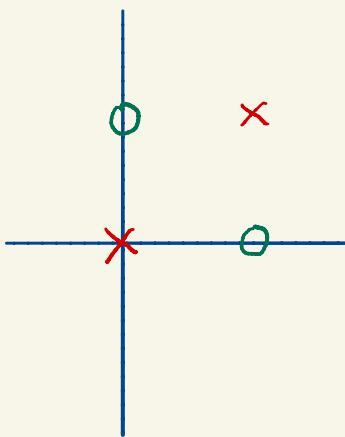




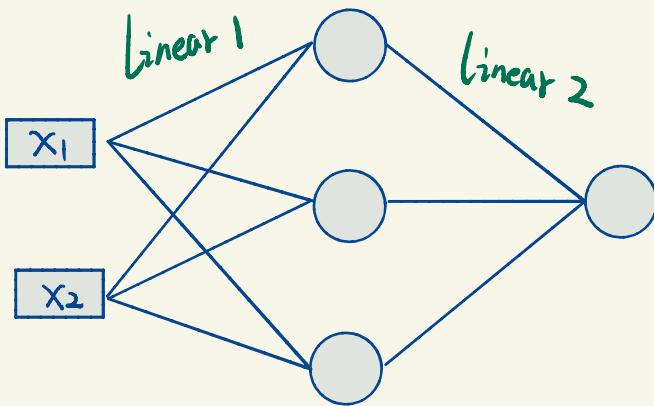
一、介绍



	\vec{x}	y
train	(0,0)	0
	(0,1)	-1
	(1,0)	-1
	(1,1)	0

$$\text{train_}X = \begin{bmatrix} 0, 0 \\ 0, 1 \\ 1, 0 \\ 1, 1 \end{bmatrix}$$

$$\text{train_}y = \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$



$$X = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \quad (4 \times 2)$$

Linear 1 { $\underline{W_1}$: (2×3)
 $\underline{b_1}$: (1×3)

Linear 2 { $\underline{W_2}$: (3×1)
 $\underline{b_2}$: (1×1)

$$\text{relu}(x) = \begin{cases} 0 & , x < 0 \\ x & , x \geq 0 \end{cases}$$

二、计算图模型

与 BP

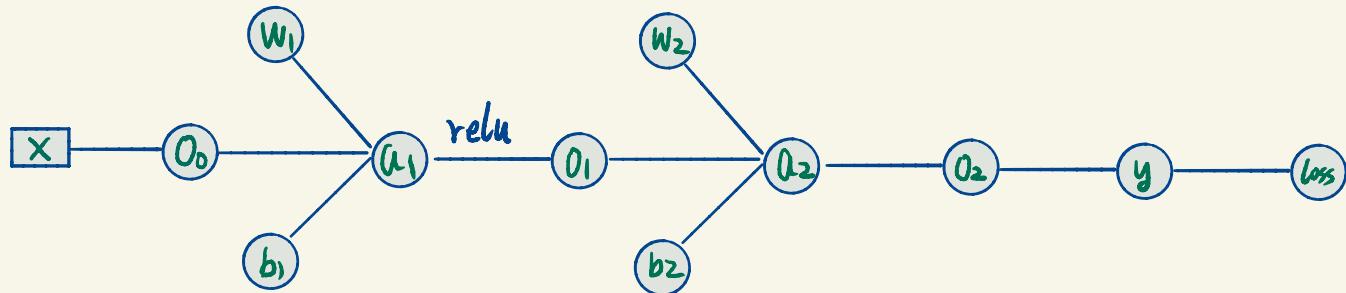
$$\hat{y} = \text{relu}(XW_1 + b_1) \cdot W_2 + b_2$$

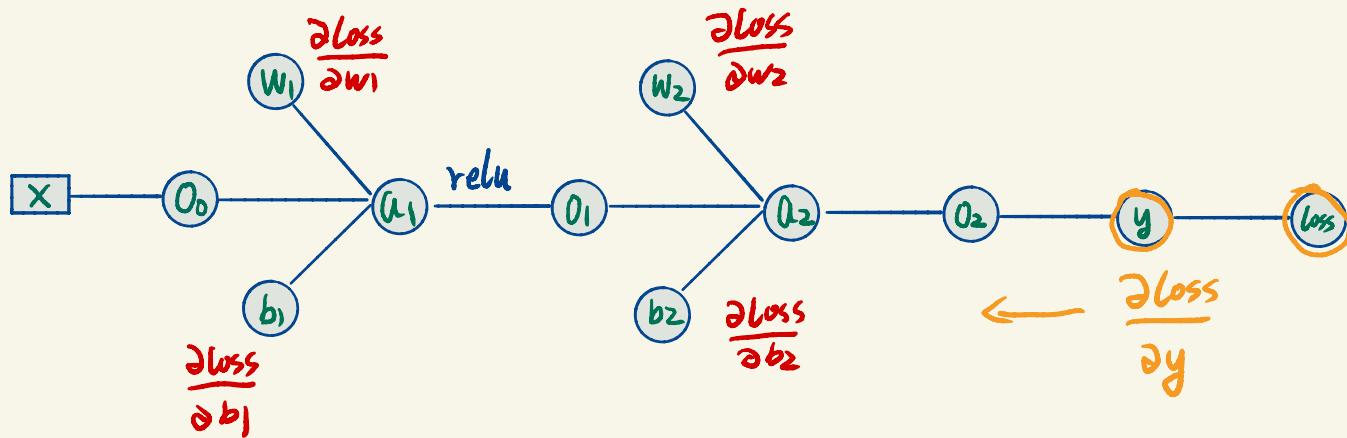
$$\text{loss} = \frac{1}{2} \cdot (y - \hat{y})^2$$

$$\left\{ \begin{array}{l} O_0 = X \\ a_1 = O_0 W_1 + b_1, \quad O_1 = \text{relu}(a_1) \\ a_2 = O_1 W_2 + b_2, \quad O_2 = a_2 \\ y = O_2 \\ \text{loss} = \frac{1}{2} (y - \hat{y})^2 \end{array} \right.$$

a_i : 第*i*层的输入

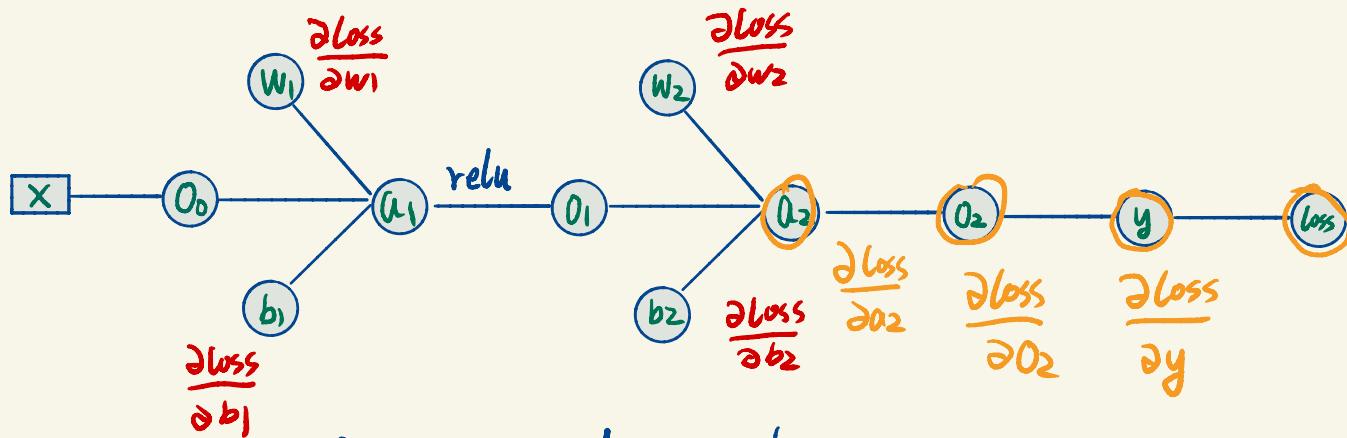
O_i : 第*i*层的输出





$$\text{loss} = \frac{1}{2} (y - \hat{y})^2$$

$$\frac{\partial \text{loss}}{\partial y} = y - \hat{y}$$

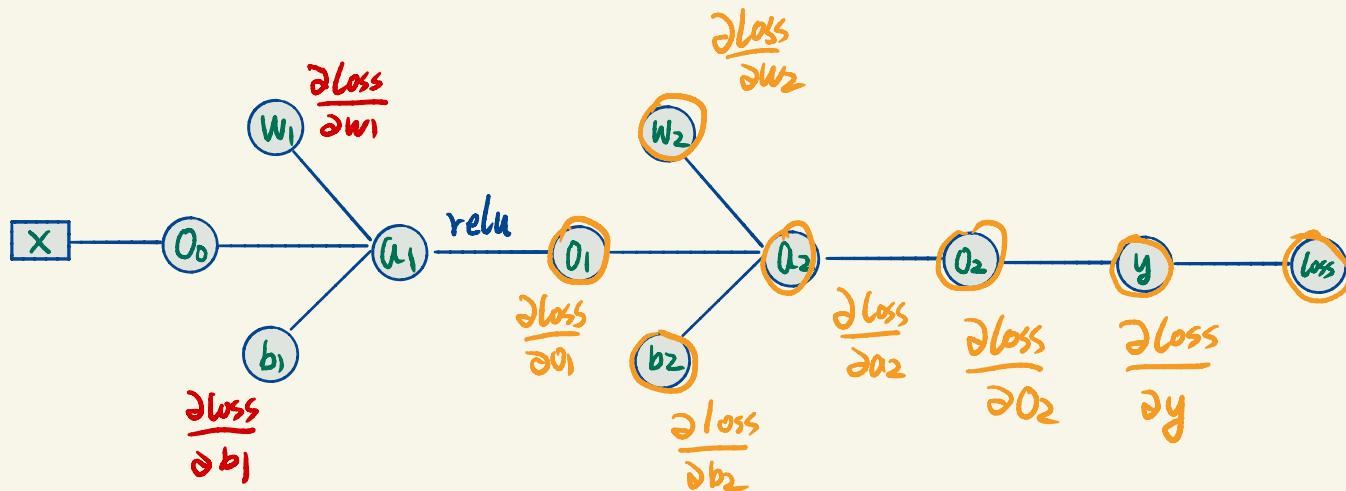


$$a_2 = O_1 W_2 + b_2$$

已知 $\frac{\partial \text{loss}}{\partial a_2}$

求

$$\left\{ \begin{array}{lcl} \frac{\partial \text{loss}}{\partial w_2} & = & \frac{\partial \text{loss}}{\partial a_2} \frac{\partial a_2}{\partial w_2} \\ \frac{\partial \text{loss}}{\partial b_2} & = & \frac{\partial \text{loss}}{\partial a_2} \frac{\partial a_2}{\partial b_2} \\ \frac{\partial \text{loss}}{\partial O_1} & = & \frac{\partial \text{loss}}{\partial a_2} \frac{\partial a_2}{\partial O_1} \end{array} \right.$$



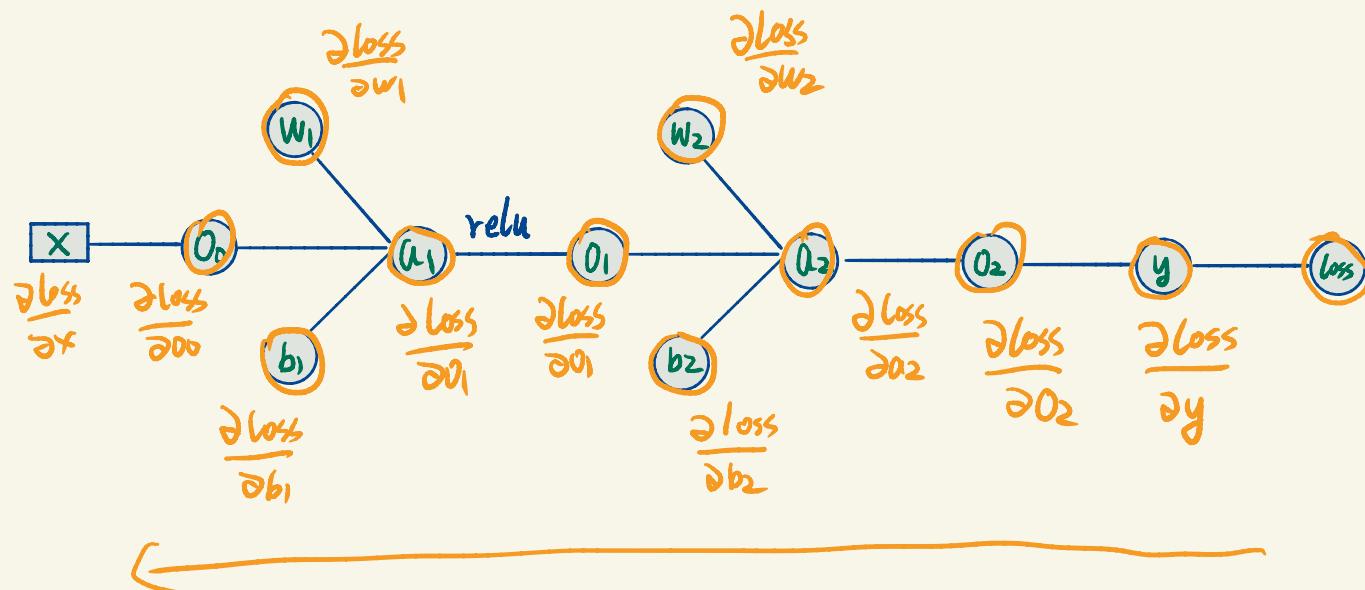
$$o_1 = \text{relu}(a_1)$$

$$\text{relu}(x) = \begin{cases} 0 & , x < 0 \\ x & , x \geq 0 \end{cases}$$

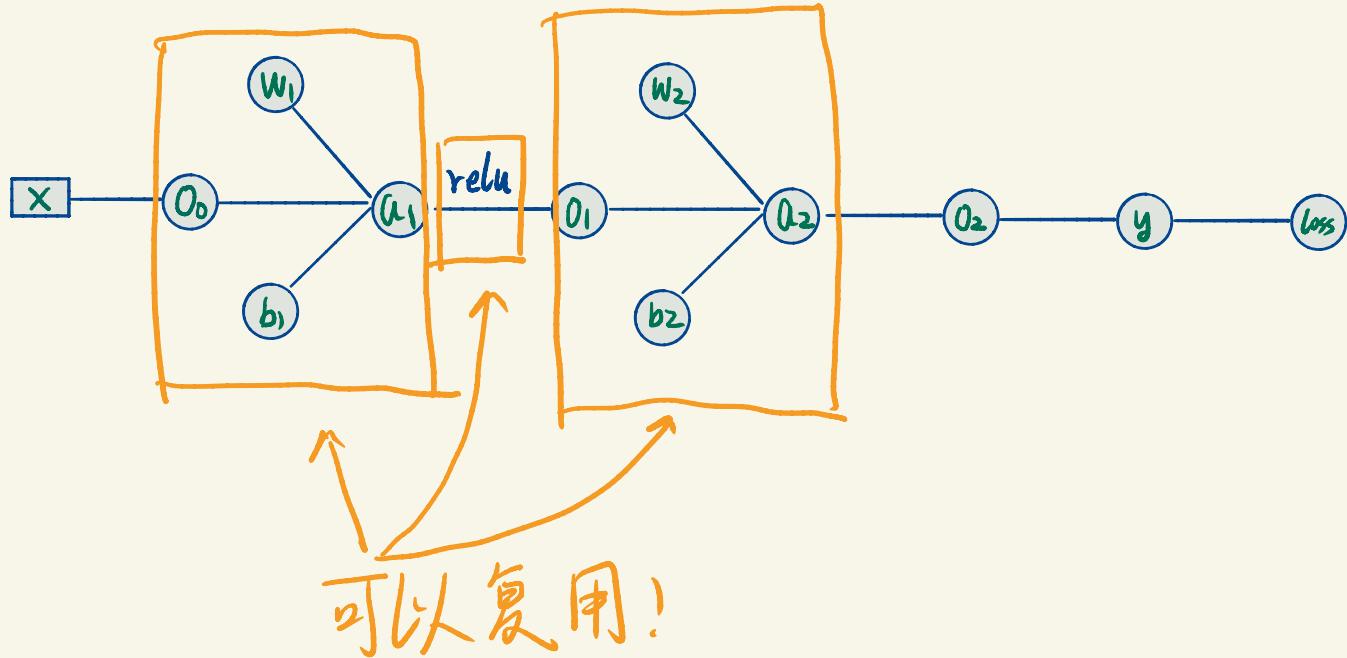
$$a_1 \begin{bmatrix} 1 & -1 \\ 2 & -5 \\ -6 & 4 \end{bmatrix}$$

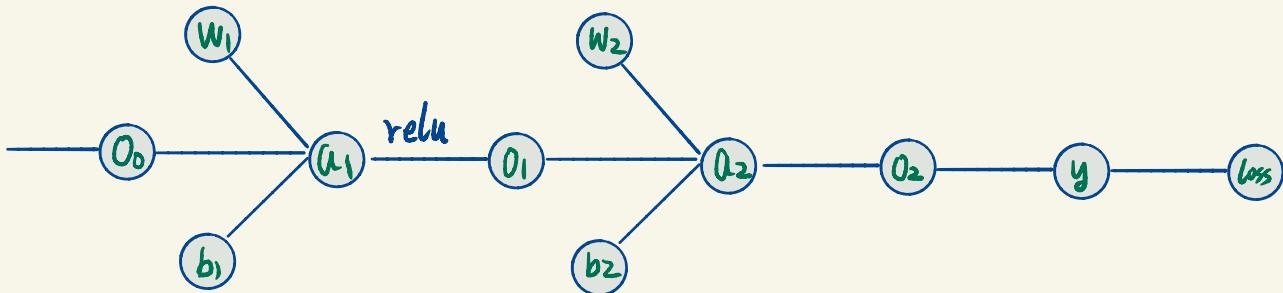
$$\frac{\partial \text{relu}(a_1)}{\partial a_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{\partial \text{relu}(x)}{\partial x} = \begin{cases} 0 & , x < 0 \\ 1 & , x \geq 0 \end{cases}$$



三、计算单元





线性层

$$Y = XW + b$$

参数：
 W ：(input-D, output-D)
 b ：(1, output-D)

方法：

创建：指定输入，输出维度

forward：传入输入，产生输出

backward：传入梯度，产生前一层梯度

$$Y = XW + b$$

$$\begin{aligned} \textcircled{1} \quad \frac{\partial \text{loss}}{\partial W} &= \frac{\partial \text{loss}}{\partial Y} \cdot \frac{\partial Y}{\partial W} \\ &= X^T \cdot \text{grad} \end{aligned}$$

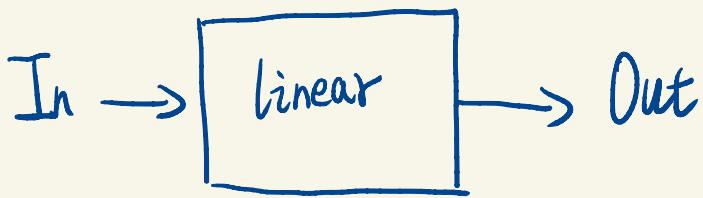
存起来

$$\begin{aligned} \textcircled{2} \quad \frac{\partial \text{loss}}{\partial b} &= \frac{\partial \text{loss}}{\partial Y} \cdot \frac{\partial Y}{\partial b} \\ &= \text{grad} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \frac{\partial \text{loss}}{\partial X} &= \frac{\partial \text{loss}}{\partial Y} \frac{\partial Y}{\partial X} \\ &= W \cdot \text{grad} \end{aligned}$$

传出去

forward:



backward

