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## 2.1 Assumptions

## 2.2 Nomenclature

# 3 Modeling Under Waves and Tides

The 3-dimensional shape constructing problem is divided into two subproblems. We first establish the model using the **Mohr-Coulomb Criterion** to decide the shape of the slope. We then construct another model with a modified version of **Cellular Automata** to determine the best shape viewed from the top,...

## 3.1 Shape of the Slope: Mohr-Coulomb Criterion

We begin by determining the Side shape of the sand castle fundation. Before Approaching the problem, we will briefly address the property of sand as a granular media.

In our assumptions, sand particle are considered as a simplified model of identical tiny spheres. If we zoom in to observe a pile of wet sand, there are the so-called liquid bridges formed between sand particles.

Various water contents produce different liquid bridge distributions, which will influence the properties of sand.

Table 1: States of Sand

| Liquid Content | State | Description | left for future use |
|---|---|---|---|
| No | Dry | 000 | 000 |
| Small | Pendular | 000 | 000 |
| Middle | Funicular | 000 | 000 |
| Almost saturated | capillary | 000 | 000 |
| More | Slurry | 000 | 000 |

When the wave hits the foundation, the surface area is in slurry state and there exists no cohesive interaction between the particles, which makes it very hard, if not impossible, to prevent sand loss. Nevertheless, collapses after the wave resides can cause more harm to the foundation, which can be avoided by alternating the shape.

For dry sand, the failure criterion is given in terms of the shear stress $\tau$, the normal compressible stress $\sigma$ and the internal friction $\mu$ as

$$\tau > \mu\sigma$$

This is simply the friction formula with different notations. Now we consider a sandpile with a normal adhesive stress $s_A$ across every plane, in addition to the stress caused by weight. The equation(1) is then modified as

$$\tau > \mu(\sigma + s_A)$$

This criterion is the so-called **Mohr-Coulumb criterion**.The stress resulting from the weight above the plane is shown in figure (1). Denote $\tau_f$ and $\mu_f$ as the shear stress and normal compressible stress at the failure plane,they can be written as

$$\tau_f = \rho g D sin\theta_c \qquad and \qquad \sigma_f = \rho g D cos\theta_c$$

where $\theta_c$ is the critical angle, $D$ is the height of the sandpile and $\rho$ is the density of sand. Therefore, to solve for $\theta_c$ is to solve the equation

$$\mu = tan\theta_c(D)\left(1 + \frac{s_A}{\rho g D cos\theta_c(D)}\right)$$

The only unknow factor is $s_A$, the adhesive stress across the plane. According to (Thomas C.H and Alex J.L -fix later)'s study, the value of $s_A$ is determined by water content and there are three regimes as a function of the added-fluid volume. We now focus only on the state where the water content is close to saturated.

### 3.2 Top View Shape

### 3.3 Calculating Results

### 3.4 Simulating Results

### 3.5 Refer to Footnote

## 4 Modeling Under Rain

## 5 Determine the Best Sand-to-water Proportion

## 6 Other Ways to Make Our Sand Castle Last Longer

## 7 Sensitivity Analysis

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## 7.1 Footnote

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## 7.2 Refer to Footnote

I'm referring to footnote 1.

[1] Hello footnote

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