better predictor of ES net ability, and was shown to have a higher correlation with ES net ability than number of connections. This shows that it is important for nodes with ES to not only have a high number of connections, but to be centrally located in a grid in order to distribute electricity efficiently, and to improve overall ES performance. The idea of comparing the average node location to grid ES net ability was tested, but there was no correlation. The idea of comparing average node location is an idea to compare a grids structure to determine which grid has a superior structure for ES performance. Further testing of this idea will need to take place to either prove or disprove this concept.

It is clear that line impedance, line capacity, and node location affects a nodes ES net ability, but it is possible there are other factors that affect a buses ES net ability. It was shown that location has an effect on a nodes ES net ability and ES performance. The results show that nodes that are centrally located in a network tend to have a higher degree of centrality and a higher ES net ability, whereas buses located along the periphery of the network tend to have lower values of centrality. Besides location, it was shown that line impedance has a major effect on ES net ability. Line impedance and grid ES net ability are inversely related, showing that a lower impedance will increase ES net ability. This shows that a higher impedance will impede power flow through a network, thus decreasing the performance of ES and the grid. The number of connections a bus has and ES net ability are strongly correlated, but this relationship weakens as more buses are added to the system. This appears to be related to an affect from bus location in a network. Buses with a lower number of connections can be centrally located, giving it a higher than expected ES net ability, while buses with a higher number of connections could be peripherally located, and have a lower than expected ES net ability. Also, it was proven that capacity will increase ES net ability. This shows that by increasing the capacity of lines in a network, power flow will increase, thus improving ES performance.

4.7 Effect of Distributed Energy on the Location of ES in Distribution Systems

The IEEE 33 bus distribution system was evaluated to determine the effect of distributed energy on the location of ES. The IEEE 33 bus system was chosen because low capacity distributed energy sources, such as wind or solar, will be installed by home owners or small businesses and then connected to the grid. As more variable energy sources are connected to the grid, the need for ES will increase, and the need to locally control the variation in electricity distribution will increase as well.

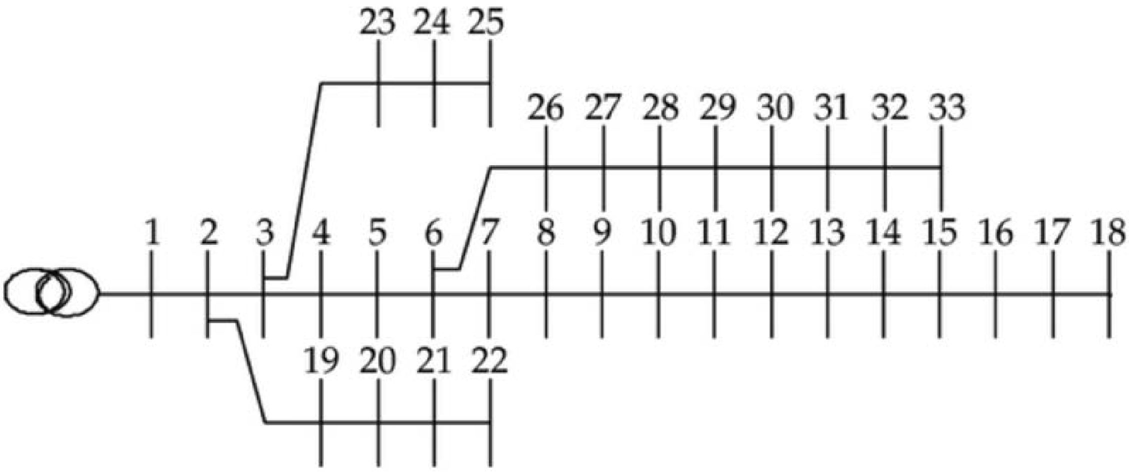

Figure 27: IEEE 33 bus system

In the IEEE 33 system, bus 1 is a generator node, similar to the connection to the grid. The remaining nodes are all loads. For the simulation, a generator will be placed at a node, and the ES net ability and betweenness of the grid will be measured. The node with the highest ES net ability and betweenness will be the optimal location for ES. For node 1, the bus system will only have one generator, which is located at node 1. For all other nodes, a generator will be placed at said node and node 1, for a total of two generators. The nodes will be ranked by which bus provides the highest ES net ability and betweenness, and the corresponding optimum ES site will be listed. The rankings are listed in tables 12 and 13.

Table 12: IEEE 33 - Ranking of optimum bus for distributed energy by grid ES net ability

IEEE 33 Bus System – ES Net Ability					
Top 10 Buses			Bottom 10 Buses		
Generator Bus	Grid ES Net Ability	ES Bus	Generator Bus	Grid ES Net Ability	ES Bus
1	53.21	2	30	37.63	2
4	45.02	3	15	36.95	2
3	44.52	2	13	36.81	2
19	44.39	2	14	36.68	2
2	43.83	3	16	36.25	2
23	43.23	3	31	36.25	2
5	43.06	3	32	35.91	2
6	41.02	2	33	35.59	2
21	40.79	2	18	35.44	2
20	40.74	2	17	35.41	2

Table 13: IEEE 33 - Ranking of optimum bus for distributed energy by betweenness

IEEE 33 Bus System – Betweenness					
Top 10 Buses			Bottom 10 Buses		
Generator Bus	Grid Betweenness	ES Bus	Generator Bus	Grid Betweenness	ES Bus
1	98.44	2	18	61.91	2
19	95.57	2	6	61.72	2
2	83.74	2	33	61.66	2
20	81.97	2	26	61.66	6
21	78.31	2	16	61.53	2
23	76.74	3	32	61.22	2
24	71.80	3	31	61.01	2
22	71.75	2	27	60.84	6
4	71.71	3	30	60.72	2
3	71.24	3	7	60.71	2

The results for both ES net ability and betweenness show that the optimum location for a generator is bus 1 with ES at bus 2. Since all of the other nodes have a generator plus a generator at node 1, the ES net ability and betweenness decreases due to the addition of a generator and ES. If distributed generation is added to the network, the optimum location for a generator according the ES net ability is bus 4, with ES located at bus 2. According to betweenness, the optimum location for a generator is bus 19, with ES at bus 2. The optimum locations for generators tend to be located at or close to buses 2 and 3, which are connected to various strings of buses, thus easily distributing electricity throughout the network. The optimum location for ES tends to be at buses 2 and 3.

In a world with a high proportion of variable energy sources connected to the grid, it is desirable to smooth the power generation before it reaches a customer. If the variable energy source is uncontrolled, where the power generation profile does not match the load, this will cause a wide-range of problems for the grid. That is why it is best to find a location in the grid where ES first regulate power flow, and then distribute the energy during periods of high demand. For this network, it is impossible to regulate variable power generators at every node, unless ES is installed at every node. It can be assumed that the generation profile from the grid will be much larger at bus 1 than any other node, since it represents the grid. If there is a large amount of capacity of variable energy sources connected beyond bus 1, then it would be optimal to install ES close to bus 1, before electricity can be delivered from the grid to the distribution network. Distributed forms of energy that are connected to the distribution network, such as wind and solar, may not have as much of an effect on the variability of the network as opposed to a grid with large installations of renewable energy. It seems reasonable to install ES at bus 2, in order to regulate the flow of electricity from the grid to the distribution network. Bus 2 also serves as a type of hub to distribute electricity efficiently throughout the network, since it is located close to many branches in the network.

5. CONCLUSION

The net ability equation from Bompard et al. and Arianos et al. was modified to account for ES within an electrical grid in order to determine optimal ES siting, and to measure the impact of grid structure on ES performance. The net ability equation was originally intended to determine the importance of each transmission line within a network, but by modifying the capacity and impedance in the net ability equation to account for ES in a grid, it can be applied to determine the optimal bus or site for ES, and decouple the grid structure from other factors to quantify which grid is better for ES performance. The betweenness equation was also intended to determine the importance of different transmission lines and nodes in a network, but did not need to be modified to determine optimal ES siting.

The results appeared to show more reliable results for grid ES net ability than for grid betweenness. The results for grid betweenness were biased towards grids with fewer nodes, and did not develop any correlation to centrality. For this reason, the results section focused on ES net ability and grid ES net ability, and the conclusion will focus on ES net ability and its revelations on how to improve grid structure for ES performance. Betweenness may need to be revised and improved in order for grid betweenness to be a viable concept. Although grid betweenness did not appear reliable, the results for betweenness in terms of individual nodes was reliable.

ES net ability and betweenness had similar rankings of buses for IEEE 30, 118, and 300. These results were able to determine which nodes in each grid are the optimum site for installing ES in order to optimize ES performance. The novel idea of ES net ability to determine optimal ES siting is different from other methods in that it optimizes ES performance, as opposed to optimizing a single criteria, such as frequency, voltage, investment profits, or operator costs.

Different factors were determined to influence the grid structures impact on ES performance. They included impedance, capacity, number of connections, and node location. If all the lines of a grid are set to the same value, then line impedance and ES net ability are inversely related. For capacity, if all lines are set to the same value, then capacity and ES net ability are directly related. This means that increasing line capacity and decreasing line impedance will improve ES performance in a grid. Increasing line capacity will reduce inefficiencies in the grid, such as reducing congestion during peak generation. Reducing impedance reduces line losses in transmission lines, thus improving ES performance. The factors of location and number of connections were tested and compared to show that location is a better predictor of ES net ability for a node. The concept of location was able to quantify a nodes location within a network, whereas number of connections solely considers its neighboring nodes. Both location and number of connections are correlated with ES net ability, but location has a much higher correlation, especially for networks with a high number of nodes. The trend of ES net ability being correlated with number of connections decreases as number of nodes are increased. This is affected by the location of the node, and how the nodes are arranged in the grid. In a smaller network, central nodes will tend to have more connections than nodes located along the periphery of the network. As more nodes are added to the network, there tends to be an equal distribution of nodes with many connections and nodes with few connections throughout the network. This leads to nodes with a small number of connections benefiting from its location, and may get a higher than expected ES net ability or betweenness. A node with many connections that is peripherally located may have a lower than expected ES net ability or betweenness because its location decreases its centrality in the network. The idea of location is that nodes more centrally located will have a higher location than nodes located along the edges of a network, leading to central nodes with a higher ES net ability, and peripheral nodes with a lower ES net ability. According to this research, it is best to establish ES at nodes with many connections, centrally located within the network, and to minimize line impedance and maximize line capacity in order to optimize ES performance.

The concept of location was taken further to test the idea of average node location leads to higher grid ES net ability. This is to say that denser and more compact networks, with more lines and connections and fewer nodes, leads to better ES performance. Unfortunately, the idea of average node location was not able to be used between networks because the more nodes a network has, the higher the average distance is for a grid, thus a higher average node location. Even though average node location was not able to produce any conclusive results, the idea of more lines per node leads to a higher ES net ability was tested and showed a slight correlation. This shows that it may be beneficial to increase the lines to node ratio for a grid to improve ES performance. The concepts of average node location and lines per node leads to higher ES net ability and ES performance will need to be tested in the future.

An idea that could be innovative would be to show a relationship average distance between every generator-node-load combination and ES net ability. Since the power flow through ES goes first from generator to ES, and then from ES to load, it would reasonable to determine the average distance between generator-node-load combinations, as opposed to finding the average distance between every node-node pair. This research could provide insight into not only determining where to install ES, but also where to install distributed generation, where to install new substations, and how to design future grid networks.

The rules for the impact of grid structure on ES performance appear to differ between standard transmission and distribution networks. All of the transmission networks had high correlations between ES net ability and location, as well as ES net ability and number of connections. This was not the case for IEEE 33 distribution network, as the coefficient of determination for these two trends was much lower than the other networks. The reason for why this is the case is unclear. Further research will need to be performed to determine correlations between ES net ability and distribution network grid structure.

For the test case of determining the best ES site for IEEE 33, both ES net ability and betweenness both selected nodes close to the grid connection for installing ES, while at the same time opting to install generators close to said bus. This conclusion matches previous assumptions about distribution networks, where it is desirable to install ES between the grid and the distribution network. This way, if the grid has a large proportion of variable renewable energy sources, ES can smooth the load profile before it gets to the distribution network. The same likewise if the distribution network has a significant amount of renewable energy source, it can smooth out the electricity before it is sent to the grid.

The novel techniques of ES net ability and betweenness are new ways to calculate optimal ES siting by optimizing grid efficiency and decouple grid structure from other factors to determine its impact on ES performance. Betweenness was limited to evaluating nodes, while ES net ability was able to quantify a grid structures effect on ES performance. The new ES net ability equation can evaluate the importance of each bus within a network. Relationships between grid structure and its grid ES net ability have been put forward to explain why a grid has a higher grid ES net ability. These findings could be used to improve the structures of existing grids, and aid in the design of future smart grids. Future research will need to be performed to confirm the trends uncovered in this paper in order to provide more evidence that can then be used to influence the designs of future electrical grids.

APPENDIX

Appendix A will be used to describe each matrix, how it was formed in Matlab, with its accompanying Matlab code. Figure 28 shows a theoretical grid system that will be used as a running example to help explain certain matrices.

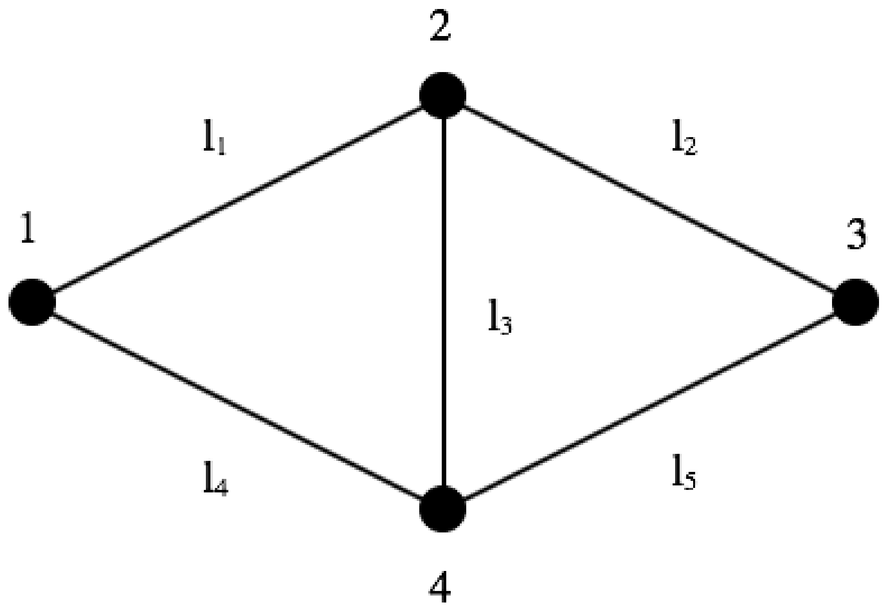

Figure 28: Generic network diagram to be used as running example in Appendix

Susceptance (B_d)

Matrix B_d is a LL diagonal matrix of each lines susceptance. Below shows how the diagonal susceptance matrix will look like.

$$\begin{pmatrix} B_1 & 0 & \cdots & 0 & 0 \\ 0 & B_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & B_{L-1} & 0 \\ 0 & 0 & \cdots & 0 & B_L \end{pmatrix}$$

Figure 29: Typical diagonal susceptance matrix (B_d)

Below shows the Matlab code used to calculate the susceptance matrix. Two for loops were used to run through every line-line combination. An if statement was used to differentiate between when i equals j, and when i does not equal j. When i equals j, the cell equals the line susceptance, and if not the cell equals 0.

```
Bd = zeros(line_max);

for i = 1:line_max
   for j = 1:line_max
      if i == j
         Bd(i,j) = line_susceptance(i);
      else
         Bd(i,j) = 0;
      end
   end
end
```

Incidence (A)

The incidence matrix describes the connections between each node. This is a LN matrix, and if nodes i and j are connected, then $A_{il} = 1$ and $A_{jl} = -1$. If i and j are not connected, $A_{il} = A_{ji} = 0$. This means that for each row l, there will only be one value equal to 1, and only one value equal to -1, with the remaining values equal to 0. It is unimportant as to which node is i and which one is j. Using figure 28 as the example grid, figure 30 shows the incidence matrix for the running example.

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Figure 30: Incidence matrix (A)

Below shows the Matlab code used to calculate the susceptance matrix. The find command will find the bus number of the "from bus" and the "to bus". The following lines will give a value of 1 to A_{il}, and -1 to A_{jl}. All the other numbers in row l equal 0.

```
A = zeros(line_max,node_max);

for i = 1:line_max
    [a,~] = find(bus(:,1)==line(i,2));
    [b,~] = find(bus(:,1)==line(i,3));
    A(i,a) = 1;
    A(i,b) = -1;
end
```

Power Line Flow (p_L) and Power Node Flow (p_N)

The equations for power line flow and power node flow are quite straightforward, and do not require any for loops or if statements. The Matlab code is shown below.

```
Pl = Bd*A;
Pn = A'*Bd*A;
```

Matrix p_n is a NN matrix, and p_l is a LN matrix. It should be noted that the matrices calculated from the above equations include the slack bus. p_n is linearly dependent, which means p_n does not have an inverse, therefore a reference node must be removed. The slack bus will be the reference node, and must be removed from both p_n and p_l matrices.

Remove Slack Bus Node

Since p_n is a NN matrix, and p_l is a LN matrix, one column and one row will be removed from p_n, and one row will be removed from p_l. The code below shows that elements of the rows and columns to be removed are set to zero. This step is unnecessary, but it was used to check if the removal and resizing of the matrices was performed correctly.

```
Pl(:,slack_bus) = 0;
Pn(:,slack_bus) = 0;
Pn(slack_bus,:) = 0;
```

After setting the slack bus elements to zero, a new set of matrices is formed, as shown below.

```
Pn_new = zeros(node_max-1);
Pl_new = zeros(line_max,node_max-1);
```

After formation of the new matrices, the old matrices are placed into the new matrices. The location of the slack bus within the matrices determines how the new matrices will be formed, and the following code reflects this.

```
if slack_bus == 1
   Pn_new = Pn(2:node_max,2:node_max);
   Pl_new = Pl(:,2:node_max);
elseif slack_bus == node_max
   Pn_new = Pn(1:node_max-1,1:node_max-1);
   Pl_new = Pl(:,1:node_max-1);
else
      Pn_new(1:slack_bus-1,slack_bus:node_max-1) = Pn(1:slack_bus-1,slack_bus+1:node_max);
      Pn_new(slack_bus:node_max-1,slack_bus:node_max-1) =
      Pn(slack_bus+1:node_max,slack_bus+1:node_max);
      Pn_new(slack_bus:node_max-1,1:slack_bus-1) = Pn(slack_bus+1:node_max,1:slack_bus-1);
   Pn_new(1:slack_bus-1,1:slack_bus-1) = Pn(1:slack_bus-1,1:slack_bus-1);
   Pl_new(:,slack_bus:node_max-1) = Pl(:,slack_bus+1:node_max);
   Pl_new(:,1:slack_bus-1) = Pl(:,1:slack_bus-1);
end
```

Add Slack Bus Node and PTDF w/ Slack Bus Node

After the slack bus has been removed from matrices p_n and p_l, the ptdf can be calculated. The inverse of matrix p_n is calculated by using the function pinv. There was no discernable difference between using the function inv or pinv, but in the end, pinv was used. The PTDF was then calculated. The PTDF without a slack bus node is a LN-1 matrix.

```
Pn_new = pinv(Pn_new);

ptdf = Pl_new*(Pn_new);
```

The PTDF needs to add the slack bus node into the PTDF, since it was removed in order to calculate the inverse of p_n. The slack bus node is added as a column of zeros, which turns the PTDF into a LN matrix. The following code reflects the above statement.

```
ptdf_adj = zeros(line_max,node_max);

if slack_bus == 1
   ptdf_adj(:,2:node_max) = ptdf;
elseif slack_bus == node_max
   ptdf_adj(:,1:node_max-1) = ptdf;
else
   ptdf_adj(:,1:slack_bus-1) = ptdf(:,1:slack_bus-1);
   ptdf_adj(:,slack_bus+1:node_max) = ptdf(:,slack_bus:node_max-1);
end
```

Determine Generator List and Load List

With the information provided from the University of Washington in the bus system data files, lists for transmission, load, and generator buses were created. The bus type, and real and reactive power at each node were used to differentiate between each type of node. The first step uses the bus type array from the bus matrix that numerically differentiates each node. A value of 1 indicates a load or transmission bus, and a value of 2 or 3 indicates a generator bus. A value of 3 specifies the slack bus. To differentiate between a transmission bus and a load bus, the real power and reactive power arrays from the bus matrix are used. Each array indicates the power injection or removal at each bus. For a transmission bus, there is no power injection or removal, therefore these elements will be equal to zero. If there is a value other than zero for either element, then it is a load bus. The code below shows the total number of transmission buses is calculated and a list of transmission buses is determined, but this step is not required.

```
t = 0;
l = 0;
g = 0;

for i = 1:node_max
   if bus(i,2) == 1
      if bus(i,3) == 0 && bus(i,4) == 0
         t = t + 1;
         tran_list(t) = bus(i,1);
      else
         l = l + 1;
         load_list(l) = bus(i,1);
      end
   else
      g = g + 1;
      gen_list(g) = bus(i,1);
   end
end
```

Distribution Factor (DF), Capacity (C), and ES Capacity (C_{es})

The distribution factor (DF) is an NNL matrix that determines the power flow between any node-node pair along any line. Based on equation 29, the following code calculates every node-node-line combination to calculate the DF.

$$a_l^{ij} = a_{li} - a_{lj} \tag{29}$$

```
a = zeros(node_max,node_max,line_max);

for i = 1:node_max
    for j = 1:node_max
        for l = 1:line_max
            a(i,j,l) = ptdf_adj(l,i) - ptdf_adj(l,j);
        end
    end
end
```

Once the DF has been calculated, the capacity can be calculated. The capacity matrix is a NN matrix. The equation to calculate shows the absolute value of the DF is performed, and the absolute value of the entire DF matrix is calculated prior to calculating the capacity.

$$C_i^j = MIN\left(\frac{P_1^{max}}{\left\|a_1^{ij}\right\|}, \cdots, \frac{P_l^{max}}{\left\|a_l^{ij}\right\|}, \cdots, \frac{P_L^{max}}{\left\|a_L^{ij}\right\|}\right) \tag{30}$$

Equation 28 shows that the minimum value is calculated, but the code shows the function max was used. Since the function max is in the denominator and that the maximum power for every line is the same, it calculates the same value as if it were to take the minimum of the entire value.

```
a = abs(a);
C = zeros(node_max,node_max);

p_max = 100;

for i = 1:node_max
    for j = 1:node_max
        C(i,j) = p_max/max(a(i,j,:));
    end
end
```

The ES capacity matrix is a GDN matrix. The matrix is a GDN matrix as opposed to a GND matrix because the author thought it might be easier to write the code in this manner. The code was written based on equation 29, shown below.

$$C_{es}^{ged} = \frac{E}{\frac{E}{C_1}+\frac{E}{C_2}} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{C_1 C_2}{C_1+C_2} \tag{31}$$

The main difference between this series of for loops and other for loops in this code is that it has to determine the elements corresponding to the capacity for each generator-node pair and node-load pair. Since the capacity matrix is an NN matrix, and ES capacity is a GDN matrix, an additional step needs to be added to complement the series of for loops to calculate the ES capacity. For each generator-node-load pair, the generator and load list are called to find the node number of each generator and load. This is done to find the correct capacity for each generator-node pair and node-load pair. After the corresponding node number has been determined for each generator node and load node, the ES capacity can be calculated. If the generator or load is equal to the node, then the capacity is set to zero, since there cannot be any capacity between the same node.

```
Ce = zeros(size_gen(1),size_load(1),node_max);

for g = 1:size_gen(1)
   for d = 1:size_load(1)
      for e = 1:node_max
         gg = gen_list(g);
         dd = load_list(d);
         if gg == e
            Ce(g,d,e) = 0;
         elseif dd == e
            Ce(g,d,e) = 0;
         else
            Ce(g,d,e) = 1 / (1/C(gg,e) + 1/C(e,dd));
         end
      end
   end
end
```

Admittance (Y), Impedance (Z), Equivalent Impedance (Z_e), and ES Impedance (Z_{es})

The admittance matrix is an NN matrix, and using the running example from figure 28, the admittance matrix will appear as shown below in figure 31. It is assumed that each transmission line has an admittance of 1.

$$\begin{pmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 & 1 \\ 1 & -3 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & 1 & 1 & -3 \end{pmatrix}$$

Figure 31: Admittance matrix (Y)

If node i equals j, then Y_{ii} is equal the negative sum of all the line admittances connected to node i. If node i does not equal j, then Y_{ij} equals the line admittance between node i and j. If the nodes are not directly connected, then Y_{ij} equals zero.

```
adm = zeros(node_max);

for i = 1:node_max
   for j = 1:node_max
      if i ~= j
         for a = 1:line_max
            if l_new(a,1) == i
               if l_new(a,2) == j
                  adm(i,j) = -l_new(a,4);
                  adm(j,i) = -l_new(a,4);
               end
            end
         end
      end
      if i == j
         for a = 1:line_max
            if l_new(a,1) == i || l_new(a,2) == i
               adm(i,i) = l_new(a,4) + adm(i,i);
            end
         end
      end
   end
end
```

After calculating the admittance matrix, the inverse of the admittance matrix equals the impedance matrix. The function pinv was required in this instance, as the function inv did not perform the inverse properly.

```
Z = pinv(adm);
```

The impedance matrix is a NN matrix, and is used to calculate the equivalent impedance between any node-node pair. The reasoning behind the equivalent impedance matrix being a NN matrix is the same reason why the DF and capacity matrices are NN matrices and not GD matrices. The equation and code to calculate the equivalent impedance are shown below.

$$Z_e^{ij} = \left(Z_{ii} - Z_{ij}\right) - \left(Z_{ij} - Z_{jj}\right) = Z_{ii} - 2Z_{ij} + Z_{jj} \tag{32}$$

```
for i = 1:node_max
   for j = 1:node_max
      Zequ(i,j) = Z(i,i) - 2*Z(i,j) + Z(j,j);
   end
end
```

The equivalent impedance is a NN matrix, and can now be used to calculate the ES impedance. The equation to calculate the ES impedance is shown below.

$$Z_{es}^{ged} = Z_g^e + Z_e^d \tag{33}$$

The ES impedance is the sum of the generator-ES and ES-load pair equivalent impedance. The ES impedance is a GDN matrix, and the same procedure to find the corresponding bus number for each generator and load node has to be determined. The code is shown below for the ES impedance.

```
Ze = zeros(size_gen(1),size_load(1),node_max);

for g = 1:size_gen(1)
   for d = 1:size_load(1)
      for i = 1:node_max
         gg = gen_list(g);
         dd = load_list(d);
         Ze(g,d,i) = 1/(Zequ(gg,i) + Zequ(i,dd));
      end
   end
end
```

It should be noted that equation 31 is the sum of the two equivalent impedances, while the code calculates the inverse of the sum. This change is rectified in the final calculation for ES net ability.

ES Net Ability (A_Y^e)

The ES net ability is a N1 matrix, and the formula is shown below.

$$A_Y^e = \frac{1}{N_G N_D} \sum_{g \in G} \sum_{d(d \neq g) \in D} C_{es}^{ged} \frac{1}{Z_{es}^{ged}} \tag{34}$$

The code functions by summing the ES impedance and ES capacity product of every generator-node-load combination. The product is used instead of the ratio because the inverse sum of the two generator-ES and ES-load impedances was used to calculate the ES impedance. After the sums have been calculated, the entire matrix is divided by the generator-load product.

```
na_sum(:,2) = zeros(node_max,1);

for e = 1:node_max
   for g = 1:size_gen(1)
      for d = 1:size_load(1)
         na_sum(e,2) = na_sum(e,2) + Ze(g,d,e) * Ce(g,d,e);
      end
   end
end

na_sum(:,2) = na_sum(:,2)/(size_gen(1)*size_load(1));
```

Betweenness *(T)*

Below is the equation for betweenness

$$T = \frac{1}{2N_G N_D} \sum_{g \in G} \sum_{d \in D} C_g^d \sum_{l \in L} \left| a_l^{gd} \right| \tag{35}$$

Below is the code for betweenness

```
sum_DF = zeros(size_gen(1),size_load(1),node_max);

for g = 1:size_gen(1)
   for d = 1:size_load(1)
      for l = 1:line_max
            gg = gen_list(g);
            dd = load_list(d);
            [node1,~] = find(bus(:,1) == branch(l,1));
            [node2,~] = find(bus(:,1) == branch(l,2));
            DF(gg,dd,l);
            sum_DF(g,d,node1) = sum_DF(g,d,node1) + abs(DF(gg,dd,l));
            sum_DF(g,d,node2) = sum_DF(g,d,node2) + abs(DF(gg,dd,l));
      end
   end
end

sum_DF;

multiplexity = zeros(size_gen(1),size_load(1),node_max);

for g = 1:size_gen(1)
   for d = 1:size_load(1)
      gg = gen_list(g);
      dd = load_list(d);
      C(gg,dd);
      multiplexity(g,d,:) = C(gg,dd) * sum_DF(g,d,:);
   end
end

multiplexity = multiplexity/(2*size_gen(1)*size_load(1));

bet = zeros(node_max,1);

for g = 1:size_gen(1)
   for d = 1:size_load(1)
      for e = 1:node_max
         bet(e) = multiplexity(g,d,e) + bet(e);
      end
   end
end
```

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