> # Лабораторная работа 3.1.
> # Вариант 1.
> # Выполнил: Кончик Денис, 153503

> # Задание 1
> # Решите уравнения и сравните с результатами, полученными в Maple. Постройте в одной системе координат несколько интегральных кривых.

> # 1.1
> # $x = y'' + e^{-y''}$

> # Решаем в параметрическом виде

> # $x = t + e^{-t}$

$x := t \to t + \mathrm{e}^{-t};$

$$x := t \to t + \mathrm{e}^{-t} \tag{1}$$

> # $y'' = t$

$y2 := t \to t;$

$$y2 := t \to t \tag{2}$$

> # $dx = (1 - e^{-t})dt$
> # $d(y') = y''dx = t(1 - e^{-t})dt$
> # $y' = \int (t - t \cdot e^{-t})\,\mathrm{d}t$

$y1 := unapply\left(simplify\left(\int \left(t - t \cdot \mathrm{e}^{-t}\right) \mathrm{d}t + \_C1\right), t\right);$

$$y1 := t \to \frac{1}{2}\, t^2 + t\,\mathrm{e}^{-t} + \mathrm{e}^{-t} + \_C1 \tag{3}$$

> # Аналогичным образом выражается
> # $y = \int y'\,\mathrm{d}x$

$y := unapply\left(simplify\left(\int y1(t) \cdot \left(1 - \mathrm{e}^{-t}\right) \mathrm{d}t + \_C2\right), t\right);$

$$y := t \to \frac{1}{6}\, t^3 + \frac{1}{2}\, \mathrm{e}^{-t}\, t^2 - \mathrm{e}^{-t} + \frac{1}{2}\, t\,\mathrm{e}^{-2t} + \frac{3}{4}\, \mathrm{e}^{-2t} + \_C1\, t + \mathrm{e}^{-t}\,\_C1 + \_C2 \tag{4}$$

> # Имеем общее решение ДУ в параметрическом виде. Построим в одной системе координат несколько интегральных кривых

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> curve1 := plot([x(t), subs(_C1 = 3, _C2 = 1, y(t)), t = -20..20], x = -20..20, y = -20..20,
        thickness = 2, color = black) :
  curve2 := plot([x(t), subs(_C1 = -4, _C2 = 15, y(t)), t = -20..20], x = -20..20, y = -20..20,
        thickness = 2, color = red) :
  curve3 := plot([x(t), subs(_C1 = -1, _C2 = -5, y(t)), t = -20..20], x = -20..20, y = -20..20,
        thickness = 2, color = coral) :
  curve4 := plot([x(t), subs(_C1 = -10, _C2 = 6, y(t)), t = -20..20], x = -20..20, y = -50..50,
        thickness = 2, color = blue) :
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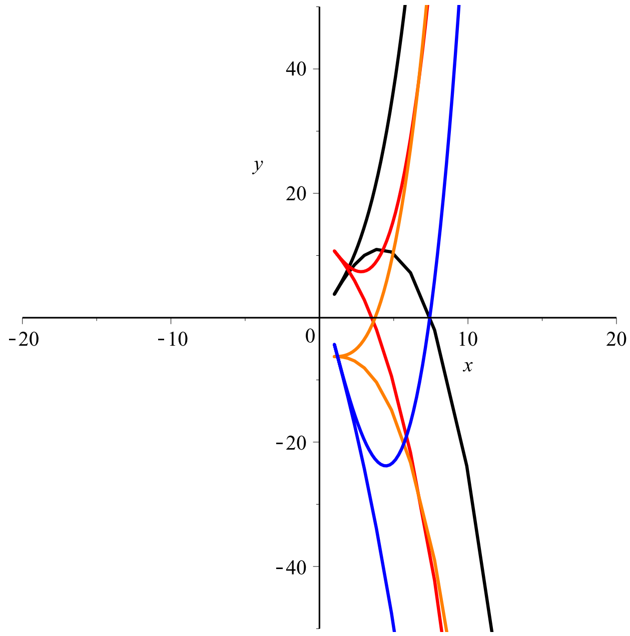
$plots[display](curve1, curve2, curve3, curve4);$



> $restart$

> # 1.2
$y \cdot y'' - y'^2 - y \cdot y' \cdot ctg(x) = 0$

> # Имеем однородное уравнение порядка $k = 2$
Замена $u = \frac{y'}{y}$, $y' = uy$
Тогда $y'' = yu^2 + yu'$
После преобразований получаем

> $\frac{d}{dx} u(x) - u(x) \cdot \cot(x) = 0;$

$$\frac{d}{dx} u(x) - u(x) \cot(x) = 0 \tag{5}$$

> $dsolve(\%);$

$$u(x) = \_C1 \sin(x) \tag{6}$$

> # И, подставляя $u = \frac{y'}{y}$, получаем общий интеграл
$\frac{\frac{d}{dx} y(x)}{y(x)} = rhs(\%);$

$$\frac{\frac{d}{dx} y(x)}{y(x)} = \_C1 \sin(x) \tag{7}$$

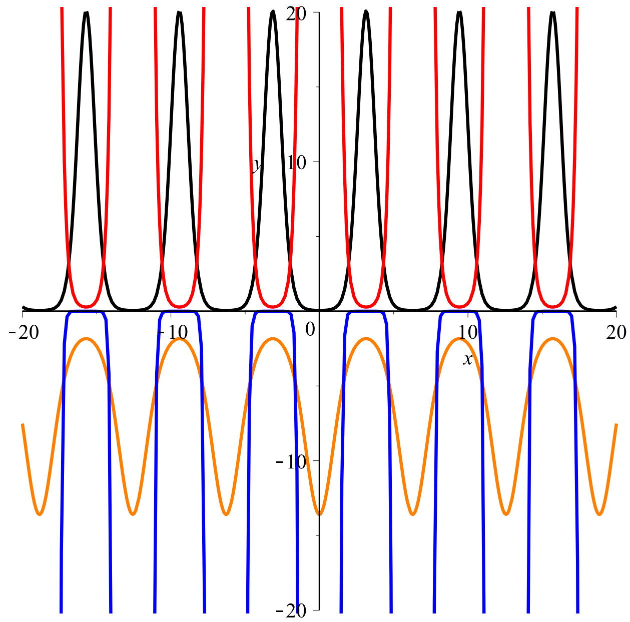
> $y := unapply(rhs(simplify(dsolve(\%))), x);$

$$y := x \rightarrow \_C2 \, e^{-\_C1 \cos(x)} \tag{8}$$

> # Имеем общее решение ДУ . Построим в одной системе координат несколько интегральных кривых

> $curve1 := plot(subs(\_C1 = 3, \_C2 = 1, y(x)), x = -20..20, y = -20..20, thickness = 2, color = black):$
$curve2 := plot(subs(\_C1 = -4, \_C2 = 15, y(x)), x = -20..20, y = -20..20, thickness = 2, color = red):$
$curve3 := plot(subs(\_C1 = -1, \_C2 = -5, y(x)), x = -20..20, y = -20..20, thickness = 2, color = coral):$
$curve4 := plot(subs(\_C1 = -10, \_C2 = -6, y(x)), x = -20..20, y = -20..20, thickness = 2, color = blue):$

$plots[display](curve1, curve2, curve3, curve4);$



> *restart*

> # 1.3
$y''(1+y^2)+y'^3=0$

> # Имеем уравнение F(y, y', y'') = 0. Ведем замену y' = u,
тогда y'' = u' y' = u' u

> $\left(\frac{d}{dy}u(y)\right)\cdot(1+y^2) =- (u(y))^2;$

$$\left(\frac{d}{dy}u(y)\right)(y^2+1) = -u(y)^2 \tag{9}$$

> dsolve(%);

$$u(y) = \frac{1}{\arctan(y) + \_C1} \tag{10}$$

> # Подставляем u(y) в y' = u
$\frac{d}{dx}y(x) = \frac{1}{\arctan(y(x)) + \_C1};$

$$\frac{d}{dx}y(x) = \frac{1}{\arctan(y(x)) + \_C1} \tag{11}$$

> # Получаем общий интеграл x(y)
dsolve(%);

$$x - y(x)\arctan(y(x)) + \frac{1}{2}\ln\left(y(x)^2+1\right) - \_C1\,y(x) + \_C2 = 0 \tag{12}$$

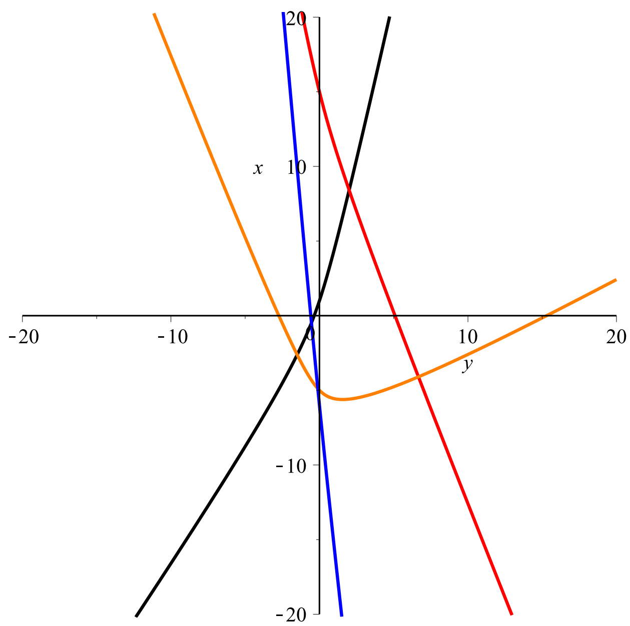
> $x := y \rightarrow y\cdot\arctan(y) - \frac{1}{2}\ln(1+y^2) + \_C1\cdot y + \_C2;$

$$x := y \rightarrow y\arctan(y) - \frac{1}{2}\ln\left(1+y^2\right) + \_C1\,y + \_C2 \tag{13}$$

> # Построим в одной системе координат несколько интегральных кривых

```
> curve1 := plot(subs(_C1 = 3, _C2 = 1, x(y)), y = -20..20,  x = -20..20, thickness = 2, color
      = black) :
curve2 := plot(subs(_C1 = -4, _C2 = 15, x(y)), y = -20..20, x = -20..20, thickness = 2, color
      = red) :
curve3 := plot(subs(_C1 = -1, _C2 = -5, x(y)), y = -20..20, x = -20..20, thickness = 2, color
      = coral) :
curve4 := plot(subs(_C1 = -10, _C2 = -6, x(y)),  y = -20..20, x = -20..20, thickness = 2, color
      = blue) :

plots[display](curve1, curve2, curve3, curve4);
```



> *restart*

> # *1.4*

y" = $3\left(\frac{y'}{x} - \frac{y}{x^2}\right) + \frac{2}{x^3}\sin\left(\frac{1}{x^2}\right)$

> $de := \frac{d^2}{dx^2}\,y(x) - \frac{3}{x}\cdot\frac{d}{dx}\,y(x) + \frac{3}{x^2}\cdot y(x) = \frac{2}{x^3}\cdot\sin\left(\frac{1}{x^2}\right);$

$$de := \frac{d^2}{dx^2}\,y(x) - \frac{3\left(\frac{d}{dx}\,y(x)\right)}{x} + \frac{3\,y(x)}{x^2} = \frac{2\,\sin\left(\frac{1}{x^2}\right)}{x^3} \qquad \textbf{(14)}$$

> # *Общее решение*

y := *unapply*(*rhs*(*dsolve*(*de*)), *x*);

$$y := x \rightarrow x^3\,\_C2 + x\,\_C1 - \frac{1}{2}\,x^3\,\sin\left(\frac{1}{x^2}\right) \qquad \textbf{(15)}$$

> # *Построим в одной системе координат несколько интегральных кривых*

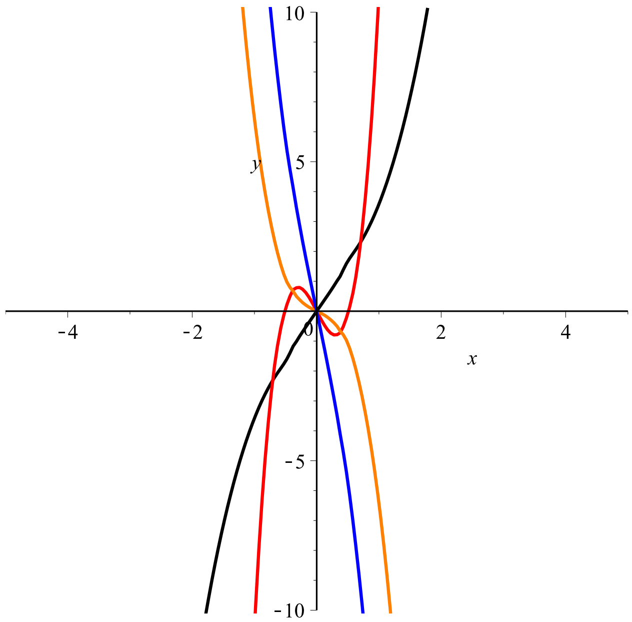
> *curve1* := *plot*(*subs*(_C1 = 3, _C2 = 1, *y*(*x*)), x = -5 ..5, y = -10 ..10, thickness = 2, color = black) :

curve2 := *plot*(*subs*(_C1 = -4, _C2 = 15, *y*(*x*)), x = -5 ..5, y = -10 ..10, thickness = 2, color = red) :

curve3 := *plot*(*subs*(_C1 = -1, _C2 = -5, *y*(*x*)), x = -5 ..5, y = -10 ..10, thickness = 2, color = coral) :

curve4 := *plot*(*subs*(_C1 = -10, _C2 = -6, y(x)), x = -5 ..5, y = -10 ..10, thickness = 2, color = blue) :

plots[*display*](*curve1*, *curve2*, *curve3*, *curve4*);



> *restart*

> *# Задание 2*
> *# Найдите общее решение уравнения и сравните с результатом, полученным в системе Maple.*
> *# y''' x·ln(x)=y''*

> $de := \frac{d^3}{dx^3}\, y(x) \cdot x \cdot \ln(x) = \frac{d^2}{dx^2}\, y(x);$

$$de := \left(\frac{d^3}{dx^3}\, y(x)\right) x \ln(x) = \frac{d^2}{dx^2}\, y(x) \qquad \textbf{(16)}$$

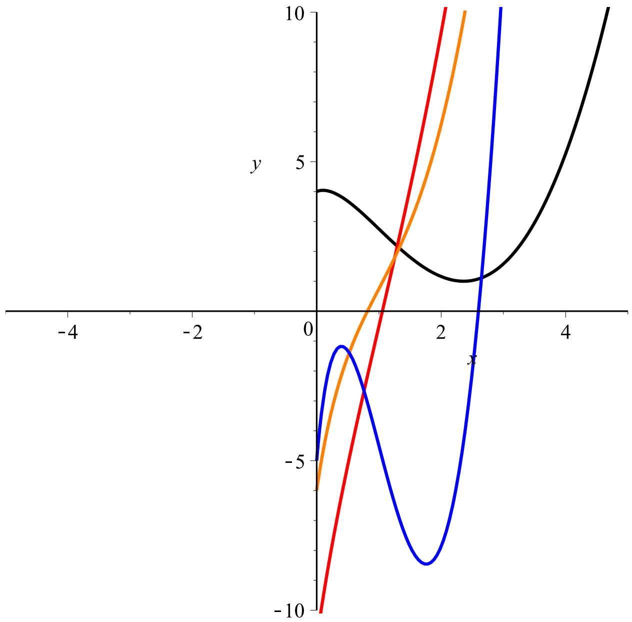
> *# Общее решение*
> $y := unapply(rhs(dsolve(de)), x);$

$$y := x \rightarrow \frac{1}{2}\,\_C1\, x^2 \ln(x) - \frac{3}{4}\,\_C1\, x^2 + \_C2\, x + \_C3 \qquad \textbf{(17)}$$

> *# Построим в одной системе координат несколько интегральных кривых*

> curve1 := plot(subs(_C1 = 3, _C2 = 1, _C3 = 4 , y(x)), x = -5 ..5, y = -10 ..10, thickness = 2, color = black) :
curve2 := plot(subs(_C1 = 6, _C2 = 15, _C3 = -11 , y(x)), x = -5 ..5, y = -10 ..10, thickness = 2, color = red) :
curve3 := plot(subs(_C1 = 11, _C2 = 15, _C3 = -6 , y(x)), x = -5 ..5, y = -10 ..10, thickness = 2, color = coral) :
curve4 := plot(subs(_C1 = 34, _C2 = 26, _C3 = -5 , y(x)), x = -5 ..5, y = -10 ..10, thickness = 2, color = blue) :

plots[display](curve1, curve2, curve3, curve4);



> *restart*

> # Задание 3
Найдите общее решение дифференциального уравнения.
$y'' + 2y' = 4e^{x}(\sin(x) + \cos(x))$

> $de := diff(y(x), x\$2) + 2\cdot diff(y(x), x\$1) = 4\cdot e^{x}\cdot(\sin(x) + \cos(x));$

$$de := \frac{d^2}{dx^2}\, y(x) + 2\left(\frac{d}{dx}\, y(x)\right) = 4\, e^{x}\,(\sin(x) + \cos(x)) \qquad \mathbf{(18)}$$

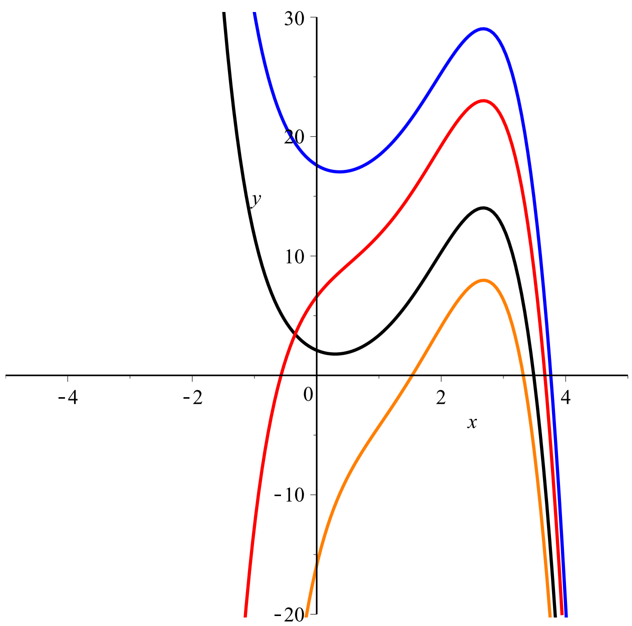
> # Общее решение
$y := unapply(rhs(dsolve(de)), x);$

$$y := x \rightarrow -\frac{2}{5}\, e^{x}\cos(x) + \frac{6}{5}\, e^{x}\sin(x) - \frac{1}{2}\, e^{-2x}\,\_C1 + \_C2 \qquad \mathbf{(19)}$$

> # Построим в одной системе координат несколько интегральных кривых

> curve1 := plot(subs(_C1 =-3, _C2 = 1 , y(x)), x =-5 ..5, y =-20 ..30, thickness = 2, color = black) :
curve2 := plot(subs(_C1 = 6, _C2 = 10 , y(x)), x =-5 ..5, y =-10 ..10, thickness = 2, color = red) :
curve3 := plot(subs(_C1 = 21, _C2 =-5 , y(x)), x =-5 ..5, y =-10 ..10, thickness = 2, color = coral) :
curve4 := plot(subs(_C1 =-4, _C2 = 16 , y(x)), x =-5 ..5, y =-10 ..10, thickness = 2, color = blue) :

plots[display](curve1, curve2, curve3, curve4);



30
20
y
10
-4
-2
0
2
4
x
-10
-20