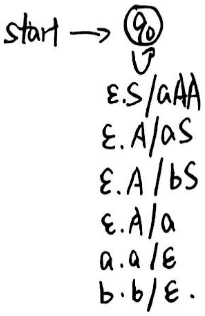
Week 8. By Deng Yufan.

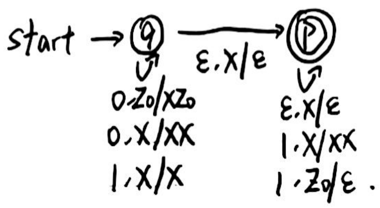
Prob 1. Convert to PDA accept by empty stack:

$S \to aAA \quad A \to aS \mid bS \mid a.$



initially, the stack contains S.

Prob 2. Convert the PDA accept by final state to CFG:



Easy to see $L(P) = 0\{0,1\}^*$, since:

if $w \in L(P)$, then since $\delta(q, 1, Z_0) = \phi$, and q is not final state. so w starts with 0.

if w starts with 0. then we have $(q, 0w', Z_0) \vdash (q, w', XZ_0)$

$\vdash^* (q, \varepsilon, X^{\#0(w')+1} Z_0)$

$\vdash (p, \varepsilon, X^{\#0(w')} Z_0)$.

Then we have CFG

$S \to 0A. \quad A \to 0A \mid 1A \mid \varepsilon.$

and $L(G) = 0\{0,1\}^* = L(P)$.

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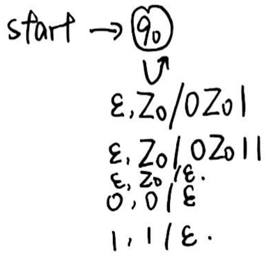
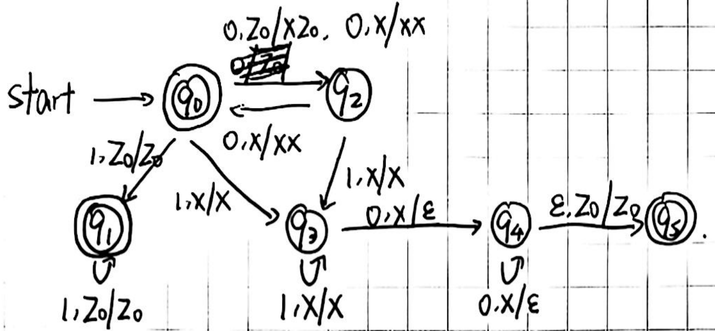
1 Prob 3. Design PDA accepts $\{0^n 1^m \mid n \le m \le 2n\}$ by empty stack.



Prob 4. Design DPDA accepts $\{0^n 1^m 0^n\}$.



Prob 5. Prove:

a. if $L = N(P)$ for DPDA P, then L is prefix free.

~~If $w \in L$, then~~

We first prove: if $(p, w, \alpha) \vdash^* (q, \varepsilon, \beta)$,
then $(p, ww', \alpha) \vdash^* (q', \varepsilon, \gamma) \iff (q, w', \beta) \vdash^* (q', \varepsilon, \gamma)$.

Induction steps taken in the condition.

As basis: when $q = p$, $w = \varepsilon$, $\beta = \alpha$, it's obviously true.
when $(p, w, \alpha) \vdash (q, \varepsilon, \beta)$, write α as $X\alpha'$.

if $w = \varepsilon$. since P is DPDA, $|\delta(p, \varepsilon, X)| = 1$, $\delta(p, a, X) = \phi$.
so (p, ww', α) only $\vdash (q, w', \beta)$. then the thm is true.
else. w is a char. likewise, $|\delta(p, w, X)| = 1$, $\delta(p, \varepsilon, X) = \phi$.
so (p, ww', α) only $\vdash (q, w', \beta)$.

As induction: we assume n and prove $n+1$.

We have $(p, w, \alpha) \vdash^n (p', y, \alpha') \vdash (q, \varepsilon, \beta)$.

~~then $(p, ww', \alpha) \vdash^* (q', \varepsilon, \gamma)$~~
~~$\iff (p', w'w', \alpha') \vdash^* (q$~~

then write w into xy. we have $(p, x, \alpha) \vdash^n (p', \varepsilon, \alpha')$.

So $(p, ww', \alpha) \vdash^* (q', \varepsilon, r) \Leftrightarrow (p', yw', \alpha') \vdash^* (q', \varepsilon, r)$

$\Leftrightarrow (q, w', \beta) \vdash^* (q', \varepsilon, r)$.

So the thm is proven.

Then if $w \in$ ~~(P)~~ $N(P)$, we have $(q_0, w, Z_0) \vdash^* (q, \varepsilon, \varepsilon)$.

So $(q_0, ww', Z_0) \vdash^* (q', \varepsilon, \varepsilon) \Leftrightarrow (q, w', \varepsilon) \vdash^* (q', \varepsilon, \varepsilon)$.

By definition, we know when $w' \neq \varepsilon$, $(q, w', \varepsilon) \vdash^* (q', \varepsilon, \varepsilon)$ takes atleast 1 step. but it requires ε can write into $X\alpha$. which is imposible.

So $ww' \notin N(P)$. then L is prefix free.)

b. If $L = N(P)$ for DPDA P. then there exists DPDA P'. $L = L(P')$.

Since P is PDA. we can have

PDA $P' = (Q \cup \{q_0', q_F\}, \Sigma, \Gamma \cup \{X_0\}, \delta', q_0', X_0, \{q_F\})$

where $\delta'(q_0', \varepsilon, X_0) = \{(q_0, Z_0X_0)\}$.

$\delta'(q, a, X) = \delta(q, a, X) \quad (q \neq q_0'. q \neq q_F, X \neq X_0)$.

$\delta'(q, \varepsilon, X_0) = \cancel{\delta(q, \varepsilon, X)} \{(q_F, X_0)\} \quad (q \neq q_0')$.

We have proven $L(P') = N(P)$. So we are about to prove P' is DPDA. ε)

Easy to see $\delta(q, a, X_0) = \phi$ where $a \neq \varepsilon$. and $|\delta(q, \varepsilon, X_0)| \leq 1$.

$\cup \delta'(q_0, a, X)$ have only 1 transition.

$\cup \delta'(q_F, a, X)$ have no transition.

So P' is DPDA.

C. If L is prefix free, $L(P^\circ) = L$ for DPDA P°. then exists DPDA P'. $L = N(P')$.

Since P is PDA. we can have

PDA $P' = (Q \cup \{q_0', q_F\}, \Sigma, \Gamma \cup \{X_0\}, \delta', q_0', X_0)$

where $\delta'(q_0', \varepsilon, X_0) = \{(q_0, Z_0X_0)\}$

$\delta'(q, a, X) = \delta(q, a, X) \quad (q \neq q_0', q \neq q_F. X \neq X_0, q \notin F \vee a \neq \varepsilon)$

$\delta'(q, \varepsilon, X) = \delta(q, \varepsilon, X) \cup \{(q_F, \varepsilon)\} \quad (q \in F. X \neq X_0)$

$\delta'(q, \varepsilon, X_0) = \{(q_F, \varepsilon)\} \quad (q \in F)$

$\delta'(q_F, \varepsilon, X) = \{(q_F, \varepsilon)\}$.

We have proven $N(P') = L(P)$. We are about to find DPDA P'' and $N(P'') = N(P)$.

c. If L is prefix free, and $L(P)=L$ for DPDA P, then exists DPDA P', $N(P')=L$.

Consider DPDA $P''=(Q,\Sigma,\Gamma,\delta',q_0,Z_0,F)$.

$$\text{where } \delta'(q,a,X)=\delta(q,a,X) \quad (q\notin F \lor a\neq\varepsilon)$$
$$\delta'(q,\varepsilon,X)=\emptyset \quad (q\in F).$$

We first prove $L(P'')=L$. Obviously $L(P'')\subseteq L$.

For all $w\in L$, $(q_0,w,Z_0)\vdash^*(q,\varepsilon,\alpha)$ $(q\in F)$.

since for all proper prefix of w, x. $x\notin L$. then $(q_0,x,Z_0)\not\vdash^*(q,\varepsilon,\beta)$ $(q\in F)$

thus $(q_0,w,Z_0)\not\vdash^*(q,y,\beta)$ $(q\in F, y\neq\varepsilon)$.

So $(q_0,w,Z_0)\vdash^*(q,\varepsilon,\alpha)$ in P''. since $\delta'=\delta$ if we ignore case $q\in F$ and $y=\varepsilon$.

So $L(P'')\supseteq L$.

Then since P'' is PDA, we can have

PDA $P'=(Q\cup\{q_0',q_F\},\Sigma,\Gamma\cup\{X_0\},\delta'',q_0',X_0)$

$$\begin{aligned}
\text{where } \delta''(q_0',\varepsilon,X_0)&=\{(q_0,Z_0X_0)\}\\
\delta''(q,a,X)&=\delta'(q,a,X) && (q\neq q_0',\ q\neq q_F,\ X\neq X_0,\ q\notin F\lor a\neq\varepsilon)\\
\delta''(q,\varepsilon,X)&=\delta'(q,\varepsilon,X)\cup\{(q_F,\varepsilon)\} && (q\in F,\ X\neq X_0)\\
&=\{(q_F,\varepsilon)\}.\\
\delta''(q,\varepsilon,X_0)&=\{(q_F,\varepsilon)\}. && (q\in F).\\
\delta''(q_F,\varepsilon,X)&=\{(q_F,\varepsilon)\}.
\end{aligned}$$

Easy to see P' is DPDA, by discussing possibilities like b.

Since $N(P')=L(P'')=L$. Then the thm is proven.