Week 1. By Deng Yufan.

Prob 1. Prove $\hat{\delta}(p, xy) = \hat{\delta}(\hat{\delta}(p, x), y)$.

Induction x. When $x = \epsilon$, according to the defination of $\hat{\delta}$, which is

$$\hat{\delta}(p, s) = \begin{cases} p & (s = \epsilon) \\ \hat{\delta}(\delta(p, w), a) & (s \neq \epsilon, \text{ can be writed as } wa), \end{cases}$$

the proof is trival.

Consider $x \neq \epsilon$, which means x can be writed as wa, where w is a charac-ter and a is a string. We have

$\hat{\delta}(p, xy) = \hat{\delta}(p, way) = \hat{\delta}(\delta(p, w), ay)$.

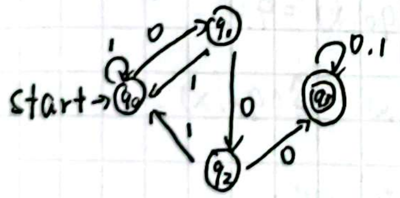
Use the induction assume, we have

$\hat{\delta}(\delta(p, w), ay) = \hat{\delta}(\hat{\delta}(\delta(p, w), a), y) = \hat{\delta}(\hat{\delta}(p, x), y)$.

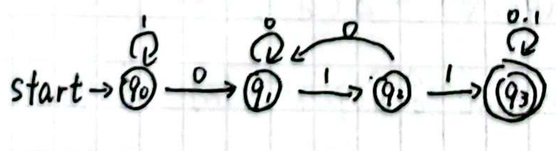
So we have $\hat{\delta}(p, xy) = \hat{\delta}(\hat{\delta}(p, x), y)$. ■.

Prob 2. Construct a DFA over alphabet $\{0, 1\}$.

a. Strings that have three consecutive 0.

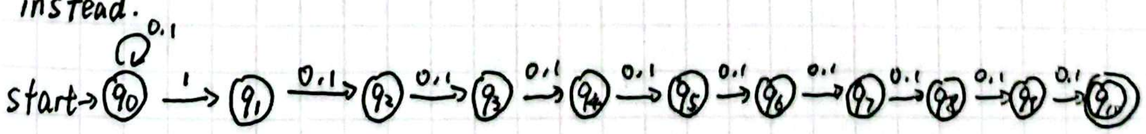


b. Strings that have substring 011.



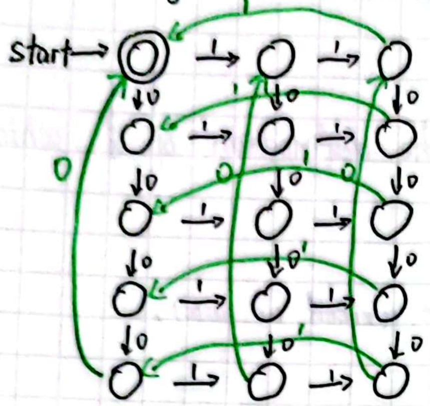
c. Strings that tenth character from the right end is 1.

Since the nodes of DFA can't be less than 2^{10}, we only construct a NFA instead.



According to the proof of DFAs and NFAs are equivalent, we can convert it into a DFA

d. Strings that number of 0 is divisible by 5. and 1 is by 3.



To make it easier to see, we use different colors.

Prob 3. Let $A=(Q,\Sigma,\delta,q_0,\{q_f\})$ be a DFA, and we have $\delta(q_0,a)=\delta(q_f,a)$.

a. Proof for all $w\neq\epsilon$, we have $\hat{\delta}(q_0,w)=\hat{\delta}(q_f,w)$.

since $w\neq\epsilon$, we can write w as ax, where a is a character and x is a string. We have

$$\hat{\delta}(q_0,w)=\hat{\delta}(q_0,ax)=\hat{\delta}(\delta(q_0,a),x)=\hat{\delta}(\delta(q_f,a),x)=\hat{\delta}(q_f,ax)=\hat{\delta}(q_f,w)$$

■.

b. Proof for all ~~L(A)~~ $x\neq\epsilon$, $x^k\in L(A)$. $(k\in N^*)$.

Since we have $x\in L(A)$, that is $\hat{\delta}(q_0,x)=q_f$.

Further we have $x\neq\epsilon$, so $q_f=\hat{\delta}(q_0,x)=\hat{\delta}(q_f,x)$.

Induction k. While $k=1$, it's trival to proof.

Consider $k>1$, we have $\hat{\delta}(q_0,x^k)=\hat{\delta}(\hat{\delta}(q_0,x^{k-1}),x)$.

Use the induction assume. we have $\hat{\delta}(\hat{\delta}(q_0,x^{k-1}),x)=\hat{\delta}(q_f,x)=q_f$.

So for all k we have $\hat{\delta}(q_0,x^k)=q_f$. ■.

Prob 4. Convert the NFA to DFA.

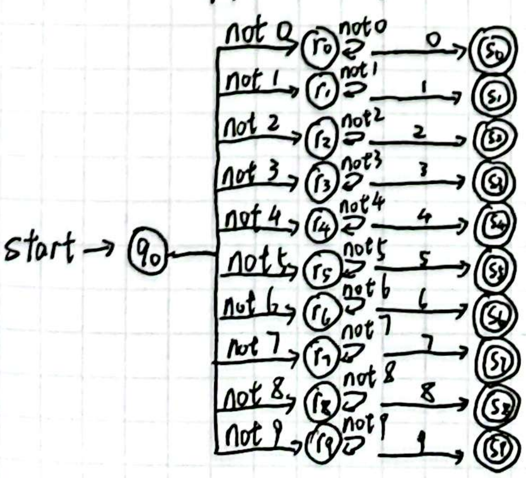
	0	1
$\to p$	$\{q,s\}$	$\{q\}$
$*q$	$\{r\}$	$\{q,r\}$
r	$\{s\}$	$\{p\}$
$*s$	$\emptyset$	$\{p\}$

We have

	0	1
$V_0=\emptyset$	V_0	V_0
$*V_1=\{s\}$	V_0	V_8
$V_2=\{r\}$	V_1	V_8
$*V_3=\{r,s\}$	V_1	V_8
$*V_4=\{q\}$	V_2	V_6
$*V_5=\{q,s\}$	V_2	V_{14}
$*V_6=\{q,r\}$	V_3	V_{14}
$*V_7=\{q,r,s\}$	V_3	V_{14}
$\to V_8=\{p\}$	V_5	V_4
$*V_9=\{p,s\}$	V_5	V_{12}
$V_{10}=\{p,r\}$	V_5	V_{12}
$*V_{11}=\{p,r,s\}$	V_5	V_{12}
$*V_{12}=\{p,q\}$	V_7	V_6
$*V_{13}=\{p,q,s\}$	V_7	V_{14}
$*V_{14}=\{p,q,r\}$	V_7	V_{14}
$*V_{15}=\{p,q,r,s\}$	V_7	V_{14}

Prob 5. Construct a NFA.

a. strings over alphabet $\{0,1,\cdots 9\}$ that the final digit has not appeared before.



Notice that we use "not x" to identify every elements in set $\Sigma \setminus \{x\}$.

b. strings over alphabet {0,1} that there are 2 '0' separated by a number of positions that is a multiple of 4.