Assignment Three

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Question 1

$P(i,j)$: 当A还需赢下 i 场，B还需赢下 j 场时，A拿下比赛的可能性。

a. 给出$P(i,j)$的递推关系式

$i>0,j>0:P(i,j)=p\times P(i-1,j)+q\times P(i,j-1)$

$i=0,j>0:P(i,j)=P(0,j)=1$

$i>0,j=0:P(i,j)=P(i,0)=0$

b. 计算：当一场比赛A的胜率为0.4时，A赢下7场制系列赛的概率

$n=7,p=0.4,q=1-p=0.6$

```
P(1, 1) = p×P(0, 1) + q×P(1, 0) = 0.400000×1.000000 + 0.600000×0.000000 =
0.400000
P(1, 2) = p×P(0, 2) + q×P(1, 1) = 0.400000×1.000000 + 0.600000×0.400000 =
0.640000
P(1, 3) = p×P(0, 3) + q×P(1, 2) = 0.400000×1.000000 + 0.600000×0.640000 =
0.784000
P(1, 4) = p×P(0, 4) + q×P(1, 3) = 0.400000×1.000000 + 0.600000×0.784000 =
0.870400

P(2, 1) = p×P(1, 1) + q×P(2, 0) = 0.400000×0.400000 + 0.600000×0.000000 =
0.160000
P(2, 2) = p×P(1, 2) + q×P(2, 1) = 0.400000×0.640000 + 0.600000×0.160000 =
0.352000
P(2, 3) = p×P(1, 3) + q×P(2, 2) = 0.400000×0.784000 + 0.600000×0.352000 =
0.524800
P(2, 4) = p×P(1, 4) + q×P(2, 3) = 0.400000×0.870400 + 0.600000×0.524800 =
0.663040

P(3, 1) = p×P(2, 1) + q×P(3, 0) = 0.400000×0.160000 + 0.600000×0.000000 =
0.064000
P(3, 2) = p×P(2, 2) + q×P(3, 1) = 0.400000×0.352000 + 0.600000×0.064000 =
0.179200
P(3, 3) = p×P(2, 3) + q×P(3, 2) = 0.400000×0.524800 + 0.600000×0.179200 =
0.317440
P(3, 4) = p×P(2, 4) + q×P(3, 3) = 0.400000×0.663040 + 0.600000×0.317440 =
0.455680

P(4, 1) = p×P(3, 1) + q×P(4, 0) = 0.400000×0.064000 + 0.600000×0.000000 =
0.025600
P(4, 2) = p×P(3, 2) + q×P(4, 1) = 0.400000×0.179200 + 0.600000×0.025600 =
0.087040
```

```
P(4, 3) = p×P(3, 3) + q×P(4, 2) = 0.400000×0.317440 + 0.600000×0.087040 =
0.179200
P(4, 4) = p×P(3, 4) + q×P(4, 3) = 0.400000×0.455680 + 0.600000×0.179200 =
0.289792
```

$i \setminus j$	**0**	**1**	**2**	**3**	**4**
0		1	1	1	1
1	0	0.400000	0.640000	0.784000	0.870400
2	0	0.160000	0.352000	0.524800	0.663040
3	0	0.064000	0.179200	0.317440	0.455680
4	0	0.025600	0.087040	0.179200	0.289792

The answer is **0.289792**.

c. 写出使用动态规划法解决上述问题的伪代码，分析其时间复杂度与空间复杂度

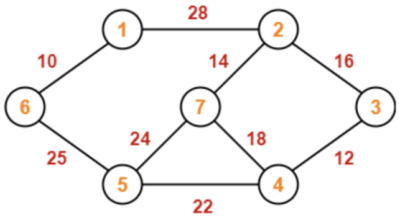
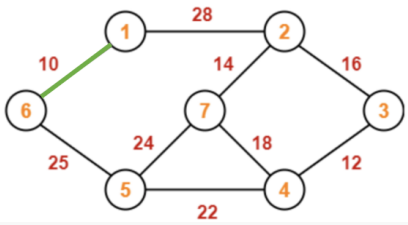
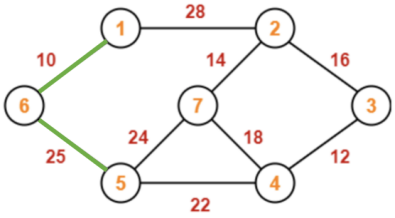
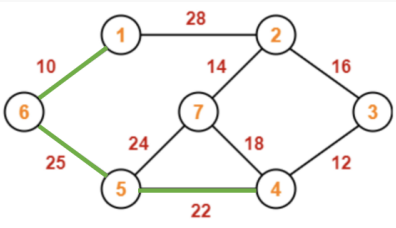
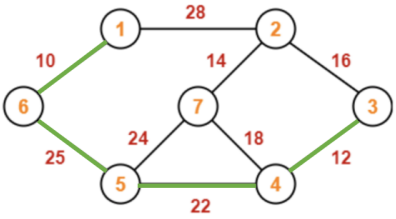
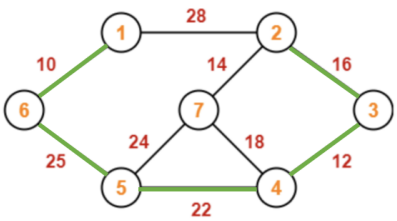
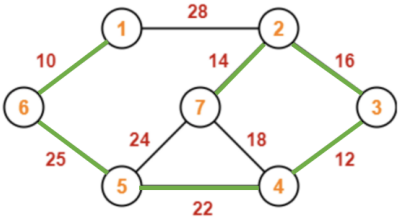
```
//参数：获胜所需赢得比赛的数量 - n
//      A单场比赛的胜率 - p
//返回：A在系列赛获胜的概率
probability_A_win (n, p)
{
    q <-- 1 - p;
    // P(0,j) = 1(j>0)
    for j <-- 1 to n do
        P[0,j] <-- 1.0
    // P(i,0) = 0(i>0)
    for i <-- 1 to n do
        P[i,0] <-- 0.0
    // P(i,j) = p * P(i-1, j) + q * P(i, j-1)
    for i <-- 1 to n do
        for j <-- 1 to n do
            P[i][j] <-- p * P[i-1, j] + q * P[i][j-1]

    return P[n][n]
}
```

时间复杂度	**空间复杂度**
$\Theta(n^2)$	$\Theta(n^2)$

Question 2

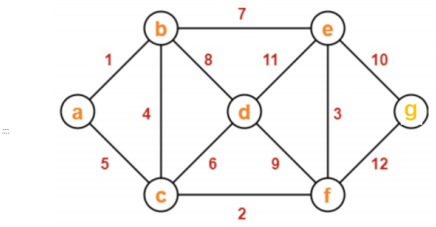
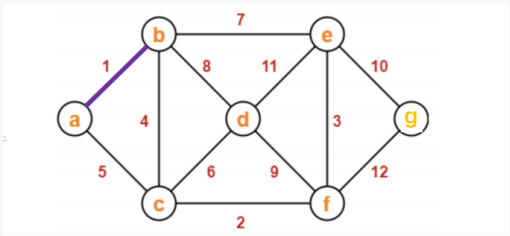
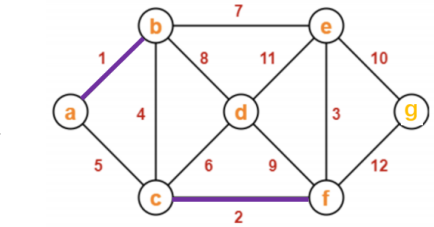
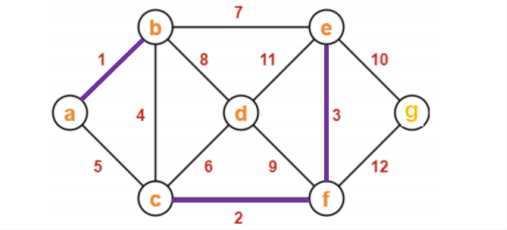
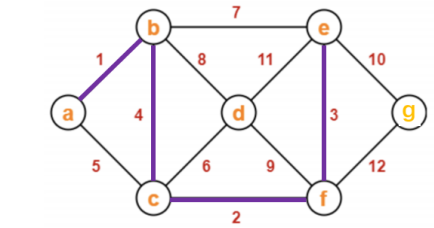
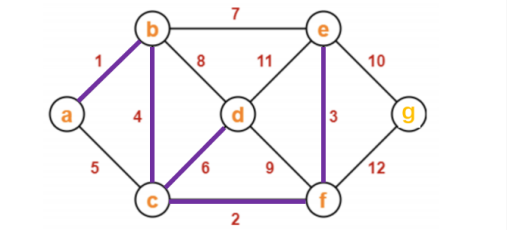
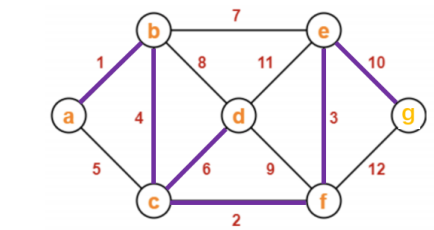
为下列给定的图构建最小生成树（MST），并计算其权重。

a. Using Prim's Algorithm

Tree vertices	Remaining vertices	diagram
$1(-,0)$	$2(1,28)\ 3(-,\infty)\ 4(-,\infty)\ 5(-,\infty)$ $6(1,10)\ 7(-,\infty)$	
$6(1,10)$	$2(1,28)\ 3(-,\infty)\ 4(-,\infty)\ 5(6,25)$ $7(-,\infty)$	
$5(6,25)$	$2(1,28)\ 3(-,\infty)\ 4(5,22)\ 7(5,24)$	
$4(5,22)$	$2(1,28)\ 3(4,12)\ 7(4,18)$	
$3(4,12)$	$2(3,16)\ 7(4,18)$	
$2(3,16)$	$7(4,14)$	
$7(4,14)$		

the cost of MST : 10+25+22+12+16+14 = **99**

b. Using Kruskal's Algorithm

Tree edges	Sorted list of edges	diagram
	ab(1) cf(2) ef(3) bc(4) ac(5) cd(6) be(7) bd(8) df(9) eg(10) de(11) fg(12)	
ab(1)	ab(1) **cf**(2) ef(3) bc(4) ac(5) cd(6) be(7) bd(8) df(9) eg(10) de(11) fg(12)	
cf(2)	ab(1) cf(2) **ef**(3) bc(4) ac(5) cd(6) be(7) bd(8) df(9) eg(10) de(11) fg(12)	
ef(3)	ab(1) cf(2) ef(3) **bc**(4) ac(5) cd(6) be(7) bd(8) df(9) eg(10) de(11) fg(12)	
bc(4)	ab(1) cf(2) ef(3) bc(4) ac(5) **cd**(6) be(7) bd(8) df(9) eg(10) de(11) fg(12)	
cd(6)	ab(1) cf(2) ef(3) bc(4) ac(5) cd(6) be(7) bd(8) df(9) **eg**(10) de(11) fg(12)	
eg(10)	ab(1) cf(2) ef(3) bc(4) ac(5) cd(6) be(7) bd(8) df(9) eg(10) de(11) fg(12)	

the cost of MST : 1+2+3+4+6+10 = **26**