Lecture 16: Measurement Devices

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0.1 Beam Splitter Example, Continued

From before, we have a four-port system with a shift operator S which takes $|mz\rangle$ to $|(m+1)z\rangle$. It takes branch a to a superposition of branches c and d: $S|0a\rangle = \frac{1}{\sqrt{2}}(|1c\rangle + |1d\rangle)$. We also have a detector operator R which adds another dimension to the Hilbert space. $R|2c, n\hat{c}\rangle = |2c, (1-n)\hat{c}\rangle$. Therefore, the time evolution operator is $T = SR$ and $|\psi_0\rangle \xrightarrow{T} |\psi_1\rangle = 1ac1\sqrt{2}(|1c, 0\hat{c}\rangle + |1d, 0\hat{c}\rangle) \xrightarrow{T} |\psi_2\rangle = \frac{1}{\sqrt{2}}(|2c, 02c, 0\hat{c}\rangle + |2d, 0\hat{c}\rangle) \xrightarrow{T} |\psi_3\rangle = \frac{1}{\sqrt{2}}(|3c, 1\hat{c}\rangle + |3d, 0\hat{c}\rangle)$.

We can think of the family of histories as a subset of $\{[mz, n\hat{c}]\}$:

$$Y^c = [\psi_0]_0 \odot [1c, 0\hat{c}]_c \odot [2c, 0\hat{c}]_c \odot [3c, 1\hat{c}] \tag{1}$$

We claim the chainket for this is nonzero. If we were to apply the projector $[3c, 0\hat{c}]$ instead, it would vanish.

$$|Y^c\rangle = [3c, 1\hat{c}]T_{32}[2c, 0\hat{c}]T_{21}[1c, 0\hat{c}]T_{09}|\psi_0\rangle = \frac{1}{\sqrt{2}}|3c, 1\hat{c}\rangle \tag{2}$$

There's another history with a non-vanishing chainket:

$$Y^d = [\psi_0]_0 \odot [1d, 0\hat{c}]_1 \odot [2d, 0\hat{c}]_2 \odot [3d, 0\hat{c}]_3 \tag{3}$$

$$Pr([1\hat{c}]_3 \mid [2c]_2) = Pr(Y^c)/Pr(Y^c) = 1 \tag{4}$$

We could also ask

$$Pr([2c]_2 \mid [1\hat{c}]_3) = 1 \tag{5}$$

0.1.1 Measurement of Spin-$\frac{1}{2}$

A Stern-Gerlach apparatus allow us to split a beam of electrons (or any spin-$\frac{1}{2}$ particle) into two branches, one for each spin. Let's imagine we start in position w, go to w' right before the apparatus, $w+$ after the detector in the spin-up branch, and $w-$ after the detector in the spin-down branch. We also have a screen behind the apparatus, so when the electron passes through the device, we can see which branch it went through, but the electron state is destroyed.

$$|z+, w\rangle \to |z+, w'\rangle \to |z+, w_+\rangle \tag{6}$$

and

$$|z-, w\rangle \to |z-, w'\rangle \to |z-, w_-\rangle \tag{7}$$

Suppose we started in the $x+$ state:

$$|psi_0\rangle = |x+, w\rangle = \frac{1}{\sqrt{2}}(|z+, w\rangle + |z-, w\rangle) \tag{8}$$

$$|x+, w\rangle \to |x+, w'\rangle \to \frac{1}{\sqrt{2}}(|z+, w+\rangle + |z-, w_0\rangle) \tag{9}$$

The unitary history is

$$U = [\psi_0]_0 \odot [\psi_1]_1 \odot [\psi_2]_2 \tag{10}$$

What can we "discuss" in this unitary history? What makes sense within our framework?

- $[I_s \otimes w+]$
- $[I_s \otimes w-]$

where I_s is the local spin operator. These topics are incompatible with the unitary history because $[\psi_2][I_s \otimes w+] \neq [I_s \otimes w+][\psi_2]$ (remember, ψ_0 is the x-polarized spin state). Let's then imagine a new family of histories for this state:

$$[\psi_0]_0 \odot [\psi_1]_1 \odot \begin{cases} [z+, w+]_2 \\ [z-, w-]_2 \end{cases} \tag{11}$$

along with $I - [psi_1]_1$ and $I - [\psi_0]_0$, to complete the history. Now surely, while we measure the state on the screen, it must have been measured in the Stern-Gerlach apparatus itself, so we believe we are detecting the state of the particle at time 1. We need a "new" new family of histories:

$$[\psi_0] \odot \begin{cases} [z+, w']_1 \odot [z+, w+]_2 \leftarrow Y_+ \\ [z-, w']_1 \odot [z-, w-]_2 \leftarrow Y_- \end{cases} \tag{12}$$

Now we can ask about some probabilities:

$$Pr([z+]_1 \mid [w+]_2) = Pr(Y_+)/Pr(Y_+) = 1 \tag{13}$$

$$Pr([z-]_1 \mid [w+]_2) = 1 \tag{14}$$