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Kingdom Built on a Pile of Sand:Slow and Steady

Contents

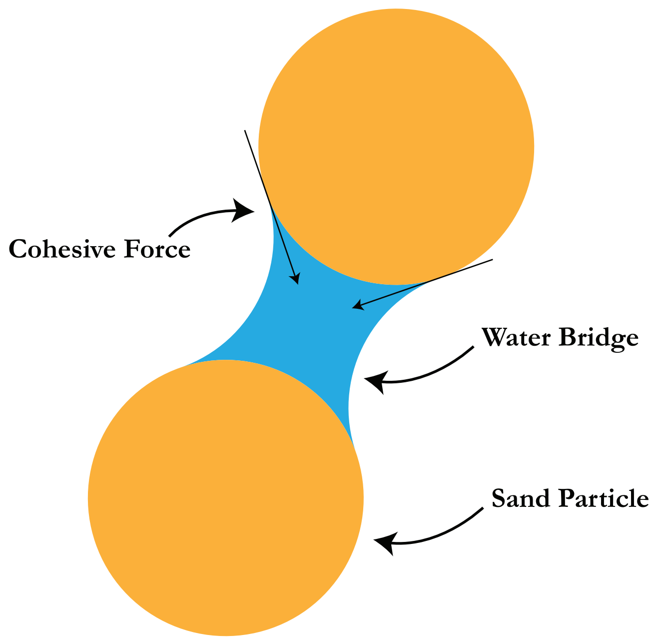

Figure 1: Water Bridge Between Sand Particle

1 Introduction

1.1 Problem Background

Sunshine, clear blue sea and golden color sand always seem to leave people in a happy state of mind. And a beach is where these three are combined, drawing people all around towards it. Sand, the granular matter formed by constant brushing of flowing water, however, can react with water in a different way, despite the fact that people refer to it as non-stable or unreliable. On a beach, where the already formed granular sand and the rise and fall of sea wave lies together, a new buff can be added to our flowing friend, a wetted state.

Magically but not randomly, sand gets sticky when combined with water, due to the most obvious physical theorem: surface tension and atmosphere pressure. From previous people's work, we've know that this buff comes from the water bridge formed between sand particles, which can significantly cluster together during the increasing water-sand portion[**pakpour2012construct**, **mitarai2006wet**, **kudrolli2008sticky**]. Kudrolli and Arshad has visualized the bridge between sand like Figure 1.

And that's the magic that glue our favorite sand castle together, which is one of the most entertainment for enthusiastic beach goers. However, being built near the water that melts mud and our wet sand, all sand castles have to face the fate that they'll get too wet to hold its own weight and the impact from sea waves. That's because the water bridge has another property of clustering together[**kudrolli2008sticky**]. When you throw a pile of sand into water, they behave just as when they're completely dry, melting down like fluid. Thus beach castle builder might want to make their sand castle last longer than those build arbitrarily than others, which is also the purpose of this article.

1.2 Our Work

Normally people will sculpt their kingdom from a pile of tedious wet sand. To simplify the model and grasp main threads, we'll focus our research on this single, nondescriptive mound of wetted sand.

2 Assumptions & Nomenclature

2.1 Assumptions

Several assumptions are made in order to simplify our model.

1) The weather condition is suitable for sand castle building, namely, there is no wind and...
2) The height of wave is the same, which we set as...
3) Our optimization goal is to acquire the largest platform size while minimizing the lost of sand.
4) Sand grains are regarded as tiny spheres. In our foundation, these grains are of the same size and closely packed.
5) The top of the foundation is a flat platform whose surface is horizontal.
6) ...

2.2 Nomenclature

Table 1: States of Sand

Symbol	Meaning
τ	the shear stress on a plane
τ_f	the shear stress on the failure plane
μ	the internal friction coefficient
σ	the normal compressible stress on a plane
σ_f	the normal compressible stree on the failure plane
ρ	the density of sand
θ_c	the critical angle of a sandpile
D	the height from the top of the sand pile
P	the pressure[]
Γ	the surface tension of water
s_A	the adhesive stress
f_A	the adhesive force
V	the total amount of fluid present per particle contact
R	the radius of particle

3 Modeling Under Waves and Tides

The 3-dimensional shape constructing problem is divided into two subproblems. We first establish the model using the **Mohr-Coulomb Criterion** to decide the shape of the slope. We then construct another model with a modified version of **a modified version of Cellular Automata** to determine the best shape viewed from the top,...

3.1 Shape of the Slope: Mohr-Coulomb Criterion

We begin by determining the Side shape of the sand castle foundation. Before Approaching the problem, we will briefly address the property of sand as a granular media.

In our assumptions, sand particles are considered as identical tiny spheres. If we zoom in to observe a pile of wet sand, there are the so-called liquid bridges formed between sand particles.

Various water contents produce different liquid bridge distributions, which will influence the properties of sand.

Table 2: States of Sand

Liquid Content	State	Description	left for future use
No	Dry	000	000
Small	Pendular	000	000
Middle	Funicular	000	000
Almost saturated	capillary	000	000
More	Slurry	000	000

When the wave gets into contact with the foundation, the surface area is in slurry state and there exists no cohesive interaction between the particles, which makes it very hard, if not impossible, to prevent sand loss. Nevertheless, collapses after the wave resides can cause more harm to the foundation, which can be avoided by alternating the shape.

For dry sand, the failure criterion is given in terms of the shear stress τ, the normal compressible stress σ and the internal friction μ as

$$\tau > \mu\sigma$$

This is simply the friction formula with different notations. For wet sand, we consider a sandpile with a normal adhesive stress s_A across every plane, in addition to the stress caused by weight. The equation(1) is then modified as

$$\tau > \mu(\sigma + s_A)$$

This is the so-called **Mohr-Coulumb criterion**. The stress resulting from the weight above the plane is shown in figure(2). Denote τ_f and μ_f as the shear stress and normal compressible stress at the failure plane, it is obvious from the schematic that they can be written as

$$\tau_f = \rho g D sin\theta_c \qquad and \qquad \sigma_f = \rho g D cos\theta_c$$

where θ_c is the critical angle, D is the height of the sandpile and ρ is the density of sand. Therefore, combine the equations above, to solve for θ_c is to solve the equation

$$\mu = tan\theta_c(D)\left(1 + \frac{s_A}{\rho g D cos\theta_c(D)}\right)$$

The only unknown factor is s_A, the adhesive stress across the plane. According to (Thomas C.H and Alex J.L -fix later)'s study, the value of s_A is determined by water content and there are three regimes as a function of the added-fluid volume. We now focus only on the state where the water content is close to saturation.

In this case, with water serves as lubricate, it makes sense to model sand particles as frictionless spheres. The pressure difference is then given by []

$$P = -\frac{\Gamma}{\sqrt{V/2\pi R}}$$

where Γ is the surface tension of the fluid, V is the total amount of fluid present per particle contact, and R the radius of the particle. The adhesive force is given by

$$f_A = 2\pi\Gamma R$$

Note that (with certain conditions like distance between grains remains constant)the term V which denotes the volume of liquid does not appear in equation(6).

Assume our foundation contains sand particles that are closely packed. Such structure means there is an average of 6 contacts per square.

3.2 Top View Shape

We used a modified version of **cellular automata** to simulate the impact of tides and waves. There are several factors that are taken into consideration:

1) ...

2) ...

3) ...

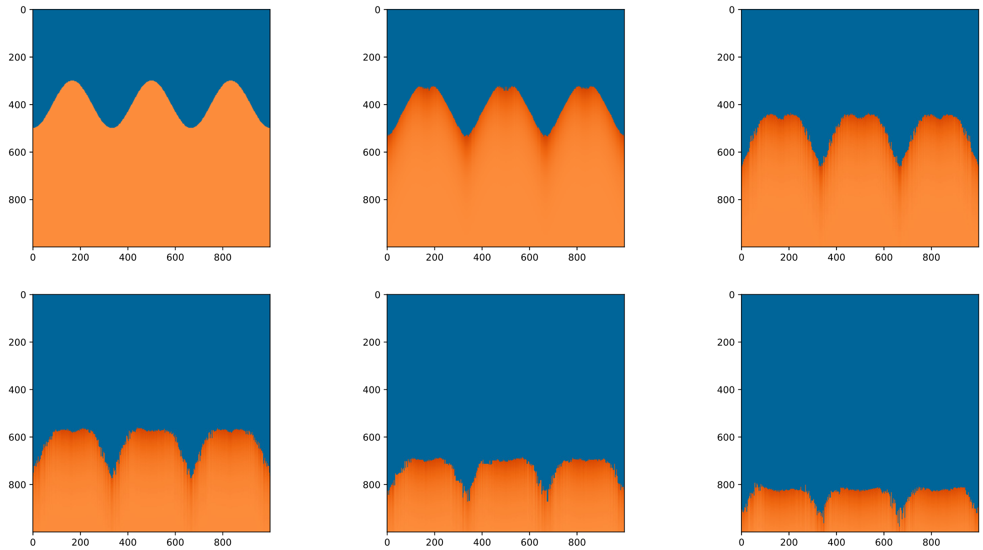

Figure 2: A Windows Terminal.

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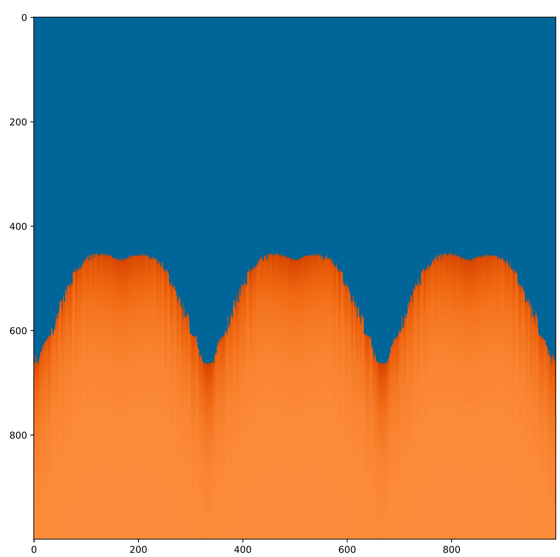

Figure 3: A Windows Terminal.

3.3 Calculate & Simulate Results

Briefly review the equation derived in section 3.1

$$tan\theta_c = \mu + \frac{\sqrt{2}\pi\mu\Gamma}{R\rho g D} sec[tan^{-1}(k)]$$

To solve for the best shape of the slope, the necessary information about sand is listed below

Table 3: States of Sand

Physical Quantities	Values	Units
Γ	72.8	mN/m
μ	0.55 0.60	-
ρ	1631	kg/m^3
R	0.05 2	mm
g	9.81	m/s^2

It is not sensible to embark on a trip to the beach on a stormy day, so we assume that the near-shore wave is gentle, with its height not exceeding 20cm, or $D \in [0, 20](unit : cm)$. We first assume that..

4 Modeling Under Rain

5 Determine the Best Sand-to-water Proportion

6 Other Ways to Make Our Sand Castle Last Longer

7 Sensitivity Analysis

8 Strengths and Weaknesses

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I'm referring to footnote 1.

[1]Hello footnote

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