Lecture 19: Interference, Continued

Wed. Oct 2 2019

0.1 Interference, Cont.

From the same interferometer we had before (using a different labeling from Monday, we now have the channels maintaining their names the whole way through both beam splitters, with 0 before the first, 1 before the mirror, 2 after the mirror, and 3 after the last beam splitter. The phase shifters are right in front of the mirrors),

$$S|0a\rangle = \frac{1}{\sqrt{2}}(|1a\rangle + |1b\rangle) \tag{1}$$

$$S|0b\rangle = \frac{1}{\sqrt{2}}(-|1a\rangle + |1b\rangle) \tag{2}$$

$$S|1a\rangle = e^{\imath\phi_a}|2a\rangle \tag{3}$$

$$S|1b\rangle = e^{\imath\phi_b}|2b\rangle \tag{4}$$

$$S|2a\rangle = \frac{1}{\sqrt{2}}(|3a\rangle + |3b\rangle) \tag{5}$$

$$S|2b\rangle = \frac{1}{\sqrt{2}}(-|3a\rangle + |3b\rangle) \tag{6}$$

$$|\psi_0\rangle = |0a\rangle \to |\psi_1\rangle = \frac{1}{\sqrt{2}}(|1a\rangle + |1b\rangle) \to |\psi_2\rangle = \frac{1}{\sqrt{2}}(e^{\imath\phi_a}|2a\rangle + e^{\imath\phi_b}|2b\rangle) \tag{7}$$

$$|\psi_3\rangle = \frac{1}{2}(e^{\imath\phi_a} - e^{\imath\phi_b}|3a\rangle + \frac{1}{2}(e^{\imath\phi_a} - e^{\imath\phi_b}|3b\rangle \tag{8}$$

so with no detector,

$$\Pr([3a]_3) = \langle\psi_3|\,[3a]\,|\psi_3\rangle = \sin(\Delta/2) \tag{9}$$

where Δ is the difference in phase.

If there is a detector $\hat{a}$ before the phase shifter on the a path,

$$\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_{\hat{a}} \tag{10}$$

$$|\Psi_0\rangle = |\psi_0, 0\hat{a}\rangle \tag{11}$$

$$|\Psi_3\rangle = \frac{1}{2}\left[e^{\imath\phi_a}(|3a,1\hat{a}\rangle + |3b,1\hat{a}\rangle) + e^{\imath\phi_b}(-|3a,0\hat{a}\rangle + |3b,0\hat{a}\rangle)\right] \tag{12}$$

Therefore,

$$\Pr([3a]_3 \otimes I_{\hat{a}}) = \langle\Psi_3|\,[3a]_3 \otimes I_{\hat{a}}\,|\Psi_3\rangle = \underbrace{\frac{1}{4}}_{(0\hat{a})} + \underbrace{\frac{1}{4}}_{(1\hat{a})} = \frac{1}{2} \tag{13}$$

Notice we lose the Δ relationship, so detection on channel a causes a loss of the interference pattern.

We can imagine a third case where there is a detector on a and b simultaneously (so we know when a particle goes through but we don't know which path):

$$\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_{\hat{a}} \tag{14}$$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}(e^{\imath\phi_a}|2a, 1\hat{a}\rangle + e^{\imath\phi_b}|2b, 1\hat{a}\rangle) \tag{15}$$

$$|\Psi_3\rangle = e^{\imath\phi_a}(|3a, 1\hat{a}\rangle + |3b, 1\hat{a}\rangle e^{\imath\phi_b}(-|3a, 1\hat{a}\rangle + |3b, 1\hat{a}\rangle \tag{16}$$

$$\Pr([3a]_3) = \sin^2(\Delta/2) \tag{17}$$

In a fourth case, we have two detectors, $\hat{a}$ and $\hat{b}$.

$$\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_{\hat{a}} \otimes \mathcal{H}_{\hat{b}} \tag{18}$$

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}(e^{\imath\phi_a}\left|2a, 1\hat{a}, 0\hat{b}\right\rangle + e^{\imath\phi_b}\left|2b, 0\hat{a}, 1\hat{b}\right\rangle) \tag{19}$$

$$Pr([3a]_3) = \frac{1}{2} \tag{20}$$

In the fifth case, we have detectors $\hat{c}$ and $\hat{d}$ on the a and b channels respectively **after** the second beam splitter.

$$[\Phi_0] \odot \begin{cases} [1a] \\ [1b] \end{cases} \odot I_2 \otimes I_3 \odot \begin{cases} [0\hat{c}] \\ [1\hat{c}] \end{cases} \tag{21}$$

Let's label the histories Y^{a0}, Y^{a1}, Y^{b0}, Y^{b1} corresponding to the branch and whether or not the detector was triggered. This family is NOT consistent. If we were to form the chainket for $\left\langle Y^{a0}\middle|\middle|Y^{b0}\right\rangle \neq 0$. This is due to the fact that we can't tell which branch we had gone through, because the beam splitters create superpositions of the states. A particle going through either branch has some probability to exit to either detector. Any sort of $[0\hat{c}]$ or $[1\hat{d}]$ (and other) combinations in the final state of this history will result in inconsistencies.

A consistent history could be

$$[\Psi_0] \odot \begin{cases} [1a] \\ [1b] \end{cases} \odot I_2 \odot I_3 \odot \begin{cases} [\hat{c}+] \\ [\hat{c}-] \end{cases} \tag{22}$$

where

$$\hat{c}\pm = \frac{1}{\sqrt{2}}(|0\hat{c}\rangle \pm |1\hat{c}\rangle) \tag{23}$$

We can show that going through one branch makes the final state $[\hat{c}\pm]$, but from this we can't tell which path was taken.

In the sixth (and final) case, we have weak detection on each channel:

$$S\left|1a, 0\hat{a}, 0\hat{b}\right\rangle = \alpha e^{\imath\phi_a}\left|2a, 1\hat{a}, 0\hat{b}\right\rangle + \beta e^{\imath\phi_a}\left|2a, 0\hat{a}, 0\hat{b}\right\rangle \tag{24}$$

The other channel would have the same scenario, for some nonzero β corresponding to the chance to miss a detection.

$$\Pr([3a]_3) = \|\beta\|^2 \sin^2(\Delta/2) + \frac{1}{2}\|\alpha\|^2 \tag{25}$$