Assignment One 说明文档

1 分析

- ***1.1 问题重述***

  分别输入 n 和 k，输出应满足如下要求：

  1. 每一个数字都是n位数，且为非负整数
  2. 相邻两数位上的值的差的绝对值等于k
  3. 每个数字不能带有前导零（例：01）
  4. 有多个数字满足要求可输出时，按升序排列，用逗号分隔

- ***1.2 参数分析***

  $n \geq 2$

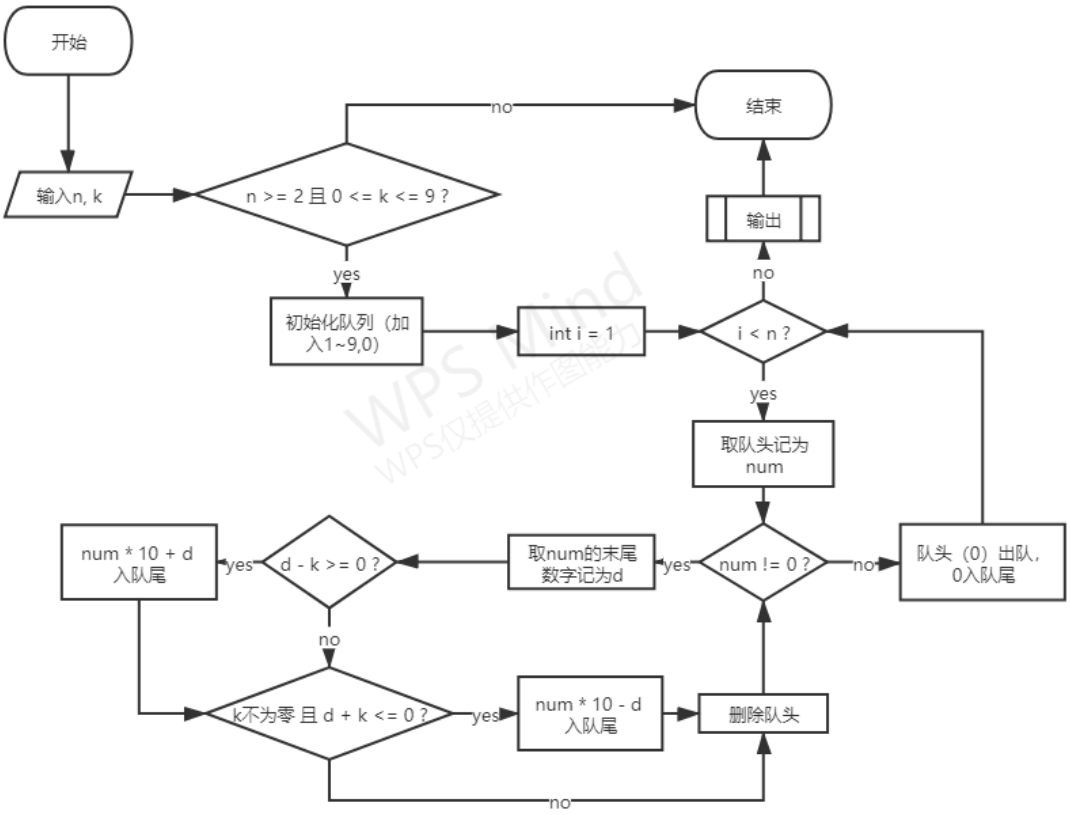
  $0 \leq k \leq 9$

2 算法设计

- **2.1 思路**

  - 采用BFS（广度优先），从首位开始，每层试着在末尾加一位数字，一个数一个数地构造符合要求的答案
  - 由于不能有前导零，第一位数字最多有 9 种选择 $\{1,2,3,4,5,6,7,8,9\}$；其他位的数字最多有 2 种选择（加k / 减k）。这意味着 n 位数最多有 $9 \times 2^{n-1}$ 种情况
  - 一个 n 位的数字可以看作 $n-1$ 位数加上最后一位数字，如果 $n-1$ 位数以数字 d 结尾，则 n 位数字将以 $d-k$ 或 $d+k$ 结尾（前提：在 $[0,9]$ 范围内）

- ## 2.2 流程图



- ## 2.3 实现

  - ### 细节说明：

    - 做**合法性判断**，只有参数合法才能进行以下过程，否则输出为空
    - 采用了数据结构**队列**，初始时依次加入**可能的**首位数字 $1,2,3,4,5,6,7,8,9$
    - 向队列插入**0**，由于0不可能成为合理的输出，在此利用这个特性将其作为每层循环的**结束符**（结束标志）
    - 利用队列先进先出（**FIFO**）的特点，每出一个队头（i 位数），则检查该队头（不为0时）能否拓展为符合要求的 $(i+1)$ 位数，若能则将拓展后的 $(i+1)$ 位数插入队尾；

      拓展方法：

      1. 记： num（当前的 i位数） d（当前的 i位数的末尾数字）；
      2. $d-k$ 或 $d+k$ 在$[0,9]$范围内， 则 $num\times 10+d-k$ 或 $num\times 10+d+k$ 就是要找的 $(i+1)$ 位数；
      3. 按升序原则，先检查$d-k$，再检查$d+k$；
      4. 在检查 $d+k$ 时，应同时检测 k 是否等于 0，因为若 k 等于 0而不进行该层判断，将会在队列中插入重复的数。
    - 当取出的队头为 0 时，表示这一层循环结束，往队尾插入 0 作为**下一层循环**的结束符，进入下一层循环
    - 如此，第 $i(1\le i\le (n-1))$ 层循环的过程，实质上是队列元素从从 i 位数到 $(i+1)$ 位数的**拓展**过程；

      第 i层循环结束后（第 $(i+1)$层循环开始前），队列中的元素都是 $(i+1)$位数（在这里不考虑结束符 0）；

$(n-1)$层循环后，队列中的元素便是符合要求的 n位数了（除去队尾元素 0），进行输出

- **输出**时，依次输出队头元素，直到队头为 0（0不输出）；最后一个数字输出时，后面不接逗号

主函数代码实现：

```
int main()
{
    //非负整数有n位，任意相邻两位上的数字之差（绝对值）为k
    int n, k;
    cout << "请输入n，k：" << endl;
    cin >> n >> k;

    if (n >= 2 && k >= 0 && k <= 9) {    //合法性判断
        queue<int> que;
        for (int i = 1; i <= 9; i++) {  //初始化队列，先加入（可能的）首位数字
            que.push(i);
        }
        que.push(0);                        //0不可能是输出，将其作为每次循环的结束符

        //构建
        int num, d;                         //当前数字以及其末尾数字
        for (int i = 1; i < n; i++) {    //循环n-1次
            while ((num = que.front()) != 0) {
                d = num % 10;
                //按大小顺序依次加入
                if (d - k >= 0) {
                    que.push(num * 10 + d - k);
                }
                if (k != 0 && d + k <= 9) {
                    que.push(num * 10 + d + k);
                }
                que.pop();
            }
            que.pop();          //队头的0出队
            que.push(0);        //0入队尾
        }

        //输出
        if (que.front() != 0) {
            cout << que.front();
            que.pop();
        }
        while (que.front() != 0) {
            cout << "," << que.front();
            que.pop();
        }
    }

    return 0;
}
```

- **2.4 复杂度分析**

时间复杂度	空间复杂度
$O(2^n)$	$O(2^n)$

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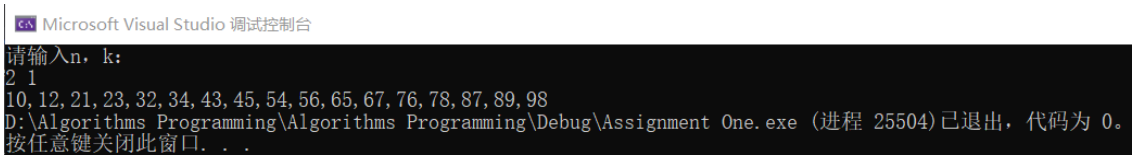
3 测试

- ***3.1 功能测试***

  case 1:

  ```
  Input: n = 2, k = 1
  Output: [10,12,21,23,32,34,43,45,54,56,65,67,76,78,87,89,98]
  ```

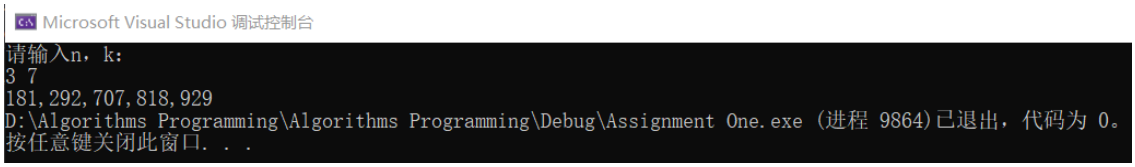
  运行结果：

  case 2:

  ```
  Input: n = 3, k = 7
  Output: [181,292,707,818,929]
  ```

  运行结果：

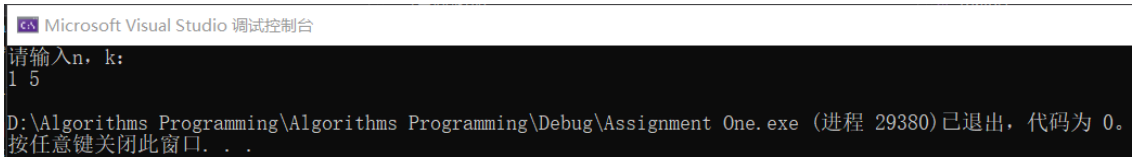
  

- ***3.2 边界测试***

  case 1:

  ```
  Input: n = 1, k = (any value)
  Output: []
  ```

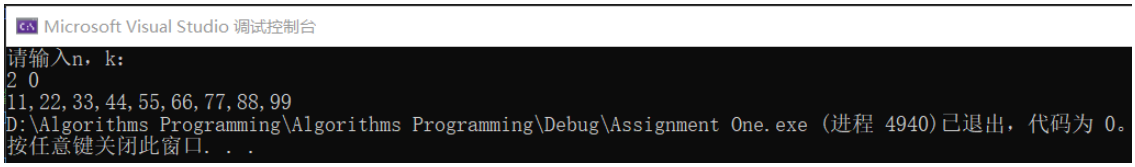
  运行结果：



case 2:

```
Input: n = 2, k = 0
Output: [11,22,33,44,55,66,77,88,99]
```

运行结果：



- ## *3.3 性能测试*

将主程序插入在如下基准程序的指定位置，来测试时间性能。

测试环境： PC机（Legion R7000 2020）

处理器（AMD Ryzen 5 4600H with Radeon Graphics 3.00 GHz）

机带RAM（16.00 GB）

操作系统（Windows 10 家庭中文版 64位）

IDE（Visual Studio 2019 社区版）

```
#include <iostream>
#include <queue>
#include <iomanip>
#include <cstdio>
#include <windows.h> //取系统时间
using namespace std;

int main()
{
    LARGE_INTEGER tick, begin, end;

    QueryPerformanceFrequency(&tick);    //获得计数器频率
    QueryPerformanceCounter(&begin);     //获得初始硬件计数器计数

    /* 此处是主程序开始 */


    /* 此处是主程序结束 */

    QueryPerformanceCounter(&end);       //获得终止硬件计数器计数

    cout<<endl;
    cout << "计数器频率 : " << tick.QuadPart << "Hz" << endl;
    cout << "计数器计数 : " << end.QuadPart - begin.QuadPart << endl;
```

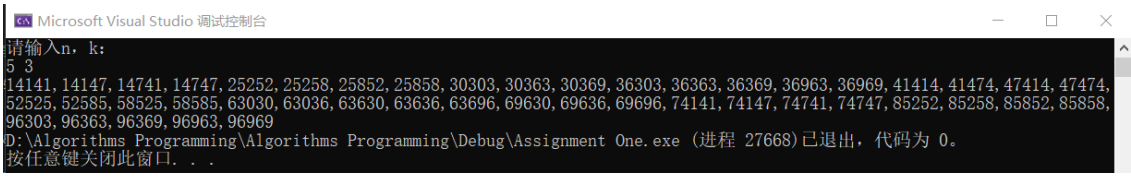
```
    cout << setiosflags(ios::fixed) << setprecision(6) <<
double(end.QuadPart - begin.QuadPart) / tick.QuadPart << "秒" << endl;

    return 0;
}
```

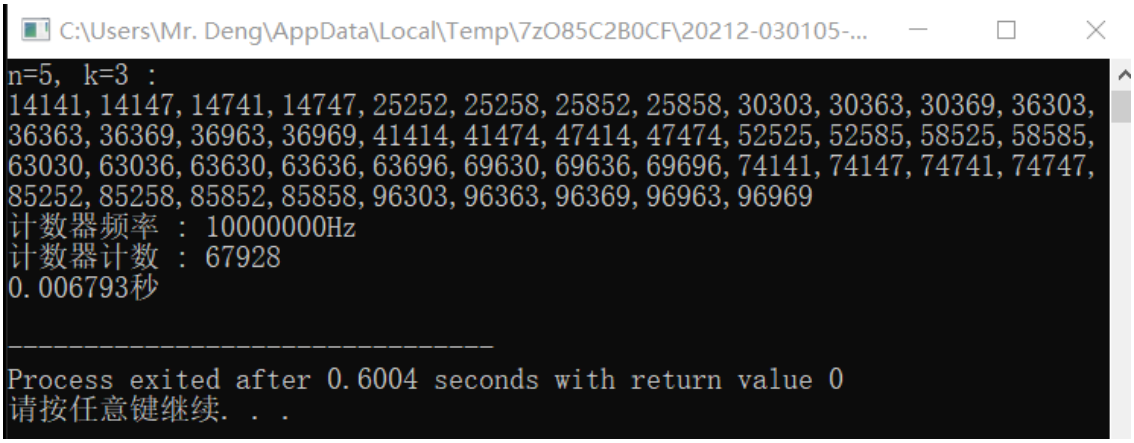
case 1:

```
Input: n = 5, k = 3
Output: (参见如下程序运行截图)
```

运行结果:



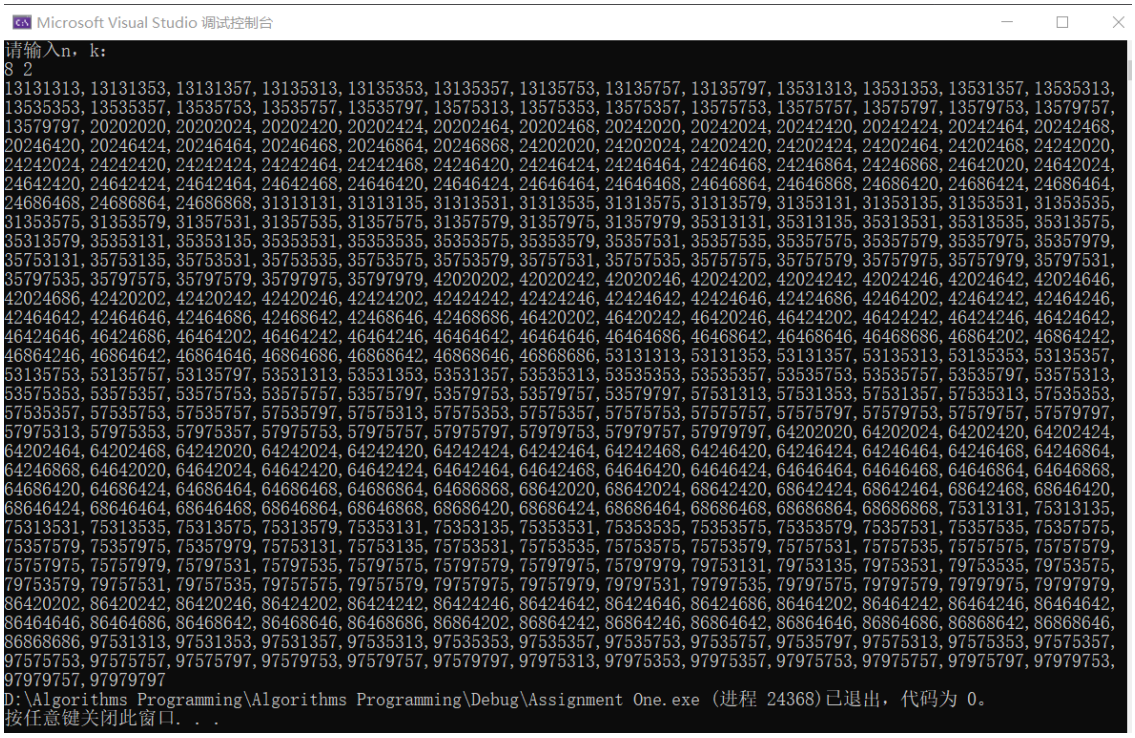
运行时间：$6.793ms$



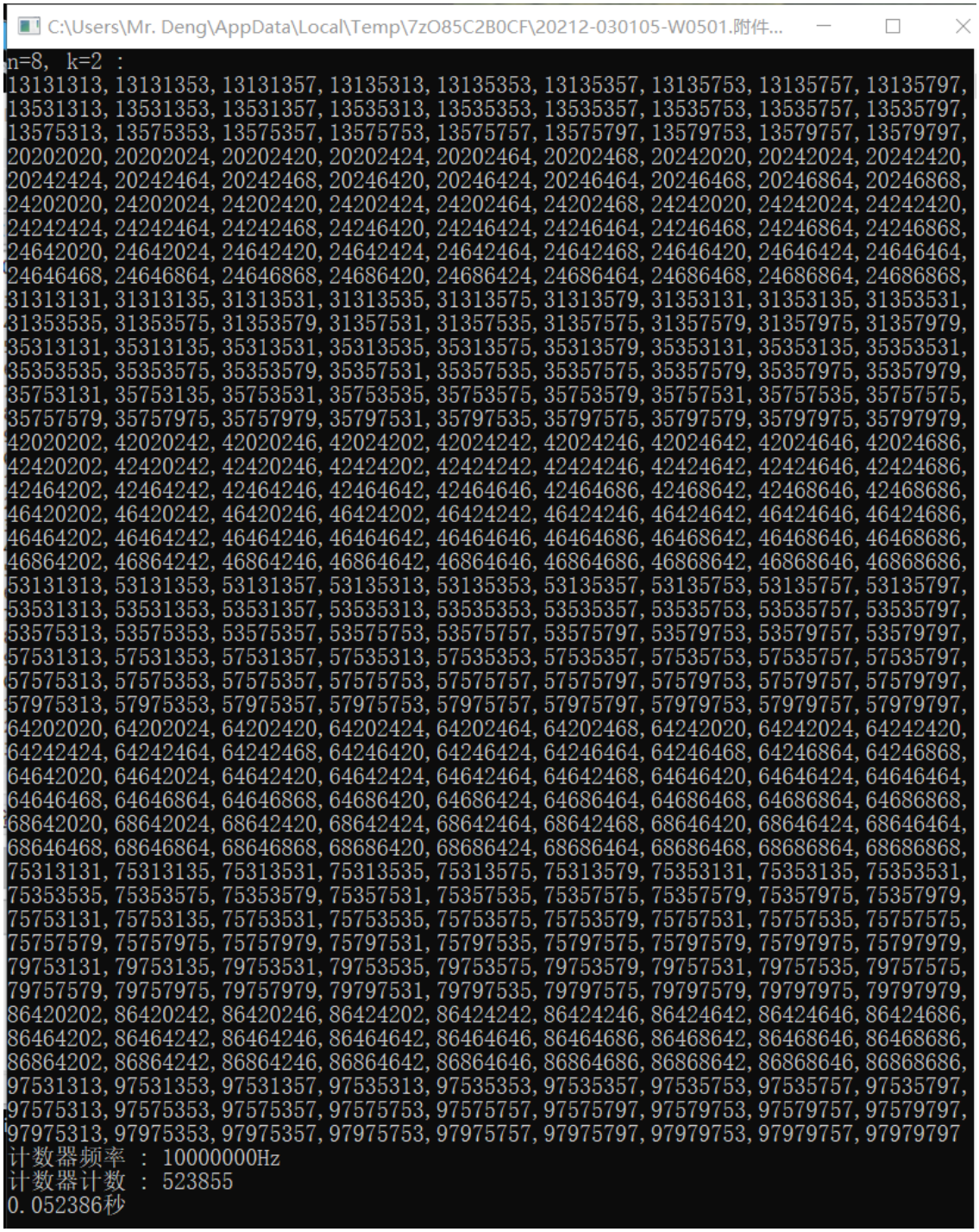
case 2:

```
Input: n = 8, k = 2
Output: (参见如下程序运行截图)
```

运行结果:



运行时间：$52.386ms$


C:\Users\Mr. Deng\AppData\Local\Temp\7zO85C2B0CF\20212-030105-W0501.附件...
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计数器频率 : 10000000Hz
计数器计数 : 523855
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