





row matrix : A 只有 1 列

列矩陣

column matrix : A 只有 1 行

行矩陣

square matrix : A 的數行數相同

方陣

diagonal matrix : 對角矩陣

除對角項外, 均為 0

 $\text{diag}(d_1, \dots, d_n)$ d_i 為第 i 個對角項

if 對角項全 1 \Rightarrow 單位矩陣 I_n
 \downarrow
 n 階

triangular matrix : A 之對角線上/下全為 0

A_{ij}

upper triangular matrix	:	$i > j$	$a_{ij} = 0$
lower "	:	$i < j$	$a_{ij} = 0$
strictly upper "	:	$i \geq j$	$a_{ij} = 0$
" lower "	:	$i \leq j$	$a_{ij} = 0$

$$\alpha(\beta A) = (\alpha\beta)A$$

$$(\alpha + \beta)A = \alpha A + \beta A$$

$$\alpha(A + B) = \alpha A + \alpha B$$

$$A = [a_{ij}]_{m \times n}$$

$$B = [b_{ij}]_{n \times p}$$

$$C = AB = [c_{ij}]_{m \times p}$$

$$c_{ij} = \sum_{k=1}^n a_{ik} b_{kj} \quad 1 \leq i \leq m \quad 1 \leq j \leq p$$

$$\star A^0 = I \quad A^{k+1} = A^k A = A A^k$$

$$\star J_n = \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}_{n \times n} \quad \text{全是 "1"}$$

☆ 矩陣乘法性質

1. 結合律 associative

$$(AB)C = A(BC)$$

2. 分配律 distributive

$$A(B+C) = AB + AC$$

$$(B+C)A = BA + CA$$

$$3. \quad c(AB) = (cA)B = A(cB)$$

↑

constant

4. 與零矩陣相乘恆為零矩陣

$$A_{m \times n} \times O_n = O_{m \times n}$$

$$O_m \times A_{m \times n} = O_{m \times n}$$

5. 與單位矩陣相乘恆為本身

$$A_{m \times n} I_n = A_{m \times n}$$

$$I_m \times A_{m \times n} = A_{m \times n}$$

6. 指數

$$A^r A^s = A^{r+s}$$

$$(A^r)^s = A^{rs}$$

7. 上(下, 對角)三角矩陣相乘必為
上(下, 對角)三角矩陣

☆ idempotent 冪等

$$H^2 = H$$

★ 矩陣乘法不恆成立之性質

1. 交換律

$$(A+B)^2 = A^2 + 2AB + B^2 \text{ 不恆成立}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$



not same

$$2. \text{ if } A \neq 0 \wedge B \neq 0$$

則 $AB \neq 0$ 不恆成立

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\text{但 } AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$3. \text{ 正整數 } n, \text{ if } A^n = 0 \text{ 則 } A = 0 \text{ 不恆成立}$$

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$4. \text{ if } A^2 = A, \text{ 則 } A = 0 \vee I \text{ 不恆成立}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = A$$

$$5. \text{ if } A^2 = I, \text{ 則 } A = I \vee -I \text{ 不恆成立}$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$6. \text{ 消去律} \quad \text{if } AB = AC, \text{ 且 } A \neq 0, \text{ 則 } B = C \\ \text{不恆成立}$$

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = AC \quad \text{但 } B \neq C$$

☆ 矩陣分割 (partition)

$$A = \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{array} \right]$$

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

A : 方塊矩陣

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} B^{(1)} \\ B^{(2)} \\ B^{(3)} \end{bmatrix}$$

$B^{(i)}$: B 的列向量

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$= \begin{bmatrix} C^{(1)} & C^{(2)} & C^{(3)} \end{bmatrix}$$

$C^{(i)}$: C 的行向量

$$\text{if } A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$\text{則 } AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

CH 3 向量空間

vector space

1. 加法結合性

$$\forall u, v, w \in V$$

