Lecture 14: Compatible Properties in Histories

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We don't want to make the mistake of discussing incompatible properties. To do this with histories, we start with some family of histories $\{Y^\alpha\}$ and we require $Y^\alpha Y^\beta = Y^\beta Y^\alpha$, $\forall \alpha\beta$. This is a complete family of histories, so $\sum_{\vec{\alpha}} Y^{\vec{\alpha}} = \tilde{I}$. Therefore, **logical negation** is $\neg Y^{\vec{\alpha}} = \tilde{I} - Y^{\vec{\alpha}}$.

Example. Coin toss: $\{(H,H),(H,T),(T,H),(T,T)\}$. The negation of (H,H) is $\neg(H,H) = \{(H,T),(T,H),(T,T)\}$ ◇

Example.

$$\neg[z+] \odot [x+] = [z-] \odot [x+] + [z+] \odot [x-] + [z-] \odot [x-] \tag{1}$$

◇

We can also have **conjunction**, $Y \wedge Y' = YY'$.

Example.

$$Y = [z+]_0 \odot I_1 \tag{2}$$

$$Y' = I_0 \odot [x+]_1 \tag{3}$$

$$YY' = [z+] \odot [x+] \tag{4}$$

◇

Additionally, **disjunction** is defined by $Y \vee Y' = Y + Y' - YY'$.

0.1 Chainket

If we start in a pure state, we can define a product history

$$Y^{\vec{\alpha}} = [\psi_0] \odot P_1^{\alpha_1} \odot \ldots \odot P_f^{\alpha_f} \in \tilde{\mathcal{H}} \tag{5}$$

Definition 1. A **chainket** is defined by

$$|\vec{\alpha}\rangle = P_f^{\alpha_f} T_{f,f-1} \ldots T_{21} P_1^{\alpha_1} T_{10} |\psi_0\rangle \tag{6}$$

Theorem 1. Generalized Born Rule:

$$Pr(\vec{\alpha}) = \langle\vec{\alpha}|\vec{\alpha}\rangle \tag{7}$$

Is it correct? Let's check some cases.

0.1.1 Two-Time History

$$Y^k = [\psi_0] \odot [\phi_1^k] \tag{8}$$

All of our states start at $|\psi_0\rangle$, so we need to define a complete set of histories by adding all the things that don't start there:

$$Z = (I - [\psi_0]) \odot I_1 \tag{9}$$

The chainket for this state is

$$|k\rangle = [\phi_1^k]T_{10}\,|\psi_0\rangle \tag{10}$$

$$\langle k|k\rangle = \langle\psi_0|\,T_{01}\,|\phi_1^k\rangle\langle\phi_1^k|\,T_{10}\,|\psi_0\rangle = |\,\langle\phi_1^k|\psi_1\rangle\,|^2 = Pr([\phi_1^k] \mid \psi_0) \tag{11}$$

N.B.

$$|\,\langle\phi_1^k|\psi_1\rangle\,|^2 = \langle\phi_1^k|\psi_1\rangle\,\langle\psi_1|\phi_1^k\rangle = \langle\psi_1|\,(\phi_1^k)^2\,|\psi_1\rangle = \langle\psi_1|\,[\phi_1^k]\,|\psi_1\rangle \tag{12}$$

0.1.2 Unitary History

$$|\psi_0\rangle \to |\psi_1\rangle = T_{10}\,|\psi_0\rangle \to |\psi_2\rangle = T_{21}\,|\psi_1\rangle = T_{20}\,|\psi_0\rangle \tag{13}$$

$$U = [\psi_0] \odot [\psi_1] \odot [\psi_2] \tag{14}$$

$$|U\rangle = |\psi_2\rangle\langle\psi_2|\,T_{21}\,|\psi_1\rangle\langle\psi_1|\,T_{10}|\psi_0\rangle \tag{15}$$

There's another way of thinking about this:

$$|U\rangle = |\psi_2\rangle\,\overset{\langle\psi_1|}{\cancel{\langle\psi_2|\,T_{21}}}|\psi_1\rangle\,\overset{\langle\psi_0|}{\cancel{\langle\psi_1|\,T_{10}}}|\psi_0\rangle \tag{16}$$

so

$$Pr(U) = \langle U|U\rangle = \langle\psi_2|\psi_2\rangle = 1 \tag{17}$$