Summary of Short-term Research Objectives

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1 Model Specification

Let L be a K-dimensional Bernoulli random variable denoting the true state. Consider the general log linear model:

$$f(l;\Theta) = \exp\{\Theta_1^T l + \Theta_2^T u_2 + \ldots + \Theta_K^T u_K - A(\Theta)\}$$

where U_k is a $\binom{K}{k} \times 1$ vector of k-way cross-products, $k = 1, \ldots, K$, and $\Theta = (\Theta_1, \ldots, \Theta_K)$ contains the the natural parameters, which is a $(2^K - 1) \times 1$ vector.

Model restrictions, let $\tilde{l} = (l, u_2, \ldots, u_K)^T$, and $S = \sum_{j=1}^K L_j = s$ has some fixed pmf

$$\pi(s) = \frac{1}{A} \sum_{\tilde{l}:S=s} \exp\{\Theta^T \tilde{l}\}\ ,\ s = 0, 1, \ldots, K \tag{1}$$

$$A = \sum_{\tilde{l}:l\in\{0,1\}^K} \exp\{\Theta^T \tilde{l}\} \tag{2}$$

2 Research Objectives

In addition to restriction (1), derive the implied prior for Θ in general **log linear model** and in **quadratic exponential model** under the following assumptions:

1. **Conditional prevalence**:

$$\eta_j := P(l_j = 1|S = s)/s = \frac{\sum_{\tilde{l}:l_j=1,S=s} \exp\{\Theta^T \tilde{l}\}}{s \sum_{\tilde{l}:S=s} \exp\{\Theta^T \tilde{l}\}} \tag{3}$$

   for $s = 1, \ldots, K-1$, and $j = 1, \ldots, K$.

2. **Conditional independence**:

$$\text{pair-wise: } P(l_{j_1} = 1, l_{j_2} = 1|S = s > 2) = \eta_{j_1}\eta_{j_2}s^2 \tag{4}$$

$$\Rightarrow \sum_{\tilde{l}:l_{j_1}=1,S=s} \exp\{\Theta^T \tilde{l}\} \sum_{\tilde{l}:l_{j_2}=1,S=s} \exp\{\Theta^T \tilde{l}\} = \sum_{\tilde{l}:S=s} \exp\{\Theta^T \tilde{l}\} \sum_{\tilde{l}:l_{j_1}=1,l_{j_2}=1,S=s} \exp\{\Theta^T \tilde{l}\}$$

$$\Rightarrow \sum_{\tilde{l}:l_{j_1}=1,l_{j_2}=0,S=s} \exp\{\Theta^T \tilde{l}\} \sum_{\tilde{l}:l_{j_1}=0,l_{j_2}=1,S=s} \exp\{\Theta^T \tilde{l}\}$$

$$= \sum_{\tilde{l}:l_{j_1}=0,l_{j_2}=0,S=s} \exp\{\Theta^T \tilde{l}\} \sum_{\tilde{l}:l_{j_1}=1,l_{j_2}=1,S=s} \exp\{\Theta^T \tilde{l}\} \tag{5}$$

3. **Competitor Model**
   Let $\tilde{l}_{+j}$ be the sub-vector of $\tilde{l}$ whose elements involve l_j.

$$P(l_j = 1|l_{-j}) = \frac{\exp\{\Theta_{+j}^T \tilde{l}_{+j}\}}{\exp\{\Theta_{+j}^T \tilde{l}_{+j}\} + 1} \tag{6}$$

   For example, $\Theta_{+j}^T \tilde{l}_{+j} = \theta_j + \sum_{k \neq j} \theta_{jk} l_k + \sum_{k,r \neq j} \theta_{jkr} l_k l_r + \ldots$