> *Since there are some problems about mathjax, I also upload the pdf version [/Network / 7 Threshold Behavior].*

We focus on simple ER model here.

What is a Threshold Behavior

Some Example

- Numbers of Triangle
  - When $np < 1$, we do not expect to see triangles;
  - When $np > 1$, we would expect to see many triangles;
- Size of Largest Component
  - When $p < \frac{1}{n}$, the largest component has size $O(\log n)$.
  - When $p < \frac{1}{n}$, then the largest component has linear size with high probability.

The DFS algorithm

DFS algorithm is also the depth-first search algorithm. It searches the vertexes of a graph. It labels the vertexes with three types in each round (One vertex moves):

- S: explored;
- T: unvisited;
- $U = \mathcal{V} \backslash (S \cup T)$: under investigation.

Algorithm

Start: $S = U = \emptyset, T = \mathcal{V}$
If $U \neq \emptyset$: Let v be the last vertex added to U.

1. If v has a neighbour u in T then delete u from T and insert it into U.
2. If v does not have a neighbour in T then v is moved from U to S.

If $U = \emptyset$: choose the first vertex of T and move it from T to U. If $U \cup T = \emptyset$: query all the remaining pairs of vertices in $S = V$, then stop.

Properties of DFS

- At each round, either from T to U, or from U to S.
- Each round may consist of multiple queries of edges to determine whether or not the edge is present.
- At any stage of the algorithm it has been revealed already that the graph G has no edges between the current set S and the current set T
- The set U always spans a path (and hence a connected component).

Threshold Behavior

Small Component

Let $\varepsilon > 0$, and let $G \sim \mathcal{G}(n, p)$, with $n \geq 3$. Let $\varepsilon < 1$ and let $p = \frac{1-\varepsilon}{n}$. Then the probability that all connected components of G are of size at most $\frac{7}{\varepsilon^2} \log n$ tends to 1 as $n \to \infty$.

Large Component

Let $\varepsilon > 0$, and let $G \sim \mathcal{G}(n, p)$, with $n \geq 3$. Let $\varepsilon < \frac{1}{10}$ and let $p = \frac{1+\varepsilon}{n}$. Then the probability that G contains a path of length at least $\frac{1}{5} \varepsilon^2 n$ tends to 1 as $n \to \infty$.