Erdős–Rényi-Gilbert Random Graph Model

Background

The Erdős–Rényi model was firstly introduced by Paul Erdős and Alfréd Rényi in 1959, while Edgar Gilbert introduced the other model contemporaneously and independently of Erdős and Rényi.

The underlying ideas for them are respectively:

- Erdős and Rényi: All graphs on a fixed vertex set with a fixed number of edges are equally likely;
- Gilbert: Each edge has a fixed probability of being present or absent, independently of the other edges.

Model

Erdős and Rényi

The network is denoted as $G(N, E)$ because they assumed the number of nodes and edges are fixed.

- An undirected graph involving N nodes and a fixed number of edges, E, chosen randomly from the $\binom{N}{2}$ possible edges in the graph.
- All $\binom{\binom{N}{2}}{E}$ graphs are equally likely.

Gilbert

The network is denoted as $G(N, E)$ because Gilbert assumed the number of nodes and the probability of each edge existing is fixed.

$$\ell(G(N,p) \text{ has } E \text{ edges } \mid p) = p^E (1-p)^{\binom{N}{2}-E}$$

Or equivalently,

$$\ell(Y \mid p) = \prod_{i \neq j} p^{Y_{ij}} (1-p)^{1-Y_{ij}}$$

Properties

When considering asymptotic behavior is the value of $\lambda = pN$:

- If $\lambda < 1$, then a graph in $G(N, p)$ will have no connected components of size larger than $O(\log N)$, a.s. as $N \to \infty$
- If $\lambda = 1$, then a graph in $G(N, p)$ will have a largest component whose size is of $O\left(N^{2/3}\right)$, a.s. as $n \to \infty$
- If λ tends to a constant $c > 1$, then a graph in $G(N, p)$ will have a unique "giant" component containing a positive fraction of the nodes, a.s. as $N \to \infty$. No other component will contain more than $O(\log N)$ nodes, a.s. as $N \to \infty$.

Reference

- A survey of statistical network
- Erdős–Rényi model