5.1 Discussion Questions

Name Matt Lizak

1. Determine whether each of the functions below is a power function. If it is, rewrite it in the form $f(x) = kx^p$ and identify k and p.

a. $f(x) = \frac{7}{3x^4}$ $= \frac{7}{3}x^{-4}$ $K = \frac{7}{3}$ $p - 4$	b. $h(u) = 17(2^x)^{-1}$ $= 17(2)^{-x}$ Not a power function	c. $h(x) = (11x^4)(\pi x^{-1})$ $= 11\pi x^3$ $K = 11\pi$ $p = 3$
d. $p(x) = 3x^2 + 7x - 5$ Not a power function	e. $g(w) = \frac{3w^6}{5\sqrt{w^3}}$ $= \frac{3}{5} \cdot \frac{w^6}{w^{3/2}}$ $w = \frac{9}{2}$ $= \frac{3}{5}w^{9/2}$ $K = \frac{3}{5}$	f. $y = \frac{1}{2}(3x\sqrt{x})^3$ $= \frac{1}{2} \cdot 27x^3 \cdot x^{3/2}$ $= \frac{27}{2}x^{9/2}$ $K = \frac{27}{2}$ $p = \frac{9}{2}$

2. Find a formula for the power function that passes through $(2, 20)$ and $(3, 45)$. $y = Kx^p$

X	2	3
f(x)	20	45

$20 = K(2)^p$
$45 = K(3)^p$

$\frac{20}{45} = \frac{K(2)^p}{K(3)^p} = \left(\frac{2}{3}\right)^p$ (5·4 / 5·9)

$\frac{4}{9} \rightarrow \frac{2^2}{3^2}$ $p = 2$

$20 = K(2)^2$
$20 = K4$
$\frac{20}{4} = \frac{K4}{4}$
$K = 5$

$\boxed{y = 5x^2}$

3. Find a formula for the power function that passes through $\left(2, -\frac{5}{4}\right)$ and $\left(5, -\frac{2}{25}\right)$.

x	2	5
f(x)	$-\frac{5}{14}$	$-\frac{2}{25}$

$K = -10$

$-\frac{40}{4} = -\frac{5}{4} \cdot \frac{8}{1}$

$-\frac{5}{4} = K(2)^{-3}$

$\frac{-\frac{5}{4}}{\frac{1}{8}} = \frac{(K\frac{1}{8})}{\frac{1}{8}}$

$-\frac{5}{4} = K(2)^p$
$-\frac{2}{25} = K(5)^p$

$\frac{-\frac{5}{4}}{-\frac{2}{25}} = \frac{(2)^p}{(5)^p} = \left(\frac{2}{5}\right)^p$

$\frac{5}{4} \cdot \frac{25}{2} = \frac{5^3}{2^3} = \left(\frac{2}{5}\right)^p \Rightarrow p = -3$

$y = -10x^{-3}$

4. Consider the following functions:

$$f(x) = \frac{1}{x} \qquad g(x) = \frac{1}{x^2}$$

a. Complete the following table.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	-10	-100	-1000	undef.	1000	100	10
$g(x)$	100	10,000	1,000,100	undef	1,000,000	10,000	100

b. Fill in the blanks:

As x approaches zero from the **right**, $\boldsymbol{f(x)}$ is approaching ∞.

That is, $\lim\limits_{x \to 0^+} f(x) = \infty$.

As x approaches zero from the **left**, $\boldsymbol{f(x)}$ is approaching $-\infty$.

That is, $\lim\limits_{x \to 0^-} f(x) = -\infty$.

As x approaches zero from the **right**, $\boldsymbol{g(x)}$ is approaching ∞.

That is, $\lim\limits_{x \to 0^+} g(x) = \infty$.

As x approaches zero from the **left**, $\boldsymbol{g(x)}$ is approaching ∞.

That is, $\lim\limits_{x \to 0^-} g(x) = \infty$.

c. Which function appears to be growing more rapidly as x goes to 0 from the right?

$g(x)$

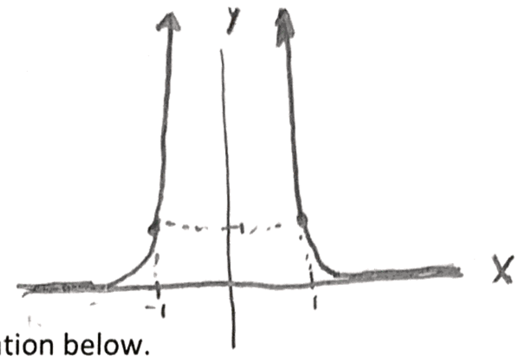
5. For the power function $F(x) = kx^p$, let $f(x) = pkx^{p-1}$.
For instance, if $F(x) = 3x^{-4/5}$, then $k = 3$ and $n = -\frac{4}{5}$, so $f(x) = -\frac{4}{5}(3)x^{-9/5}$.
Find $f(x)$ for the function $F(x)$ given below:

$$F(x) = \frac{7}{\sqrt[3]{x}} \qquad 7x^{-\frac{1}{3}} \qquad k = 7 \quad p = -\frac{1}{3}$$

$$f(x) = -\frac{1}{3}(7)\,x^{-\frac{1}{3}-1}$$

$$f(x) = -\frac{1}{3}(7)x^{-\frac{4}{3}}$$

6. a. Let $f(x) = x^{-10}$. Sketch a graph of this function. It doesn't need to be precise, but the general shape should be correct.

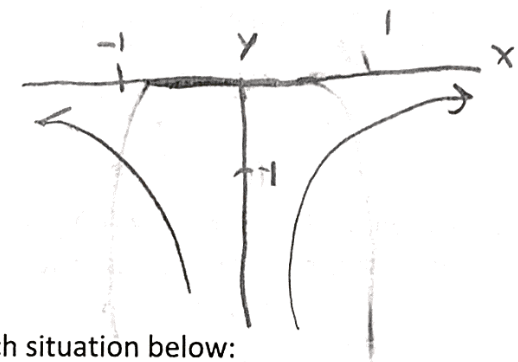


b. Describe the behavior of $f(x) = x^{-10}$ for each situation below.

$\lim_{x\to\infty} f(x) =$ 0 $\lim_{x\to 0^+} f(x) =$ ∞

$\lim_{x\to -\infty} f(x) =$ 0 $\lim_{x\to 0^-} f(x) =$ ∞

c. Let $g(x) = -x^{10}$. Sketch a graph of this function. It doesn't need to be precise, but the general shape should be correct.



d. Describe the behavior of $g(x) = -x^{10}$ for each situation below:

$\lim_{x\to\infty} g(x) =$ 0 $\lim_{x\to 0^+} g(x) =$ $-\infty$

$\lim_{x\to -\infty} g(x) =$ 0 $\lim_{x\to 0^-} g(x) =$ $-\infty$

7. Consider the power functions $f(x) = \frac{x^4}{100}$ and $g(x) = x^3$.

a. For what values of x is $f(x) > g(x)$?

$\frac{x^4}{100} > x^3$

$x^3 > 0$ for $x > 0$

$\frac{x}{100} - 1 > 0$ for $x > 100$

$x^4 - x^3 > 0$

$x^3(\frac{x}{100} - 1) > 0$

$\boxed{f(x) > 0 \text{ for } x > 100 \text{ and } x < 0}$

b. Which function dominates, $f(x)$ or $g(x)$?

$f(x)$ because $f(x) > g(x)$ for large values of x

8. In a microwave oven, cooking time is inversely proportional to the amount of power used. It takes 6.5 minutes to heat a frozen dinner at 750 watts.

   a. Write a formula for the cooking time, t, as a function of power level, w.

   $t = \frac{k}{w}$ $6.5 = \frac{k}{750} \Rightarrow k = 4875$

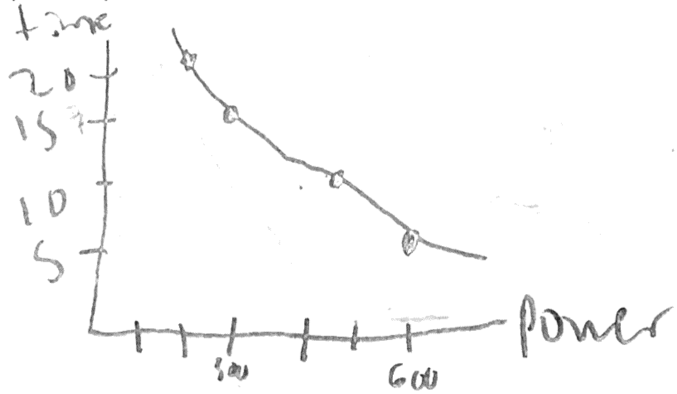
   $t = \frac{4875}{w}$

   b. Fill in the table below with the cooking times needed to heat the frozen dinner at various power levels. *(Hint: For 650 watts, the cooking time should be 7.5 minutes.)*

Power, w (watts)	250	300	500	650
Time, t (min)	19.5	16.25	9.75	7.5

   $\frac{4875}{250}$ $\frac{4875}{300}$ $\frac{4875}{500}$ $\frac{4875}{650}$

   c. Graph the function $t = f(w)$. Be sure that your graph has the correct shape, and label at least two specific points on it.



   d. If it takes 2 minutes to heat a rhubarb crumble at 250 watts, how long will it take at 500 watts? *(Hint: This is a re-do of part (a) with new values. Answer: 1 minute. Show how to get this.)*

   $2 = \frac{k}{250} \Rightarrow k = 500$

   $t = \frac{500}{500} = 1$ minute

   Each food has different k-value.