Summary of Short-term Research Objectives

Detian Deng

March 18, 2015

1 Model Specification

Let L be a K-dimensional Bernoulli random variable denoting the true state. Consider the general log linear model:

$$f(l;\Theta) = \exp\{\Theta_1^T l + \Theta_2^T u_2 + \ldots + \Theta_K^T u_K - A^*(\Theta)\}$$

where U_k is a $\binom{K}{k} \times 1$ vector of k-way cross-products, $k = 1, \ldots, K$, and $\Theta = (\Theta_1, \ldots, \Theta_K)$ contains the the natural parameters, which is a $(2^K - 1) \times 1$ vector.

Model restrictions, let $\tilde{l} = (l, u_2, \ldots, u_K)^T$, and $S = \sum_{j=1}^K L_j = s$ has some fixed pmf

$$\pi(s) = \frac{1}{A(\Theta)} \sum_{\tilde{l}:S=s} \exp\{\Theta^T \tilde{l}\}\ ,\ s = 0, 1, \ldots, K \tag{1}$$

$$A(\Theta) = \sum_{\tilde{l}:l \in \{0,1\}^K} \exp\{\Theta^T \tilde{l}\} \tag{2}$$

2 Research Objectives

Consider using the QE model, observed data are $\tilde{l}_i, i = 1, \ldots, n$ and $\tilde{S}_i, i = n+1, \ldots, n+m$, what is the MLE of θ?

$$
\begin{aligned}
P(\theta; \tilde{L}_n, \tilde{S}_m) =& \frac{1}{A^n(\theta)} \prod_{i=1}^{n} \exp[\theta^T \tilde{l}_i] \frac{1}{A^m(\theta)} \prod_{i=n+1}^{n+m} \left\{ \sum_{j:\tilde{l}_j^T 1 = S_i} \exp[\theta^T \tilde{l}_j] \right\} \\
\log P(\theta) =& \sum_{i=1}^{n} \theta^T \tilde{l}_i + \sum_{i=n+1}^{n+m} \log \left\{ \sum_{j:\tilde{l}_j^T 1 = S_i} \exp[\theta^T \tilde{l}_j] \right\} - (n+m) \log A(\theta) \\
=& \sum_{i=1}^{n} \theta^T \tilde{l}_i + \sum_{s=0}^{K} m_s \log A_s(\theta) - (n+m) \log A(\theta) \\
\text{where } m_s =& \sum_{i=n+1}^{n+m} 1(S_i = s) \\
A_s(\theta) =& \sum_{j:\tilde{l}_j^T 1 = s} \exp\{\theta^T \tilde{l}_j\}
\end{aligned}
$$