Week 2. By Deng Yufan.

Prob 1. Consider the following ε-~~D~~NFA.

	ε	a	b	c
→ p	{q, r}	∅	{q}	{r}
q	∅	{p}	{r}	{p, q}
* r	∅	∅	∅	∅

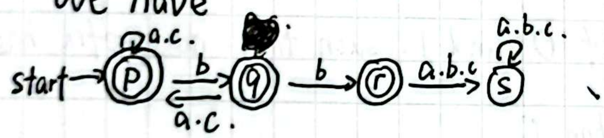
a. Give all the strings of length 3 or less accepted by the automaton.

ε. a. b. c. aa. ab. ac. ba. bb. bc. ca. cb. cc. aaa. aab. aac. aba. abb. abc. aca. acb. acc. baa. bab.

bac. bca. bcb. bcc. caa. cab. cac. cba. cbb. cbc. cca. ccb. ccc.

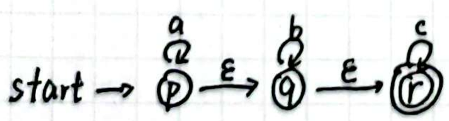
b. Convert the automaton to a DFA.

We have

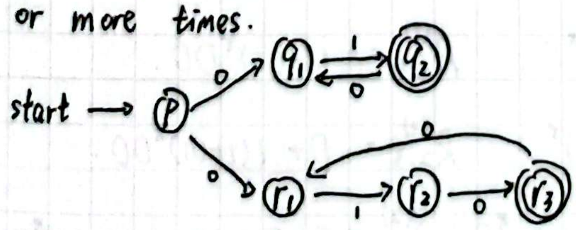


Prob 2. Design ε-~~D~~NFA.

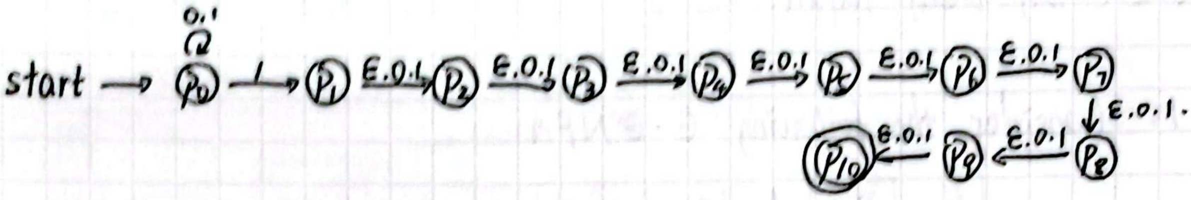
a. Strings consisting of zero or more a, followed by zero or more b, followed by zero or more c.



b. Strings consisting of 01 repeated ~~one~~ 1 or more times or 010 repeated 1 or more times.



c. Strings of 0 and 1 that at least one of the last 10 positions is 1.



Prob #3. Write regular expressions.

a. Strings of 0 and 1 whose 10th symbol from the right end is 1.

$(0+1)^*1(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$.

b. Strings of 0 and 1 with at most 1 pair of consecutive 1.

$(\varepsilon+1)(00^*100^*)^*(\varepsilon+1) + (\varepsilon+1)(00^*100^*)^*11(00^*100^*)^*(\varepsilon+1)$.

c. Strings of 0 and 1 whose number of 0 is ~~divd~~ divisible by 5.

$(1^*01^*01^*01^*01^*01^*)^*$.

d. Strings of 0 and 1 not containing 101.

$0^* + 0^*11^*(000^*11^*)^*0^*$.

e. Strings with an equal number of 0 and 1, such that no prefix has 2 more 0 than 1, nor 2 more 1 than 0.

$(01+10)^*$.

Prob 4. Consider the DFA

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$* q_3$	q_3	q_2

a. Give all $R_{ij}^{(2)}$.

$R_{1,1}^{(2)} = (1+01)^*$. $R_{1,2}^{(2)} = (1+01)^*0$. $R_{1,3}^{(2)} = (1+01)^*00$.

$R_{2,1}^{(2)} = 1(1+01)^*$ $R_{2,2}^{(2)} = \varepsilon + 1(1+01)^*0$. $R_{2,3}^{(2)} = 0 + 1(1+01)^*00$.

$R_{3,1}^{(2)} = 11(1+01)^*$. $R_{3,2}^{(2)} = 1 + 11(1+01)^*0$. $R_{3,3}^{(2)} = \varepsilon + 0 + 10 + 11(1+01)^*00$.

b. Give a regular expression for the language of DFA.

~~$(1+01)^*00$~~ $(1+01)^*00(0+10+11(1+01)^*00)^*$.

Prob 5. Suppose L_1 and L_2 are two languages accepted by finite automata M_1 and M_2. Prove that $L_1 \cap L_2$ is also accepted by a finite automata.

We can assume M_1 and M_2 are DFAs. $(Q_1, \Sigma, \delta_1, q_1, F_1)$ and $(Q_2, \Sigma, \delta_2, q_2, F_2)$, respectively. Consider the DFA $M_3 = (Q_1 \times Q_2, \Sigma, \delta, (q_1, q_2), \{(i,j) \mid i \in F_1, j \in F_2\})$, in which $\delta((i,j), a) = (\delta_1(i,a), \delta_2(j,a))$. Now we prove ~~$L(M_3) = L(M_1) \cap L(M_2)$~~. $\hat{\delta}((q_1,q_2), w) = (\hat{\delta}_1(q_1, w), \hat{\delta}_2(q_2, w))$.

~~Induction the length of accepted string. It's obvious that $\varepsilon \in L(M_3) \Leftrightarrow \varepsilon \in L(M_1) \cap L(M_2)$. Now we assume for all n-length string w, $w \in L(M_3) \Leftrightarrow w \in L(M_1) \cap L(M_2)$, and consider $n+1$-length string wa, where w is a n-length string and $a \in \Sigma$.~~

~~We have $\hat{\delta}((q_1,q_2), w) \in \{(i,j) \mid i \in F_1, j \in F_2\} \Leftrightarrow w \in L(M_3) \Leftrightarrow w \in L(M_1) \cap L(M_2) \Leftrightarrow \hat{\delta}_1(q_1, w) \in F_1 \land \hat{\delta}_2(q_2, w) \in F_2$.~~

Induction the length of w. It's obvious that $\hat{\delta}((q_1,q_2), \varepsilon) = (q_1, q_2) = (\hat{\delta}_1(q_1, \varepsilon), \hat{\delta}_2(q_2, \varepsilon))$.
Now we assume for all n-length string w, $\hat{\delta}((q_1,q_2), w) = (\hat{\delta}_1(q_1, w), \hat{\delta}_2(q_2, w))$, and consider $(n+1)$-length string wa, where w is a n-length string and $a \in \Sigma$. We have

$$\hat{\delta}((q_1,q_2), wa) = \delta(\hat{\delta}((q_1,q_2), w), a) = \delta((\hat{\delta}_1(q_1,w), \hat{\delta}_2(q_2,w)), a)$$
$$= (\delta_1(\hat{\delta}_1(q_1,w), a), \delta_2(\hat{\delta}_2(q_2,w), a)) = (\hat{\delta}_1(q_1, wa), \hat{\delta}_2(q_2, wa)).$$

So for all w we have $\hat{\delta}((q_1,q_2), w) = (\hat{\delta}_1(q_1, w), \hat{\delta}_2(q_2, w))$. Thus

$$w \in L(M_3) \Leftrightarrow \hat{\delta}((q_1,q_2), w) \in \{(i,j) \mid i \in F_1, j \in F_2\} \Leftrightarrow \hat{\delta}_1(q_1, w) \in F_1 \land \hat{\delta}_2(q_2, w) \in F_2$$
$$\Leftrightarrow w \in L(M_1) \land w \in L(M_2).$$

So $L(M_3) = L(M_1) \cap L(M_2)$.