Concentration Inequality

> *Since there are some problems about mathjax, I also upload the pdf version /Large Sample / 1 Concentration Inequality.*

Chernoff Bound

Given r.v. X, if $\varphi(\lambda) = \mathbb{E}\left\{e^{\lambda(X-\mu)}\right\}, \forall\lambda \in [0, b]$, then by

- applying Markov Inequality to $Y = e^{\lambda(X-\mu)}$
- Optimize λ.

we can get

$$\log[\mathbb{P}\{(X-\mu) \geq t\}] \leq -\sup_{\lambda\in[0,b]}\left(\lambda t - \log\left[\mathbb{E}\left\{e^{\lambda(X-\mu)}\right\}\right]\right)$$

Hoeffding Bound

Hoeffding bound is obtained by controlling the moment generating function via sub-gaussian, getting an explicit expression of the conjugate function and optimize λ.

Subgaussian

X is a sub-gaussian with parameter σ^2 if it satisfies

$$\mathbb{E}\left\{e^{\lambda(X-\mathbb{E}X)}\right\} \leq \exp\left(\frac{\lambda^2\sigma^2}{2}\right), \forall\lambda \in \mathbb{R}$$

Hoeffding's Inequality

If X_i are sub-gaussian with parameter σ_i^2 , then

$$\mathbb{P}\left\{\frac{1}{n}\sum_{i=1}^{n}(X_i - \mathbb{E}X_i) \geq t\right\} \leq \exp\left(-\frac{n^2t^2}{2\sum_{i=1}^{n}\sigma_i^2}\right)$$

Bernstein Bound

Hoeffding bound only uses the information of the first moment. If we use the higher moment information, then we can get a shaper (up to a constant) bound than it (For bounded random variable).

Bernstein Inequality

Consider $X_1, \cdots, X_n$ are independent mean 0 random variable. If $X_i \leq 1$ a.s., then for $\epsilon > 0$, we have

$$\mathbb{P}\left(\frac{1}{n}\sum_{i=1}^{n} X_i > \epsilon\right) \leq \exp\left\{\frac{n\epsilon^2}{2\left(\sigma^2 + \epsilon/3\right)}\right\}$$

Reference

- Concentration Inequality: Lecture Notes