

浙江大學



The Project Report of Experiment 1

Author Name: Zong Wei Xu

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Chapter 1 Problem Introduction

The problem given in Laboratory project 1 is Performance Measurement (POW). We could use different Algorithms to compute X^N (N is a positive integer). However, different Algorithms have different complexities. To measure the performance of a function, we use C's standard library `time.h` to measure the performance of each Algorithms and evaluate the efficiency of each Algorithm.

Chapter 2 Algorithm Specification

We design three Algorithms in total to solve this problem.

Algorithm 1

the Implementation of Algorithm 1

According to the definition of power, It's easy to implement by using $N - 1$ multiplications.

```

1 double result = 1;
2 for(int i = 0; i < n; i++){
3     result = x * result;
4 } // the result we finally get is the Nth power of x.

```

the Complexity of Algorithm 1

It's obvious that the complexity of Algorithm 1 is $O(N)$.

Algorithm 2

the Implementation of Algorithm 2

We can use recursion to solve this problem. If N is even, $X^N = X^{N/2} * X^{N/2}$. If N is odd, $X^N = X^{(N-1)/2} * X^{(N-1)/2} * X$. Consequently, we define $X = 0$, $X = 1$ as **base case**. Then we can get the value of X^N by calculate the value of $X^{N/2}$.(making progress)

the Complexity of Algorithm 2

If we use M times division. We can conclude that M must be less than $\log_2 N$ (when $2^M = N$). Meanwhile, we do almost the same times multiplication. Consequently, the complexity of Algorithm 2 is $O(\log N)$.

```

1 # Base case: n = 0, n = 1
2 if (n == 0) return 1;
3 if (n == 1) return 0;
4 # making progress:
5 if (n % 2) {
6     double re = PowRecursive(x, n / 2);
7     return re * re * x;
8 } else {
9     double re = PowRecursive(x, n / 2);
10    return re * re;
11 }

```

Note: You can see that our code is different from that mentioned in the textbook. Both of the methods use the same times multiplications and divisions. Because we use variable **re** to save the result of recursive function, our complexity is still $O(\log N)$.

Algorithm 3

The Adventure of Algorithm 3

we can say that the Algorithm 2 is an effective Algorithm. However, when we do recursion, the computer push the function into stack which contains not only the parameters we use, but also the necessary information about the function. for example:

```

1 ;we use the example in assembly language
2 ;when we push the function into stack
3 push ebp ;stack base
4 mov esp, ebp ;initialize the top of stack
5 ... ;deal with the function by using the stack,such as the information of the
   function...
6 mov esp, ebp
7 pop ebp
8 ret ;the recursion finish

```

And obviously that it leads to unnecessary time and space allocation. Therefore, we can create a stack to record the parity of N during each circulation. As we only store the "state" of N , we don't need to spare time and space to deal with the function.

the Implementation of Algorithm 3

First of all, we create a stack to store the information. It's safe that the biggest number we can deal is $X^{2^{100}}$. During the process, we mod 2 and push the remainder into stack to record the parity of N . It keeps to compute until $n = 0$.

```

1 while (n) {
2     *(p++) = n % 2;
3     n /= 2;
4 }

```

Then we get the result by pop the result in stack, which determine the way of multiplication.

```

1 while (p > stack) {
2     if (*(--p)) result = result * result * x;
3     else result *= result;
4 }

```

And we don't need to explicitly consider the base case of $n = 0$ and $n = 1$.

the Complexity of Algorithm 3

The Algorithm 3 do the multiplication and division as much as Algorithm 2. Thus, the complexity is still $O(\log N)$. However, we assume that Algorithm 3 is more efficient than 2. The iterative one optimized the memory consumption - only the data are loaded into memory.

Chapter 3 Testing Results

the Test Introduction

To confirms our hypothesis' validity, we can use clock to record the ticks when we run the function. We choose the condition: $X = 1.0001$ and $N = 1000, 5000, 10000, 20000, 40000, 60000, 80000, 100000$. As the function takes less than a tick to finish, we repeat the function for K times to obtain a total run time, and then divide the total time by K to obtain a more accurate duration for a single run of the function. According to the total time we test, we choose the K as 10000 for Algorithm 1 and 1000000 for the others. We guarantee the K is big enough so that the tick is at least 10.

the Test Code

```
1 static clock_t start, stop;
2 static clock_t ticks[8];
3 static int iternum[3] = {10000, 1000000, 1000000};
4 static int aryN[8] = {1000, ... ,100000};
```

We use the array `aryN` to save the `N` we use to test. `iternum` save the `K` times we run the function. And `ticks` save the time during we run the function for `K` times.

Then, we can run the three function and save the ticks when we run the function for `K` times. After that, we define function *printResult* to print the time for function. (Mark represent the index of each function.) We export the result as markdown files. The detail about *printResult* is omitted.

```
1 for (int loopFunc = 0; loopFunc < 3; loopFunc++) { //loop for 3 times to
test the 3 algorithms
2     for (int i = 0; i < 8; i++) {
3         start = clock();
4         for (int j = 0; j < iternum[loopFunc] - 1; j++)
5             pFunc[loopFunc](testBase, aryN[i]); //pFunc[] is an array of
function pointer
6         result = pFunc[loopFunc](testBase, aryN[i]);
7         stop = clock();
8         printf("%g\n", result);
9         ticks[i] = stop - start;
10    }
11    printResult(loopFunc + 1);
12 }
```

the Test Result

We test to get result under different compilation environment.

Debugx64_clang

	N	1000	5000	10000	20000	40000	60000	80000	100000
Algorithm1	Iterations(K)	10000	10000	10000	10000	10000	10000	10000	10000
	Ticks	38	164	339	700	1113	1615	2214	2614
	Total Times(millisecond)	38.000000	164.000000	339.000000	700.000000	1113.000000	1615.000000	2214.000000	2614.000000
	Duration(nanosecond)	3800.000000	16400.000000	33900.000000	70000.000000	111300.000000	161500.000000	221400.000000	261400.000000
Algorithm2	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	94	124	134	146	190	168	178	179
	Total Times(millisecond)	94.000000	124.000000	134.000000	146.000000	190.000000	168.000000	178.000000	179.000000
	Duration(nanosecond)	94.000000	124.000000	134.000000	146.000000	190.000000	168.000000	178.000000	179.000000
Algorithm3	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	104	145	152	177	177	178	180	181
	Total Times(millisecond)	104.000000	145.000000	152.000000	177.000000	177.000000	178.000000	180.000000	181.000000
	Duration(nanosecond)	104.000000	145.000000	152.000000	177.000000	177.000000	178.000000	180.000000	181.000000

Debugx64_gcc

	N	1000	5000	10000	20000	40000	60000	80000	100000
Algorithm1	Iterations(K)	10000	10000	10000	10000	10000	10000	10000	10000
	Ticks	40	172	268	548	1342	1615	2019	2569
	Total Times(milisecond)	40.000000	172.000000	268.000000	548.000000	1342.000000	1615.000000	2019.000000	2569.000000
	Duration(nanosecond)	4000.000000	17200.000000	26800.000000	54800.000000	134200.000000	161500.000000	201900.000000	256900.000000
Algorithm2	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	65	95	90	96	101	105	109	112
	Total Times(milisecond)	65.000000	95.000000	90.000000	96.000000	101.000000	105.000000	109.000000	112.000000
	Duration(nanosecond)	65.000000	95.000000	90.000000	96.000000	101.000000	105.000000	109.000000	112.000000
Algorithm3	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	52	61	64	79	78	89	101	93
	Total Times(milisecond)	52.000000	61.000000	64.000000	79.000000	78.000000	89.000000	101.000000	93.000000
	Duration(nanosecond)	52.000000	61.000000	64.000000	79.000000	78.000000	89.000000	101.000000	93.000000

Debugx64_VisualStudio

	N	1000	5000	10000	20000	40000	60000	80000	100000
Algorithm1	Iterations(K)	10000	10000	10000	10000	10000	10000	10000	10000
	Ticks	38	144	320	599	1102	1613	2147	2611
	Total Times(milisecond)	38.000000	144.000000	320.000000	599.000000	1102.000000	1613.000000	2147.000000	2611.000000
	Duration(nanosecond)	3800.000000	14400.000000	32000.000000	59900.000000	110200.000000	161300.000000	214700.000000	261100.000000
Algorithm2	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	185	250	281	288	301	322	329	325
	Total Times(milisecond)	185.000000	250.000000	281.000000	288.000000	301.000000	322.000000	329.000000	325.000000
	Duration(nanosecond)	185.000000	250.000000	281.000000	288.000000	301.000000	322.000000	329.000000	325.000000
Algorithm3	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	78	88	95	110	104	104	107	109
	Total Times(milisecond)	78.000000	88.000000	95.000000	110.000000	104.000000	104.000000	107.000000	109.000000
	Duration(nanosecond)	78.000000	88.000000	95.000000	110.000000	104.000000	104.000000	107.000000	109.000000

Debugx86_VisualStudio

	N	1000	5000	10000	20000	40000	60000	80000	100000
Algorithm1	Iterations(K)	10000	10000	10000	10000	10000	10000	10000	10000
	Ticks	37	140	333	561	1047	1570	2179	2623
	Total Times(milisecond)	37.000000	140.000000	333.000000	561.000000	1047.000000	1570.000000	2179.000000	2623.000000
	Duration(nanosecond)	3700.000000	14000.000000	33300.000000	56100.000000	104700.000000	157000.000000	217900.000000	262300.000000
Algorithm2	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	214	269	291	333	338	336	370	349
	Total Times(milisecond)	214.000000	269.000000	291.000000	333.000000	338.000000	336.000000	370.000000	349.000000
	Duration(nanosecond)	214.000000	269.000000	291.000000	333.000000	338.000000	336.000000	370.000000	349.000000
Algorithm3	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	73	88	90	98	104	111	105	105
	Total Times(milisecond)	73.000000	88.000000	90.000000	98.000000	104.000000	111.000000	105.000000	105.000000
	Duration(nanosecond)	73.000000	88.000000	90.000000	98.000000	104.000000	111.000000	105.000000	105.000000

Releasex64_VisualStudio

	N	1000	5000	10000	20000	40000	60000	80000	100000
Algorithm1	Iterations(K)	10000	10000	10000	10000	10000	10000	10000	10000
	Ticks	18	76	118	264	455	705	953	1140
	Total Times(milisecond)	18.000000	76.000000	118.000000	264.000000	455.000000	705.000000	953.000000	1140.000000
	Duration(nanosecond)	1800.000000	7600.000000	11800.000000	26400.000000	45500.000000	70500.000000	95300.000000	114000.000000
Algorithm2	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	20	31	34	35	38	43	39	41
	Total Times(milisecond)	20.000000	31.000000	34.000000	35.000000	38.000000	43.000000	39.000000	41.000000
	Duration(nanosecond)	20.000000	31.000000	34.000000	35.000000	38.000000	43.000000	39.000000	41.000000
Algorithm3	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	18	23	28	29	31	32	33	33
	Total Times(milisecond)	18.000000	23.000000	28.000000	29.000000	31.000000	32.000000	33.000000	33.000000
	Duration(nanosecond)	18.000000	23.000000	28.000000	29.000000	31.000000	32.000000	33.000000	33.000000

Releasex86_VisualStudio

	N	1000	5000	10000	20000	40000	60000	80000	100000
Algorithm1	Iterations(K)	10000	10000	10000	10000	10000	10000	10000	10000
	Ticks	15	77	126	253	517	759	922	1146
	Total Times(millisecond)	15.000000	77.000000	126.000000	253.000000	517.000000	759.000000	922.000000	1146.000000
	Duration(nanosecond)	1500.000000	7700.000000	12600.000000	25300.000000	51700.000000	75900.000000	92200.000000	114600.000000
Algorithm2	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	24	31	35	38	42	42	43	44
	Total Times(millisecond)	24.000000	31.000000	35.000000	38.000000	42.000000	42.000000	43.000000	44.000000
	Duration(nanosecond)	24.000000	31.000000	35.000000	38.000000	42.000000	42.000000	43.000000	44.000000
Algorithm3	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
	Ticks	22	27	28	30	30	31	33	40
	Total Times(millisecond)	22.000000	27.000000	28.000000	30.000000	30.000000	31.000000	33.000000	40.000000
	Duration(nanosecond)	22.000000	27.000000	28.000000	30.000000	30.000000	31.000000	33.000000	40.000000

Chapter 4 Analysis and Comments

the result Analysis

First of all, from the result(Duration) we get, we can evaluate the complexity of the three Algorithms. Under VisualStudioDebugx64 condition, for example, We can see that as N grow in multiples, the duration of Algorithm 1 increase in linear while Algorithm 2 and Algorithm 3 in logarithmic growth. The function images in Figure 4.1 show the growth trends of each Algorithm. (Note: Algorithm 1's time unit is 10^2 ns)

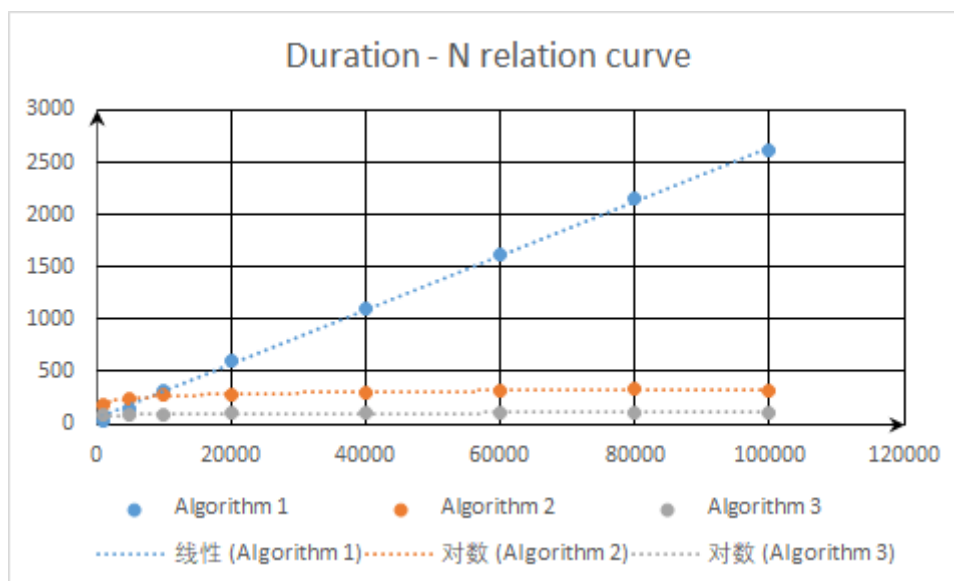


figure 4.1

Then, from the image above, we can compare the result between different Algorithm. It's obvious that as N become big enough, it takes a long time for Algorithm 1 to compute the result of X^N . Compare with Algorithm 1, Algorithm 2 and Algorithm 3 seems to grow in much slower trends. As we assume, we can see that the Algorithm 3 is more efficient than the other two Algorithms. We assume that Algorithm 3 is faster than 2 as it consume less space and time. However, during our test, we find that the condition different when we run the function under clangDebugx64 condition.(show in figure 4.2)

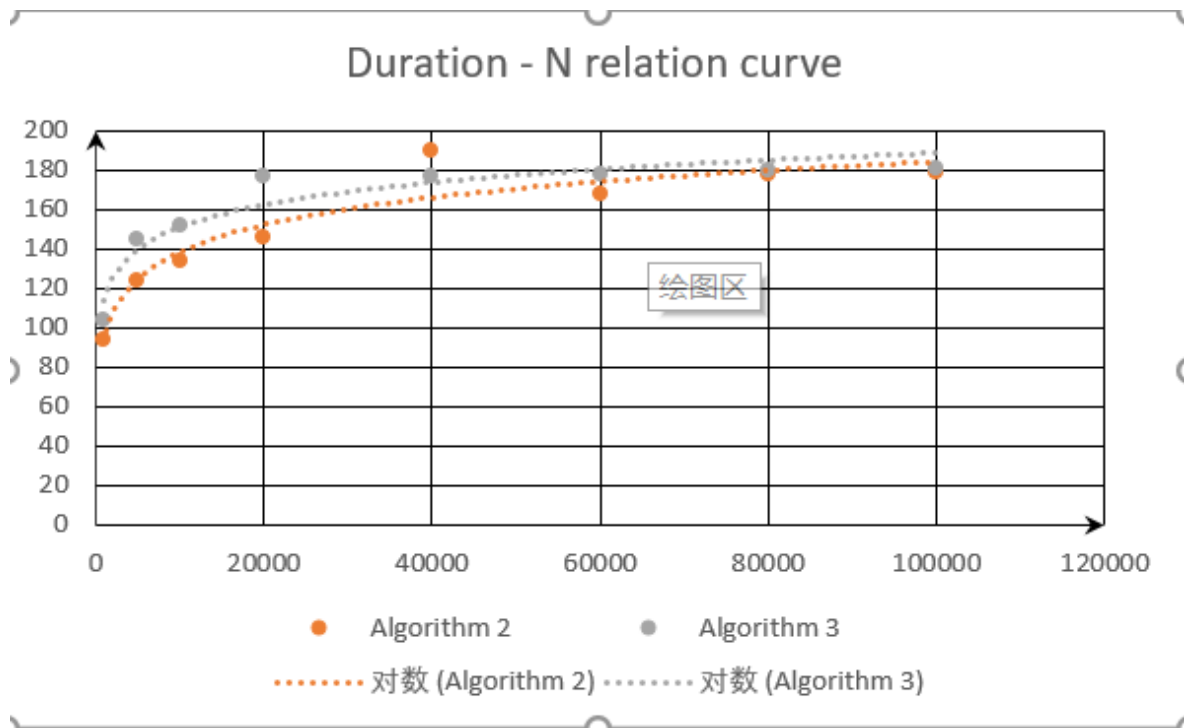


figure 4.2

We can see that if we use clang to compile, the advantage of Algorithm 3 is not so obvious. In other words, it sometimes even slower than Algorithm 2. We can't make out the exact reason of this. We assume that maybe clang do some optimization when it do recursion.

And obviously, the duration we get from test varies with the compiler environment we use. For example, under the debug mode, gcc performs faster than the other compilers.

Improvement

Our Advantage

We use features in markdown to print out table automatically so as to make high magnitude of TEST possible. For example, we can print out the result under certain compilation enviroment in format as follow:

```

1 | | N | 1000 | 5000 | 10000 | 20000 | 40000 | 60000 | 80000 | 100000 |
2 | :--: | :--: | :--: | :--: | :--: | :--: | :--: | :--: | :--: |
3 | Algorithm1 | Iterations(K) | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
4 | Classic | Ticks | 4219 | 14796 | 29018 | 58639 | 112193 | 170814 | 218157 |
5 | | Total Times(ms) | 4.219 | 14.796 | 29.018 | 58.639 | 112.193 | 170.814 | 218.157 |
6 | | Duration(ns) | 4219 | 14796 | 29018 | 58639 | 112193 | 170814 | 218157 |
7 | Algorithm2 | Iterations(K) | 1000000 | 1000000 | 1000000 | 1000000 | 1000000 | 1000000 |
8 | Recursive | Ticks | 93505 | 129289 | 152644 | 171364 | 189981 | 174747 |
9 | | Total Times(ms) | 93.505 | 129.289 | 152.644 | 171.364 | 189.981 | 174.747 |

```


10			Duration(ns)		93.505		129.289		152.644		171.364		189.981		174.747
					182.554		179.645								
11		Algorithm3		Iterations(K)		1000000		1000000		1000000		1000000		1000000	
						1000000		1000000		1000000		1000000			
12		Iterative		Ticks		105405		138466		149387		162006		171425	
						181395		183903							
13			Total Times(ms)		105.405		138.466		149.387		162.006		171.425		
						171.617		181.395		183.903					
14			Duration(ns)		105.405		138.466		149.387		162.006		171.425		
						171.617		181.395		183.903					

Then a markdown builder can turn the .md file into a beautified table and make the result easier to read an compare.

Further Improvement

In fact, We design Algorithm 4, which is a more efficient implementation of Iteration. As we know, if we want to use iteration to replace recursion, we always can use "stack" to implement, which is sometimes better than iteration. However, for this problem, it's obvious that we should reduce the complexity by do less operation.(multipation and division) And if we use a variable **ans** to store the result of multiplication instead of stack while we do division of n, it will cost less space than Algorithm 3. We also test this result. Because our code is structured, our optimization goes smoothly. Here is the test result.

compiled by clang

	N	1000	5000	10000	20000	40000	60000	80000	100000
Algorithm1	Iterations(K)	1000	1000	1000	1000	1000	1000	1000	1000
Classic	Ticks	0	1	1	3	6	7	10	13
	Total Times(ms)	0	1	1	3	6	7	10	13
	Duration(ns)	0	1000	1000	3000	6000	7000	10000	13000
Algorithm2	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
Recursive	Ticks	22	27	30	33	34	37	38	49
	Total Times(ms)	22	27	30	33	34	37	38	49
	Duration(ns)	22	27	30	33	34	37	38	49
Algorithm3	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
Iterative	Ticks	18	22	23	25	26	28	27	29
	Total Times(ms)	18	22	23	25	26	28	27	29
	Duration(ns)	18	22	23	25	26	28	27	29
Algorithm4	Iterations(K)	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000
IterativeBetter	Ticks	8	10	10	12	13	13	14	13
	Total Times(ms)	8	10	10	12	13	13	14	13
	Duration(ns)	8	10	10	12	13	13	14	13

We can see that the Algorithm4 performs better than other Algorithm. And the complexity of Algorithm 4 is still $O(n)$.(show in figure 4.3)

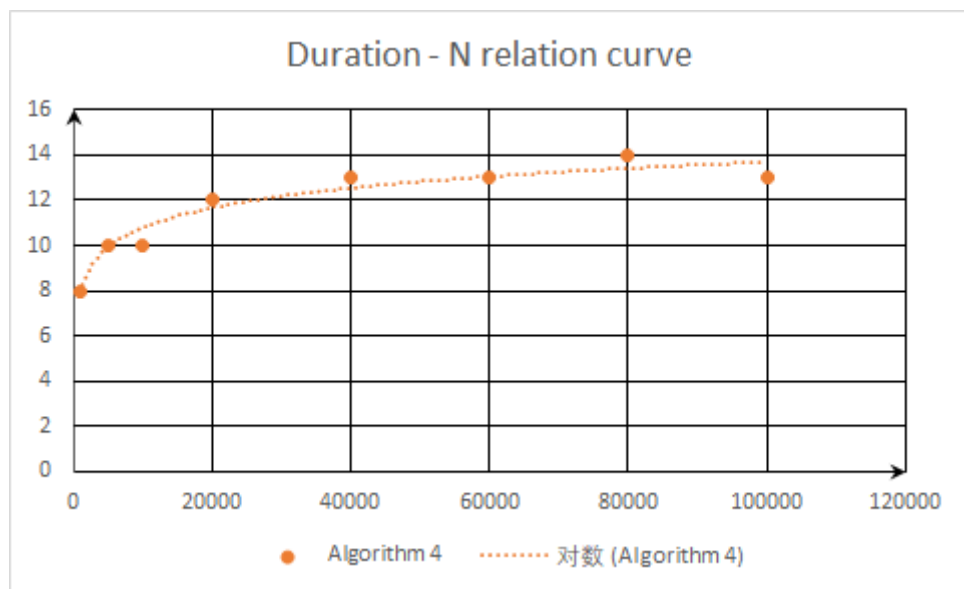


figure 4.3

Declaration

We hereby declare that all the work done in this project titled "The project report of experiment 1" is of our independent effort as a group.

Duty Assignment

programmer: Xu Zhen 3180105504

Tester: Chen Xiyao 3180103012

Report Writer: Zong Weixu 3180102776