DOT project

October 24, 2020

1 DIFFUSE OPTICAL MICROSCOPY PROJECT

1.1 Phase 1: Time domain DOT

1.1.1 Objectives:

- Obtain Formula for absorption
- Study Contrast function

1.1.2 Formula on Contrast :

1. write fluence $\phi_0(\mu_a, \mu_s', t)$ for an homogeneous medium:

$$\phi_0(\mu_a, \mu_s', t) = \frac{c}{(4\pi cDt)^{3/2}} \cdot exp(-c\mu_a t)$$

2. write fluence perturbation $\delta\phi_0(\mu_a, \mu_s', t, \delta\mu_a, V, \vec{r})$:

$$\delta\phi_0(\mu_a, \mu_s', t, \delta\mu_a, V, \vec{r}) = -\frac{c^2}{(4\pi Dc)^{5/2} t^{3/2}} \cdot exp(-\mu_a ct) \int_{V_i} \delta\mu_a(\vec{r_p}) \left(\frac{1}{\rho_{12}} + \frac{1}{\rho_{23}}\right) exp\left\{-\frac{(\rho_{12} + \rho_{23})^2}{4cDt}\right\} d^3\vec{r_p}$$

3. write $C(t) \equiv \delta\phi_0/\phi_0$:

$$C(t) = -\frac{1}{4\pi D} \cdot \delta\mu_a V \left(\frac{1}{\rho_{12}} + \frac{1}{\rho_{23}}\right) \cdot exp\left\{-\frac{(\rho_{12} + \rho_{23})^2}{4cDt}\right\}$$

1.1.3 Plots

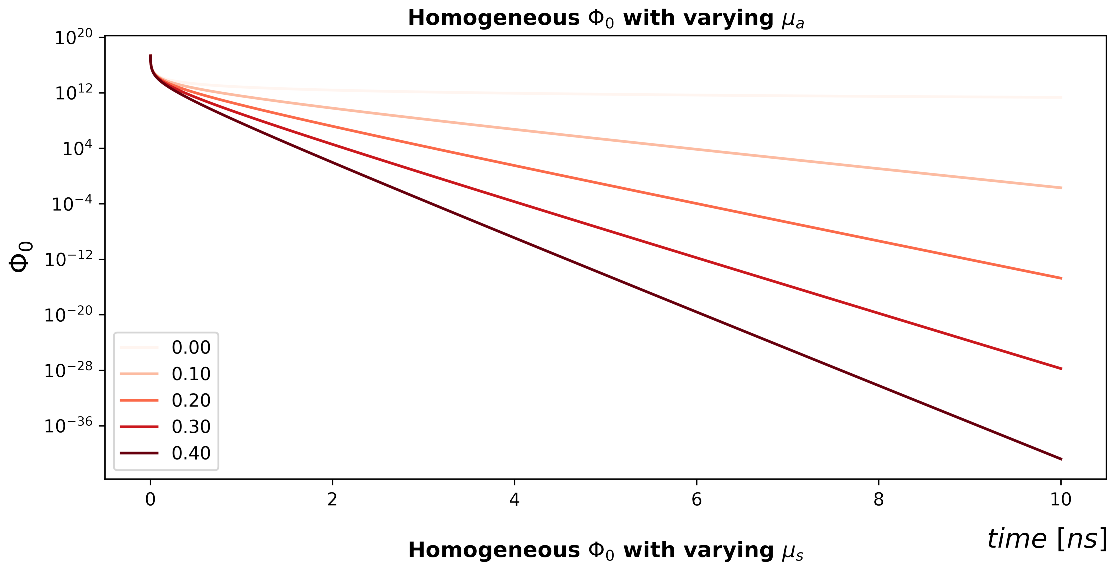
[1]:
```
#%matplotlib widget
#%matplotlib inline

import matplotlib.pyplot as plt
import matplotlib.colors as mcolors
from matplotlib import ticker
from matplotlib import cm
import numpy as np
```

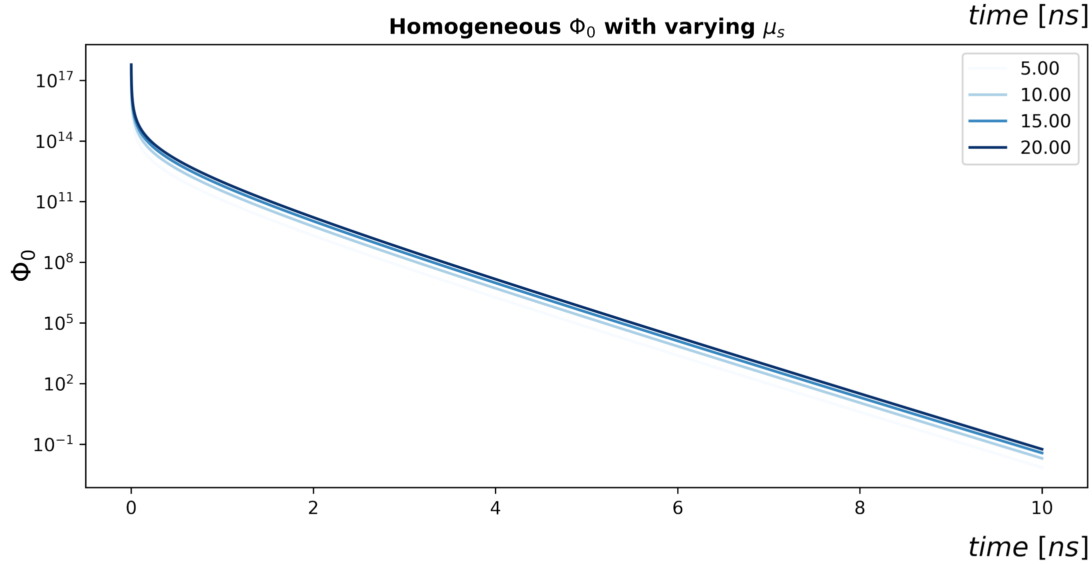
[2]:
```
dim=75
```

```
## Problem data

n=1 #refraction index
c= 30/n # cm/ns
mus= 10 #cm^-1
mua=0.1 # cm^-1
Dmua=0.1 #cm^-1
D=1/(3*mus) #cm

t=np.linspace(0,10,10000)[1:] # ns

V=1 #cm^3 perturbation dimension

RP= (V*3/4)**(1/3) #cm perturbation radius


rs=np.array([0,0,0])
rd= np.array([0,0,0])
rp=np.array([0, 0 ,2])

r= np.linalg.norm(rd)
Phi0= 1e13*(c*((4*np.pi*c*D*t)**(-3/2)))*np.exp(-c*mua*t)

def perturbation(t,rs,rd,rp,contrast=True):

    r= np.linalg.norm(rd)
    xs,ys,zs= rs
    xd,yd,zd= rd
    xp,yp,zp = rp


    def sumInt(tt):
        rho12 = np.linalg.norm(rp-rs)
        rho23 = np.linalg.norm(rp-rd)
        return (V)*Dmua*(1/rho12 + 1/rho23)* np.exp(- (rho12 +rho23)**2 /
 ↪(4*c*D*t))

    if contrast:
        contrast= -1/(4*np.pi*D) * np.array(sumInt(t))
        return contrast
    else:
        delPhi0= -1e13*(c**2/(4*np.pi*c*D)**(5/2))*(t**-3/2)*np.exp(-c*mua*t) *␣
 ↪np.array(sumInt(t))
        return delPhi0
```

[3]:
```
# Plot 1 Fluence
plt.close()
```

```
plt.rcParams['figure.dpi'] = 600
plt.rcParams["figure.figsize"] = [10,10]
fig,axs= plt.subplots(2,1,sharey=False)

axs[0].set_yscale('log')
axs[1].set_yscale('log')


muas=np.linspace(0,0.4,5)

for iteration,mua in enumerate(muas):
    #fixing mus
    mus=10
    D=1/(3*mus)

    colors = plt.cm.get_cmap('Reds', len(muas))
    Phi0=1e13* (c*((4*np.pi*c*D*t)**(-3/2)))*np.exp(-c*mua*t)
    axs[0].plot(t,Phi0,color=colors(iteration),label=format(mua, '.2f'))
    axs[0].set_title("Homogeneous $\Phi_0$ with varying $\mu_a$",fontweight="bold")
    axs[0].set_ylabel("$\Phi_0$",fontsize=15, labelpad=5,loc="center")
    axs[0].set_xlabel("$time\ [ns] $",fontsize=15, labelpad=10,loc="right")
    axs[0].legend()


muss= np.arange(5,20.1,5)

for iteration,mus in enumerate(muss):
    #fixing mua
    mua= 0.1

    D=1/(3*mus)
    colors = plt.cm.get_cmap('Blues', len(muss))
    Phi0=1e13*(c*((4*np.pi*c*D*t)**(-3/2)))*np.exp(-c*mua*t)
    axs[1].plot(t,Phi0,color=colors(iteration),label=format(mus, '.2f'))
    axs[1].set_title("Homogeneous $\Phi_0$ with varying $\mu_s$",fontweight="bold")
    axs[1].set_ylabel("$\Phi_0$",fontsize=15, labelpad=5,verticalalignment="center")
    axs[1].set_xlabel("$time\ [ns] $",fontsize=15, labelpad=10,loc="right")
    axs[1].yaxis.set_tick_params(which='both', labelbottom=True)

    axs[1].legend()
```



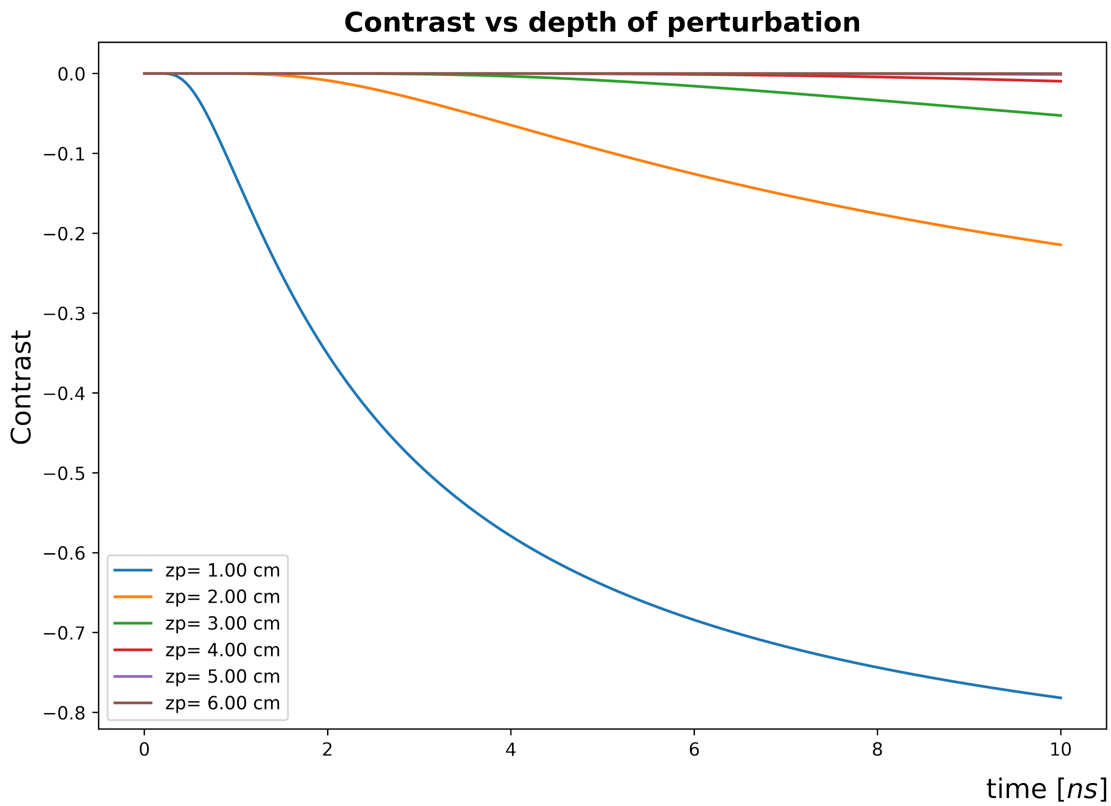


1) Fluence time response with varying absorption coefficient μ_a

2) Fluence time response with varying diffusion coefficient μ_s

[4]:
```
zp=np.arange(1,6.1)
plt.rcParams["figure.figsize"] = [10,7]
fig2= plt.figure()

for iteration,zp in enumerate(zp):
    rs=np.array([0,0,0])
    rd= np.array([0,0,0])
    rp=np.array([0, 0 ,zp])

    Contrast= perturbation(t,rs,rd,rp)
```

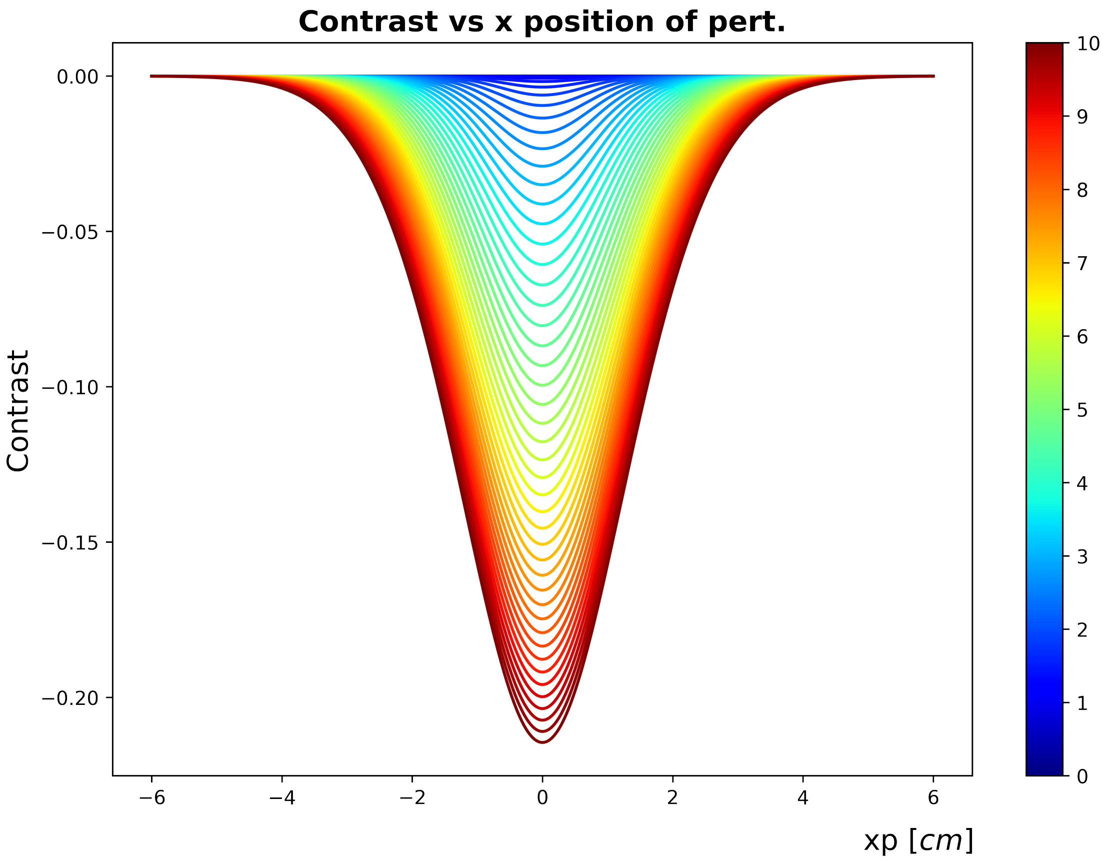
```
    plt.plot(t,Contrast,label="zp= "+format(zp, '.2f')+" cm")
    plt.title("Contrast vs depth of perturbation",fontsize=15,␣
↪fontweight="bold")
    plt.ylabel("Contrast", fontsize=15)
    plt.xlabel("time$\ [ns] $",fontsize=15, labelpad=10,loc="right")
    plt.legend()


#fig2.show()
```



Contrast time repsonse at several perturbation depth z_p, with $\mu_s = 10\ cm^{-1}$, $\mu_a = 0.1\ cm^{-1}$ and $\delta\mu_a = 0.1\ cm^{-1}$

[5]:
```
tt=np.linspace(0,10,50)
xp= np.linspace(-6,6,1000)
fig2= plt.figure()
normalize = mcolors.Normalize(vmin=tt.min(), vmax=tt.max())

colormap =cm.jet
```

```python
for iteration,t in enumerate(tt[1:]):
    Contrast=[]
    Phi0= 1e13*(c*((4*np.pi*c*D*t)**(-3/2)))*np.exp(-c*mua*t)
    for x in xp:

        rs=np.array([0,0,0])
        rd= np.array([0,0,0])
        rp=np.array([x, 0 ,2])

        cc= perturbation(t,rs,rd,rp)
        Contrast.append(cc)
    plt.plot(xp,Contrast,color=colormap(normalize(t)))
    plt.title("Contrast vs x position of pert.",fontsize=15, fontweight="bold")
    plt.ylabel("Contrast", fontsize=15)
    plt.xlabel("xp$\ [cm] $",fontsize=15, labelpad=10,loc="right")

scalarmappaple = cm.ScalarMappable(norm=normalize, cmap=colormap)
scalarmappaple.set_array(tt)
cb=plt.colorbar(scalarmappaple)
tick_locator = ticker.MaxNLocator(nbins=10)
cb.locator = tick_locator
cb.update_ticks()
```



Contrast time evolution as a function of perturbation off-axis position x_p , with perturbation depth $z_p = 2\ cm$, $\mu_s = 10\ cm^{-1}$, $\mu_a = 0.1\ cm^{-1}$ and $\delta\mu_a = 0.1\ cm^{-1}$