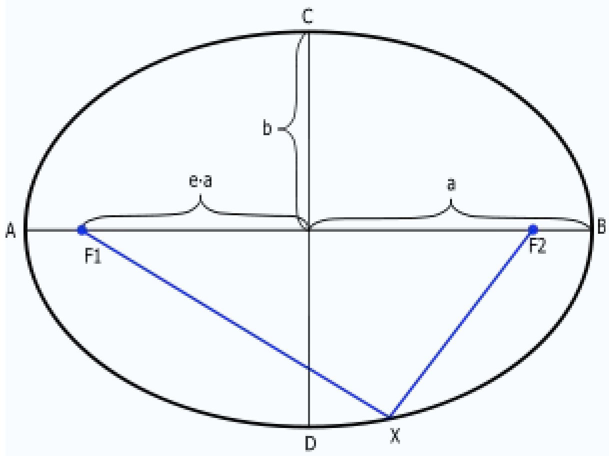

The ellipse and some of its mathematical properties.

In mathematics, an **ellipse** (from the Greek for *absence*) is the locus of points on a plane where the sum of the distances from any point on the curve to two fixed points is constant. The two fixed points are called **foci** (plural of **focus**).

An ellipse is a type of conic section: if a conical surface is cut with a plane which does not intersect the cone's base, the intersection of the cone and plane is an ellipse. For a short elementary proof of this, see Dandelin spheres.

Algebraically, an ellipse is a curve in the Cartesian plane defined by an equation of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

such that $B^2 < 4AC$, where all of the coefficients are real, and where more than one solution, defining a pair of points (x, y) on the ellipse, exists.

An ellipse can be drawn with two pins, a loop of string, and a pencil. The pins are placed at the foci and the pins and pencil are enclosed inside the string. The pencil is placed on the paper inside the string, so the string is taut. The string will form a triangle. If the pencil is moved around so that the string stays taut, the sum of the distances from the pencil to the pins will remain constant, satisfying the definition of an ellipse.

The line segment AB, that passes through the foci and terminates on the ellipse, is called the **major axis**. The major axis is the longest segment that can be obtained by joining two points on the ellipse. The line segment CD, which passes through the center (halfway between the foci), at right angles to the major axis, and terminates on the ellipse, is called the **minor axis**. The **semimajor axis** (denoted by *a* in the figure) is one half the major axis: the line segment from the center, through a focus, and to the edge of the ellipse. Likewise, the **semiminor axis** (denoted by *b* in the figure) is one half the minor axis.

If the two foci coincide, then the ellipse is a circle; in other words, a circle is a special case of an ellipse, one where the eccentricity is zero.

An ellipse centered at the origin can be viewed as the image of the unit circle under a linear map associated with a symmetric matrix $A = PDP^T$, D being a diagonal matrix with the eigenvalues of A, both of which are real positive, along the main diagonal, and P being a real unitary matrix having as columns the eigenvectors of A. Then the axes of the ellipse will lie along the eigenvectors of A, and the eigenvalues are the lengths of the **semimajor** and **semiminor** axes.

An ellipse can be produced by multiplying the *x* coordinates of all points on a circle by a constant, without changing the *y* coordinates.