Deadlines:

For all Groups: December 6 2021

Grading system:

One problem is one point.

- 1 problem - 1

- 2 problems - 2

- 3 problems - 3

- 4 problems - 4

- 5 problems - 5

- 6 problems - 6

- 7 problems - 7

- 8 problems - 8

- 9 problems - 9

- 10 problems - 10

1. Old crooked Seidenberg

I see that you liked the betting problems from the last laboratories so lets play a new betting game. The rules are that I have 3 dice. I have to roll them a few times. If at least one of my rolled dice fulfills the special condition, then I win otherwise the opponent wins. The condition is: **the sum of my dice has to be 14**. Now remains to solve a small problem that I'm dealing with. What is the number of times I have to roll the dice such that the game would seem fair but at the same time would tip the odds in my favor (naturally)? But you have to be very careful. If the number of rolls is too great, then everybody will understand that the game is a sham hence nobody will play. But if the number of rolls is too small then I'll go bankrupt very soon.

The task is to find how many times I need to roll the dice to make the game playable but at the same time to bring profit (winning rate should be a tad smaller than 0.5). Try simulating the game for various numbers of roles. For better proving your decision, draw me a plot that shows the winning probability in relation to the number of rolls. Remember that you'll obtain a higher probability convergence by increasing the number of simulations, around 10k should be enough.

2. Densities

Take a stick of unit length and break it into two pieces, choosing the breakpoint at random. Now break the longer of the two pieces at a random point. What is the probability that the three pieces can be used to form a triangle?

Write a computer program that would simulate the experiment. Explain your results.

3. Densities

Three points are chosen at random on a circle of unit circumference. What is the probability that the triangle defined by these points as vertices has three acute angles?

4. Densities

Choose independently two numbers B and C at random from the interval [-1, 1] with uniform distribution, and consider the quadratic equation $x^2 + Bx + C = 0$. Find the probability that the roots of this equation

- are both real
- are both positive

5. Densities

At the Ziua Vinului, a coin toss game works as follows. Coins of 25 bani are tossed onto a checkerboard. The management keeps all the coins, but for each coin landing entirely within one square of the checkerboard the management pays 1 leu. Assume that the edge of each square is twice the diameter of a coin, and that the outcomes are described by coordinates chosen at random. Is this a fair game?

6. Densities

Write a program to carry out the following experiment. A coin is tossed 100 times and the number of heads that turn up is recorded. This experiment is then repeated 1000 times. Have your program plot a bar graph for the proportion of the 1000 experiments in which the number of heads is n, for each n in the interval [35, 65]. Does the bar graph look as though it can be fit with a normal curve?

7. Densities

At a mathematical conference, 10 participants are randomly seated around a circular table for meals. Using simulation, estimate the probability that no two people sit next to each other at both lunch and dinner. Can you make an intelligent conjecture for the case of n participants when n is large?

8. Random Variables

A game is played as follows: A random number X is chosen uniformly from [0, 1]. Then a sequence Y1, Y2, . . . of random numbers is chosen independently and uniformly from [0, 1]. The game ends the first time that Yi > X. You are then paid (i - 1) dollars. What is a fair entrance fee for this game?

9. Random Variables

An insurance company has 1000 policies on men of age 50. The company estimates that the probability that a man of age 50 dies within a year is 0.01. Estimate the number of claims that the company can expect from beneficiaries of these men within a year.

10. Important Distributions

Jora Petrovici never pays the *troleibuz* (cost = 2 lei in Chisinau). He assumes that there is a probability of 0.05 that he will be caught by the *controlor*. The first offense costs 50 lei, the second costs 150 lei, and subsequent offenses cost 300 lei. There's also an empirically proven probability of 0.02 that the *taxatoarea* is a hairy muscular guy, which means that he'll have to pay the 2 lei anyway. How does the expected cost of riding the *troleibuz* for a year (two times a day) without paying compare to the cost that is paid by us, law-abiding students?