Elias Omega Proof

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Let n ∈ Z+ and m ≠ n ∈ Z+

Let Cn be the Elias Omega codeword for n and Cm be the encoded Elias Omega codeword for m.

Cn and Cm are composed of a length encoding and a data encoding component.

Let Ln be the length component of Cn and Lm be the length component of Cm.

Let Dn be the data component of Cn and Dm be the data component Cm.

Let us assume Cm and Cn are Elias Omega codewords, |Cm| > |Cn| and Cn is a prefix of Cm. Hence, Cm can be rewritten as Cnx where x is a string of bits.

Let us now consider the decoding of Cnx (Cm).

Given the structure of Elias Omega encoding, the length segment Ln is composed of one or more recursive encodings of the next code segment length minus 1. These length segments have had their most significant bits flipped to be 0 instead of 1. Additionally, the data component Dn encodes the actual integer value of n in binary and starts with a 1, distinguishing it from the length components in Ln.

Given Cn is an Elias Omega codeword, we decode the length segments by using the current known length segments until we reach a component which has a 1 in the most significant bit indicating we have reached a data component.

Assuming |Cn| < |Cm|, Ln < Lm because the length of Elias Omega codewords depends on the number of bits used to represent the positive integer. Therefore, as we decode, we will encounter Dn before Dm as Ln dictates how we decode the codeword. As a result, the decoder will output Dn instead of Dm, which was our intended target.

Therefore, Cm cannot be a valid Elias Omega encoding, thus contradicting our assumptions. Hence, our assumption that there exists two Elias Omega codewords where one is a prefix of another, is false. Finally, we can conclude that Elias Omega encoding is prefix-free.