Summary of Short-term Research Objectives

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1 Model Specification

Let L be a K-dimensional Bernoulli random variable denoting the true state. Consider the Quadratic Exponential Family, or pairwise log linear model:

$$f(l;\Theta) = \exp\{\Theta_a^T l + \Theta_b^T u - A(\Theta)\}$$

where $U = (L_1L_2, \ldots, L_{K-1}L_K)$ is a $K(K-1)/2 \times 1$ vector of two-way cross-products and $\Theta = (\Theta_a, \Theta_b)$ contains the the natural parameters, which is a $K(K+1)/2 \times 1$ vector.

For the prior, let $S = \sum_{j=1}^{K} L_j = s$ has some fixed pmf $\pi(s)$, $s = 1, \ldots, K$, and let each possible outcome of L given $S = s$ occur with probability $\frac{\pi(s)}{\binom{K}{s}}$.

2 Research Objectives

1. Given the above model specification, what is the implied prior for Θ?

2. Using the implied prior calculated above, simulate measurement data Y from the following hierarchical model:

$$\begin{aligned} \Theta &\sim [\Theta|\pi, K] \\ L &\sim [L|\Theta] \\ Y &\sim [Y|L] \end{aligned}$$

   where $[Y|L]$ is defined by $Pr(Y_j = 1|L_j = 1) = \psi_j$ and $Pr(Y_j = 0|L_j = 0) = \phi_j$ with Y_j being conditional independent with each other given L.

3. Based on the data simulated above, estimate the individual etiology $P(L_i|Y)$ and the population etiology $P(\Theta|Y)$.