Elias Omega Proof

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Let $n \in Z^+$ and $m \neq n \in Z^+$
Let Cn be the Elias Omega codeword for n and Cm be the encoded Elias Omega codeword for m.

C_n and C_m are composed of a length encoding and a data encoding component.
Let Ln be the length component of C_n and L_m be the length component of C_m.
Let D_n be the data component of C_n and D_m be the data component C_m.

Let us assume C_m and C_n are Elias Omega codewords, $|C_m| > |C_n|$ and C_n is a prefix of C_m. Hence, C_m can be rewritten as C_nx where x is a string of bits.

Let us now consider the decoding of C_nx (C_m).

Given the structure of Elias Omega encoding, the length segment L_n is composed of one or more recursive encodings of the next code segment length minus 1. These length segments have had their most significant bits flipped to be 0 instead of 1. Additionally, the data component D_n encodes the actual integer value of n in binary and starts with a 1, distinguishing it from the length components in L_n.

Given C_n is an Elias Omega codeword, we decode the length segments by using the current known length segments until we reach a component which has a 1 in the most significant bit indicating we have reached a data component.

Assuming $|C_n| < |C_m|$, $L_n < L_m$ because the length of Elias Omega codewords depends on the number of bits used to represent the positive integer. Therefore, as we decode, we will encounter D_n before D_m as L_n dictates how we decode the codeword. As a result, the decoder will output D_n instead of $D_{m,}$ which was our intended target.

Therefore, C_m cannot be a valid Elias Omega encoding, thus contradicting our assumptions. Hence, our assumption that there exists two Elias Omega codewords where one is a prefix of another, is false. Finally, we can conclude that Elias Omega encoding is prefix-free.