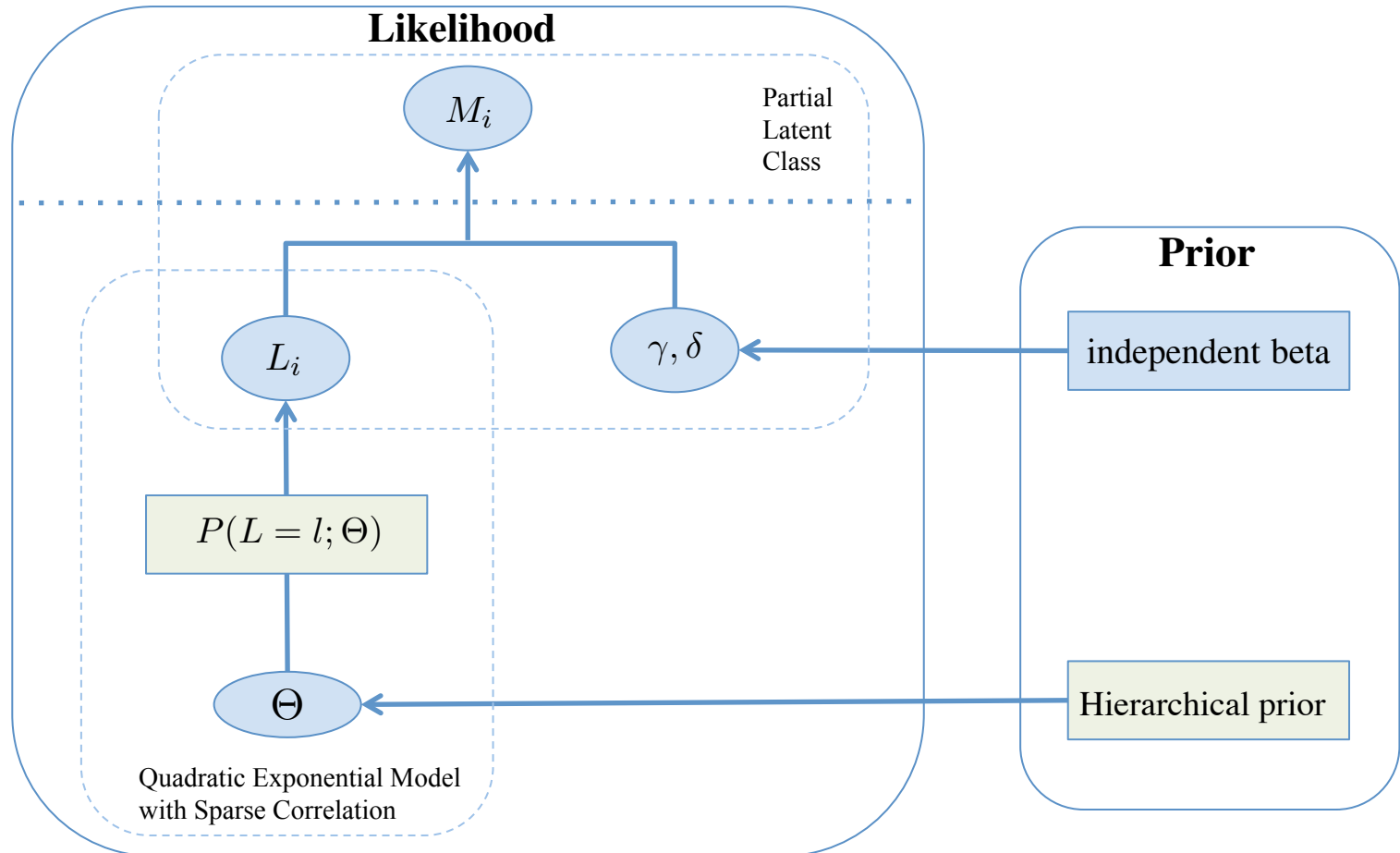
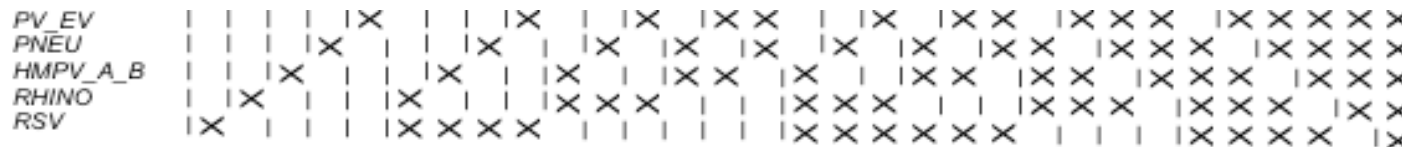


# Model Hierarchy



# Latent Variable



Regular Quadratic Exponential Model:

$$P(L = l; \Theta) = \exp\{\Theta_1^T l + \Theta_2^T u_2\} / A(\Theta)$$

$$\text{with } A(\Theta) = \sum_{l^* \in \{0,1\}^K} \exp\{\Theta^T l^*\}$$

$$\theta_{ik}^{(1)} = X_i^T \beta_k, \text{ where } X_i \text{ is the vector of covariates.}$$

Sparse Correlation:

$$\Theta_2 = \theta_2 \cdot (I_1, \dots, I_{\binom{K}{2}})$$

Hierarchical Prior:

$$I_j \sim \text{Bernoulli}(p)$$

$$p \sim \text{Beta}(a, b)$$



# Simulation Study

5 candidate etiological pathogens.  
BS are available for each of them.  
500 cases and 1000 controls in each data set.  
200 independent data sets simulated.  
Two binary covariates: age and severity.

## Simulation I-a:

Allow multiple pathogen infect lung at the same time.  $\Pr(2 \text{ or more}) \approx 0.35$   
All pathogens have SS measurements.  
Relatively low-quality data:  
 $\text{SS TPR} \approx 0.1$   $\text{BS TPR} \approx 0.6$   $\text{BS FPR} \approx 0.45$

## Simulation I-b:

Only pathogen D, E have SS measurements.

## Simulation II:

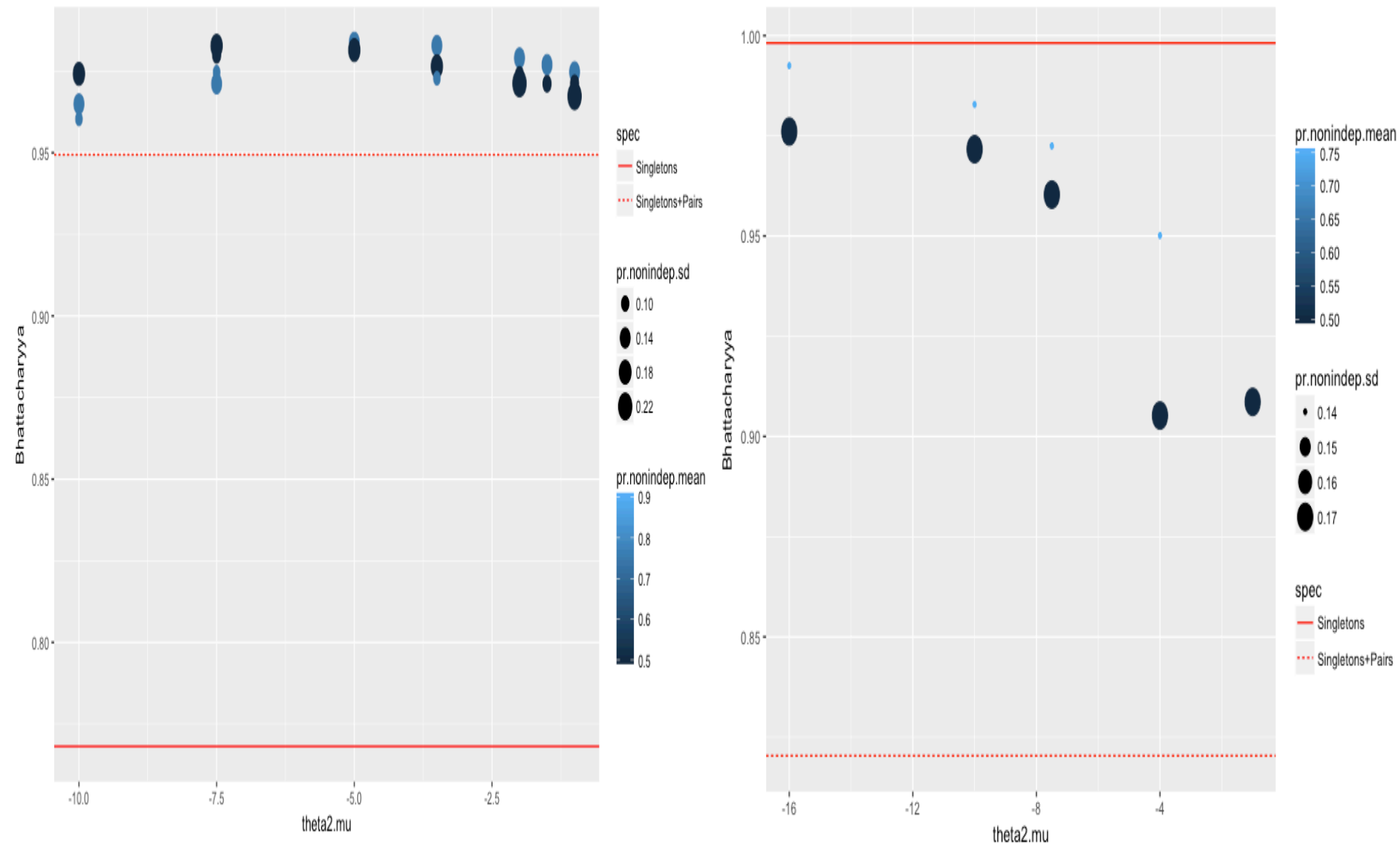
Relatively high-quality data:  
 $\text{SS TPR} \approx 0.8$   $\text{BS TPR} \approx 0.9$   $\text{BS FPR} \approx 0.05$

## Simulation III:

Only single-pathogen infection is possible.

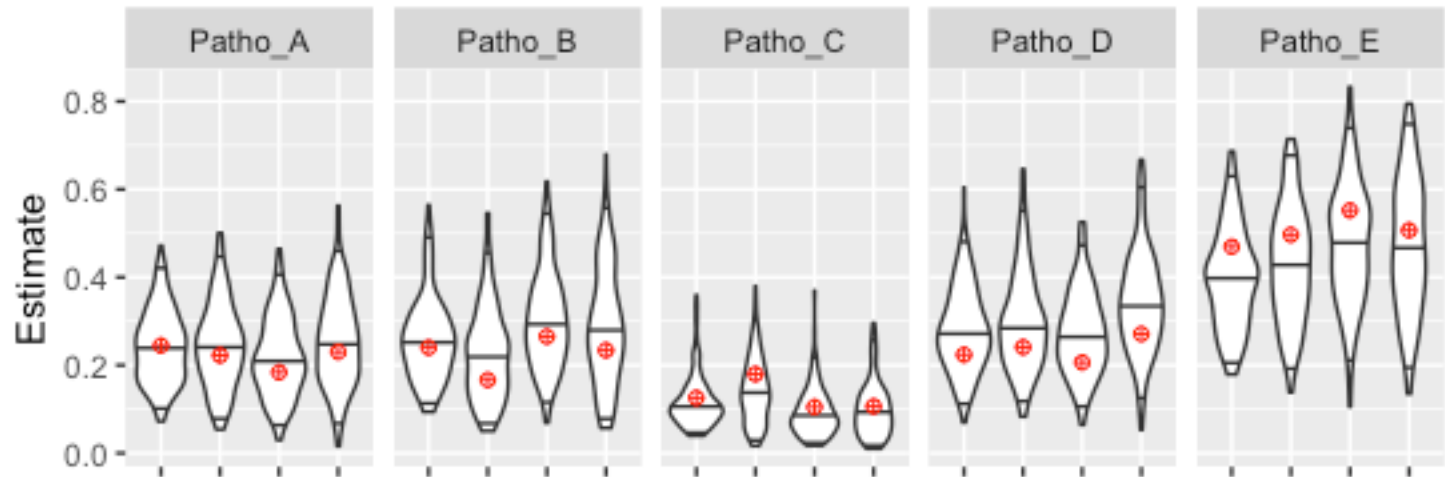


# Simulation I and III vs. pLCM

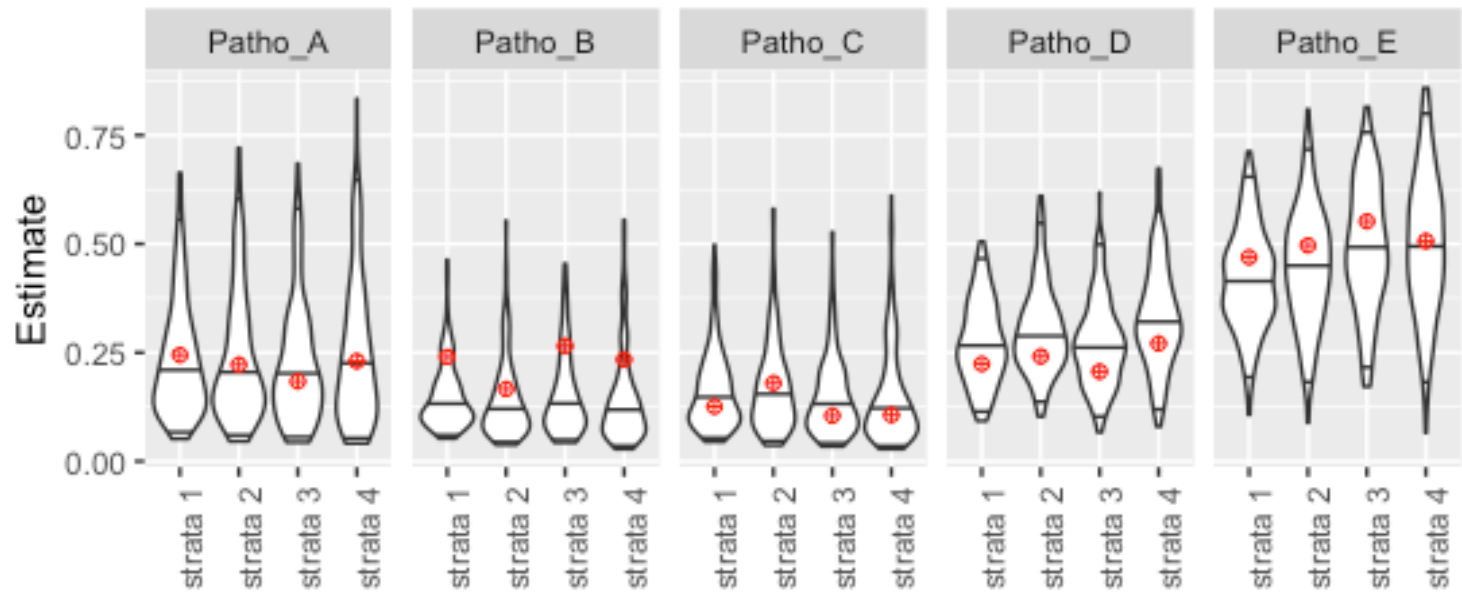


# Simulation I-a vs. I-b

SS available  
for all five  
pathogens:

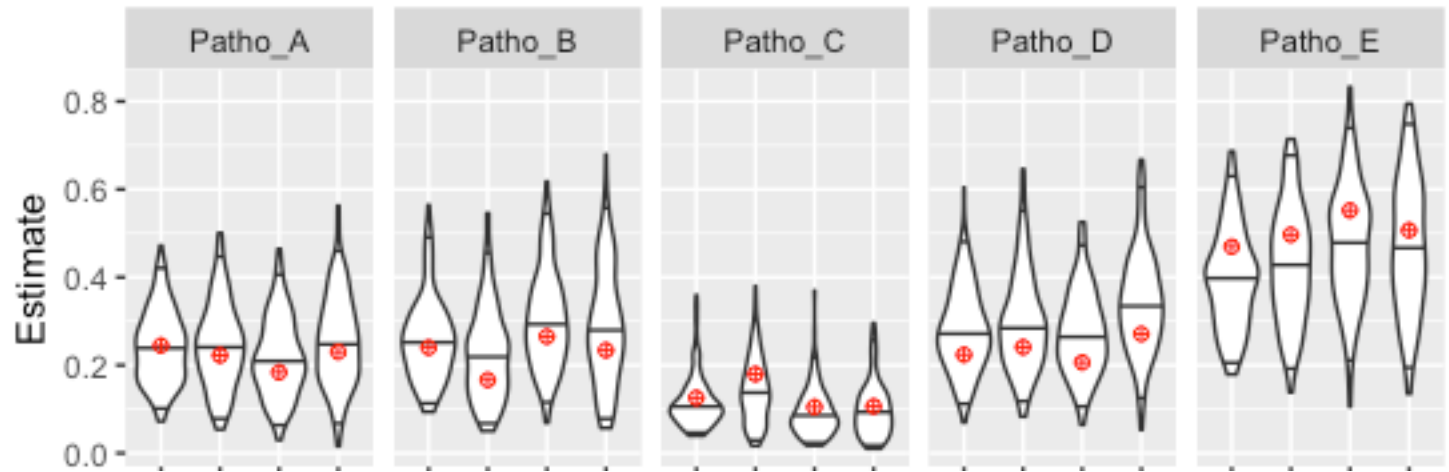


SS available  
for pathogen  
D and E only:

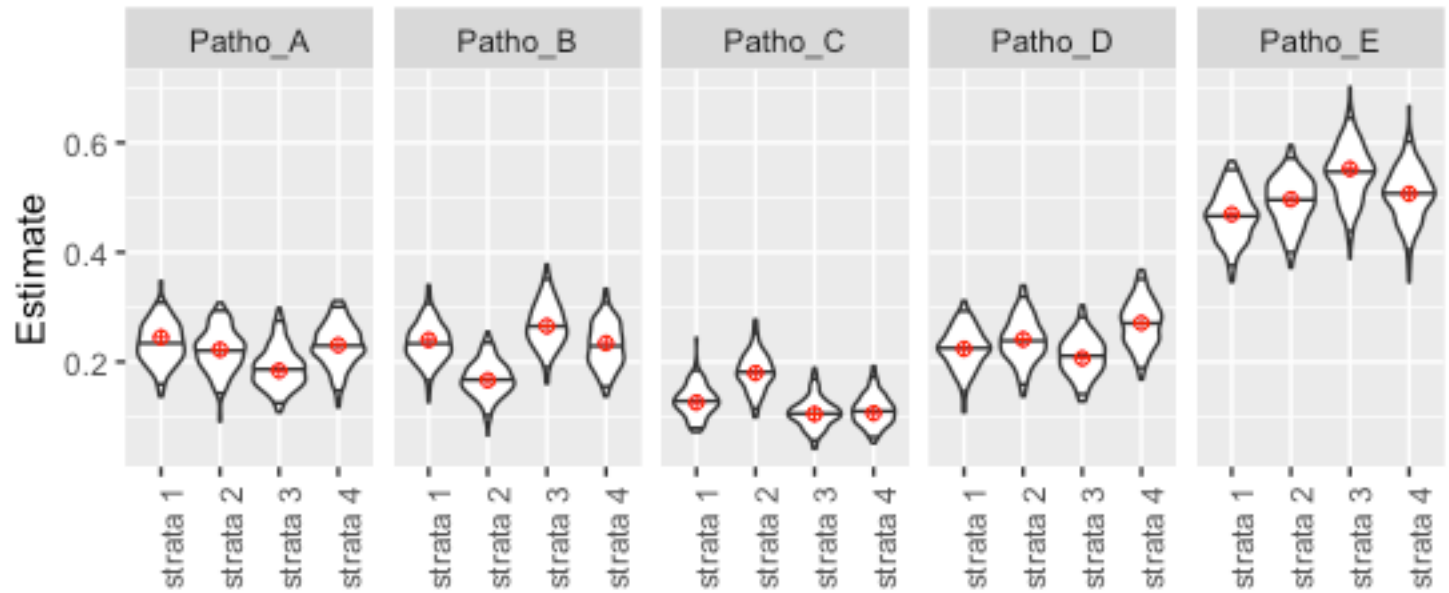


# Simulation I-a vs. II

Low-quality  
data:



High-quality  
data:



# Appendix



$\beta_0$	0.210	-0.282	-0.835	-0.205	1.068
$\beta_{\text{age}}$	-0.1	-0.5	0.5	0.2	0.1
$\beta_{\text{sev}}$	-0.3	0.2	-0.2	-0.1	0.3
$\beta_{\text{a:s}}$	0.4	0.3	-0.4	0.2	-0.2

$$\Theta_2 = (-1.5, -1.5, -1.5, -1.5, 0.0, 0.0, -1.5, -1.5, -1.5, -1.5)$$

Mu	[1]	[2]	[3]	[4]	[5]
00	0.245	0.240	0.126	0.224	0.469
10	0.222	0.166	0.180	0.241	0.497
01	0.184	0.265	0.105	0.206	0.552
11	0.230	0.234	0.106	0.271	0.507

Pr (s)	[0]	[1]	[2]	[3]	[4]	[5]
00	0.090	0.550	0.329	0.031	0.000	0
10	0.085	0.555	0.328	0.031	0.000	0
01	0.081	0.558	0.329	0.032	0.000	0
11	0.076	0.536	0.351	0.036	0.001	0

