Simple Linear Regression — Math Summary

1. Model (Hypothesis)

$$\hat{y}_i = b_0 + b_1 x_i$$

2. Error (Residual)

$$e_i = y_i - \hat{y}_i$$

3. Cost Function (Mean Squared Error)

$$J(b_0, b_1) = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - (b_0 + b_1 x_i)\right)^2$$

4. Means

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

5. Variance of x

$$\mathrm{Var}(x) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

6. Covariance of x and y

$$\mathrm{Cov}(x, y) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

7. Optimal Slope

$$b_1 = \frac{\mathrm{Cov}(x, y)}{\mathrm{Var}(x)}$$

8. Optimal Intercept

$$b_0 = \bar{y} - b_1 \bar{x}$$

9. Prediction

$$\hat{y}_{\text{new}} = b_0 + b_1 x_{\text{new}}$$

10. Feature Scaling (Standardization)

$$z = \frac{x - \mu}{\sigma} \qquad \hat{y} = b_0 + b_1 z$$

11. Mean Squared Error

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

12. Coefficient of Determination (R^2)

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$