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Kingdom Built on a Pile of Sand:Slow and Steady

Contents

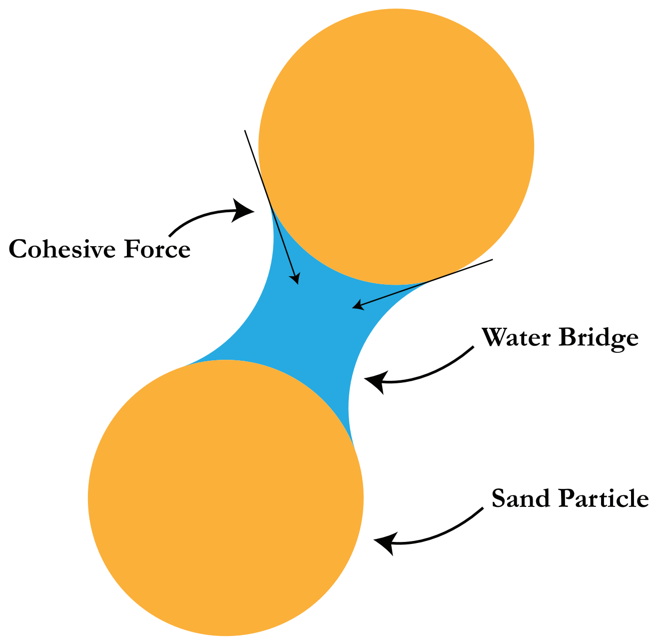

Figure 1: Water Bridge Between Sand Particle

1 Introduction

1.1 Problem Background

Sunshine, clear blue sea and golden color sand always seem to leave people in a happy state of mind. And a beach is where these three are combined, drawing people all around towards it. Sand, the granular matter formed by constant brushing of flowing water, however, can react with water in a different way, despite the fact that people refer to it as non-stable or unreliable. On a beach, where the already formed granular sand and the rise and fall of sea wave lies together, a new buff can be added to our flowing friend, a wetted state.

Magically but not randomly, sand gets sticky when combined with water, due to the most obvious physical theorem: surface tension and atmosphere pressure. From previous people's work, we've know that this buff comes from the water bridge formed between sand particles, which can significantly cluster together during the increasing water-sand portion[1]–[3]. Kudrolli and Arshad has visualized the bridge between sand like Figure 1.

And that's the magic that glue our favorite sand castle together, which is one of the most entertainment for enthusiastic beach goers. However, being built near the water that melts mud and our wet sand, all sand castles have to face the fate that they'll g et too wet to hold its own weight and the impact from sea waves. That's because the water bridge has another property of clustering together[3]. When you throw a pile of sand into water, they behave just as when they're completely dry, melting down like fluid. That is, sand castles lasts for only a period of time. Thus beach castle builder might want to make their sand castle last longer than those build arbitrarily than others, which is also the purpose of this article.

1.2 Our Work

Normally people would sculpt their kingdom from a pile of tedious wet sand. To simplify the model and grasp main threads, we'll focus our research on this single, nondescriptive mound of wetted sand. In this article, we develop a method for determining the most long-lasting three dimensional shape for a tedious sand castle, which will be later referred to as *sand castle foundation*.

Our model addresses the problem of constructing a three dimensional structure by projection, which makes the construction of individual part of our model easy to implement. Firstly we determine the optimal *side view* of our *sand castle foundation* by analysing and extracting information from similar research on wet granular material[2]. Then we construct the best *top view* of the *sand castle foundation* by constructing a **Cellular Automata**.

Side Slope Determination You've probably seen questions about the slope of a pile of dry sand in junior high school practice books. In our research, we address the problem about dry sand as a normal junior high student would. However, the analysis of wet sand requires some technique beyond low level physics. In our research, we establish this model using **Mohr-Coulomb Criterion**.

Top View Determination Our model analyses the impact of sea wave flow from a top view, or a *slice* of the *sand castle foundation*. Then we can combine the sliced impaction result together with the *side slope* analysis to form a possible 3D object. Inside our model we construct a **Cellular Automata** since the shape of the sand slice edge isn't as obvious the side slope may be, that is, it's merely possible to construct continuous function on the shape.

Other Factors It's worth noting that the interaction between sand and sea wave is far a lot more complex than direct force impact or sand-to-water mixture proportion change. Some other factors include: gravity of sea water that submerge the sand underneath, influx of weaker sea wave and the gravity of water inside the wetted sand. During the assembly of two views, we'll address these other factors that may influence a *sand castle foundation*'s life span.

2 Assumptions & Nomenclature

2.1 Assumptions

2.2 Nomenclature

3 Modeling Under Waves and Tides

The 3-dimensional shape constructing problem is divided into two sub-problems. We first establish the model using the **Mohr-Coulomb Criterion** to decide the shape of the slope. We then construct another model with a modified version of **Cellular Automata** to determine the best shape viewed from the top,...

3.1 Shape of the Slope: Mohr-Coulomb Criterion

We begin by determining the side shape of the sand castle foundation. Before Approaching the problem, we will briefly address the property of sand as a granular media.

In our assumptions, sand particle are considered as a simplified model of identical tiny spheres. If we zoom in to observe a pile of wet sand, there are the so-called liquid bridges formed between sand particles.

Various water contents produce different liquid bridge distributions, which will influence the properties of sand.

Table 1: States of Sand

Liquid Content	State	Description	left for future use
No	Dry	000	000
Small	Pendular	000	000
Middle	Funicular	000	000
Almost saturated	capillary	000	000
More	Slurry	000	000

When the wave hits the foundation, the surface area is in slurry state and there exists no cohesive interaction between the particles, which makes it very hard, if not impossible, to prevent sand loss. Nevertheless, collapses after the wave resides can cause more harm to the foundation, which can be avoided by alternating the shape.

For dry sand, the failure criterion is given in terms of the shear stress τ, the normal compressible stress σ and the internal friction μ as

$$\tau > \mu\sigma$$

This is simply the friction formula with different notations. Now we consider a sandpile with a normal adhesive stress s_A across every plane, in addition to the stress caused by weight. The equation(1) is then modified as

$$\tau > \mu(\sigma + s_A)$$

This criterion is the so-called **Mohr-Coulomb criterion**.The stress resulting from the weight above the plane is shown in figure (1). Denote τ_f and μ_f as the shear stress and normal compressible stress at the failure plane,they can be written as

$$\tau_f = \rho g D sin\theta_c \qquad and \qquad \sigma_f = \rho g D cos\theta_c$$

where θ_c is the critical angle, D is the height of the sandpile and ρ is the density of sand. Therefore, to solve for θ_c is to solve the equation

$$\mu = tan\theta_c(D)\left(1 + \frac{s_A}{\rho g D cos\theta_c(D)}\right)$$

The only unknown factor is s_A, the adhesive stress across the plane. According to (Thomas C.H and Alex J.L -fix later)'s study, the value of s_A is determined by water content and there are three regimes as a function of the added-fluid volume. We now focus only on the state where the water content is close to saturated.

3.2 Top View Shape

3.3 Calculating Results

3.4 Simulating Results

4 Modeling Under Rain

5 Determine the Best Sand-to-water Proportion

6 Other Ways to Make Our Sand Castle Last Longer

7 Sensitivity Analysis

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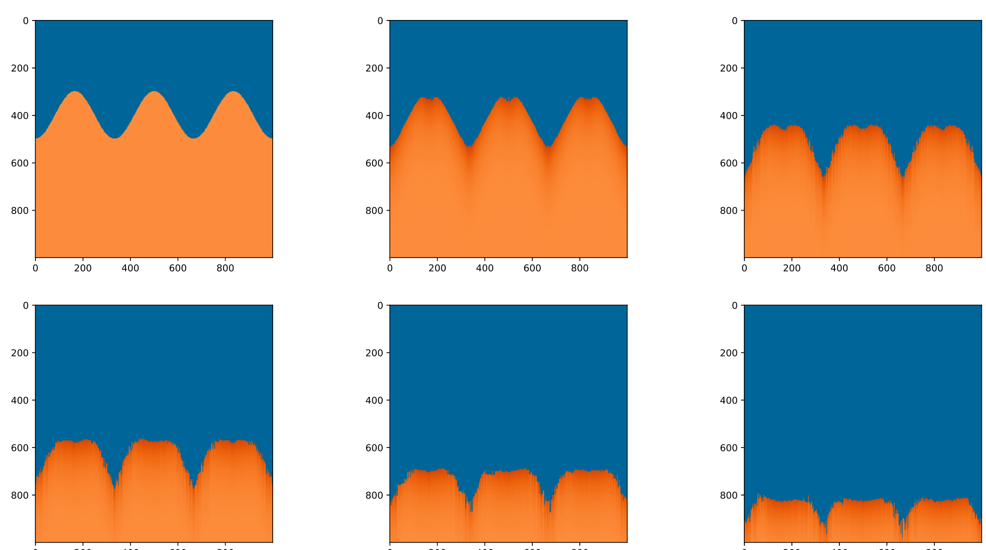

Figure 2: A Windows Terminal.

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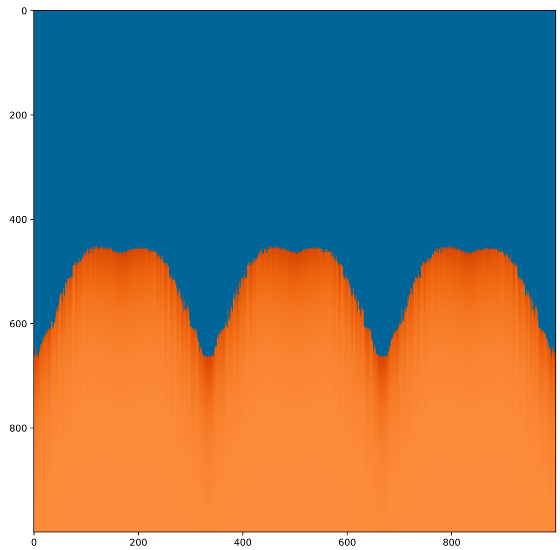

Figure 3: A Windows Terminal.

This is some example text[1].
I'm referring to footnote 1.

[1]Hello footnote

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References

[1] M. Pakpour, M. Habibi, P. Møller, and D. Bonn, "How to construct the perfect sandcastle," *Scientific reports*, vol. 2, no. 1, pp. 1–3, 2012.

[2] N. Mitarai and F. Nori, "Wet granular materials," *Advances in Physics*, vol. 55, no. 1-2, pp. 1–45, 2006.

[3] A. Kudrolli, "Sticky sand," *Nature materials*, vol. 7, no. 3, pp. 174–175, 2008.