
P1.1. Decorated cubic lattices (*cf.* A&M p. 92) Find a Bravais lattice and (if needed) a basis for each of these arrangements of points:

a) Base-centered cubic, that is, a simple cubic lattice with additional points in the centers of the horizonal faces of the cubic cell.
b) Side-centered cubic, that is, simple cubic with additional points in the centers of the *vertical* faces.
c) Edge-centered, with points added at the center of each edge joining nearest neighbors.

P1.2. Skipping alternate lattice points (*cf.* A&M prob 2, page 80) What is the name of the lattice formed by all points with Cartesian co-ordinates (n_1, n_2, n_3) if

a) the n_i are all even?
b) all odd?
c) The sum of the n_i is required to be even?
d) The sum is required to be odd?
You don't have to prove anything. Just work it out by drawing a few lattice points. Some of these may not be Bravais lattices with standard names.

P1.3. optimal tavern problem As Chapter 4 says, the face-centered cubic lattice has the highest density for packing spheres. There is a complementary property that is sometimes of interest. For a given lattice there are certain points that are as far as possible from the nearest lattice site. We denote this maximal distance as x. For example, in a simple cubic lattice of side a, $x = \frac{1}{2}a\sqrt{3}$ and the point is at the middle of the cube. Consider an FCC and a BCC lattice each having a volume v per site.

a) Find the maximal distance x for each lattice and describe its position(s) in the Wigner-Seitz cell.
b) which lattice has the lesser maximal distance, and by how much? As it turns out, this lattice has the smallest maximal distance of any Bravais lattice. This lattice solves the "optimal tavern problem" for drinkers in three dimensions. If there is a fixed number of taverns in a given region, no drinker, wherever situated, should be further than necessary from a tavern. One cubic Bravais lattice minimizes this distance.

P1.4. morphs in reciprocal space Suppose you have a generic lattice of points in front of you. Its primitive lattice vectors point in no particular directions. There is a specific reciprocal lattice corresponding to this direct lattice.

a) What happens to the reciprocal lattice as you rotate the direct lattice by an angle θ about a vertical axis? The direction is clockwise as you look downward. That is, how do the reciprocal lattice points move?
b) What happens to the reciprocal lattice if you squeeze the direct lattice by a factor λ in the vertical direction? Justify your answer using the defining properties of the reciprocal lattice.

P1.5. density of points in a lattice plane (*cf.* A&M p. 94) The number of (Bravais) lattice points per unit area ρ_a in a lattice plane varies greatly depending on the plane chosen. If a given family of planes is separated by a distance d, and the volume per lattice point is v, one can relate ρ_a to d and v in a simple way.

a) Derive this relation. This is simple and requires no lattice vectors.
b) Using a) and the connection between d, the Miller indices, and the reciprocal lattice vector, find the Miller indices for the planes with the highest ρ_a in an FCC lattice.
c) Find this for a BCC lattice.
