

Day Two: Linear Models

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review

while everyone is getting settled, load these data into R and run through all the things we talked about yesterday.

```
dat <- read.csv('data/primate_energetics.csv')
```

1. Are the data clean?
2. Are there troubling patterns in missingness?
3. Are the data tidy?
4. What is this table about?
5. Are there any obvious relationships to look at?

1. Mean testing

introduction

a picture is worth 1,000 words, but a p-value is worth a dissertation

basically, inferential statistics is the application of probability theory to decide what is real and what isn't

we'll start by trying to tell whether differences between group summaries are real

t.test with two vectors (default method)

```
t.test(dat$RMR, dat$TEE)
```

```
##
##  Welch Two Sample t-test
##
## data:  dat$RMR and dat$TEE
## t = -8.5423, df = 212.87, p-value = 2.533e-15
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1066.1348  -666.3539
## sample estimates:
## mean of x mean of y
##  410.7926 1277.0369
```

note that R takes care of the defaults for you - what it is really computing is `t.test(datRMR, datTEE, alternative = "two.sided", paired = FALSE, var.equal = FALSE, mu = 0, conf.level = 0.95)`

how would you find this out for yourself?

t.test with subsets of one vector (default method)

```
t.test(dat$W[dat$Clade == "Strepsirrhini"], dat$outside.W[dat$Clade == "Platyrrhini"])

##
## One Sample t-test
##
## data: dat$W[dat$Clade == "Strepsirrhini"]
## t = 7.6434, df = 42, p-value = 1.772e-09
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.8580563 1.4737158
## sample estimates:
## mean of x
##  1.165886
```

recall that we mentioned inconsistency on day one - here it is, and in a big way

t.test with S3 method

```
t.test(W ~ Clade, data = dat, subset = dat$Clade %in% c("Strepsirrhini", "Platyrrhini"))

##
## Welch Two Sample t-test
##
## data: W by Clade
## t = 0.91554, df = 35.664, p-value = 0.3661
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -0.5309705 1.4043317
## sample estimates:
## mean in group Platyrrhini mean in group Strepsirrhini
## 1.602567 1.165886
```

aov

first, you would think anova would be called by `anova`, but that's reserved for conducting F-tests on lm objects

second, you really shouldn't be using anova, but if you must do it in R, the syntax looks like this

```
aov(W ~ Clade, data = dat)

## Call:
## aov(formula = W ~ Clade, data = dat)
##
## Terms:
## Clade Residuals
## Sum of Squares 139574.10 72924.24
```

```
## Deg. of Freedom      4      190
##
## Residual standard error: 19.59111
## Estimated effects may be unbalanced
```

this isn't particularly helpful, but remember that it is an object, and we can call other, more helpful functions, on that object

remember our old friend `summary`? it works on almost everything

```
model.1 <- aov(W ~ Clade, data = dat)
summary(model.1)
```

```
##              Df Sum Sq Mean Sq F value Pr(>F)
## Clade         4 139574    34894   90.91 <2e-16 ***
## Residuals    190  72924      384
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

post-hoc testing

that's a little better - but it doesn't tell us where that difference really is

unfortunately, the only built-in test for multiple comparisons is Tukey's

```
TukeyHSD(model.1)
```

```
##    Tukey multiple comparisons of means
##      95% family-wise confidence level
##
## Fit: aov(formula = W ~ Clade, data = dat)
##
## $Clade
##              diff          lwr          upr          p adj
## Hominoidea-Cercopithecoidea    29.6957071    16.69211    42.699299 0.0000000
## Homo-Cercopithecoidea         56.4497379    44.79633    68.103143 0.0000000
## Platyrrhini-Cercopithecoidea   -6.5835444   -19.92205     6.754959 0.6543474
## Strepsirrhini-Cercopithecoidea -7.0202251   -19.20942     5.168972 0.5080599
## Homo-Hominoidea              26.7540309    14.78934    38.718718 0.0000000
## Platyrrhini-Hominoidea       -36.2792515   -49.89055   -22.667949 0.0000000
## Strepsirrhini-Hominoidea     -36.7159321   -49.20306   -24.228803 0.0000000
## Platyrrhini-Homo             -63.0332824   -75.36114   -50.705428 0.0000000
## Strepsirrhini-Homo           -63.4699630   -74.54411   -52.395811 0.0000000
## Strepsirrhini-Platyrrhini     -0.4366806   -13.27220    12.398843 0.9999824
```

other post-hoc tests

these require the `multcomp` package:

```
library(multcomp)
```

```
## Loading required package: mvtnorm
## Loading required package: survival
## Loading required package: TH.data
```

then, we give a model and specify the method for multiple comparisons

```
summary(glht(model.1, linfct=mcp(Clade = "Dunnett")))
```

```
##
## Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: Dunnett Contrasts
##
##
## Fit: aov(formula = W ~ Clade, data = dat)
##
## Linear Hypotheses:
##
## Hominoidea - Cercopithecoidea == 0      Estimate Std. Error t value Pr(>|t|)
## Homo - Cercopithecoidea == 0            56.450     4.231  13.341  <0.001
## Platyrrhini - Cercopithecoidea == 0     -6.584     4.843   -1.359    0.454
## Strepsirrhini - Cercopithecoidea == 0   -7.020     4.426   -1.586    0.318
##
## Hominoidea - Cercopithecoidea == 0      ***
## Homo - Cercopithecoidea == 0            ***
## Platyrrhini - Cercopithecoidea == 0
## Strepsirrhini - Cercopithecoidea == 0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```

```
summary(glht(model.1, linfct=mcp(Clade = "Williams")))
```

```
##
## Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: Williams Contrasts
##
##
## Fit: aov(formula = W ~ Clade, data = dat)
##
## Linear Hypotheses:
##
## Estimate Std. Error t value Pr(>|t|)
## C 1 == 0   -7.020     4.426   -1.586    0.194
## C 2 == 0   -6.841     3.990   -1.715    0.153
## C 3 == 0   19.781     3.702    5.343  <0.001 ***
## C 4 == 0   21.839     3.616    6.040  <0.001 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```

2.0 linear models

mean tests are really just a subset of linear models where your predictor is a category

`cor.test` (Pearson)

earlier, we were looking at differences between the means of two variables

but those variables were both continuous, so we can ask whether they are related

```
cor.test(dat$W, dat$BrainW)
```

```
##
## Pearson's product-moment correlation
##
## data: dat$W and dat$BrainW
## t = 22.439, df = 167, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## 0.8233139 0.8998182
## sample estimates:
## cor
## 0.8665701
```

okay, so they're related - now what?

`lm`

this is probably the closest you will get to building a linear model by hand

this means `lm` is a powerful tool, but you have to know what you're doing

the basic call is the S3 method

```
model.1 <- lm(W ~ BrainW, data = dat)
summary(model.1)
```

```
##
## Call:
## lm(formula = W ~ BrainW, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -25.023  -4.269  -2.915   0.373  97.062
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.912142   1.607194   1.812   0.0718 .
## BrainW       0.047452   0.002115  22.439 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 15.95 on 167 degrees of freedom
## (26 observations deleted due to missingness)
## Multiple R-squared: 0.7509, Adjusted R-squared: 0.7495
## F-statistic: 503.5 on 1 and 167 DF, p-value: < 2.2e-16
```

R automatically one-hot encodes your categories

```
model.2 <- lm(W ~ BrainW + Clade, data = dat)
summary(model.2)
```

```
##
## Call:
## lm(formula = W ~ BrainW + Clade, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -23.636  -4.636  -0.013   0.331  97.364
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -0.90223    4.70281  -0.192   0.8481
## BrainW         0.10507    0.02359   4.453 1.57e-05 ***
## CladeHominioidea -8.36965    7.30339  -1.146   0.2535
## CladeHomo      -74.19993   29.02793  -2.556   0.0115 *
## CladePlatyrrhini  0.02628    5.23210   0.005   0.9960
## CladeStrepsirrhini 0.54245    5.09236   0.107   0.9153
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15.43 on 163 degrees of freedom
## (26 observations deleted due to missingness)
## Multiple R-squared: 0.7724, Adjusted R-squared: 0.7654
## F-statistic: 110.6 on 5 and 163 DF, p-value: < 2.2e-16
```

2.1 Contrasts

contrasts are useful for testing specific hypotheses

With an F-test, you can learn that a variable is causing a difference, but not what that difference is. You can think about contrasts as a way to look at the levels of a variable and ask which level, or groups of levels, are causing the difference. Maybe all of the levels are involved, but each one only a little bit. Contrasts allow you to test this possibility too.

the default contrast set for a factor is `contr.treatment`

This is the same thing as ‘one-hot encoding’. R treats the first level of the variable as the basis for comparison, and then every subsequent level is represented as a binary ‘dummy’ variable.

```
contr.treatment(4)
```

```
##    2 3 4
## 1 0 0 0
## 2 1 0 0
## 3 0 1 0
## 4 0 0 1
```

if your data don't follow a treatment logic, you can try the sum or Helmert contrasts

```
contr.helmert(4)
```

```
##    [,1] [,2] [,3]
## 1    -1    -1    -1
## 2     1    -1    -1
## 3     0     2    -1
## 4     0     0     3
```

```
contr.sum(4)
```

```
##    [,1] [,2] [,3]
## 1     1     0     0
## 2     0     1     0
## 3     0     0     1
## 4    -1    -1    -1
```

if your data have inherent order, try the polynomial contrast set

```
contr.poly(4)
```

```
##           .L    .Q           .C
## [1,] -0.6708204  0.5 -0.2236068
## [2,] -0.2236068 -0.5  0.6708204
## [3,]  0.2236068 -0.5 -0.6708204
## [4,]  0.6708204  0.5  0.2236068
```

This produces linear and quadratic contrasts. It's a bit hard to see that though, right? This is because R is making all of the contrasts orthogonal for you. Non-orthogonal contrasts risk explaining the same bit of variance twice, and make you think that you are predicting more than you actually are.

assigning contrasts to a factor is easy

The syntax looks like this:

```
my.factor = factor(c('Dillon', 'Andrew', 'Shinhye', 'Patty'))
contrasts(my.factor) <- contr.helmert
contrasts(my.factor)
```

```
##           [,1] [,2] [,3]
## Andrew    -1   -1   -1
## Dillon     1   -1   -1
## Patty      0    2   -1
## Shinhye    0    0    3
```

but you don't have to let R do it for you

The benefit to using R's built-ins is that they are guaranteed to be correct. But, sometimes, they are not really what you want

```
my.contrast <- matrix(c(
  -2, -1, 1, 2,
  0, 0, 1, 1,
  0, 1, 0, 1
), nrow=4, ncol=3)
contrasts(my.factor) <- my.contrast
contrasts(my.factor)
```

```
##           [,1] [,2] [,3]
## Andrew    -2    0    0
## Dillon    -1    0    1
## Patty      1    1    0
## Shinhye    2    1    1
```

Your turn!

Let's load in some real data from the data folder. This is a dataset containing information about different primates - how large they are, how large their brains are, and how much energy they use in a day.

```
dat <- read.csv('data/primate_energetics.csv')
```

Look at the variable `clade`. This variable contains five levels:

—————|————— Strepsirrhini | lemurs and lorises Platyrrhini | New World monkeys like capuchins and howlers Cercopithecoidea | Old World monkeys like macaques and babboons Hominoidea | apes like gibbons and gorillas Homo | people like you

How would you construct a contrast for this variable?

2.2 Robusticity

Some assumptions of the linear model

1. Errors are normally distributed
2. Errors are independent
3. Variation is homoscedastic
4. X is measured without error

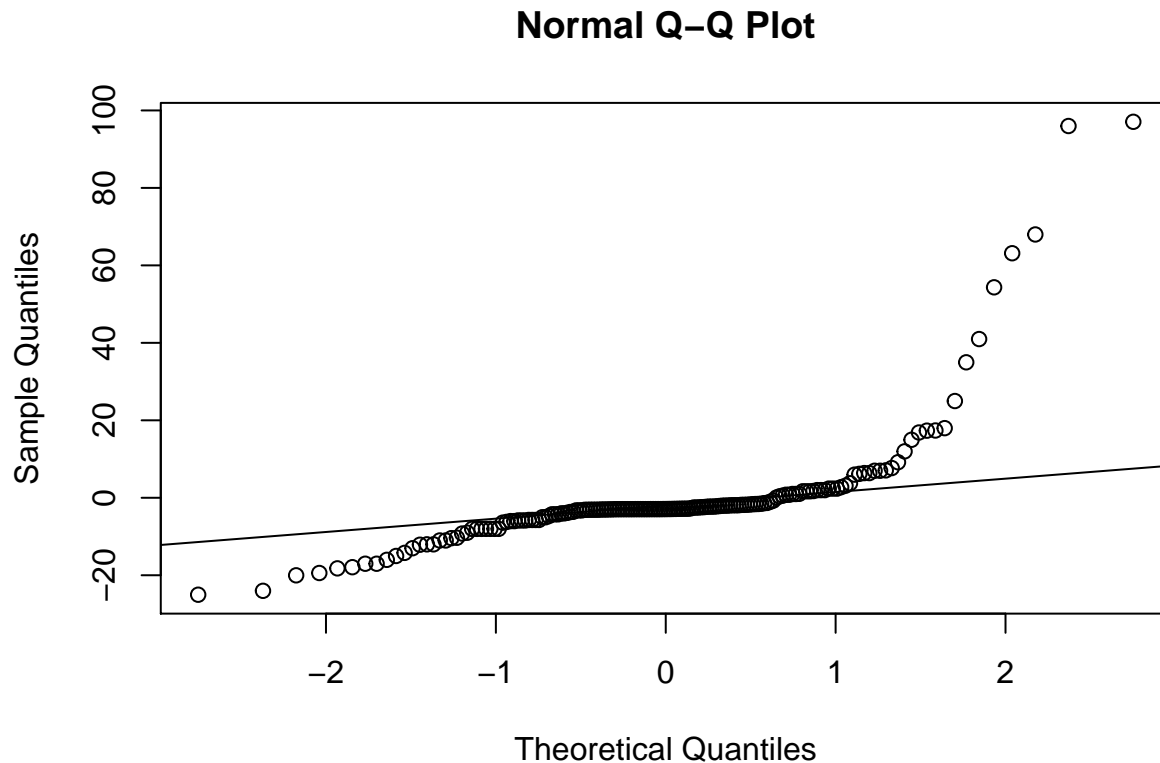
checking normality of errors

Suppose we were interested in predicting body size from brain size. We could construct a model like this:

```
model.1 <- lm(W ~ BrainW, data=dat)
```

And then plot the residuals from the model against a normal distribution

```
qqnorm(model.1$residuals)
qqline(model.1$residuals)
```



checking independence of errors

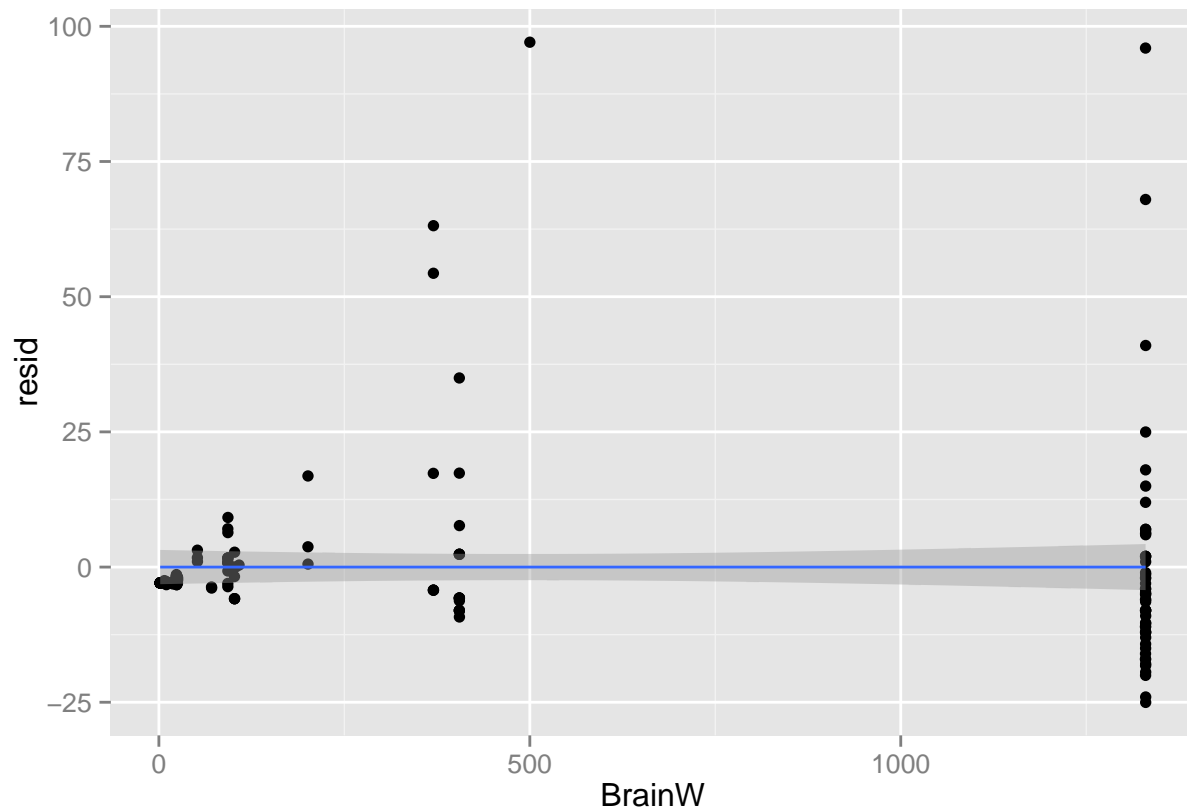
This one is pretty easy - you can simply regress your predictors on your residuals (or plot them)

```
summary(lm(model.1$resid ~ BrainW, data=dat[!(is.na(dat$BrainW)), ]))
```

```
##
## Call:
## lm(formula = model.1$resid ~ BrainW, data = dat[!(is.na(dat$BrainW)),
##      ])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -25.023  -4.269  -2.915   0.373  97.062
##
## Coefficients:
```

```
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.813e-16  1.607e+00      0      1
## BrainW       2.649e-19  2.115e-03      0      1
##
## Residual standard error: 15.95 on 167 degrees of freedom
## Multiple R-squared:  1.798e-34, Adjusted R-squared:  -0.005988
## F-statistic: 3.002e-32 on 1 and 167 DF, p-value: 1
```

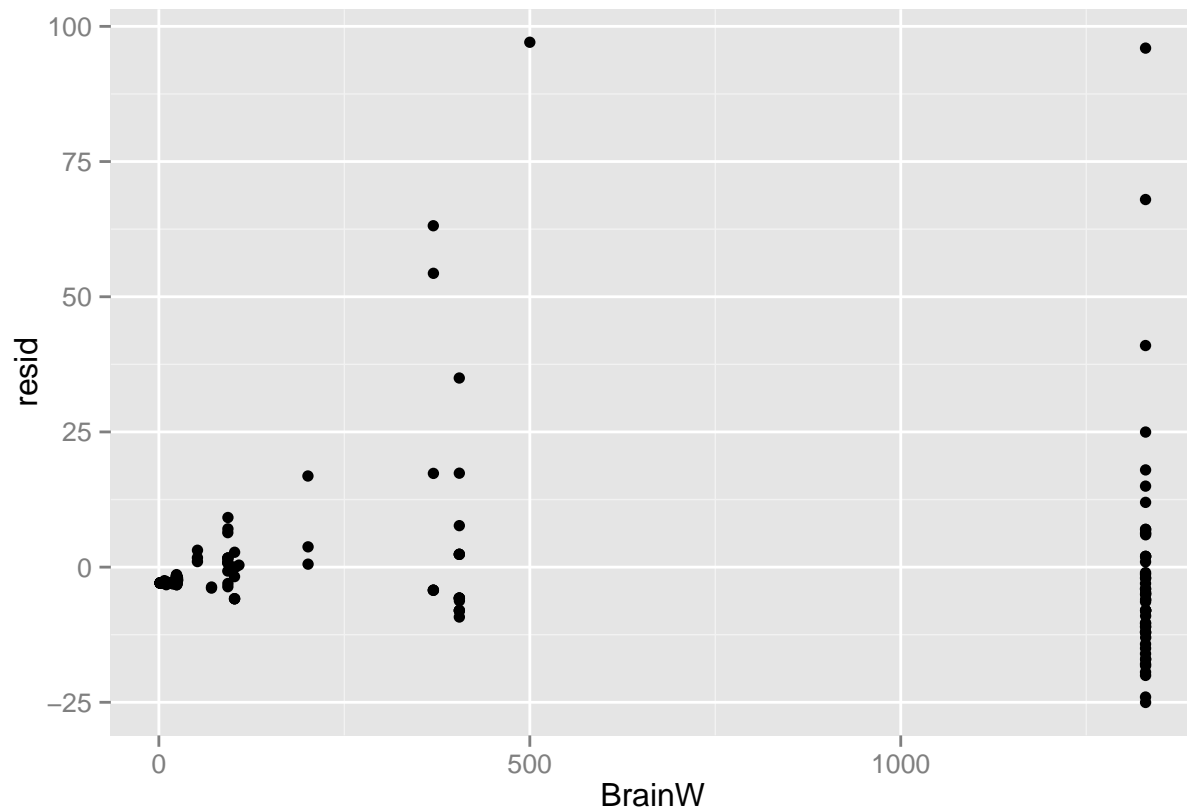
```
library(ggplot2)
dat.1 <- dat[!(is.na(dat$BrainW)), ]
dat.1$resid <- model.1$residuals
ggplot(data=dat.1, aes(x=BrainW, y=resid)) + geom_jitter() + stat_smooth(method='lm')
```



checking homoscedasticity of errors

You can eyeball this by plotting your residuals against your predictor, and looking at rolling estimate of variance

```
ggplot(data=dat.1, aes(x=BrainW, y=resid)) + geom_jitter() + stat_quantile()
```



A more formal test, called the Breusch-Pagan test, is found in the `lmtest` package. Essentially, it is just squaring the residuals and regressing your predictors on it.

```
library(lmtest)
```

```
## Loading required package: zoo
##
## Attaching package: 'zoo'
##
## The following objects are masked from 'package:base':
##
##   as.Date, as.Date.numeric
```

```
bptest(model.1)
```

```
##
## studentized Breusch-Pagan test
##
## data: model.1
## BP = 4.2271, df = 1, p-value = 0.03978
```

checking for error in X

There isn't really a way to establish statistically that your predictors have been measured without error. The good news is that linear models are fairly robust to violations of this assumption already. The better news is

that if you suspect your predictors have very large margins of error, you can use major axis regressions.

```
library(lmodel2)
model.2 <- lmodel2(W ~ BrainW, data=dat, nperm=10)
```

```
## RMA was not requested: it will not be computed.
```

```
model.2
```

```
##
## Model II regression
##
## Call: lmodel2(formula = W ~ BrainW, data = dat, nperm = 10)
##
## n = 169    r = 0.8665701    r-square = 0.7509437
## Parametric P-values:    2-tailed = 2.767738e-52    1-tailed = 1.383869e-52
## Angle between the two OLS regression lines = 0.8989437 degrees
##
## Permutation tests of OLS, MA, RMA slopes: 1-tailed, tail corresponding to sign
## A permutation test of r is equivalent to a permutation test of the OLS slope
## P-perm for SMA = NA because the SMA slope cannot be tested
##
## Regression results
##   Method Intercept      Slope Angle (degrees) P-perm (1-tailed)
## 1    OLS  2.9121421 0.04745212      2.716768      0.09090909
## 2     MA  2.8947748 0.04748750      2.718791      0.09090909
## 3    SMA -0.6740652 0.05475855      3.134304              NA
##
## Confidence intervals
##   Method 2.5%-Intercept 97.5%-Intercept 2.5%-Slope 97.5%-Slope
## 1    OLS      -0.2608942      6.085178 0.04327719 0.05162705
## 2     MA       0.8436461      4.945092 0.04331025 0.05166641
## 3    SMA      -2.8012460      1.297108 0.05074254 0.05909240
##
## Eigenvalues: 339487 252.3885
##
## H statistic used for computing C.I. of MA: 1.737758e-05
```

linear models are very susceptible to outliers

This isn't really an assumption that we are violating so much as a consequence of parameterizing with means (highly susceptible to outliers) and variance (squared outliers). You can test for outliers with various measures of leverage, including:

1. DFBetas
2. DFFit
3. Cook's Distance

These can be calculated with:

```
influence.measures(model.1)
```

```
## Influence measures of
## lm(formula = W ~ BrainW, data = dat) :
##
##      dfb.1_  dfb.BrnW    dffit cov.r   cook.d    hat inf
## 8      0.000306 -0.000171  0.000308 1.021 4.77e-08 0.00855
## 13     0.009798 -0.005569  0.009848 1.021 4.88e-05 0.00870
## 14    -0.004242  0.002411 -0.004264 1.021 9.15e-06 0.00870
## 15     0.009798 -0.005569  0.009848 1.021 4.88e-05 0.00870
## 16     0.009798 -0.005569  0.009848 1.021 4.88e-05 0.00870
## 17     0.005703 -0.003241  0.005732 1.021 1.65e-05 0.00870
## 18     0.037312 -0.021206  0.037503 1.019 7.07e-04 0.00870
## 19     0.053728 -0.030537  0.054003 1.017 1.46e-03 0.00870
## 20    -0.021211  0.012056 -0.021320 1.020 2.29e-04 0.00870
## 21    -0.017700  0.010060 -0.017791 1.021 1.59e-04 0.00870
## 22     0.004533 -0.002576  0.004556 1.021 1.04e-05 0.00870
## 23     0.004533 -0.002576  0.004556 1.021 1.04e-05 0.00870
## 24     0.041413 -0.023538  0.041626 1.019 8.71e-04 0.00870
## 26     0.088935 -0.040901  0.091534 1.006 4.19e-03 0.00739
## 27     0.002893 -0.001330  0.002978 1.020 4.46e-06 0.00739
## 28     0.019729 -0.009073  0.020306 1.019 2.07e-04 0.00739
## 39     0.401156  0.008417  0.532604 0.615 1.11e-01 0.00592  *
## 40    -0.033962  0.019024 -0.034175 1.019 5.87e-04 0.00857
## 41    -0.033962  0.019024 -0.034175 1.019 5.87e-04 0.00857
## 42    -0.033962  0.019024 -0.034175 1.019 5.87e-04 0.00857
## 43    -0.010165  0.005694 -0.010229 1.021 5.26e-05 0.00857
## 45    -0.033391  0.005749 -0.039294 1.015 7.75e-04 0.00605
## 46    -0.033391  0.005749 -0.039294 1.015 7.75e-04 0.00605
## 47    -0.033391  0.005749 -0.039294 1.015 7.75e-04 0.00605
## 48    -0.023818  0.004101 -0.028029 1.017 3.95e-04 0.00605
## 49    -0.023818  0.004101 -0.028029 1.017 3.95e-04 0.00605
## 50    -0.023818  0.004101 -0.028029 1.017 3.95e-04 0.00605
## 51    -0.023818  0.004101 -0.028029 1.017 3.95e-04 0.00605
## 52     0.009847 -0.001695  0.011588 1.018 6.75e-05 0.00605
## 53     0.009847 -0.001695  0.011588 1.018 6.75e-05 0.00605
## 54     0.009847 -0.001695  0.011588 1.018 6.75e-05 0.00605
## 55     0.072430 -0.012471  0.085235 1.004 3.63e-03 0.00605
## 56     0.031890 -0.005491  0.037528 1.015 7.07e-04 0.00605
## 57     0.147449 -0.025389  0.173515 0.960 1.47e-02 0.00605  *
## 58    -0.038390  0.006610 -0.045177 1.014 1.02e-03 0.00605
## 59    -0.025899  0.004459 -0.030477 1.016 4.67e-04 0.00605
## 60    -0.018550  0.004302 -0.021101 1.017 2.24e-04 0.00617
## 61    -0.018550  0.004302 -0.021101 1.017 2.24e-04 0.00617
## 62    -0.018550  0.004302 -0.021101 1.017 2.24e-04 0.00617
## 63     0.288170 -0.066822  0.327795 0.835 4.89e-02 0.00617  *
## 64     0.075552 -0.017520  0.085941 1.004 3.69e-03 0.00617
## 65     0.244716 -0.056746  0.278366 0.881 3.63e-02 0.00617  *
## 69     0.015941 -0.008929  0.016041 1.020 1.29e-04 0.00857
## 70     0.016795 -0.142496 -0.173246 1.011 1.50e-02 0.01829
## 71     0.021046 -0.178566 -0.217099 1.000 2.34e-02 0.01829
## 72     0.005364 -0.045515 -0.055336 1.029 1.54e-03 0.01829
## 73     0.011901 -0.100978 -0.122769 1.021 7.54e-03 0.01829
```

## 74	0.005030	-0.042678	-0.051887	1.029	1.35e-03	0.01829	
## 75	0.015019	-0.127429	-0.154927	1.015	1.20e-02	0.01829	
## 76	0.020193	-0.171328	-0.208299	1.003	2.15e-02	0.01829	
## 77	0.010893	-0.092423	-0.112367	1.023	6.33e-03	0.01829	
## 78	0.010137	-0.086012	-0.104573	1.024	5.48e-03	0.01829	
## 79	-0.005325	0.045182	0.054932	1.029	1.52e-03	0.01829	
## 80	0.012574	-0.106688	-0.129710	1.020	8.42e-03	0.01829	
## 81	0.001105	-0.009373	-0.011396	1.031	6.53e-05	0.01829	
## 82	0.003944	-0.033461	-0.040682	1.030	8.32e-04	0.01829	
## 83	0.008712	-0.073917	-0.089868	1.026	4.05e-03	0.01829	
## 84	0.008628	-0.073206	-0.089004	1.026	3.97e-03	0.01829	
## 85	0.015272	-0.129579	-0.157541	1.015	1.24e-02	0.01829	
## 86	0.016287	-0.138187	-0.168006	1.012	1.41e-02	0.01829	
## 87	-0.005158	0.043763	0.053207	1.029	1.42e-03	0.01829	
## 88	0.014259	-0.120984	-0.147092	1.017	1.08e-02	0.01829	
## 89	0.006702	-0.056867	-0.069139	1.028	2.40e-03	0.01829	
## 90	-0.012535	0.106353	0.129303	1.020	8.36e-03	0.01829	
## 91	0.002524	-0.021415	-0.026036	1.030	3.41e-04	0.01829	
## 92	-0.034913	0.296225	0.360149	0.950	6.26e-02	0.01829	*
## 93	-0.060171	0.510524	0.620691	0.815	1.72e-01	0.01829	*
## 94	-0.090756	0.770028	0.936194	0.626	3.44e-01	0.01829	*
## 95	-0.001650	0.013999	0.017020	1.031	1.46e-04	0.01829	
## 96	0.006702	-0.056867	-0.069139	1.028	2.40e-03	0.01829	
## 97	0.005030	-0.042678	-0.051887	1.029	1.35e-03	0.01829	
## 98	0.006702	-0.056867	-0.069139	1.028	2.40e-03	0.01829	
## 99	0.006702	-0.056867	-0.069139	1.028	2.40e-03	0.01829	
## 100	0.001689	-0.014331	-0.017424	1.031	1.53e-04	0.01829	
## 101	-0.021006	0.178226	0.216686	1.000	2.33e-02	0.01829	
## 102	-0.005827	0.049438	0.060106	1.029	1.82e-03	0.01829	
## 103	-0.010014	0.084966	0.103301	1.024	5.35e-03	0.01829	
## 104	0.001689	-0.014331	-0.017424	1.031	1.53e-04	0.01829	
## 105	-0.001650	0.013999	0.017020	1.031	1.46e-04	0.01829	
## 106	-0.001650	0.013999	0.017020	1.031	1.46e-04	0.01829	
## 107	-0.004991	0.042345	0.051483	1.029	1.33e-03	0.01829	
## 108	0.009215	-0.078184	-0.095056	1.025	4.53e-03	0.01829	
## 109	0.007539	-0.063968	-0.077772	1.027	3.04e-03	0.01829	
## 110	-0.015064	0.127809	0.155389	1.015	1.21e-02	0.01829	
## 111	0.003359	-0.028500	-0.034651	1.030	6.04e-04	0.01829	
## 112	-0.000815	0.006916	0.008409	1.031	3.56e-05	0.01829	
## 113	0.004194	-0.035588	-0.043267	1.030	9.41e-04	0.01829	
## 114	0.003359	-0.028500	-0.034651	1.030	6.04e-04	0.01829	
## 115	0.004194	-0.035588	-0.043267	1.030	9.41e-04	0.01829	
## 116	0.013416	-0.113832	-0.138396	1.018	9.58e-03	0.01829	
## 117	0.010054	-0.085300	-0.103707	1.024	5.39e-03	0.01829	
## 118	0.009215	-0.078184	-0.095056	1.025	4.53e-03	0.01829	
## 119	0.014259	-0.120984	-0.147092	1.017	1.08e-02	0.01829	
## 120	0.010054	-0.085300	-0.103707	1.024	5.39e-03	0.01829	
## 121	0.000854	-0.007248	-0.008813	1.031	3.91e-05	0.01829	
## 122	-0.005827	0.049438	0.060106	1.029	1.82e-03	0.01829	
## 123	0.018959	-0.011453	0.018987	1.021	1.81e-04	0.00930	
## 124	0.006199	-0.003745	0.006208	1.022	1.94e-05	0.00930	
## 125	0.010573	-0.006387	0.010589	1.021	5.64e-05	0.00930	
## 126	-0.018011	0.011392	-0.018014	1.022	1.63e-04	0.00986	
## 127	0.002152	-0.001193	0.002167	1.021	2.36e-06	0.00849	

```

## 128 -0.019484 0.012294 -0.019488 1.022 1.91e-04 0.00983
## 129 -0.018256 0.011634 -0.018257 1.022 1.68e-04 0.00996
## 130 -0.019390 0.012357 -0.019392 1.022 1.89e-04 0.00996
## 131 -0.017871 0.011389 -0.017873 1.022 1.61e-04 0.00996
## 132 -0.018124 0.011550 -0.018125 1.022 1.65e-04 0.00996
## 133 -0.017764 0.011321 -0.017765 1.022 1.59e-04 0.00996
## 134 -0.017745 0.011309 -0.017747 1.022 1.58e-04 0.00996
## 135 -0.017840 0.011417 -0.017841 1.022 1.60e-04 0.01002
## 136 -0.023178 0.013625 -0.023245 1.021 2.72e-04 0.00901
## 137 -0.021983 0.012923 -0.022047 1.021 2.44e-04 0.00901
## 138 -0.020316 0.012738 -0.020322 1.022 2.08e-04 0.00975
## 139 -0.019319 0.012113 -0.019325 1.022 1.88e-04 0.00975
## 140 -0.018584 0.011858 -0.018585 1.022 1.74e-04 0.00998
## 141 -0.020521 0.013094 -0.020522 1.022 2.12e-04 0.00998
## 142 -0.019121 0.012200 -0.019122 1.022 1.84e-04 0.00998
## 143 -0.019468 0.012422 -0.019469 1.022 1.91e-04 0.00998
## 144 -0.018837 0.012019 -0.018838 1.022 1.78e-04 0.00998
## 145 -0.020254 0.012699 -0.020260 1.022 2.06e-04 0.00975
## 146 -0.018648 0.011898 -0.018648 1.022 1.75e-04 0.00998
## 147 -0.019468 0.012422 -0.019469 1.022 1.91e-04 0.00998
## 148 -0.017991 0.011480 -0.017992 1.022 1.63e-04 0.00998
## 149 -0.019398 0.012377 -0.019399 1.022 1.89e-04 0.00998
## 150 -0.018383 0.011729 -0.018384 1.022 1.70e-04 0.00998
## 151 -0.017083 0.010900 -0.017084 1.022 1.47e-04 0.00998
## 152 -0.018389 0.011733 -0.018390 1.022 1.70e-04 0.00998
## 153 -0.014243 0.008919 -0.014248 1.022 1.02e-04 0.00973
## 154 -0.015675 0.009816 -0.015680 1.022 1.24e-04 0.00973
## 155 -0.014555 0.009114 -0.014559 1.022 1.07e-04 0.00973
## 156 -0.015115 0.009465 -0.015120 1.022 1.15e-04 0.00973
## 157 -0.012003 0.007516 -0.012006 1.022 7.25e-05 0.00973
## 158 -0.018601 0.011648 -0.018607 1.022 1.74e-04 0.00973
## 159 -0.014990 0.009387 -0.014995 1.022 1.13e-04 0.00973
## 160 -0.015675 0.009816 -0.015680 1.022 1.24e-04 0.00973
## 161 -0.012003 0.007516 -0.012006 1.022 7.25e-05 0.00973
## 162 -0.013247 0.008296 -0.013252 1.022 8.83e-05 0.00973
## 163 -0.012812 0.008023 -0.012816 1.022 8.26e-05 0.00973
## 164 -0.014243 0.008919 -0.014248 1.022 1.02e-04 0.00973
## 165 -0.011391 0.007145 -0.011395 1.022 6.53e-05 0.00975
## 166 -0.011454 0.007184 -0.011457 1.022 6.60e-05 0.00975
## 167 -0.011952 0.007497 -0.011956 1.022 7.19e-05 0.00975
## 168 -0.011204 0.007028 -0.011208 1.022 6.32e-05 0.00975
## 169 -0.010145 0.006363 -0.010148 1.022 5.18e-05 0.00975
## 170 -0.009958 0.006246 -0.009961 1.022 4.99e-05 0.00975
## 171 -0.009584 0.006011 -0.009587 1.022 4.62e-05 0.00975
## 172 -0.010332 0.006481 -0.010335 1.022 5.37e-05 0.00975
## 173 -0.014133 0.008865 -0.014137 1.022 1.01e-04 0.00975
## 174 -0.012139 0.007614 -0.012143 1.022 7.42e-05 0.00975
## 175 -0.008649 0.005425 -0.008652 1.022 3.77e-05 0.00975
## 176 -0.013697 0.008591 -0.013701 1.022 9.44e-05 0.00975
## 177 -0.015818 0.010122 -0.015818 1.022 1.26e-04 0.01002
## 178 -0.018680 0.012038 -0.018680 1.022 1.75e-04 0.01012
## 179 -0.018686 0.012042 -0.018686 1.022 1.76e-04 0.01012
## 180 -0.018635 0.012009 -0.018635 1.022 1.75e-04 0.01012
## 181 -0.018630 0.012006 -0.018630 1.022 1.75e-04 0.01012

```

```
## 182 -0.018635 0.012009 -0.018635 1.022 1.75e-04 0.01012
## 183 -0.018690 0.012045 -0.018690 1.022 1.76e-04 0.01012
## 184 -0.018610 0.011993 -0.018610 1.022 1.74e-04 0.01012
## 185 -0.018610 0.011993 -0.018610 1.022 1.74e-04 0.01012
## 186 -0.018654 0.012021 -0.018654 1.022 1.75e-04 0.01012
## 187 -0.018673 0.012034 -0.018673 1.022 1.75e-04 0.01012
## 188 -0.018591 0.011981 -0.018591 1.022 1.74e-04 0.01012
## 189 -0.018635 0.012009 -0.018635 1.022 1.75e-04 0.01012
## 190 -0.018413 0.011866 -0.018413 1.022 1.71e-04 0.01012
## 191 -0.018616 0.011997 -0.018616 1.022 1.74e-04 0.01012
## 192 -0.018673 0.012034 -0.018673 1.022 1.75e-04 0.01012
## 193 -0.018578 0.011972 -0.018578 1.022 1.74e-04 0.01012
## 194 -0.018575 0.011970 -0.018575 1.022 1.74e-04 0.01012
## 195 -0.018727 0.012069 -0.018727 1.022 1.76e-04 0.01012
```

If you find your model contains high leverage cases, you can either pluck them out by hand (preferably by selecting against something like Cook's Distance), or try running a regression build to be robust to outliers.

```
library(robust)
```

```
## Loading required package: fit.models
## Loading required package: lattice
## Loading required package: MASS
## Loading required package: robustbase
##
## Attaching package: 'robustbase'
##
## The following object is masked from 'package:survival':
##
##     heart
##
## Loading required package: rrcov
## Scalable Robust Estimators with High Breakdown Point (version 1.3-8)
```

```
model.3 <- lmRob(W ~ BrainW, data=dat)
summary(model.3)
```

```
##
## Call:
## lmRob(formula = W ~ BrainW, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.6681  -0.8612  -0.4966   3.5102 103.3319
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.7390757  0.3309017   2.234  0.0268 *
## BrainW      0.0435557  0.0005608  77.665  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```



```
## Residual standard error: 2.749 on 167 degrees of freedom
## Multiple R-Squared: 0.3895
##
## Test for Bias:
##           statistic p-value
## M-estimate      6.112 0.04707
## LS-estimate      6.212 0.04477
## 26 observations deleted due to missingness
```

Excluding the outliers in this case has brought the coefficient of our predictor close to zero. Can you guess why? If you aren't sure, plot it.

your turn!

So far we've been predicting body weight from brain weight. Try predicting body weight from brain weight and clade. Your base model should look like this:

```
<- lm(W ~ BrainW + Clade, data=dat)
```

Rerun your regression diagnostics. What do you find?

2.3 Stepwise Models and Uncertainty Reduction

fundamental constraints in prediction

We'll start with the bad news. Any kind of predicting method is constrained in how many independent variables you can include in the model (this is why machine learning algorithms always start with a dimensionality reduction method like PCA).

Let's do a little test to show how much of a problem this is. Run the code below a couple of times. How often do you get significant results? Keep in mind that these are randomly generated data, so we already know that there *shouldn't be* any significant predictors.

```
dat.toy <- data.frame(value=runif(4), var1=runif(4), var2=runif(4))
summary(lm(value ~ var1 + var2, data=dat.toy))
```

```
##
## Call:
## lm(formula = value ~ var1 + var2, data = dat.toy)
##
## Residuals:
##      1      2      3      4
## 0.166883 -0.148095  0.001025 -0.019812
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.6462      0.8538  -0.757   0.588
## var1          2.8372      1.8866   1.504   0.374
## var2         -2.4964      1.7361  -1.438   0.387
##
```

```
## Residual standard error: 0.224 on 1 degrees of freedom
## Multiple R-squared:  0.709, Adjusted R-squared:  0.1269
## F-statistic: 1.218 on 2 and 1 DF,  p-value: 0.5395
```

R won't let us run `lm` with four variables, but if it did, we would be able to 100% accurately predict four random number from four sets of other random numbers. Overfitting, combined with p hacking and the file drawer problem, are responsible for the giant world of false findings and irreproducible science that you keep hearing about. Generally speaking, it is bad science to throw predictors haphazardly into a model and see what comes out; however, there are times when this approach is either required or useful in its own right.

weeding out predictors the smart way

It may happen to you in your life that you have no theory, hypothesis, or intuitions about how your predictors could be related to your outcome variable. In this case, you can weed out predictors using what is called a stepwise regression.

```
library(MASS)
dat.omit <- na.omit(dat)
model.base <- lm(W ~ ., data=dat.omit)
model.4 <- stepAIC(model.base)

## Start:  AIC=10.23
## W ~ Clade + Genus + Species + Group + N + BrainW + RMR + TEE +
##       Source + Method
##
##
## Step:  AIC=10.23
## W ~ Clade + Genus + Species + Group + N + BrainW + RMR + TEE +
##       Source
##
##
## Step:  AIC=10.23
## W ~ Clade + Genus + Species + Group + N + BrainW + RMR + TEE
##
##
## Step:  AIC=10.23
## W ~ Clade + Genus + Species + Group + N + RMR + TEE
##
##
## Step:  AIC=10.23
## W ~ Clade + Genus + Group + N + RMR + TEE
##
##
## Step:  AIC=10.23
## W ~ Genus + Group + N + RMR + TEE
##
##
```

	Df	Sum of Sq	RSS	AIC
- N	1	0.576	57.225	8.976
<none>			56.648	10.227
- RMR	1	13.813	70.461	24.373
- TEE	1	26.708	83.356	36.810
- Genus	8	178.782	235.430	99.644

```
## - Group 2 230.458 287.106 126.328
##
## Step: AIC=8.98
## W ~ Genus + Group + RMR + TEE
##
##           Df Sum of Sq      RSS      AIC
## <none>             57.225    8.976
## - RMR      1      13.952    71.177   23.122
## - TEE      1      30.698    87.922   38.757
## - Genus    8     221.384   278.609  110.105
## - Group    2     230.098   287.322  124.384
```

```
summary(model.4)
```

```
##
## Call:
## lm(formula = W ~ Genus + Group + RMR + TEE, data = dat.omit)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.0820 -0.1801  0.0071  0.2316  3.6508
##
## Coefficients: (1 not defined because of singularities)
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -3.478e+00  6.041e-01  -5.758 3.11e-07 ***
## GenusCebus    3.158e+00  8.792e-01   3.592 0.000662 ***
## GenusEulemur  1.925e+00  6.803e-01   2.829 0.006334 **
## GenusHomo     4.979e+01  2.210e+00  22.527 < 2e-16 ***
## GenusLemur    2.151e+00  6.646e-01   3.236 0.001973 **
## GenusMacaca   7.444e+00  6.170e-01  12.065 < 2e-16 ***
## GenusMarmoset 3.306e+00  8.044e-01   4.110 0.000122 ***
## GenusMicrocebus 2.717e+00  6.089e-01   4.461 3.64e-05 ***
## GenusPan      1.017e+01  8.714e-01  11.673 < 2e-16 ***
## GenusTamarin  3.270e+00  6.737e-01   4.853 9.02e-06 ***
## GroupEcuador  -1.347e+01  1.053e+00 -12.800 < 2e-16 ***
## GroupEvenki   -1.465e+01  1.007e+00 -14.555 < 2e-16 ***
## GroupRussian      NA           NA      NA      NA
## RMR            7.939e-03  2.076e-03   3.825 0.000314 ***
## TEE            4.167e-03  7.344e-04   5.673 4.28e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9766 on 60 degrees of freedom
## Multiple R-squared:  0.9976, Adjusted R-squared:  0.997
## F-statistic: 1885 on 13 and 60 DF, p-value: < 2.2e-16
```

Doing it this way isn't particularly helpful - genera tend to be around the same body size, so really we are just recapitulating taxonomies here.

specifying forward stepwise models

Part of the problem is that backwards models tend to keep predictors that aren't particularly useful. The other part of the problem is that we threw every variable we had into the model.

```

model.base <- lm(W ~ 1, data=dat.omit)
model.5 <- stepAIC(model.base, scope=list(upper = ~ W + BrainW + RMR + TEE), direction='forward')

## Start:  AIC=428.05
## W ~ 1

## Warning in model.matrix.default(Terms, m, contrasts.arg = object
## $contrasts): the response appeared on the right-hand side and was dropped

## Warning in model.matrix.default(Terms, m, contrasts.arg = object
## $contrasts): problem with term 1 in model.matrix: no columns are assigned

##           Df Sum of Sq    RSS    AIC
## + BrainW   1     22726   697.2 169.98
## + RMR       1     22445   978.2 195.04
## + TEE       1     20070  3352.9 286.20
## <none>                23422.7 428.05
##
## Step:  AIC=169.98
## W ~ BrainW

## Warning in model.matrix.default(Terms, m, contrasts.arg = object
## $contrasts): the response appeared on the right-hand side and was dropped

## Warning in model.matrix.default(Terms, m, contrasts.arg = object
## $contrasts): problem with term 2 in model.matrix: no columns are assigned

##           Df Sum of Sq    RSS    AIC
## + RMR       1     257.52 439.64 137.86
## + TEE       1     133.83 563.33 156.21
## <none>                697.16 169.98
##
## Step:  AIC=137.86
## W ~ BrainW + RMR

## Warning in model.matrix.default(Terms, m, contrasts.arg = object
## $contrasts): the response appeared on the right-hand side and was dropped

## Warning in model.matrix.default(Terms, m, contrasts.arg = object
## $contrasts): problem with term 3 in model.matrix: no columns are assigned

##           Df Sum of Sq    RSS    AIC
## <none>                439.64 137.86
## + TEE       1     10.833 428.81 138.01

summary(model.5)

##
## Call:
## lm(formula = W ~ BrainW + RMR, data = dat.omit)

```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -6.5206  0.0926  0.3106  0.4514  9.7928
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.897984   0.400010  -2.245   0.0279 *
## BrainW       0.025710   0.002757   9.326 5.89e-14 ***
## RMR          0.016591   0.002573   6.449 1.18e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.488 on 71 degrees of freedom
## Multiple R-squared:  0.9812, Adjusted R-squared:  0.9807
## F-statistic: 1856 on 2 and 71 DF,  p-value: < 2.2e-16
```

comparing homogeneous models

Stepwise regression is evaluated by maximizing the likelihood that a model will reduce your uncertainty about the dependent variable. This can't be assessed in an exact sense, but models that differ only the inclusion of a parameter can be compared by relative likelihood (this is sometimes called comparing 'nested models'). This can be calculated with:

```
p <- exp((model.5$anova$AIC[3] - model.5$anova$AIC[2]) / 2)
p
```

```
## [1] 1.06087e-07
```

To put it another way, the probability that the complex model is better is 0.9999999.

your turn!

So far, we've been predicting body weight from brain weight. What happens if you run the model the other way around? E.g., you try:

```
BrainW ~ .
```

2.4 Comparing Heterogeneous Models

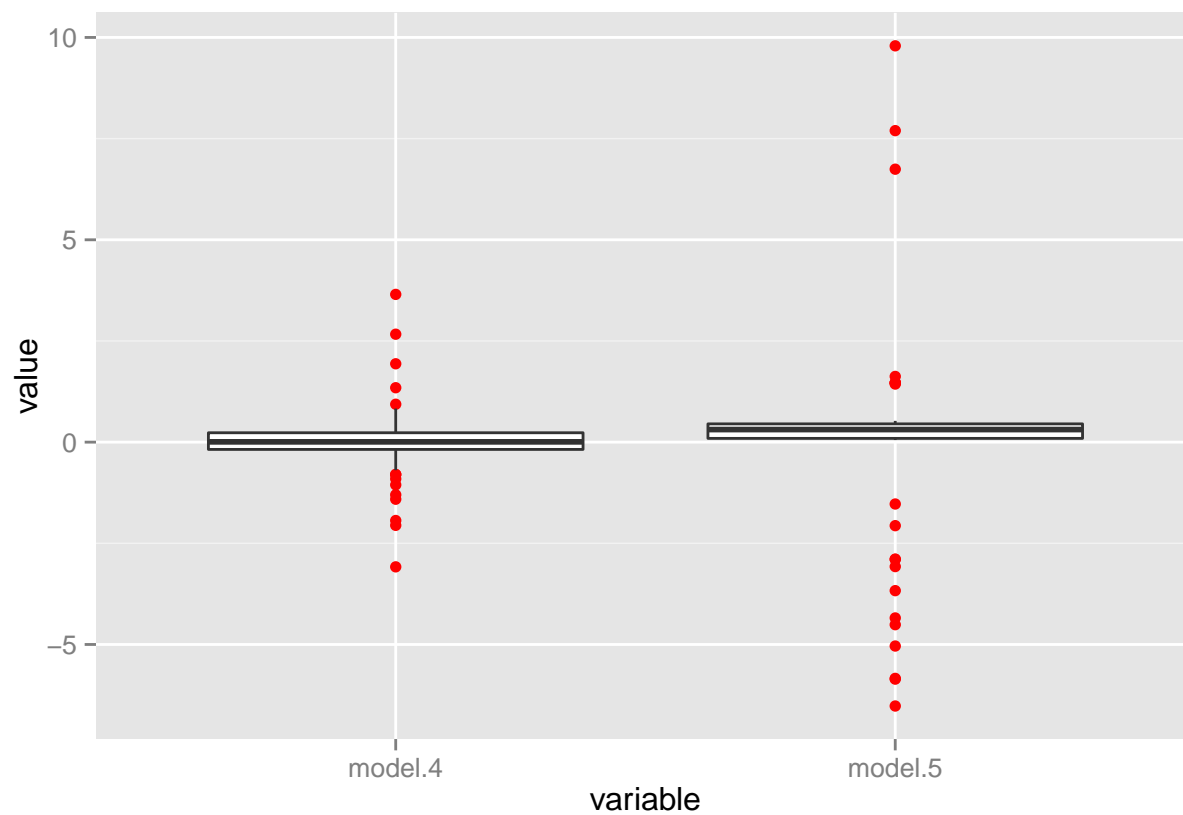
visualizing prediction error

One easy way to visualize this is to use make a dataframe out of your error terms and plot it.

```
library(reshape2)
errors <- data.frame(model.4 = model.4$residuals, model.5 = model.5$residuals)
errors <- melt(errors)
```

```
## No id variables; using all as measure variables
```

```
ggplot(data=errors, aes(x=variable, y=value)) + geom_boxplot(outlier.colour='red')
```



quantifying prediction error

- The good news: the math used to quantify prediction error is simple
- The bad news: R will not do it for you

Cautionary note - none of these methods take model complexity into account

RMSE

The root mean square error is the preferred mean-based way to quantify model fit. The math looks like $\sqrt{\text{mean}(\text{error}^2)}$

```
sqrt(mean(model.4$residuals**2))
```

```
## [1] 0.8793775
```

```
sqrt(mean(model.5$residuals**2))
```

```
## [1] 2.437431
```

MdAE

The median absolute error is the preferred median-based way to quantify model fit. It is particularly useful when a model's fit is unduly influenced by one or a few outliers. The math looks like `median(abs(error))`

```
median(abs(model.4$residuals))
```

```
## [1] 0.1880393
```

```
median(abs(model.5$residuals))
```

```
## [1] 0.3898049
```

side note - this is sometimes abbreviated **MAD** for 'median absolute deviation', but it could also mean 'mean absolute deviation'

Your turn!

You're going to compare two heterogeneous models for explaining body weight. Try using `TEE ~ W` and `TEE ~ RMR`. Which is a better fit?

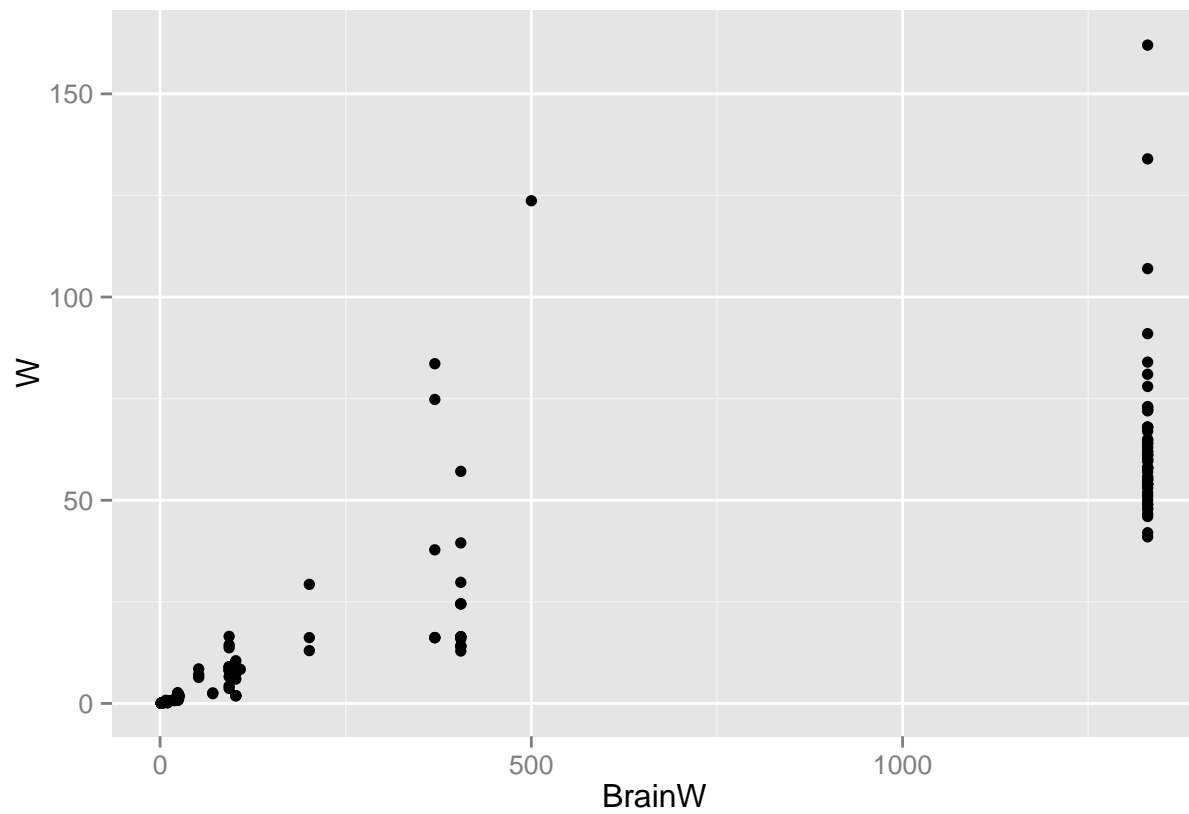
2.5 Generalized Models

lm assumes linear relationships

The linear model we have been using is assuming a couple of things that we haven't talked much about. One of these is that the relationship between the predictors and the outcome is linear (as opposed to quadratic, for example). This assumption can often be met by nonlinear transforms of the data. For example, we've been using data that are usually analyzed after they've been log transformed.

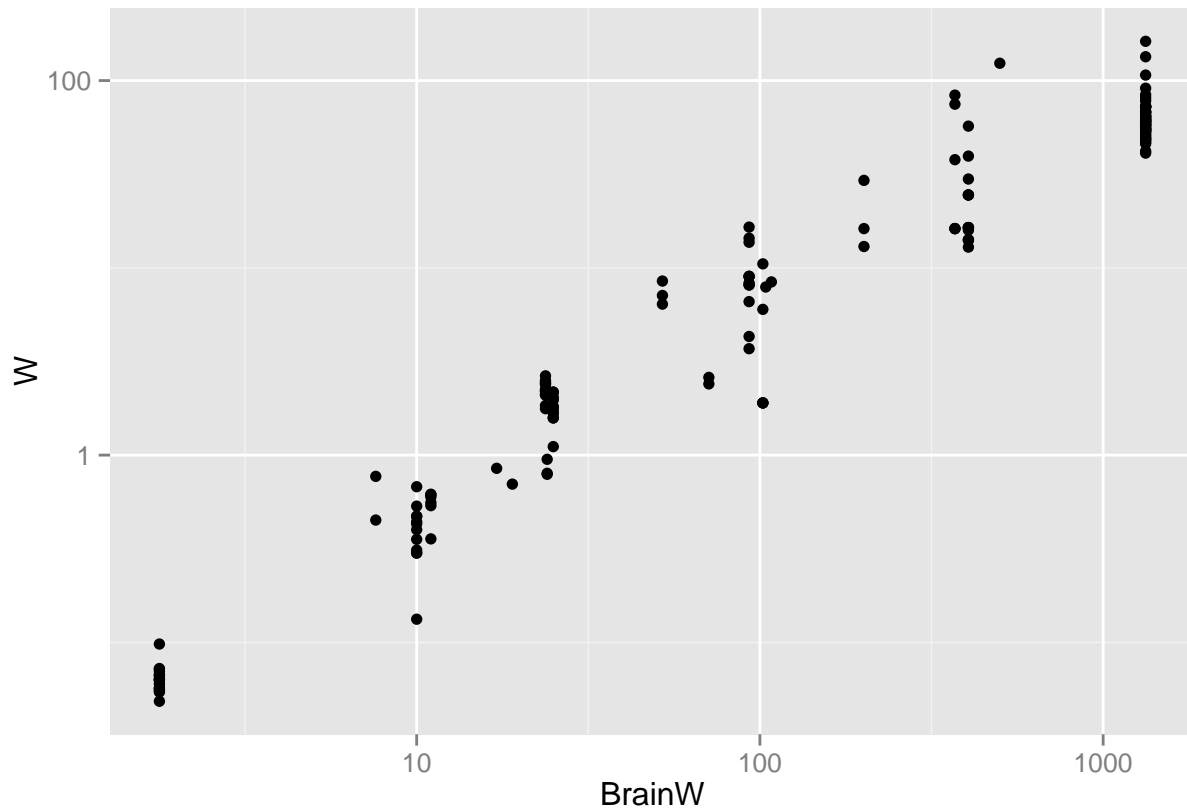
```
qplot(data=dat, x=BrainW, y=W)
```

```
## Warning: Removed 26 rows containing missing values (geom_point).
```



```
qplot(data=dat, x=BrainW, y=W, log='xy')
```

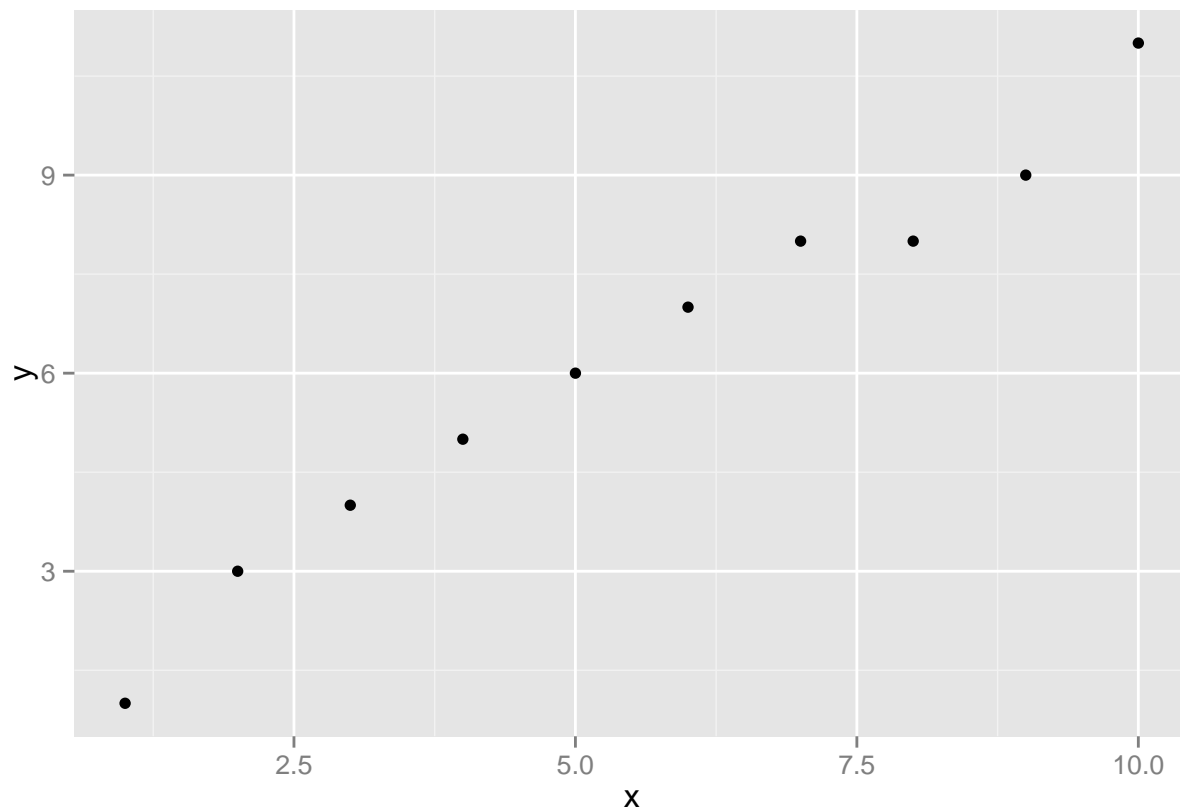
```
## Warning: Removed 26 rows containing missing values (geom_point).
```

lm also assumes normal variance

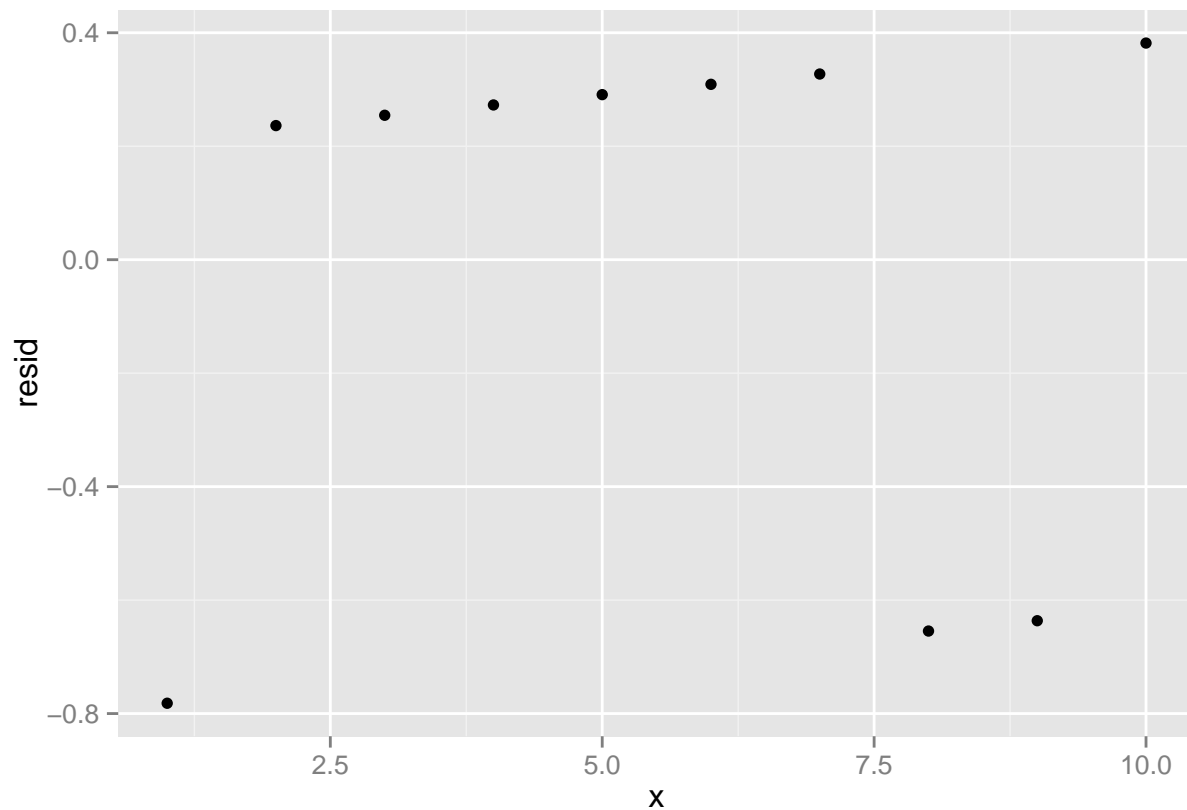
This one is a little trickier. To show why this is a problem, let's make another toy dataset.

```
dat.toy <- data.frame(x=1:10, y=1:10+rbinom(10, 1, .5))  
qplot(data=dat.toy, x=x, y=y)
```



If we predict y from x and plot the residuals against, x, we get:

```
model.toy <- lm(y ~ x, data=dat.toy)
dat.toy$resid <- model.toy$residuals
qplot(data=dat.toy, x=x, y=resid)
```



The actual prediction equation here should be $y = 0.5 + x$, but because the errors around y are non-normal, linear regression produces an incorrect solution something like $0.8 + 0.9818182 x$. This *usually* cannot be corrected by nonlinear transforms.

the generalized linear model

The good news is that `lm` is a subset of `glm` or generalized linear models, where the link is linear and the variance is normal. `lm` in R is really just a convenience function that (behind the scenes) is running:

```
summary(glm(y~x, data=dat.toy, family=gaussian))

##
## Call:
## glm(formula = y ~ x, family = gaussian, data = dat.toy)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.7818  -0.4182   0.2636   0.3045   0.3818
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.80000    0.34772   2.301  0.0504 .
## x            0.98182    0.05604  17.520 1.15e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 0.2590909)
```

```
##
## Null deviance: 81.6000 on 9 degrees of freedom
## Residual deviance: 2.0727 on 8 degrees of freedom
## AIC: 18.642
##
## Number of Fisher Scoring iterations: 2
```

You will typically only encounter two types of non-linear, linear models

the binomial family

The binomial distribution models binary variables. Typically, this is presented as modeling coin flips, with a certain probability.

```
dat.toy <- data.frame(x=1:10, y=rbinom(10,1,0.5))
summary(glm(y~x, data=dat.toy, family=binomial))
```

```
##
## Call:
## glm(formula = y ~ x, family = binomial, data = dat.toy)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.4700  -0.8379   0.0000   0.7210   1.6701
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   2.4413     1.7998   1.356   0.175
## x             -0.4439     0.2981  -1.489   0.136
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 13.863 on 9 degrees of freedom
## Residual deviance: 10.867 on 8 degrees of freedom
## AIC: 14.867
##
## Number of Fisher Scoring iterations: 3
```

the poisson family

The poisson distribution models count variables. Typically, this is presented as something like ‘how many people enter a store per minute’.

```
data.toy <- data.frame(x=1:10, y=rpois(10,1))
summary(glm(y~x, data=dat.toy, family=poisson))
```

```
##
## Call:
## glm(formula = y ~ x, family = poisson, data = dat.toy)
##
## Deviance Residuals:
```

```
##      Min      1Q   Median      3Q      Max
## -1.0727 -0.7679 -0.3137   0.2952   1.0866
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    0.2190     0.8146   0.269   0.788
## x              -0.1929     0.1702  -1.133   0.257
##
## (Dispersion parameter for poisson family taken to be 1)
##
##      Null deviance: 6.9315  on 9  degrees of freedom
## Residual deviance: 5.5275  on 8  degrees of freedom
## AIC: 19.527
##
## Number of Fisher Scoring iterations: 5
```

your turn!

The binomial family glm is more commonly known as the logistic regression. In logistic regression, each 0 and 1 outcome is modeled as the odds ratio that a row will be equal to 0 or 1 based on the predictor variables. This is the engine behind most classification algorithms.

Let's go back to the `Clade` variable, and make a new binary variable based on it.

```
dat$bin <- 0
dat$bin[dat$Clade == 'Homo'] <- 1
```

Using logistic regression and the other variables in the dataset, find the best way to predict whether a case is from a human or not.

3. Nonparametric

parametric refers to using means, deviations, and other estimates of population parameters

BUT what if you don't want to make assumptions about the structure of the population?

ranked variables

a simple case is where means don't have meaning, such as likert variables

all Likerts are really rank variables, which means they don't act like actual number-y numbers

in the real world, a 6 foot tall person is twice as tall as a 3 foot tall person

but is a level '6' really twice as many barriers to access as a '3'?

we know that 6 is more than 3, but can't really say how much - in that sense then, a scale of 1-7 is exactly the same thing as a scale of a-g.

median testing ranks

we use Mann-Whitney sums to test that the ranks are centered the same way

```
likert.1 <- c(6,6,7,7,5,4)
likert.2 <- c(5,5,4,7,1,2)
wilcox.test(likert.1, likert.2, alternative = "two.sided", paired = FALSE, mu = 0, conf.level = 0.95)
```

```
## Warning in wilcox.test.default(likert.1, likert.2, alternative =
## "two.sided", : cannot compute exact p-value with ties
```

```
##
## Wilcoxon rank sum test with continuity correction
##
## data: likert.1 and likert.2
## W = 27.5, p-value = 0.1424
## alternative hypothesis: true location shift is not equal to 0
```

see how this setup looks exactly like a t-test? that's not an accident

correlating ranks

this is just like the `cor.test` you did above, but with `method` set to equal 'spearman' instead of pearson

```
cor.test(likert.1, likert.2, method = 'spearman')
```

```
## Warning in cor.test.default(likert.1, likert.2, method = "spearman"):
## Cannot compute exact p-value with ties
```

```
##
## Spearman's rank correlation rho
##
## data: likert.1 and likert.2
## S = 10.967, p-value = 0.1319
## alternative hypothesis: true rho is not equal to 0
## sample estimates:
##      rho
## 0.6866436
```

chi-Squared

what if both of your variables are categories? we can test their counts with R's built in `chisq.test` function

```
chisq.test(dat$Method, dat$Clade)
```

```
## Warning in chisq.test(dat$Method, dat$Clade): Chi-squared approximation may
## be incorrect
```

```
##
## Pearson's Chi-squared test
##
## data: dat$Method and dat$Clade
## X-squared = 160.17, df = 12, p-value < 2.2e-16
```

your turn!

look through the primate dataset, and run an example of each non-parametric test (on variables that make sense!)

```
str(dat)
```

```
## 'data.frame': 195 obs. of 12 variables:
## $ Clade : Factor w/ 5 levels "Cercopithecoidea",...: 1 1 1 1 1 1 1 1 1 1 ...
## $ Genus : Factor w/ 26 levels "Allenopithecus",...: 1 9 9 9 9 9 9 10 10 17 ...
## $ Species: Factor w/ 34 levels "", "albifrons",...: 21 32 32 32 32 32 32 3 14 9 ...
## $ Group : Factor w/ 11 levels "", "!kung", "Ache",...: 1 1 1 1 1 1 1 1 1 1 ...
## $ N : num 1 0.3 0.3 0.3 0.3 0.3 0.3 0.3 NA NA 16 ...
## $ W : num 7.9 3.4 3.4 3.4 4.1 4.1 4.1 7.9 7 5.7 ...
## $ BrainW : num NA NA NA NA NA NA NA 104 NA NA ...
## $ RMR : num NA 172 172 170 195 196 202 327 265 573 ...
## $ TEE : num 524 NA NA NA NA NA NA 428 329 884 ...
## $ Source : Factor w/ 13 levels "CAN 2003", "Coehlo 1986",...: 7 2 2 2 2 2 2 3 3 1 ...
## $ Method : Factor w/ 4 levels "Doubly Labeled Water",...: 1 4 4 4 4 4 4 2 2 2 ...
## $ bin : num 0 0 0 0 0 0 0 0 0 0 ...
```