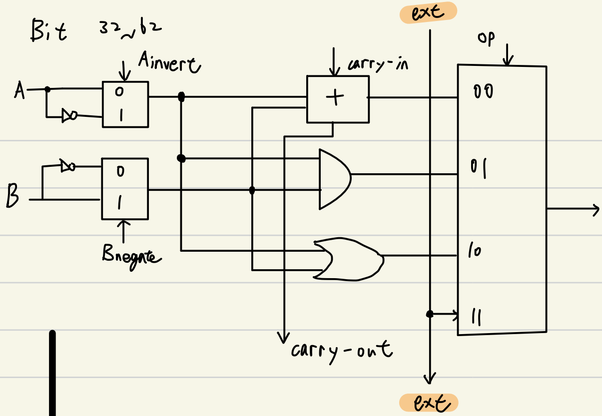
Name : 鄧弘利

ID : 111062107

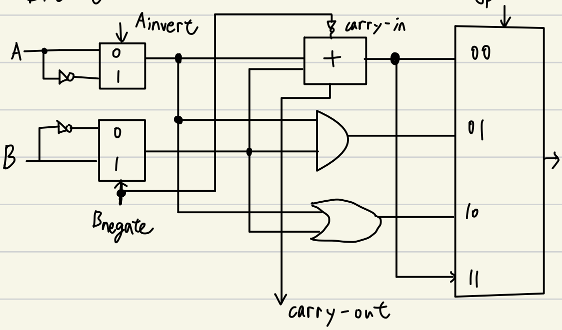
1.

Ainvert	Bnegate	Operation	Function
0	1	01	AND
0	1	10	OR
0	1	00	add
0	0	00	sub
1	0	01	NOR
0	1	11	add-ext
0	0	11	sub-ext

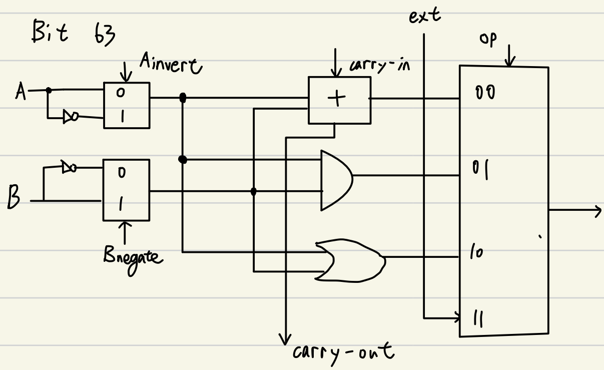
Bit 32~62



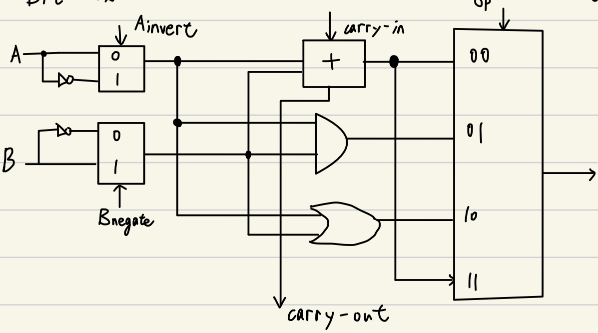
Bit 0



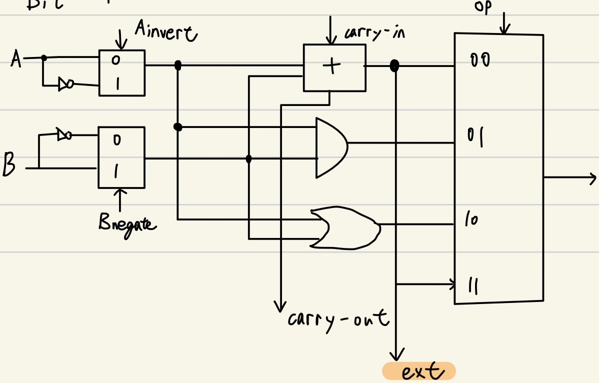
Bit 63



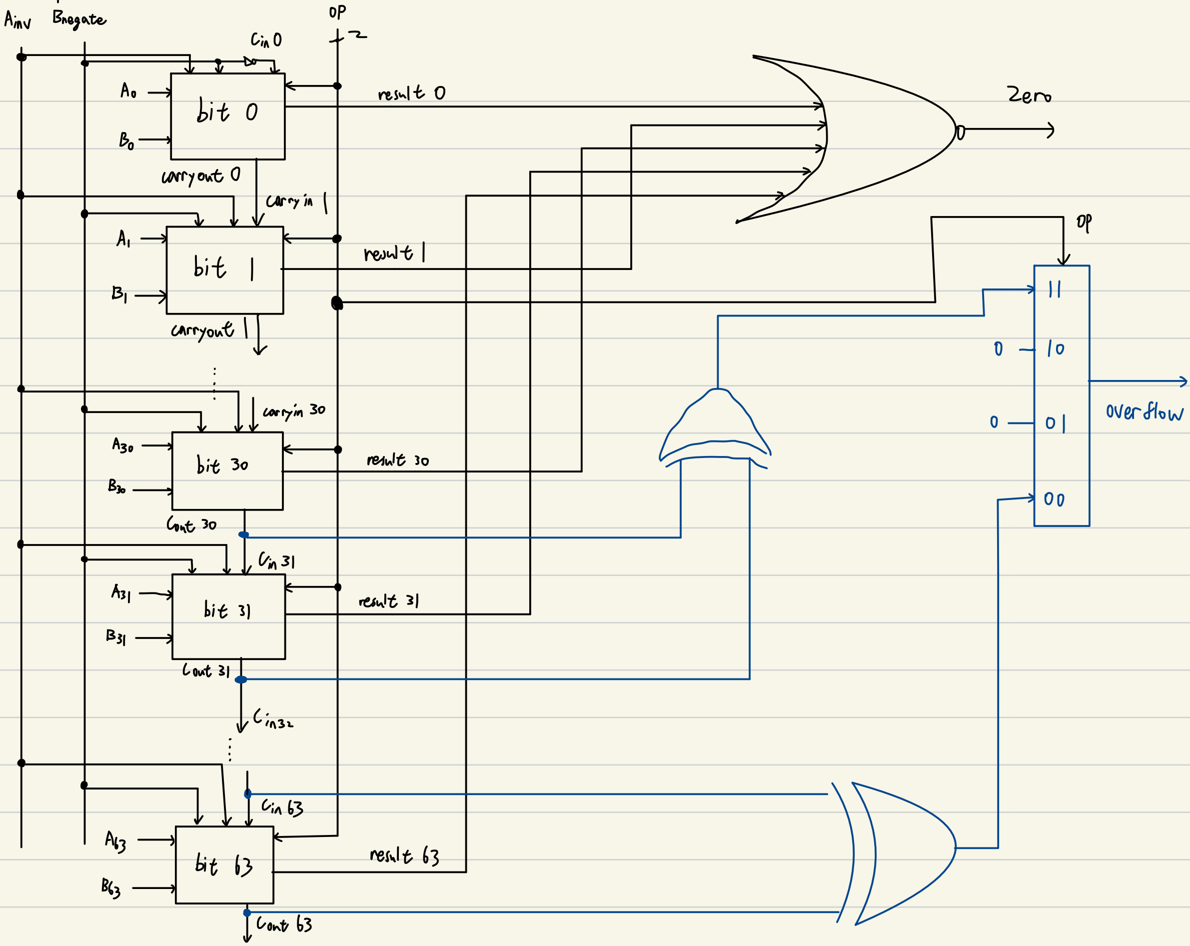
Bit 1~30



Bit 31



Complete 64-bit ALU



2. (a)

Multiplier	Multiplicand	Product
1001	0000 1110	0000 0000 ⊕
→ 0100	← 0001 1100	0000 1110
→ 0010	← 0011 1000	0000 1110
→ 0001	← 0111 0000	0000 1110 ⊕
→ 0000	← 1110 0000	0111 1110
		Ans

(b)

Multiplicand	Product	
1110	0000 1001	
1110	1110 1001	shift
1110	0111 0100	"
1110	0011 1010	"
1110	0001 1101	
1110	1111 1101	"
1110	0111 1110	
	Ans	

3. (a)

Quot	Divisor	Remainder (Dividend)	
0000	0101 0000 ⊖	0000 0111	
0000	0101 0000	1011 0111	→ <0
0000	0101 0000	0000 0111	restore
← 0000	→ 0010 1000 ⊖	0000 0111	$\boxed{\text{shift}}$ Quot : ← Divisor : →
0000	0010 1000	1101 1111	→ <0
0000	0010 1000	0000 0111	restore
← 0000	→ 0001 0100	0000 0111	$\boxed{\text{Shift}}$
0000	0001 0100	1111 0011	→ <0
0000	0001 0100	0000 0111	restore
0000	0000 1010 ⊖	0000 0111	$\boxed{\text{Shift}}$
0000	0000 1010	1111 1101	→ <0
0000	0000 0101 ⊖	0000 0111	restore, shift
0000	0000 0101	0000 0010	→ >0
0001	0000 0010 →	0000 0010	

Quot : 0001

Remainder : 0010

(b)

	Remainder	Divisor	
	0000 0111	0101	
0	0000 1110	0101 ⊖	shift Remainder left
1.1	1011 1110	0101	→ <0
1.2b	0001 1100	0101 ⊖	Restore, shift, set 0
2.1	1100 1100	0101	→ <0
2.2b	0011 1000	0101 ⊖	
3.1	1110 1000	"	→ <0
3.2b	0111 0000	" ⊖	
4.1	0010 0000	"	→ >0
4.2a	0100 0001	"	Shift, set 1
end	→ 0010 0001	0101	shift left half →

Quot : 0001

Remainder : 0010

4. (a)

$$\overset{}{0}\ \ 5\ \ 9\ \ 4\ \ 8\ \ \overset{13}{D}\ \ \overset{14}{E}\ \ \overset{12}{C}_{16}$$

0000 0101 1001 0100 1000 1101 1110 1100

$= 0\times16^7 + 5\times16^6 + 9\times16^5 + 4\times16^4 + 8\times16^3 + 13\times16^2 + 14\times16^1 + 12\times16^0$

$= 93621740_{10}$ (in 2's complement)

For unsigned number, $05948DEC_{16} = 93621740_{10}$

So, we can see that $05948DEC_{16}$ has the <u>same</u> value for 2's complement and unsigned representation. #

---

sign $\underline{1}111$ ($\therefore <0$)

(b) $FA6B7214_{16}$ (2's complement)

= 1111 1010 0110 1011 0111 0010 0001 0100

0000 0101 1001 0100 1000 1101 1110 1011 (1100)

$= -93621740_{10}$

If it's an unsigned number, it should be

$15\times16^7 + 10\times16^6 + 6\times16^5 + 11\times16^4 + 7\times16^3 + 2\times16^2 + 1\times16^1$

$= 4201345556_{10} > 0$ (unsigned number always ≥ 0)

and in 2's complement $FA6B7214_{16}$ is negative, so the result is <u>different</u>. #

(c) $05948DEC_{16}$

$= \underbrace{0}_{\text{sign}}\underbrace{000\ 0101\ 1}_{\text{exponent}\ 11}\underbrace{001\ 0100\ 1000\ 1101\ 1110\ 1100}_{\text{fraction}}{}_2$

bias

$\Rightarrow 1.001\,0100\,1000\,1101\,1110\,1100 \times 2^{11-127}$

$= 1.001\,0100\,1000\,1101\,1110\,1100 \times 2^{-116}$

$= 1.16058111190795898438_{10} \times 2^{-116}$

$\approx 1.3969987_{10} \times 10^{-35}$ #

<0 $FA6B7214_{16}$

$= \underline{1}111\ 1010\ 0110\ 1011\ 0111\ 0010\ 0001\ 0100_2$

~~4+16+32+64+128~~ ~~20~~ ~~52~~ ~~116~~ 244 117

$= -1.1101011\,0111\,0010\,0001\,0100 \times 2^{244-127}$

$= -1.1101011\,0111\,0010\,0001\,0100 \times 2^{117}$

$= -1.83941888809204101563_{10} \times 2^{117}$

$\approx -3.0562589_{10} \times 10^{35}$ #

5. (a) $X = 88.4375$

$Y = -7.3125$

$88.4375 \to 88 \Rightarrow 1011000$

$0.4375 \Rightarrow 0111$

$\underbrace{1011000}.0111 = 1.0110000111 \times 2^6$

2	88	
2	44	0
2	22	0
2	11	0
2	5	1
2	2	1
2	1	0
	0	1

$0.4375 \times 2 = 0.8750$ 0

$0.8750 \times 2 = 1.7500$ 1

$0.75 \times 2 = 1.5$ 1

$0.5 \times 2 = 1$ 1

$6 + 127 = 133$

2	133	
2	66	1
2	33	0
2	16	1
2	8	0
2	4	0
2	2	0
2	1	0
	0	1

133

0 | 1000 0101 | 011 0000 111 0000 0000 0000 0

sign | exponent 8 bits | fraction 23 bits

$-7.3125 \to -7 \to 7 \Rightarrow 111$ (sign bit: 1)

$0.3125 \Rightarrow 0101$

$\underbrace{111}.0101 \Rightarrow 1.110101 \times 2^2$

2	7	
2	3	1
2	1	1
	0	1

$0.3125 \times 2 = 0.6250$ 0

$0.6250 \times 2 = 1.2500$ 1

$0.25 \times 2 = 0.5$ 0

$0.5 \times 2 = 1$ 1

$2 + 127 = 129$

1	0	0	0	0	0	0	1
128	64	32	16	8	4	2	1

$2 + 127 = 129$

1 | 1000 0001 | 110101 00000000000000000

sign | exponent | mantisa

(b) $X \times (-Y)$ → only care the absolute value first

$133-127$ $\qquad\qquad$ $129-127$

$$= 1.01110000111 \times 2^{6} \times 1.110101 \times 2^{2}$$

$$= (1.01110000111 \times 1.110101) \times 2^{8}$$

$$= \underline{10.1000011010110011} \times 2^{\cancel{8}\,9}$$

$$\begin{array}{r}
1.01110000111 \\
\times\ 1.1101010000 \\
\hline
1011100001110000 \\
101110000111 \\
10111000011 \\
10111000011 \\
10111000011 \\
\hline
10.1000011010110011\,0000
\end{array}$$

since $\oplus \times \ominus = \ominus$

1	1000 1000	010 0001 1010 1100 1100 0000
sign	exponent	fraction

$9 + 127 = 136$

128 . . . 8 4 2 1

1 0 0 0 1 0 0 0

$$X \ Y = (-1) \times (1.010\ 0001\ 1010\ 1100\ 1100\ 0000_2) \times 2^{9}$$

bias: 255

6. (a) $\underset{\text{sign}}{\underline{0}}$ $\underset{\text{exponent}}{\underline{0000\ 0000\ 1}}$ $\underset{\text{fraction}}{\underline{000\ 000}}$

9 bits 6 bits

since positive

$$a_0 = 1.\underset{\text{fraction}}{\underline{000\,000}} \times 2^{\cancel{exp}\,1-255}$$

$$= 1.000000_2 \times 2^{-254}$$

(b) 111 110 second largest

$\underset{\text{sign}}{\underline{0}}$ $\underset{\text{exponent}}{\underline{0000\ 0000\ 0}}$ $\underset{\text{fraction}}{\underline{111\ 111}}$ largest

since denormalized

$$a_1 = 0.111111 \times 2^{-254} = 1.111110_2 \times 2^{-255}$$ (⊕, largest, denormalized)

$$a_2 = 0.111110 \times 2^{-254} = 1.111100_2 \times 2^{-255}$$ (⊕, 2nd largest, denormalized)

(c) $|a_0 - a_1|$ $\quad 10 \times 2^{-255}$

$$= a_0 - a_1 = 1 \times 2^{-254} - 1.11111 \times 2^{-255}$$

$$= 0.00001 \times 2^{-255} \times 2^5 \times 2^{-5}$$

$$= 1 \times 2^{-255} \times 2^{-5}$$

$$= 1.000000_2 \times 2^{-260} \quad \#$$

$|a_1 - a_2|$

$$= a_1 - a_2 = 1.11111 \times 2^{-255} - 1.11110 \times 2^{-255}$$

$$= 0.00001 \times 2^{-255}$$

$$= 1.000000 \times 2^{-260} \quad \#$$

(d) $\underset{<0}{\overset{=}{1}}\ \underline{01111\ 0110}\ \underline{100111}$

~~192~~ ~~224~~ ~~240~~ ~~244~~ 246

~~128~~ ~~64~~ ~~32~~ ~~16~~ ~~4~~ ~~2~~

$$-1.100111 \times 2^{246-255}$$

$$= -1.100111_2 \times 2^{-9} \quad \#$$

$$\begin{array}{ccccccccc} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 256 & 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \end{array}$$

$0 + 255 = 255$

(e) $1.31 \to 1$

$0.31 \approx 010100$) $\frac{1}{4}$ $1.010100 \times 2^0 \Rightarrow \underset{\text{sign}}{\underline{0}}\ \underset{\text{exponent}}{\underline{01111\,111\,11}}\ \underset{\text{fraction}}{\underline{010\,100}}$ #

U

0.31×2	=	0.62	0
0.62×2	=	1.24	1
0.24×2	=	0.48	0
0.48×2	=	0.96	0
0.96×2	=	1.92	1
0.92×2	=	1.84	1
0.84×2	=	1.68	1
0.68×2	=	1.36	1
0.36×2	=	0.72	0

6 bits

$+1$

$010011\,\underline{110}$ round up

$\overset{+1}{\Rightarrow} 010100$

$16+4+1$

$1 + \frac{1}{4} + \frac{1}{16}$

$= \frac{21}{16} = 1.3125$ #

$$\begin{array}{r} 1.3125 \\ 16 \overline{)21} \\ 16 \\ \hline 50 \\ 48 \\ \hline 20 \\ 16 \\ \hline 40 \\ 32 \\ \hline 80 \end{array}$$

U's decimal value

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7. (a) if $X = 5$ $\begin{array}{l} 0101 \\ 1000 \end{array}$

$(5+3) >> 2$

$= 0010$) not the same

$5/4 = 1$

$\therefore (X+3) >> 2 \neq X/4$ #

Also, there will be overflow problem if X is too large.

No

$-X = \overline{X} + 1$

(b) if X is negative

$\overline{(\boxed{\overline{X}+1})} >> 2 + 1$

$= (\boxed{\overline{-X}}) >> 2 + 1$

Ex. $-7 \div 4$

$1001 \div 0100$ $(X \div 4)$

$1^{0} \Rightarrow 0111 \div 0100 = 0001$ $((\overline{X}+1)/4)$

$0001 \to 1110 + 1$

$\overline{(\overline{X}+1)/4} + 1$

same

$\because X = \overline{-X} + 1$

$\therefore (X+3) >> 2$

$= ((\overline{-X}+1)+3) >> 2$

$= (\overline{-X}) >> 2 + 1$

$\therefore$ when $X < 0$, $X/4 = (X+3) >> 2$

if X is positive or 0 Yes

then $X / 4 = X >> 2$ is correct

so $((X>=0))\ ?\ X>>2 : (X+3)>>2$ provide the correct result for $(X/4)$

which means $(X>=0)\ ?\ X>>2 : (X+3)>>2 = X/4$ #

(c) if $X = -5$ $0101 \rightarrow 1010+1$ 1011

$-5/4 = -1$ not same No

$1011 >> 2 = 1110$ (-2) $\therefore X>>2 \neq X/4$ #

1111...111 $x<0$

0000...000 $x \geq 0$

(d) $((X >> 31)\ \&\ 00...011) \Rightarrow$ sign bit

0000..0011

$(X+3) >> 2$ (if X is negative)

0000...0000

$(X+0) >> 2$ (if X is positive)

then this is equal to (b) Yes

which we prove that it provides

the correct result for $(X/4)$

$\therefore (X + (X>>31)\ \&\ 3) >> 2 = X/4$ #