Week 9. By Deng Yufan.

Prob 1. Consider grammar $S \to ASB \mid \varepsilon$
$A \to aAS \mid a$
$B \to SbS \mid A \mid bb$

a. eliminate ε.

nullable virables: S.

then
$S \to ASB \mid AB$
$A \to aAS \mid aA \mid a$
$B \to SbS \mid Sb \mid bS \mid b \mid A \mid bb$.

b. eliminate unit.

unit pairs: (S,S). (A,A). (B,B). (B,A).

then
$S \to ASB \mid AB$
$A \to aAS \mid aA \mid a$
$B \to SbS \mid Sb \mid bS \mid b \mid bb \mid aAS \mid aA \mid a$.

c. eliminate useless.

generating: S. A. B.
then grammar stays the same.

reachable: S. A. B.
then grammar stays the same.

$S \to ASB \mid AB$
$A \to aAS \mid aA \mid a$
$B \to SbS \mid Sb \mid bS \mid b \mid bb \mid aAS \mid aA \mid a$.

d. into normal form.

$S \to A[SB] \mid AB$
$A \to [a][AS] \mid [a]A \mid a$
$B \to S[bS] \mid S[b] \mid [b]S \mid b \mid [b][b] \mid [a][AS] \mid [a]A \mid a$

$[a] \to a$.
$[b] \to b$.
$[AS] \to AS$
$[SB] \to SB$.

Prob 2.

We first prove for all virable X. $S \Rightarrow^{n} \alpha X \beta \rightarrow X$ can be found by algo.
induction n. as basis, for $n=0$. we only have $S \Rightarrow^{0} S$, and S can be found.
as induction: we assume n and prove $n+1$.

Since $S \Rightarrow^{n+1} \alpha X \beta$. then $S \Rightarrow^{n} \alpha' Y \beta'$. and $\alpha' Y \beta' \Rightarrow \alpha X \beta$ applying production of Y: $Y \rightarrow r$.
which is $\alpha' r \beta' = \alpha X \beta$.
① α' or β' consists X.

then $S \Rightarrow^{n} \alpha' Y \beta'$, which is $S \Rightarrow^{n} \alpha'' X \beta''$, X can be found by assumption.
② r consists X.

since $S \Rightarrow^{n} \alpha' Y \beta'$. Y can be found by assumption. and $Y \rightarrow r$ which is $Y \rightarrow r_1 X r_2$. So X can be found by the def of algo.

Then we prove for all virable X. X can be found $\xrightarrow{\text{in } n \text{ step}}$ $S \Rightarrow^{*} \alpha X \beta$.
induction n. as basis. for $n=0$. we only have S can be found. and $S \Rightarrow^{*} S$.
as induction: we assume n and prove $n+1$.

For all X found in $n+1$ step, we need to have $Y \rightarrow \alpha' X \beta'$, and Y found in n step.
then $S \Rightarrow^{*} \alpha'' Y \beta'' \Rightarrow \alpha'' \alpha' X \beta' \beta''$.

Prob 3. Prove following lang. is not CFL.

a. $L = \{a^n b^n c^i \mid i \leq n\}$.

Let n_0 be the constante in Pumping Lemma. Consider $a^{n_0} b^{n_0} c^{n_0} \in L$.
write $a^{n_0} b^{n_0} c^{n_0}$ into $uvwxy$.
Since $|vwx| \leq n_0$, it's impossible that v consist a and x consist c.
① v consist a. x not consist c. (then v not consist c).
since $uwy \in L$, but $\#a(uwy) < \#a(uvwxy) = \#c(uvwxy) = \#c(uwy)$. then $uwy \notin L$. contradiction.

② v not consist a (then x not consist a). x consist c.
since $uv^2wx^2y \in L$, but $\#a(uv^2wx^2y) = \#a(uvwxy) = \#c(uvwxy) < \#c(uv^2wx^2y)$. then $uv^2wx^2y \notin L$. contradiction.

③ v not consist a. x not consist c. (then vx consist b; x not consist a).
since $uv^0wx^2y \in L$. but $\#b(uv^2wx^2y) > \#b(uvwxy) = \#a(uvwxy) = \#a(uv^2wx^2y)$. then $uv^3wx^3y \notin L$. contradiction.

b. $L = \{0^i 1^j \mid j = i^2\}$.

Let n be the constance in PL. Consider $0^n 1^{n^2} \in L$. write $0^n 1^{n^2}$ into $uvwxy$. Let $s = \#0(vx)$. $t = \#1(vx)$.

Since $\#1(uv^2wx^2y) = n^2 + t = (n+s)^2 = (\#0(uv^2wx^2y))^2$. then $t = 2ns + s^2$.
$n^2 + 2t = (n+2s)^2$. $t = 2ns + 2s^2$. $\Rightarrow s = t = 0$. $vx = \varepsilon$. contradiction.

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c. $L = \{ww^Rw \mid w \in \{0,1\}^*\}$.

Let n be the constance in PL. we have $01^n001^n001^n0 = (01^n)(1^n0)(01^n) \in L$.
write $01^n001^n001^n0$ into $uvwxy$. since $|vwx| \le n$, $\#0(vwx) \le 2$.

Easy to see $w \in L \Rightarrow 3 \mid \#0(w)$. if $3 \nmid \#0(vx)$. then $3 \nmid \#0(uwy) = 3 - \#0(vx)$.
thus $uwy \notin L$. contradiction.

Then $3 \mid \#0(vx)$. and $\#0(vx) \le 2$. so $\#0(vx) = 0$. Let $v = 1^s$. $x = 1^t$. then
$uwy = 01^{n-s-t}001^n001^n0$ or $01^n001^{n-s-t}001^n0$ or $01^n001^n001^{n-s-t}0$.
or $01^{n-s}001^{n-t}001^n0$ or $01^n001^{n-s}001^{n-t}0$.
in summary, $uwy = 01^a001^b001^c0$. where $a \ne b$, $a \ne c$, or $b \ne c$.

We know $uwy \in L$. then uwy can write into zz^Rz. since $\#0(uwy) = 6$. then $\#0(z) = 2$.
Since uwy start and end with 0, then z start and end with 0. so $z = 01^d0$,
thus $uwy = 01^d001^d001^d0$. but we know $uwy = 01^a001^b001^c0$, where $a \ne b$, $b \ne c$ or $a \ne c$.
Contradiction.

Prob 4. Prove L is CFL $\to$ $L \backslash a$ is CFL.

Consider PDA P that $L(P) = L$. Let PDA

$P' = (Q \cup \{q_F\}, \Sigma, \Gamma, \delta', q_0, Z_0, \{q_F\})$. (if $a \notin \Sigma$, obviously $L\backslash a = \emptyset$ is CFL. so we assume $a \in \Sigma$)

where $\delta'(q, b, X) = \delta(q, b, X)$ $(b \ne \varepsilon)$.
$\delta'(q, \varepsilon, X) = \delta(q, \varepsilon, X) \cup \bigcup_{q' \in F} \bigcup_{(q,a,X) \vdash^* (q',\varepsilon,\alpha)} \{(q_F, \alpha)\}$.
$\delta'(q_F, b, X) = \emptyset$.

We have $(q_0, w, Z_0) \vdash^* (q_F, \varepsilon, \alpha)$ in P'.
$\Leftrightarrow (q_0, w, Z_0) \vdash^* (q, \varepsilon, X\beta)$ in P'. $q' \in F$. $(q,a,X) \vdash^* (q',\varepsilon,\beta')$. $\alpha = \beta'\beta$.
$\Leftrightarrow (q_0, wa, Z_0) \vdash^* (q, a, X\beta)$ in P. $q' \in F$. $(q,a,X) \vdash^* (q',\varepsilon,\beta')$. $\alpha = \beta'\beta$.
$\Leftrightarrow (q_0, wa, Z_0) \vdash^* (q', \varepsilon, \alpha)$ in P. $q' \in F$.
So $L(P') = L(P)\backslash a$.

Prob 5. Use CYK algo to determine $aabab \in L(G)$.

$S \to AB \mid BC$. $A \to BA \mid a$. $B \to CC \mid b$. $C \to AB \mid a$.

~~∅~~ {S,C}				
~~∅~~ {S,A,C}	~~∅~~ {B}			
{B}	~~∅~~ {B}	~~∅~~ {S,C}		
{B}	{S,C}	{S,A}	{S,C}	
{A,C}	{A,C}	{B}	{A,C}	{B}
a	a	b	a	b

~~Then aabab ∉ L(G)~~. Then $aabab \in L(G)$.