Useful Equations in Electrodynamics

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Green's Functions

$$Lu(x) = f(x)$$

has solutions

$$u(x) = \int G(x,s) f(s) \, \mathrm{d}s$$

$$\int LG(x,s) f(s) \, \mathrm{d}s = \int \delta(x-s) f(s) \, \mathrm{d}s = f(x)$$

Green's Identities:

$$\int_V (\phi \nabla^2 \psi + \vec{\boldsymbol{\nabla}} \phi \cdot \vec{\boldsymbol{\nabla}} \psi) \, \mathrm{d}^3 x = \oint_S \phi \frac{\partial \psi}{\partial n} \, \mathrm{d}a$$

$$\int_V \left(\phi \nabla^2 \psi - \psi \nabla^2 \phi \right) \mathrm{d}^3 x = \oint_S \left[\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n} \right] \mathrm{d}a$$

L	G
∇^2_{3D}	$\frac{1}{\lvert\vec{\mathbf{x}}-\vec{\mathbf{x}}'\rvert}$
$\nabla^2_{\mathrm{3D}} + k^2$	$\frac{e^{-\imath k \lvert\vec{\mathbf{x}}-\vec{\mathbf{x}}'\rvert}}{\lvert\vec{\mathbf{x}}-\vec{\mathbf{x}}'\rvert} = \imath k h_0^{(2)}(k\lvert\vec{\mathbf{x}}-\vec{\mathbf{x}}'\rvert)$
$\Box = \frac{1}{c^2}\partial_t^2 - \nabla^2_{\mathrm{3D}}$	$\frac{\delta\left(t - \frac{\lvert\vec{\mathbf{x}}-\vec{\mathbf{x}}'\rvert}{c}\right)}{\lvert\vec{\mathbf{x}}-\vec{\mathbf{x}}'\rvert}$

Table 0.1: Table of Green's Functions