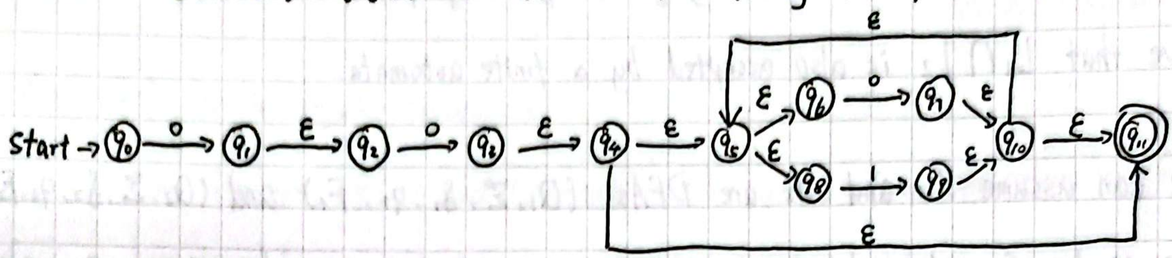
Week 3. By Deng Yufan

Prob 1. Convert $00(0+1)^*$ to ε-NFA using Thompson's method.



Prob 2. Let $A = \{Q, \Sigma, \delta, q_0, \{q_f\}\}$ be a ε-NFA, st. no trans into q_0 or out of q_f.

Describe the language using $L = L(A)$.

a. A adding an ε-trans from q_f to q_0.

LL^*.

b. A adding an ε-trans from q_0 to every state reachable from q_0.

A suffix of every string in L, which can be ε.

c. A adding an ε-trans from every state reachable ~~from~~ to q_f to q_f.

A prefix of every string in L, which can be ε.

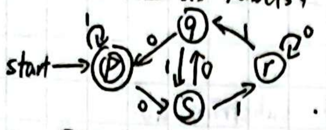
d. A doing both b. and c.

A range (or continuous substring) of every string in L. which can be ε.

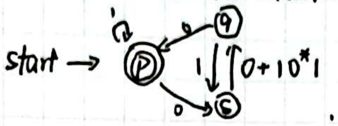
Prob 3. Convert the DFA to RE using state-elimination.

	0	1
$\rightarrow * p$	s	p
q	p	s
r	r	q
s	q	r

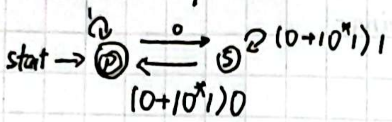
Step 1. Use RE as labels, we have



Step 2. Eliminate r. that is



Step 3. Eliminate q. that is



Step 4. Eliminate s. that is

start → (p) $1+0\left[(0+10^*1)1\right]^*(0+10^*1)0$

Step 5. Find answer.

$$\left[1+0\left[(0+10^*1)1\right]^*(0+10^*1)0\right]^*$$

Prob 4. Prove or disprove following statements.

a. $(\varepsilon+R)^* = R^*$. (true).

We are about to prove $(\varepsilon+a)^* = a^*$. First we have $\varepsilon+a = a+\varepsilon$, so $(\varepsilon+a)^n = \bigcup_{i=0}^{n} \varepsilon^{n-i}a^{i} = \bigcup_{i=0}^{n} a^i$.

Then $(\varepsilon+a)^* = \bigcup_{n=0}^{+\infty}(\varepsilon+a)^n = \bigcup_{n=0}^{+\infty}\bigcup_{i=0}^{n} a^i = \bigcup_{i=0}^{+\infty} a^i = a^*$.

b. $(RS+R)^*R = R(SR+R)^*$. (true).

We are about to prove $(ab+a)^*a = a(ba+a)^*$.

First we have $(b+\varepsilon)(ab+a) = bab+ba+ab+a = (ba+a)(b+\varepsilon)$. So $(b+\varepsilon)(ab+a)^n = (ba+a)(b+\varepsilon)(ab+a)^{n-1} = \cdots = (ba+a)^n(b+\varepsilon)$.

Then for $i \geq 1$, $(ab+a)^i a = a(b+\varepsilon)(ab+a)^{i-1}a = a(ba+a)^{i-1}(b+\varepsilon)a = a(ba+a)^i$.

So $(ab+a)^*a = \bigcup_{i=0}^{+\infty}(ab+a)^i a = \bigcup_{i=0}^{+\infty} a(ba+a)^i = a(ba+a)^*$.

c. $(R+S)^*S = (R^*S)^*$. ~~true~~. (false).

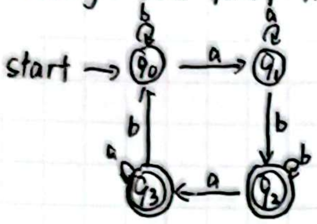
We are about to prove $(a+b)^*b \neq (a^*b)^*$.

~~We are about to prove $(a+b)^*b$ and $(a^*b)^*$ both refer to all strings end with~~

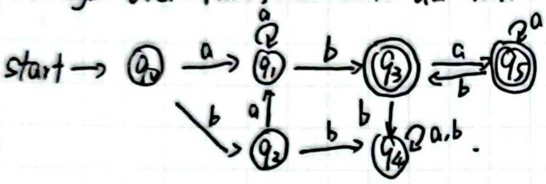
Obviously $\varepsilon \notin (a+b)^*b$, since ε don't end with b. but $\varepsilon \in (a^*b)^*$. So $(a+b)^*b \neq (a^*b)^*$.

Prob 5. Construct DFA.

a. Strings over $\{a,b\}$ have and only have odd number of substring ab.



b. Strings over $\{a,b\}$ contain ab but bb as substring.



Prob 6. Construct RE.

a. $\{xwx^R \mid x,w \in (a+b)^+\}$

$\cancel{a(a+b)^*a + b(a+b)^*b}$. $a(a+b)(a+b)^*a + b(a+b)(a+b)^*b$.

b. $\{w \mid w \in \{0,1\}^*$, has at least 3 1 and the third from the last position is a 1$\}$.

$(0+1)^*111 + (0+1)^*10^*1(01+10) + (0+1)^*10^*10^*100$.