#### First-principles Computational Material Research

# Lecture Notes Calculations on Optical Properties

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#### **VASP Home Page**

http://cms.mpi.univie.ac.at/vasp/vasp/vasp.html

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{eff}(\vec{r}) \right] \psi_i(\vec{r}) = \varepsilon_i \psi_i(\vec{r}) \qquad ; \quad \rho(\vec{r}) = \sum_i^{occ} \left| \psi_i(\vec{r}) \right|^2$$

$$V_{eff}(\vec{r}) = V_{ext}(\vec{r}) + V_{H}(\vec{r}) + V_{xc}(\vec{r}) + V_{ps}(\vec{r})$$

$$V_H(\vec{r}) = \int \frac{\rho(\vec{r})}{|\vec{r} - \vec{r}'|} d^3r; \quad V_{xc}(\vec{r}) = V_{xc}(\rho(\vec{r})) \quad LDA$$

$$V_{eff}(\vec{r}+\vec{R}) = V_{eff}(\vec{r})$$

$$\psi_k(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u(\vec{r})$$

$$V_{eff}(\vec{r} + \vec{R}) = V_{eff}(\vec{r})$$

$$\psi_{k}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u(\vec{r})$$

$$u_{k}(\vec{r}) = \sum_{G_{n} < G_{\text{max}}} C_{n}e^{i\vec{G}_{n}\cdot\vec{r}}$$

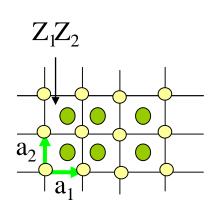
$$\underbrace{\mu(\vec{k})C - \lambda SC = 0}_{\vec{k}} = 0$$

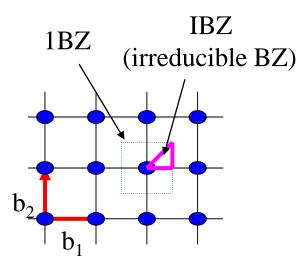
$$\underline{H}(\vec{k})\underline{C} - \lambda \underline{S}\underline{C} = 0$$

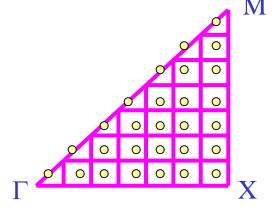
$$\overrightarrow{\mathcal{T}}_{i} \rightarrow \begin{matrix} \rho_{in} & \longrightarrow & V_{eff}(r) & \longrightarrow & \varepsilon_{i}; \psi_{i} & \longrightarrow & \rho_{out} \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & \\ & & \\ & & \\ & & \\ & \\ & & \\ & \\ & & \\ & & \\ & \\ & & \\ & \\ & \\ & \\ &$$

$$oldsymbol{ar{ au}}_{_{i}}=oldsymbol{\eta}ar{F}_{_{i}}$$

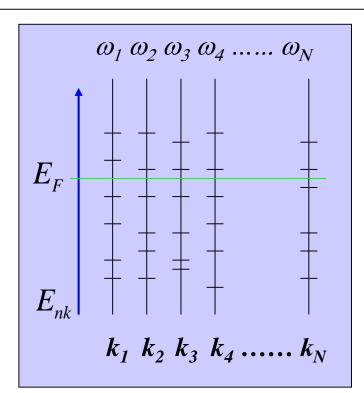
#### Example: 2 D







IBZ (irreducible BZ)



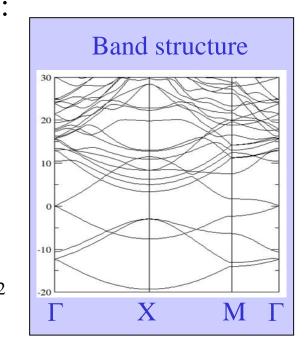
Weight of the k points:

$$\omega_{k} = \frac{2A_{k}}{\sum_{k} A_{k}}$$

$$\sum_{n} \sum_{k} \omega_{k} = Z_{1} + Z_{2}$$

$$E_{nk} \leq E_{F}$$
charge density
$$\rho(\vec{r}) = \sum_{n} \sum_{k} \omega_{k} |\psi_{nk}|^{2}$$

 $E_{nk} \leq E_F$ 



#### Example: 3 D

Copper (Cu): fcc structure

$$\vec{a}_1 = \frac{a}{2} \left( \vec{j} + \vec{k} \right)$$

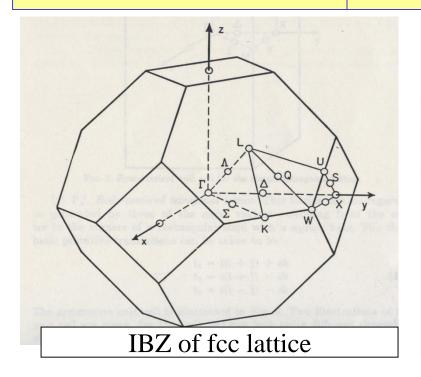
$$\vec{a}_2 = \frac{a}{2} \left( \vec{i} + \vec{k} \right)$$

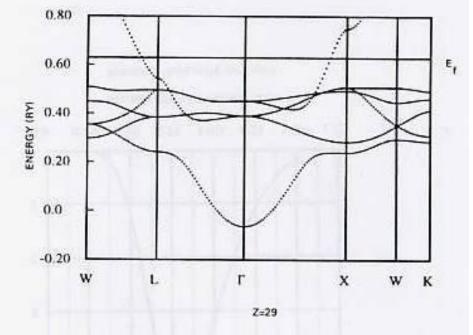
$$\vec{a}_3 = \frac{a}{2} \left( \vec{i} + \vec{j} \right)$$

$$\vec{b}_1 = \frac{4\pi}{a} \frac{1}{2} \left( -\vec{i} + \vec{j} + \vec{k} \right)$$

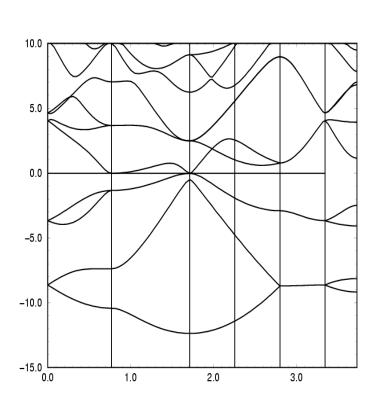
$$\vec{b}_2 = \frac{4\pi}{a} \frac{1}{2} \left( \vec{i} - \vec{j} + \vec{k} \right)$$

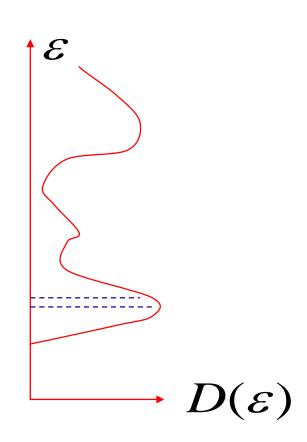
$$\vec{b}_3 = \frac{4\pi}{a} \frac{1}{2} \left( \vec{i} + \vec{j} - \vec{k} \right)$$





#### Density of states

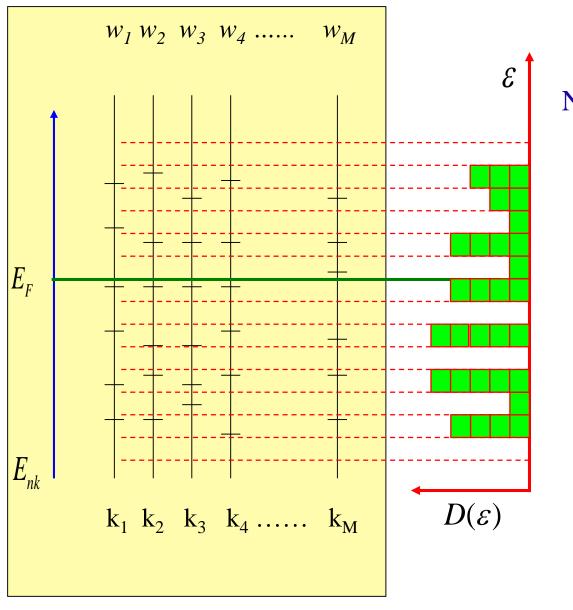




$$\Delta N = D(\varepsilon) \Delta \varepsilon$$

number of state from  $\varepsilon$  to  $\varepsilon + \Delta \varepsilon$ 

 $D(\varepsilon)$ : Density of states



$$D(\varepsilon_i)\Delta\varepsilon$$

Number of state within $\Delta \varepsilon$ 

$$\omega_k = \frac{2A_k}{\sum_k A_k}$$

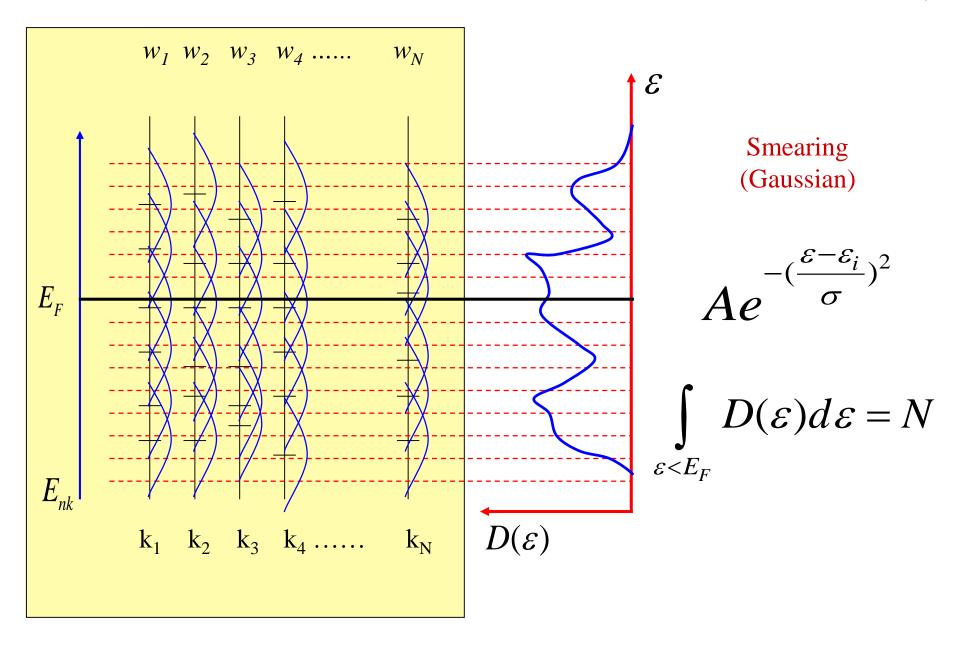
$$\sum_{k} \omega_{k} = 2$$

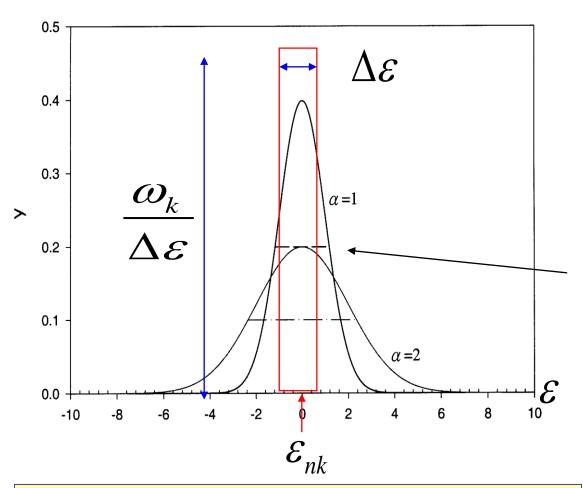
$$D(\varepsilon_{i}) = \frac{1}{\Delta \varepsilon} \sum_{k} \omega_{k}$$

$$N = \frac{1}{\Lambda \varepsilon} \sum_{k} \omega_{k} f_{k}$$

where

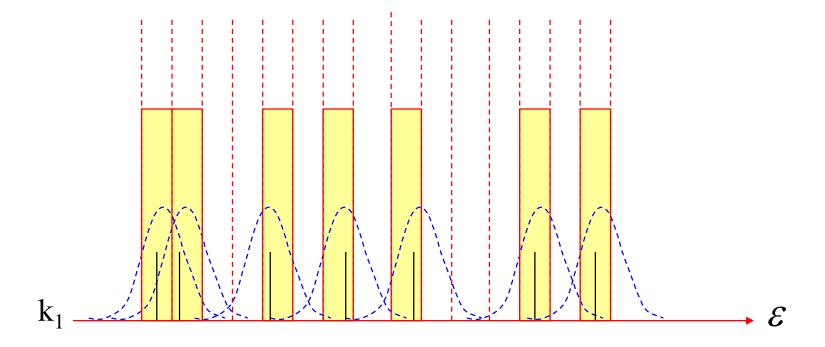
$$f_k = \begin{cases} 1 & occupied \\ 0 & otherwise \end{cases}$$





They have the same area.

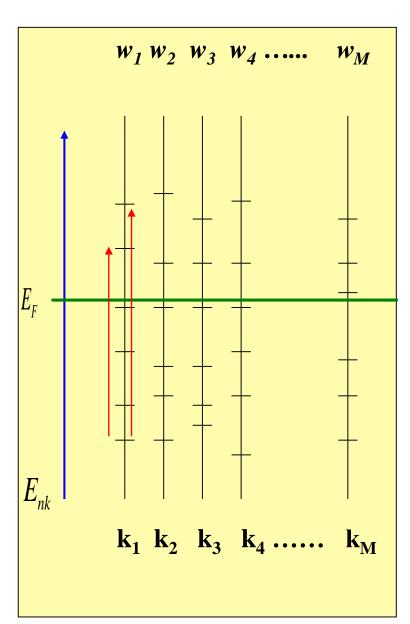
$$D_{nk}(\varepsilon) = \frac{\omega_k}{\sqrt{2\pi\alpha}} e^{-\frac{(\varepsilon - \varepsilon_{nk})^2}{2\alpha^2}}$$



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# How to calculate the absorption spectrum of a crystal?

#### How to calculate the absorption spectrum of a crystal?



$$\psi(t=0) = \phi_{vi}$$

$$P_{cj}(t) = \left| \left\langle \phi_{cj} \left| \psi(t) \right\rangle \right|^2 = \left| c_{cj}(t) \right|^2$$

Transition probability per unit time:

$$\Gamma_{vi \to cj} = \frac{P_{vi,cj}(t)}{t}$$

**Transition rate W:** 

$$W = \sum_{\substack{\text{valence conduction} \\ \textit{band}}} \sum_{\vec{k}} w_{\vec{k}} \Gamma_{\textit{vi} \rightarrow \textit{cj}}$$

**Absorption Constant (α):** 

$$\frac{\hbar\omega \times \text{Transition rate (W)}}{\text{Incident flux (I)}}$$

#### **Time-Dependence Perturbation Theory**

$$i\hbar \frac{\partial \psi(t)}{\partial t} = \left[ H_{_{0}} + \lambda V(t) \right] \psi(t) \qquad ; \quad H_{_{0}}\phi_{_{n}} = E_{_{n}}^{0}\phi_{_{n}} ; \quad \left\langle \phi_{_{m}} \middle| \phi_{_{n}} \right\rangle = \mathcal{S}_{mn}$$

$$\psi(t) = \sum_{n} c_{_{n}}(t) \exp(-iE_{_{n}}^{0}t/\hbar)\phi_{_{n}} \qquad (if \quad V(t) = 0 \quad \Rightarrow \quad C_{_{n}}(t) = const.)$$

$$i\hbar \frac{\partial}{\partial t} \left[ \sum_{_{n}} c_{_{n}}(t) \exp(-iE_{_{n}}^{0}t/\hbar)\phi_{_{n}} \right] = \left[ H_{_{0}} + \lambda V(t) \right] \left[ \sum_{_{n}} c_{_{n}}(t) \exp(-iE_{_{n}}^{0}t/\hbar)\phi_{_{n}} \right]$$

$$\sum_{_{n}} \left[ i\hbar \frac{dc_{_{n}}(t)}{dt} + E_{_{n}}^{0}c_{_{n}}(t) \right] \exp(-iE_{_{n}}^{0}t/\hbar) \phi_{_{n}} = \sum_{_{n}} \left[ E_{_{n}}^{0} + \lambda V(t) \right] c_{_{n}}(t) \exp(-iE_{_{n}}^{0}t/\hbar) \phi_{_{n}}$$

$$i\hbar\sum_{n} \frac{dc_n(t)}{dt} \exp(-iE_n^0 t/\hbar)\phi_n = \lambda\sum_{n} c_n(t)V(t) \exp(-iE_n^0 t/\hbar) \phi_n \dots (1)$$

$$\langle \phi_m | \exp(iE_m^0 t/\hbar) \times (1)$$

$$i\hbar \frac{dc_m(t)}{dt} = \lambda \sum_n c_n(t) \exp \left[i(E_m^0 - E_n^0)t/\hbar\right] \langle \phi_m | V(t) | \phi_n \rangle$$

$$i\hbar \frac{dc_m(t)}{dt} = \lambda \sum_n c_n(t) \exp \left[i(E_m^0 - E_n^0)t/\hbar\right] \langle \phi_m | V(t) | \phi_n \rangle$$

initial condition at t = 0:  $\psi(t = 0) = \phi_k \rightarrow c_n(0) = \delta_{nk}$ 

We want to find  $C_n(t)$ 

for  $m \neq k$ 

$$i\hbar \frac{dc_m(t)}{dt} = \lambda \exp\left[i(E_m^0 - E_k^0)t/\hbar\right] \langle \phi_m | V(t) | \phi_k \rangle$$

$$c_{m}(t) = \frac{\lambda}{i\hbar} \int dt \exp \left[i(E_{m}^{0} - E_{k}^{0})t'/\hbar\right] \langle \phi_{m} | V(t') | \phi_{k} \rangle$$

the probability  $P_n(t) = \left| \left\langle \phi_n \left| \psi(t) \right\rangle \right|^2 = \left| c_n(t) \right|^2$ 

#### **Absorption of Light**

$$H = \frac{\left[\overrightarrow{p} + e\overrightarrow{A}(\overrightarrow{r}, t)\right]^{2}}{2m} + V(r)$$

(1) Choose Coulomb gauge

$$\vec{p}, \vec{A} = 0 \Rightarrow \vec{p} \cdot \vec{A} = \vec{A} \cdot \vec{p}$$

(2) Neglect A<sup>2</sup> term



$$A(r,t) = A_0 \vec{\mathcal{E}} \cos(\vec{k} \cdot \vec{r} - \omega t)$$

$$= \frac{1}{2} A_0 \vec{\mathcal{E}} \exp(i\vec{k} \cdot \vec{r}) \exp(-i\omega t) + c.c.$$

$$c_n(t) = \frac{eA_0}{2m} \langle \phi_n | e^{i\vec{k}\cdot\vec{r}}\vec{\varepsilon}\cdot\vec{p} | \phi_k \rangle \int_0^t dt' e^{i(E_n^0 - E_k^0 - \hbar\omega)t'/\hbar}$$

$$+rac{eA_{0}}{2m}\langle\phi_{n}\left|e^{-i\vec{k}\cdot\vec{r}}\vec{\varepsilon}\cdot\overrightarrow{p}\left|\phi_{k}
ight
angle\int_{0}^{t}dt'e^{i(E_{n}^{0}-E_{k}^{0}+\hbar\omega)t'/\hbar}$$

$$\left|\int_{0}^{t} dt' e^{i(E_{n}^{0} - E_{k}^{0} \pm \hbar\omega)t'/\hbar}\right|^{2} = \left|\int_{0}^{t} dt' e^{i\Omega t'}\right|^{2} = \left|\frac{2}{\Omega} e^{i\Omega t/2} \sin\frac{\Omega t}{2}\right|^{2} = \frac{4}{\Omega^{2}} \sin^{2}\frac{\Omega t}{2}$$

where 
$$\Omega = \frac{E_n^0 - E_k^0 \pm \hbar \omega}{\hbar}$$

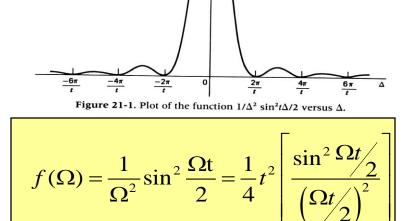
$$\int_{-\infty}^{\infty} f(\Omega) \frac{4}{\Omega^2} \sin^2 \frac{\Omega t}{2} d\Omega$$

$$= f(0) \int_{-\infty}^{\infty} d\Omega \frac{4}{\Omega^2} \sin^2 \frac{\Omega t}{2}$$

$$=2tf(0)\int_{-\infty}^{\infty}dy\frac{1}{y^2}\sin^2y$$

$$=2\pi tf(0)$$

#### For large t



$$\frac{4}{\Omega^2}\sin^2\frac{\Omega t}{2} \to 2\pi t \delta(\Omega) = 2\pi t \hbar \delta(E_n^0 - E_k^0 \pm \hbar \omega)$$

$$P_{n}(t) = |C_{n}(t)|^{2}$$

$$= \frac{2\pi}{\hbar} t \left(\frac{eA_{0}}{m}\right)^{2} \left| \left\langle \phi_{n} \left| \exp(\pm i\vec{k} \cdot \vec{r})\vec{\varepsilon} \cdot \vec{p} \right| \phi_{k} \right\rangle \right|^{2} \delta(E_{n}^{0} - E_{k}^{0} \pm \hbar \omega)$$

Optical properties : energy of photon 0.1 eV - 20 eV;

 $\exp(\pm i\vec{k}\cdot\vec{r}) \sim 1$ 

Transition probability per unit time:

$$\Gamma_{k\to n} = \frac{2\pi}{\hbar} \left(\frac{eA_0}{m}\right)^2 \left| \left\langle \phi_n \left| \vec{\varepsilon} \cdot \vec{p} \right| \phi_k \right\rangle \right|^2 \delta(E_n^0 - E_k^o \pm \hbar \omega)$$

For absorption process:

$$\Gamma_{k\to n} = \frac{2\pi}{\hbar} \left(\frac{eA_0}{m}\right)^2 \left| \left\langle \phi_n \left| \vec{\varepsilon} \cdot \vec{p} \right| \phi_k \right\rangle \right|^2 \delta(E_n^0 - E_k^o - \hbar\omega)$$

**Transition rate W:** 

$$\vec{M}_{cv} = \left\langle \phi_c \mid \overrightarrow{p} \mid \phi_v \right\rangle$$

$$W(\omega) = \frac{2\pi}{\hbar} \left(\frac{eA_0}{m}\right)^2 \frac{V}{\left(2\pi\right)^3} \sum_{v} \sum_{c} \int_{B_z} d^3k \left| \vec{\varepsilon} \cdot \overrightarrow{M}_{cv}(\vec{k}) \right|^2 \delta \left[ E_c(\vec{k}) - E_v(\vec{k}) - \hbar\omega \right]$$

Absorption constant  $(\alpha) =$ 

$$\hbar\omega \times \text{Transition rate (W)}$$

#### Incident flux (I)

$$I = v\overline{u} = \frac{c}{n} \frac{1}{2} \varepsilon \omega^2 A_0^2 = \frac{1}{2} nc\varepsilon_0 \omega^2 A_0^2$$

$$\alpha(\omega) = \frac{4\pi e^2}{m^2 n c \varepsilon_0 \omega} \frac{V}{(2\pi)^3} \sum_{v} \sum_{c} \int_{B_z} d^3k \left| \vec{\varepsilon} \cdot \overrightarrow{M}_{cv}(\vec{k}) \right|^2 \delta \left[ E_c(\vec{k}) - E_v(\vec{k}) - \hbar \omega \right]$$

$$G_{cv}(\omega) = \frac{V}{(2\pi)^3} \int_{R_c} d^3k \delta \left[ E_c(\vec{k}) - E_v(\vec{k}) - \hbar \omega \right]$$
 Joint Density of States

$$= \frac{V}{\left(2\pi\right)^{3}} \int_{Bz} \frac{dS_{E}}{\vec{\nabla}_{\vec{k}} \left[ E_{c}(\vec{k}) - E_{v}(\vec{k}) - \hbar\omega \right]} = \sum_{T_{i}} G_{cv}^{i}(\omega)$$

$$\alpha(\omega) = \frac{4\pi e^2}{m^2 n c \varepsilon_0 \omega} \sum_{T_c} \sum_{v} \sum_{c} \left| \vec{\varepsilon} \cdot \overrightarrow{M}_{cv}(\vec{k}) \right|^2 G^i_{cv}(\omega)$$

#### **Calculation of Dielectric Constant**

$$\nabla^{2}\vec{E} - \frac{\mu\varepsilon}{c^{2}} \frac{\partial^{2}\vec{E}}{\partial t^{2}} = 0 \qquad assume \qquad \mu = 1 \qquad \text{(Jackson P297)}$$

$$\vec{E} = \vec{E}_{0}e^{i(kz-\omega t)} \quad ; \quad k^{2} = \varepsilon \frac{\omega^{2}}{c^{2}} \qquad \begin{vmatrix} \vec{E} = \vec{E}_{0}e^{i(k_{1}z-\omega t)-\frac{1}{2}\alpha z} \\ I \sim E^{2} \sim I_{0}e^{-\alpha z} \end{vmatrix}$$

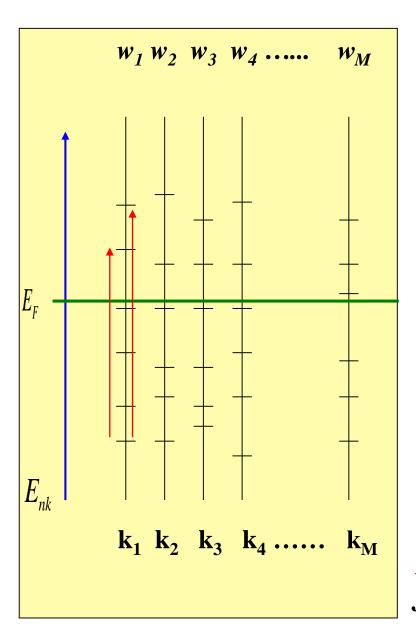
$$\varepsilon = \varepsilon_{1} + i\varepsilon_{2} \qquad \qquad \begin{cases} k_{1}^{2} - \frac{\alpha^{2}}{4} = \frac{\omega^{2}}{c^{2}}\varepsilon_{1} \\ k_{1}\alpha = \frac{\omega^{2}}{c^{2}}\varepsilon_{2} \end{cases}$$

$$\alpha = \sqrt{2}\frac{\omega}{c} \left[ -\varepsilon_{1} \pm \sqrt{\varepsilon_{1}^{2} + \varepsilon_{2}^{2}} \right]^{\frac{1}{2}} \qquad \qquad \alpha = \frac{\omega^{2}}{c^{2}k_{1}}\varepsilon_{2} = \frac{\omega}{nc}\varepsilon_{2}$$

$$\varepsilon_{2}(\omega) = \frac{4\pi e^{2}}{m^{2}\omega^{2}} \sum_{v} \sum_{c} \left| \vec{\varepsilon} \cdot \overrightarrow{M}_{cv}(\vec{k}) \right|^{2} G_{cv}(\hbar \omega)$$

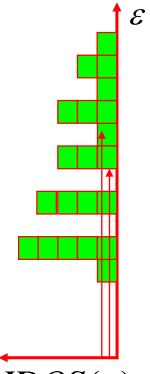
Kramers-Kronig relations  $\varepsilon_1(\omega) = 1 + \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\varepsilon_2(\omega') - 1}{\omega' - \omega} d\omega'$ 

#### **Joint Density of State**



$$G_{cv}(\omega) = \frac{V}{(2\pi)^3} \int_{Bz} d^3k \delta \left[ E_c(\vec{k}) - E_v(\vec{k}) - \hbar \omega \right]$$

$$\varepsilon_{2}(\omega) = \frac{4\pi e^{2}}{m^{2}\omega^{2}} \sum_{v} \sum_{c} \left| \vec{\varepsilon} \cdot \overrightarrow{M}_{cv}(\vec{k}) \right|^{2} G_{cv}(\hbar\omega)$$



$$\vec{M}_{cv} = \left\langle \phi_c \mid \overrightarrow{p} \mid \phi_v \right\rangle$$

#### The fermi dirac function

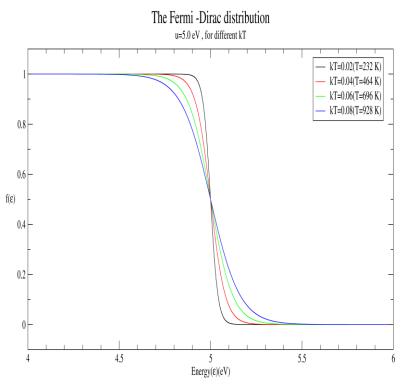
$$f(\varepsilon) = \frac{1}{1 + e^{(\varepsilon - \mu)/kT}}$$

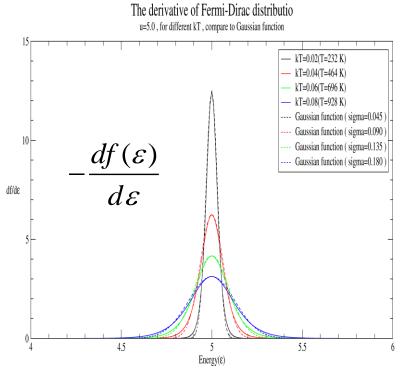
$$\mathbf{At} \; \mathbf{T} = \mathbf{0}$$

$$-\frac{df(\varepsilon)}{d\varepsilon} = \delta(\varepsilon - \mu)$$

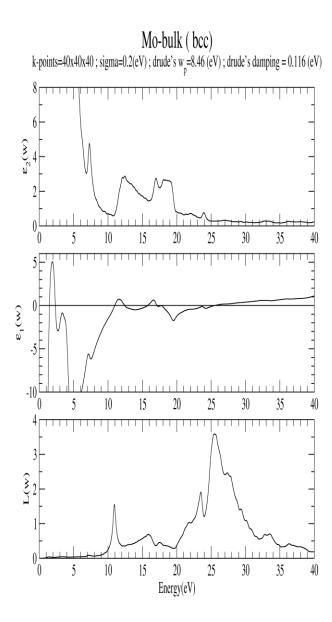
#### For small T, it behaves like Gaussian function

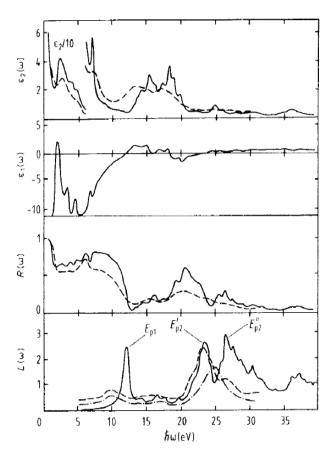
$$f(\varepsilon) = \frac{1}{\alpha \sqrt{\pi}} e^{-(\frac{\varepsilon - \mu}{\alpha})^2}$$





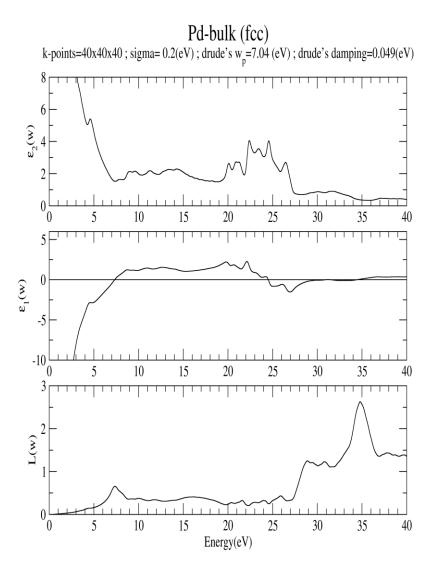
$$\varepsilon_{2}^{\text{int } ra}(\omega) = \frac{4\pi e^{2}}{m^{2}\omega^{2}} \sum_{nk} \left| \vec{\varepsilon} \cdot \overrightarrow{M}_{nn}(\vec{k}) \right|^{2} \left( -\frac{\partial f_{0}}{\partial \varepsilon} \right)_{\varepsilon_{nk}} D_{n}(\hbar \omega)$$

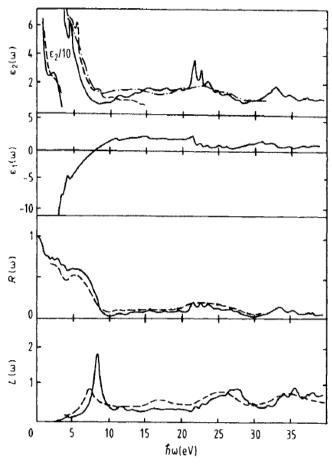




**Figure 6.** Optical properties of molybdenum: full curves, calculations; broken curves, measurements (from Mayevskii *et al* 1981); chain curve, measurements of  $L(\omega)$  (from Weaver *et al* 1974) (arbitrary units).

#### J.Phys.F:Met.Phys.18(1988) 833-849





**Figure 4.** Optical properties of palladium: full curves, calculations; broken curves, measurements of  $\varepsilon_2(\omega)$  (from Weaver 1973),  $R(\omega)$  (from Vehse *et al* 1970) and  $L(\omega)$  (from Daniels 1969); chain curve, measurements of  $\varepsilon_2(\omega)$  (from Vehse *et al* 1970) (arbitrary units).

#### J.Phys.F:Met.Phys.18(1988) 833-849

### Step by Step Tutorial

#### Calculations on Optical Properties

講師:中正大學物理系梁贊全教授

協助人員: 姚華凱同學

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{eff}(\vec{r}) \right] \psi_i(\vec{r}) = \varepsilon_i \psi_i(\vec{r}) \qquad ; \quad \rho(\vec{r}) = \sum_i^{occ} \left| \psi_i(\vec{r}) \right|^2$$

$$V_{eff}(\vec{r}) = V_{ext}(\vec{r}) + V_{H}(\vec{r}) + V_{xc}(\vec{r}) + V_{ps}(\vec{r})$$

$$V_H(\vec{r}) = \int \frac{\rho(\vec{r})}{|\vec{r} - \vec{r}'|} d^3r; \quad V_{xc}(\vec{r}) = V_{xc}(\rho(\vec{r})) \quad LDA$$

$$V_{eff}(\vec{r}+\vec{R}) = V_{eff}(\vec{r})$$

$$\psi_k(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u(\vec{r})$$

$$V_{eff}(\vec{r} + \vec{R}) = V_{eff}(\vec{r})$$

$$\psi_{k}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u(\vec{r})$$

$$u_{k}(\vec{r}) = \sum_{G_{n} < G_{\text{max}}} C_{n}e^{i\vec{G}_{n}\cdot\vec{r}}$$

$$\underbrace{\mu(\vec{k})C - \lambda SC = 0}_{\vec{k}} = 0$$

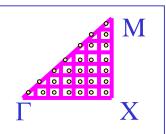
$$\underline{H}(\vec{k})\underline{C} - \lambda \underline{S}\underline{C} = 0$$

$$\vec{\overline{\tau}}_{i} \rightarrow \begin{matrix} \rho_{in} & \longrightarrow & V_{eff}(r) & \longrightarrow & \mathcal{E}_{i}; \psi_{i} & \longrightarrow & \rho_{out} \\ \rho_{in}^{n+1} = (1-\alpha)\rho_{in}^{n} + \alpha\rho_{out}^{n} & \longrightarrow & \vec{F}_{i} \\ \Delta \vec{\tau}_{i} = \eta \vec{F}_{i} \end{matrix}$$

Structure of the system (R,  $\tau$ ): **POSCAR** 

Pseudopotential of the atoms : **POTCAR** 

Number of K- points in the IBZ : **KPOINTS** 



$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \sum_{G_n < G_{\text{max}}} C_n e^{i\vec{G}_n\cdot\vec{r}}$$

$$\rho(\vec{r}) = \sum_{\vec{G}} \rho(\vec{G}) e^{i\vec{G}\cdot\vec{r}}$$

$$V(\vec{r}) = \sum_{\vec{G}} V(\vec{G}) e^{i\vec{G}\cdot\vec{r}}$$

$$G_{\text{max}}, \text{NGX}, \text{NGY}, \text{NGZ}$$

*NGX,NGY,NGZ*: controls the number of grid-points in the FFT-mesh into the direction of the three lattice-vectors.

Break condition for electronic SC-loop and ionic relaxation loop.

Max. number of electronic step and ionic step.

How the ions are updated and moved (conjugate-gradient method).

How the partial occupancies  $f_{nk}$  are set for each wavefunction.



#### **INPUT FILES**

POSCAR POTCAR KPOINTS INCAR

#### **Stop VASP during the program execution**

#### **STOPCAR**

LSTOP = .TRUE. (Ionic step)
LABORT = .TRUE. (Electronic step)

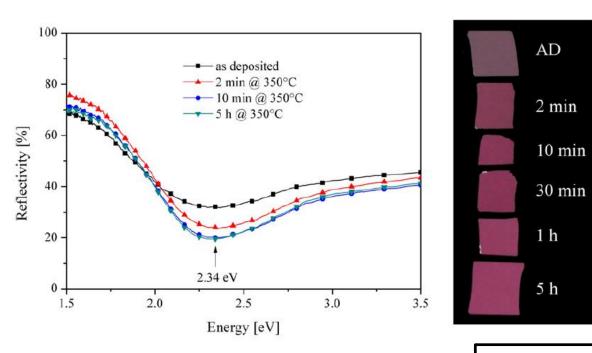
VASP will check the existence of STOPCAR for every electronic step and ionic step.
Print out WAVECAR and CHGCAR before stop.

#### **OUTPUT FILES**

OUTCAR
OSZICAR
CONTCAR
CHGCAR
WAVECAR
EIGENVAL
PROCAR
MME
MME

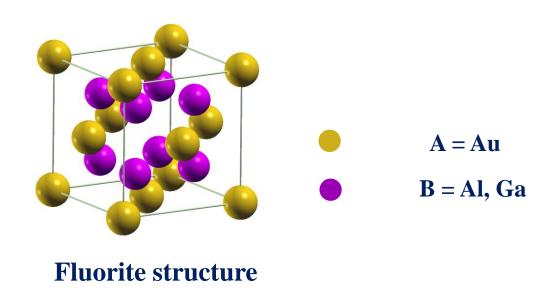
vsub\_opitcs name bkill jobid

#### The unusual purple color in AuAl<sub>2</sub>



Acta Mat. 61, 2874 (2013)

#### Lattice structure of AuAl<sub>2</sub>



#### **POSCAR**

AuAl2 6.500000 0.000000 0.500000 0.500000 0.500000 0.000000 0.500000 0.500000 0.500000 0.000000 1 2 Direct

Direct 0.000000 0.000000 0.000000 0.250000 0.250000 0.250000 0.750000 0.750000 0.750000

#### Set up the environment in optics calculaiton

 $cd \\ cp /home/s11gyg00/cp-1218.sh . \\ ./cp-1218.sh \\$ 

#### **How to run optics**

#### 1. Calculate the lattice constant of AuAl2

cdw cd Hands-on-1218 Hands-on-1218> ls 1-Au 2-W 3-AuAl2 4-AuGa2 5-AuAl2-results cd 3-AuAl2 mkdir 1-stress mkdir 2-optics cd 1-stress cat ~/pot/POTCAR.Au.LDA > POTCAR cat ~/pot/POTCAR.Al.LDA >> POTCAR vi INCAR / POSCAR / KPOINTS 1-stress> ls 1-stress> vsub\_optics 3-stress JOBID = 7344 JOB = 3-stress This script is for running a single VASP job How manyf CPUs do you want to run your job? 1сри, 4сри, 8сри, 12сри, 24сри, 48сри, 96сри, 192сри? [1сри] 12cpu If more than 48 cpus were selected, specify the number of cores per node:

6, 12, 24, or 48?

Job <544761> is submitted to queue <12cpu>j

(Enter)

#### **INCAR**

System=AuAl2
PREC=high
ISIF=3
IBRION=2
NSW=100
EDIFF=0.000001
EDIFFG = -0.01
RWIGS=1.503 1.402

#### **KPOINTS**

#### **POSCAR**

AuAl2
6.500000
0.000000 0.500000 0.500000
0.500000 0.000000 0.500000
0.500000 0.500000 0.000000
1 2
Direct
0.000000 0.000000 0.000000
0.250000 0.250000 0.250000
0.750000 0.750000 0.750000

#### 2. Calculate the matrix element of AuAl2

1-stress> cd ../2-optics

2-optics> mkdir 10k

2-optics> cd 10k

cp ../../1-stress/CONTCAR POSCAR

cp ../../1-stress/POTCAR .

cp ../../1-stress/INCAR .

cp ../../1-stress/KPOINTS .

cp ../../1-stress/CHGCAR .

System=AuAl2 RWIGS=1.503 1.402 NPAR=1 ISMEAR=0 SIGMA = 0.2 LOPTICS=.TRUE. NBANDS=30

System=AuAl2 with spin-orbit coupling
RWIGS=1.503 1.402
NPAR=1
ISMEAR=0
SIGMA = 0.2
MAGMOM=0 0 0.0000001 0 0 0.000001 0 0
0.000001
LSORBIT=.TRUE.
SAXIS=0 0 1
GGA\_COMPAT=.FALSE.
LOPTICS=.TRUE.
NBANDS=60

AuAl2

6.50000

 $\begin{array}{ccccc} 0.0000000000000000 & 0.4583121851601681 & 0.4583121851601681 \\ 0.4583121851601681 & 0.00000000000000 & 0.4583121851601681 \\ 0.4583121851601681 & 0.4583121851601681 & -0.0000000000000000 \end{array}$ 

Au Al

2

**Direct** 

0.0000000E+00 0.0000000E+00 0.0000000E+00

0.00000000E+00 0.00000000E+00 0.00000000E+00

0.00000000E+00 0.00000000E+00 0.00000000E+00

2-optics> ls

#### CHGCAR INCAR KPOINTS POSCAR POTCAR

2-optics> vsub\_optics optics

#### 3. Calculate the optical properties of AuAl2

```
2-optics > ls (only important output are shown in the following)
        OUTCAR EPSIM.dat
MME
                          PROCAR
                                      WAVEDER
MME2
        OUTCAR EPSRE.dat OPTIC
                                      STRUC
s11gyg00@alps6:/work/s11gyg00/Hands-on-1218/3-AuAl2/2-optics> grep fermi OU*
OUTCAR: ISMEAR = 0; SIGMA = 0.20 broadening in eV -4-tet -1-fermi 0-gaus
OUTCAR: E-fermi: 7.8818 XC(G=0): -10.8076 alpha+bet:-13.7653
2-optics > optics.sh AuAl2
2-optics > cd optics
optics> ls
AuAl2.mme AuAl2.mme2 AuAl2.opticin AuAl2.procar AuAl2.strucin opticpack.def
```

```
2, 1
                        ! intra (1/2 no/yes), imme (1/2)
2, 2, 0.1
                       ! islot,ikk,kkshift (for interband transition)
             ! idrude,drude_damping,wpshift (not use for intra =1)
2, 0.07, 0
1, 7.8818, 1, 1, 0, 0 ! ival, ef, imetal, ispin, iso, itr
                                                                                    AuAl2.opticin
0.0, 20.0, 0.02, 0.2, 30 ! emin, erange, de, sigma, nbcal0
                        ! isci
1
0.0
                        ! eshift
3
                        ! itot
1
                        ! icom(1~itot 1:xx, 2:yy 3:zz)
3
```

optics> optics-2017.x

#### AuAl2.opticin

```
2, 1
                         ! intra (1/2 no/yes), imme (1/2)
2, 2, 0.1
                        ! islot,ikk,kkshift (for interband transition)
2, 0.07, 0
                        ! idrude,drude_damping,wpshift (not use for intra =1)
1, 7.8818, 1, 1, 0, 0
                    ! ival,ef,imetal,ispin,iso,itr
0.0, 20.0, 0.02, 0.2, 30 ! emin, erange, de, sigma, nbcal0
                         ! isci
1
0.0
                         ! eshift
3
                         ! itot
                        ! icom(1~itot 1:xx, 2:yy 3:zz)
3
```

For intraband transitions

idrude=1 ( formula from exciting code ) :

$$\varepsilon_{ab,a=b} = \delta_{aa} + \frac{4\pi i\sigma(\omega)}{\omega + i\eta(\omega)} = 1 - \frac{\omega_p^2}{(\omega + i\eta(\omega))^2} = 1 - \frac{\omega_p^2(\omega^2 - \eta^2)}{(\omega^2 + \eta^2)^2} + i(\frac{2\omega_p^2\omega\eta}{(\omega^2 + \eta^2)^2})$$

Where  $\eta$  is sigma, the Gaussian smear factor.

( Don't use the damping term )

idrude= else ( formula from drude-model (default) ) :

$$\varepsilon_{ab,a=b} = \varepsilon_1(\omega) + i\varepsilon_2(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\Gamma\omega} = (1 - \frac{\omega_p^2}{\omega^2 + \Gamma^2}) + i\frac{\omega_p^2\Gamma}{\omega(\omega^2 + \Gamma^2)}$$

where  $\Gamma$  is the damping term (unit in eV)

wpshift: Shift the calculated plasma energy (unit in eV),

```
intra : 1= false, otherwise = true (include the intraband transition)
imme: 1-read mme file(case.mme), otherwise-read mme2 file (case.mme2 : vasp's mme)
ifstatic: if calculate static value only ( default = 0 )
islot: 1 is closed, other: open
ikk: 1 is closed, other: open
kkshift : useful for ikk \neq 1 ( default : 0.1 )
ival: number of valence bands (not used for metal)
ef: fermi energy (eV) ( not used for a non-metal)
imetal: 1 for metal, otherwise for a non-metal)
ispin: 1 or 2 ( 2 for spin-polarized)
iso: 1 for spin-orbit coupling (default=0)
itr: 1 without symmetry ( default = 0)
emin: minimum energy
erange: maximum energy=emin+erange
de: ∧energy
sigma: Gaussian smear factor (unit in eV)
nbcal0: number of bands to be considered, could be determined by mme file
       ( equal to NBANDS from INCAR)
isci: 0 - do not use
       1 – native form, only shift energy
       2 – also renormalize momentum matrix
       (default:1)
eshift: energy shift (default : 0.0)
itot: 1~6
1 - xx 2 - yy 3—zz 4—yz(zy) 5—zx(xz) 6—xy(yx)
icom: icom=icom(itot)
```

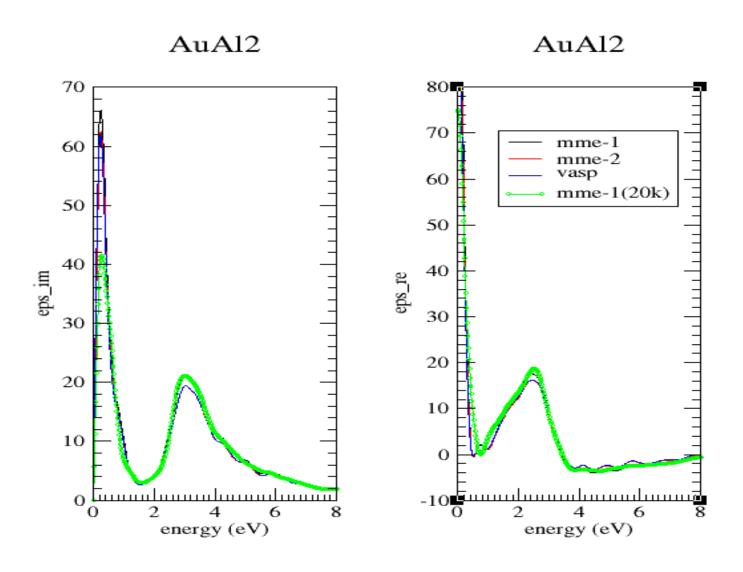
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# output of optics-2017.x

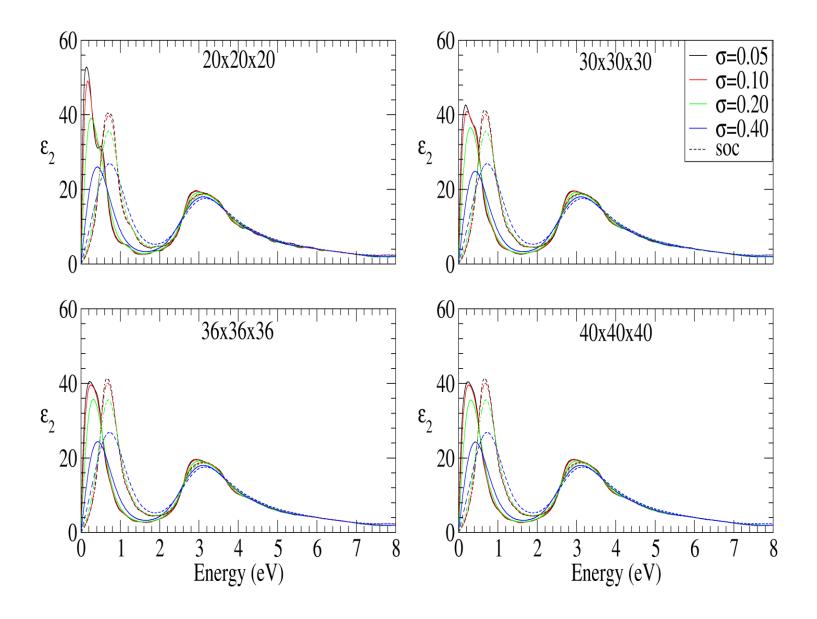
AuAl2.opticout	AuAl2.jdos	AuAl2.alpha1
AuAl2.epsre_intra	AuAl2.epsim_intra	
AuAl2.epsre_inter	AuAl2.epsim_inter	
AuAl2.epsre	AuAl2.epsim	AuAl2.epsabs
AuAl2.refrare	AuAl2.refraim	
AuAl2.sigre	AuAl2.sigim	
AuAl2.eels_intra	AuAl2.eels_inter	AuAl2.eels
AuAl2.wplas		

#### optics.sh name

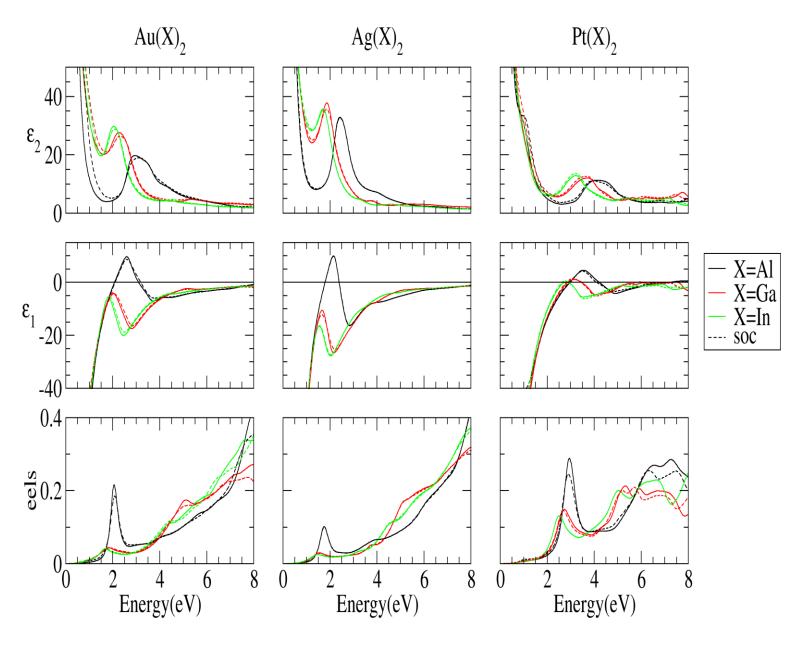
```
mkdir optics
cd optics
cp ../MME $1.mme
cp ../MME2 $1.mme2
cp ../STRUC $1.strucin
cp ../PROCAR $1.procar
cp ~/bin/X.opticin $1.opticin
cat >opticpack.def<<!
$1
!
```



interband contribution only



interband contribution only



interband + intraband

#### **Homework in Optics Properties**

- 1. Find the joint density of states, dielectric function, energy loss function and absorption constant of Au and W ( see example ) . Find the spin-orbit interaction effect and strain effect on the optical properties. ( 4%, -2%, 0%, 2%, 4%)
- 2. Find the joint density of states, dielectric function, energy loss function and absorption constant of  $AuAl_2$  and  $AuGa_2$ . Find the spin-orbit interaction effect and strain effect on the optical properties. (-4%, -2%, 0%, 2%, 4%)

## **END**