## **Structural Distance**

Structural distance captures local changes. It suites to data where local changes can induce radical changes.

## **Hamming Distance**

$$d_H(G, ilde{G}) = \sum_{i,j} rac{\left|A_{ij} - ilde{A}_{ij}
ight|}{N(N-1)} = rac{1}{N(N-1)} \|A - ilde{A}\|_1$$

#### **Jaccard Distance**

Jaccard Distance includes a normalization with respect to the volume of the union graph.

$$egin{aligned} d_{ ext{Jaccard}}\left(G, ilde{G}
ight) &= rac{|G \cup ilde{G}| - |G \cap ilde{G}|}{|G \cup ilde{G}|} \ &= rac{\sum_{i,j} \left|A_{ij} - ilde{A}_{ij}
ight|}{\sum_{i,j} \max\left(A_{i,j}, ilde{A}_{ij}
ight)} \ &= rac{\|A - ilde{A}\|_1}{\|A + ilde{A}\|_*}, \end{aligned}$$

It is the metric induced by Steinhaus transformation  $\delta(x,y)=rac{2d(x,y)}{d(x,c)+d(y,c)+d(x,y)}$  with empty graph c.

## **Four Aspects of Dissimilarity Scores**

- **Edge-Importance**: modifications of the graph structure yielding disconnected components should be penalized more.
- **Edge-submodularity**: a specific change is more important in a graph with a few edges than in a denser graph on the same nodes.
- **Weight awareness**: the impact on the similarity measure increases with the weight of the modified edge

• **Focus awareness**: random changes in graphs are less important than targeted changes of the same extent

# Reference

• Tracking network dynamics: A survey using graph distances