DIFFICULTY:

TBD

PREREQUISITES:

None

PROBLEM:

There are N points lying on the coordinate axes.
In one second, you can draw some segments joining pairs of these points, as long as no two segments intersect non-trivially.

How many seconds do you need to draw every segment?

EXPLANATION:

Since all the points lie on the axes. every line segment we draw will be one of two types:

- Either it will lie completely on one of the coordinate axes; or
- It will lie entirely in one of the four quadrants.

Now, note that:

- Two line segments in different quadrants can never intersect nontrivially
- A line segment on an axis and another one in a quadrant can never intersect nontrivially

This gives us an idea to solve the problem: separately find the minimum time required to draw line segments in each quadrant, and along the axes. The final answer is the maximum of these times.
Let's look at them separately.

▾ Quadrants

Let's consider the first quadrant, i.e, points (x, y) with $x, y > 0$.

Suppose there are k_1 points on the positive y-axis and k_2 points on the positive x-axis.

Then, it can be seen that you need $\min(k_1, k_2)$ seconds to draw all pairs of segments between them.

▾ Proof

So, just knowing the number of points on the axes for each quadrant will allow us to compute the answer for each one.

▾ Axes

Segments on the axes are a bit trickier to handle.
There are two types of segments: ones lying on the x-axis, and ones lying on the y-axis.
Note that this time, there can be intersection between these different types of segments: but this intersection can only happen at the origin.

First, let's look at segments that cross the origin.

Notice that any step can contain at most one segment crossing the origin along the x-axis and at most one segment crossing the origin along the y-axis; but it also can't contain both of these.

In other words, each step can contain at most one segment crossing the origin. So, let's compute the total number of these.

Suppose there are c_1 segments crossing the origin along the x-axis and c_2 along the y-axis. The answer is at least $c_1 + c_2$.
Computing c_1 and c_2 is easy: if there are p points with **positive** x-coordinate, and n with **negative**, then $c_1 = p \cdot n$.
c_2 can be computed similarly.

Now, let's look at all segments on the axes.

Suppose there are m points on the x-axis. Then, we need a minimum of $\left\lfloor \frac{m}{2} \right\rfloor \cdot \left\lceil \frac{m}{2} \right\rceil$ seconds to draw all pairs of segments between them.

▾ Proof

So, compute this value for the x-axis and y-axis separately, and take the maximum of both.

Note that while the latter computation does include segments that cross the origin, it doesn't matter: the value we computed is a clear lower bound for the answer, and it is also achievable.
Whenever we choose an origin-crossing x-segment, we can combine it with non-origin-crossing y-segments and vice-versa.
If non-crossing segments on one side run out, then we are limited by $c_1 + c_2$ from above anyway, so it causes no issues.

When we choose a y-segment that cross the origin, we can

The final answer is then the maximum value among everything computed above.

The only thing we did was count the number of points on the positive/negative x/y axes, which can be done in $\mathcal{O}(N)$.